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Abstract

The coming energy transition in residential neighbourhoods is the result of the increasing cost of CO_2 emission and the decreasing costs of solar PhotoVoltaics (PV) and alternative techniques of residential heating, namely Combined Heat and Power (CHP) and heat pump. The optimal transition is found by minimizing the total discounted social costs of residential energy consumption and generation. Social costs include the cost of CO_2 emission and the investment in the electric network. The model integrates economics and the electric constraints based on the Alternating Current (AC) network power flow.

The results indicate that in the optimal transition nearly all houses are going to use an air-to-water heat pump with auxiliary gas heating. This shift from gas to electricity depends very little on the future CO_2 price or the network costs. Solar PV is not yet socially profitable at this moment.

The "business case" for a household, using private costs, includes taxes and excludes CO_2 costs and uses a higher discount rate. In the resulting optimum no heat pumps are used. However, reducing the ratio of the electricity tax versus the gas tax moves the private optimum to the social optimum.

Contents

1 Introduction

In the coming decades, the cost of CO_2 emission is expected to increase. Also, the costs of alternative forms of residential heating equipment are expected to decrease and the cost of solar PV is expected to continue to decrease. All this might induce a residential energy transition consisting of a change in the type of heating or an increase of solar PV, or both.

The nature of this transition is uncertain: which type of residential heating will be chosen, and how much solar PV will be installed? The social cost of this transition is a function of these choices. This function might be relevant for the energy policy of a government.

In this paper a model is described which gives the social costs of the transition as a function of the choice of heating method, the amount of solar PV and the strength of the electric network. The model is solved by minimizing the discounted sum of the social costs over half a century. The result is called the socially optimal transition.

The neighbourhood used in this study is derived from the imaginary residential neighbourhood "Meekspolder" (or "Meekswijk"), which was designed some years ago as a general framework for studies of residential neighbourhoods in the Netherlands. The electrical network is modelled both with and without the exact physics of the AC power flow; see appendix F.6 for more details. The results in the paper are given for the exact model.

The results indicate that in the optimal transition nearly all houses are going to use an air-to-water heat pump with auxiliary gas heating. Very little mCHP cogeneration will be used.

After a discussion of the literature, the model is described in section 3. The details of various components of the model, such as heating and solar PV, are described in section 4. This section includes also the various numerical assumptions in the model and some simple computations based on these assumptions. Section 5 describes the results of the model, including alternative parameter scenarios showing the robustness (or the lack of it) of the results. Section 6 concludes.

All prices and costs are without VAT. Technical details are in appendices. Computer files are supplied with the paper. Unless specified otherwise, technical and economic data were obtained by private communication with the members of the EOS-TREIN team.

Table 1: Terminology

HP	Heat Pump (with auxiliary gas use, unless otherwise specified)
mCHP	micro Combined Heat and Power boiler
CB	Conventional high-efficiency boiler
solar PV	solar Photo Voltaics
reference solution	the solution of the model using the assumed parameter values, to be compared with solutions based on alternative parameter values

2 Literature

No study was found which models simultaneously the social optimum for the choice of residential heating method and the amount of solar PV and the electric network strength, in a neighbourhood with a gas network, over the lifetime of the electric network. Following are some papers, each discussing one of the three following subjects which play a role in the present paper: the cost of various heating methods, the choice of heating method, and the electrical network.

The present private cost of residential heating methods, including gas-fired equipment, is studied by Audenaert et al. (2011), Van der Veken et al. (2006) and Monahan and Powell (2011). In Gustavsson and Joelsson (2007) and Demmel and Alefeld (1994) there is no network of natural gas.

Many studies of residential heating focus on the determinants of the household's choice between the various types of heating equipment, such as whether the household owns the house. See Vaage (2000), Rehdanz (2007), Meier and Rehdanz (2010), Braun (2010) and Goto et al. (2011). Similarly, Manning and Rees (1982), Rowlands (2005), Parker (2008) and Macintosh and Wilkinson (2011) study incentives to invest in residential solar PV. Scarpa and Willis (2010) and Mozumder et al. (2011) estimate the willingness to pay for renewable energy.

There are several studies of the effect of the transition on the network. Paatero and Lund (2007) analyze the effect of large-scale solar PV on the middle-voltage network; they find that 2 kWp per house creates problems. Said (2010) studies the optimal low-voltage network with heat pumps and electric cars.

3   The general structure of the model

The model variables are indexed as follows: t is a year, s is an extreme moment within a year, n is a node in the electric network and ℓ is a link in the electric network.

The yearly quantities are included in the cost calculations, while the extreme moments are used to make sure that the operational limits of the electrical network are respected. The extreme moments are not a time series within a year.

I use a star in a subscript as a wildcard character, indicating an array. For instance $S_{*t} = S_{1t}, S_{2t}, \ldots$ and S_{**} is the entire two-dimensional array.

3.1   The control variables

The control variables ("decision variables") of the model are: the type of heating, the amount of solar PV and the strength of the network.

The fraction of heating type h among the houses at node n in year t is indicated by H_{hnt}. The heating types are: a conventional boiler, micro CHP and heat pump. Of course we have:

$$H_{hnt} \geq 0 \quad \text{for all } h, n, t \qquad \text{and} \qquad \sum_h H_{hnt} = 1 \quad \text{for all } n, t \qquad (1)$$

The amount of solar PV per house (in kWp) is indicated by S_{nt}. This is restricted by:

$$0 \leq S_{nt} \leq S_t^{\max} \quad \text{for all } n, t \qquad (2)$$

where the given maximum $S_t^{\max}$ is the available roof surface translated to the maximum PV output. The output/surface ratio increases over time t.

The strength of the network is indicated by x, such that a doubling of x allows a double load. For details, see appendix E.3. The x is normalized such that a standard network, discussed below, has $x = 1$.

3.2   The other variables

Let C_{nt} be the house-related social costs at network node n in year t:

$$C_{nt} = N_n f(H_{*nt}, S_{nt}; P_t) \qquad \text{for all } n, t \qquad (3)$$

where N_n and P_t are given parameters. The N_n is the number of houses at node n. The P_t is a vector containing the prices of electricity and natural gas and CO_2 emission and heating equipment and solar equipment, all in

year t. Hence this includes both investment costs and operational costs. Given P_t, the function f is a linear function of H_{*nt} and S_{nt}. Since P_t is given, there is no feedback from the electricity consumption to the price of electricity.

Let c be the variable investment cost of the network. Assuming a linear investment cost function, we have:

$$c = \gamma x \tag{4}$$

where $\gamma > 0$. The fixed costs are not relevant, since they do not change with a change of a decision variable.

The voltages V_{nts} and electric currents $E_{\ell ts}$ depend on the network strength and on the net power consumption W_{nts} at all nodes n at that moment[1]:

$$\{V_{*ts}, E_{*ts}\} = v(x, W_{*ts}) \quad \text{for all } t, s \tag{5}$$

The function v contains the engineering power flow model[2], where each W_{nts} is a two-dimensional vector containing both real and reactive power. This function might be the exact AC power flow model or a novel linear Taylor power flow approximation. For details, see appendix F.5.

The W_{nts} depend on the equipment and on the heat-unrelated exogenous electricity consumption such as lighting and TV use:

$$W_{nts} = N_n w_s(H_{*nt}, S_{nt}; U_t) \quad \text{for all } n, t, s \tag{6}$$

The parameter U_t is the yearly heat-unrelated electricity consumption per house. For any given moment s and parameter U_t, the function w_s is linear.

3.3 The technical restrictions

We have the following two technical restrictions:

$$V^{\min} \leq V_{nts} \leq V^{\max} \qquad \text{for all } n, t, s \tag{7}$$

$$E_{\ell ts} \leq E^{\max} \qquad \text{for all } \ell, t, s \tag{8}$$

In practice, voltages and currents in electric networks are restricted all the time. (See appendix E for details.) However, in order to prevent the model to become far too large, the s contain only a few extreme moments in a year.

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[1] In physics and engineering, the letter I is used for electric current. However, in order to prevent confusion with the I for investment in economics, the E was chosen here.

[2] Deviations from the steady state sine wave, such as harmonics and transients, are outside the scope of this paper.

These moments are obtained from simulations with the so-called Profile Generator, discussed in section 4.6.

The voltage limits V^{min} and V^{max} are given parameters. The maximum electric current E^{max} depends on the strength of the network:

$$E^{\text{max}} = \beta x \qquad (9)$$

with $\beta > 0$. See appendix E.2 for details.

3.4   The objective function

The social costs include the network costs c and the costs C_{nt} summed over the nodes n and over the years t, with a discount rate r:

$$\underset{H_{***},\, S_{**},\, x}{\text{minimize}} \quad c + \sum_{t=1}^{T} \sum_{n} C_{nt}(1+r)^{-(t-1)} \qquad (10)$$

$$\text{subject to } (1), \ldots, (9)$$

Given the control variables in (10), all other variables can be computed from the equations (3), (4), (5), (6) and (9). These equations are in recursive order when (6) is computed before (5). See appendix I for more computational aspects of the model. An investment is called *profitable* if it reduces the minimand of (10), possibly conditional on the other control variables.

3.5   The time pattern of investments

The choice of equipment is made at the moment of investment. This does not show in the cost equation (3), in order to reduce the complexity of the notation.

The frequency of the investment moments depends on the life time of the equipment. The time period of the model, T, is 48 years; in this period fits one lifetime of the electric network, two lifetimes of solar PV (each 24 years) and four lifetimes of heating equipment (each 12 years). In a new neighbourhood, investment in the network occurs in year 1 and investment in heating equipment occurs only in years 1 and 13 and 25 and 37.

In practice, investments in solar PV start when they have become profitable. However, in order to reduce the computational burden substantially, I assume that these investments (if any) occur only at fixed moments in time, separated by their lifetime, similar to the heating equipment.

Table 2: Discounting with $r = 5.5\%$ per year

		number of years T		
		12	24	48
		heating	solar PV	network
discount factor	$(1 + r)^{-T}$	0.53	0.28	0.08
discounted stream (years)	$\sum_{t=0}^{T-1}(1 + r)^{-t}$	9.1	13.9	17.7

In this way, at the end of the network lifetime the heating equipment and the solar equipment (if any) are also at the end of their lifetime. Hence no residual value is left. No subsequent network is modelled.

Since the data have been collected mainly in 2010, the 48 years of the model run from 2010 to 2057.

4 The components of the model

4.1 The costs

The costs are social costs, referring to the society as a whole. Social costs are without taxes, since their payment by the household and their receipt by the government cancel each other. Hence all prices in the model are without energy tax[3] and without VAT. See appendix B for details.

The costs are discounted with $r = 5.5\%$ per year. This is the sum of a 2.5% long-run risk-free interest rate and a 3% risk premium. It is the standard for social cost-benefit studies in the Netherlands; see Netherlands Ministry of Finance (2009). Table 2 shows some figures derived from this discount rate, used at various places in the text below.

4.2 The standard electric networks

The electric network of the Meekspolder neighbourhood contains five residential sub-networks with a connection to the grid. One of these sub-networks contains a school. There is no path through the network from one sub-network to another. Each sub-network has one or more forks but no loops, like a tree. The Meekspolder contains also a shopping mall and apartment buildings, which I ignore; they have their own connection to the grid.

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[3] An exception is the "business case" for the household in section 5.1.3.

Three variant neighbourhoods are modelled: two new and one old. One of the new neighbourhoods uses natural gas and the other is all-electric. The old neighbourhood is 24 years old. It has on average 43 houses per connection to the grid (apart from the connection which feeds the sub-network with the school). The new neighbourhoods have thicker electric cables, with 75 houses per connection to the grid[4]. I call these networks the *standard networks* for these neighbourhoods. In the new neighbourhoods, the network strength is endogenous: the variable x in model (10). Its value is expressed as a fraction of the strength of the standard network. In the old neighbourhood, the network is fixed at the standard strength: $x = 1$.

The variable investment cost of the network includes the price of the cables and the digging and installation. A value of 40 euro/meter has been chosen, for both types of cable. With the Meekspolder network of 2850 meter this gives the variable cost $\gamma = 114,000$ euro.

In practice a stronger (or weaker) network is implemented by increasing (reducing) the number of connections with the grid. See appendix E.3 for details of the implementation in the model. The number of houses is 319.

In order to reduce the computing time of the model, houses along a street are clustered together. This reduces the number of network nodes, while the effect on the power flow is negligible. For details, see appendix G.

The network cannot be updated after it has been built. Since there is no uncertainty, this is only relevant when the discounting more than offsets the cost increase of later updating.

The network for natural gas is not modelled, since its investment cost depends very little on its strength.

4.3 CO_2 emission

4.3.1   The emission price

The social cost of CO_2 emission is defined as the CO_2 emission permit price[5]. This price is assumed to increase from 20 to 90 euro/ton CO_2 from 2010 to 2057, which is at the end of the model time period. This is based on the CO_2 permit price series[6] in figure 3 of Durand-Lasserve et al. (2010), with the

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[4] Considering the maximum allowed electric currents per phase in table 15 in appendix E, the old network is stronger than the new network: $(205/43)/(260/75) = 1.4$. The old network has the number of houses of the original Meekspolder design.

[5] See for example Ten Cate (2010) for relations between (a) this price and (b) the costs of emission reduction and (c) the damage of emission, in a simple static setting.

[6] The title of figure 3 is "European Union CO_2 price", but the graph is meant to be more general than only the ETS price. (Private communication)

Table 3: Characteristics of grid energy in the Netherlands, 2010

	unit	CO_2 emission (kg)	price without CO_2 (eurocent)	emission costs at 20 euro/ton (eurocent)	energy tax (eurocent)	tax per ton CO_2 (euro)
natural gas	m^3	1.8	30	3.6	17	94
natural gas	kWh	0.20	3.4	0.4	1.9	94
electricity	kWh	0.57	5.9	1.1	11	193

The second row was computed by multiplying the first row with 3.6 divided by 31.7 MJ/m^3.
The taxes are for small consumers.

"hard" emission cap. In their model the price rises temporarily between 2010 and 2100, due to banking of permits. Ignoring this, I interpolate linearly over this time period. Conversion to euros is done with 1.3 dollar/euro.

This result is derived as follows, starting at 20 euro (26 dollar) in 2010. In Durand-Lasserve et al. (2010) the price rises to 200 dollar in 2100. Hence at the end of my model period, after 48 years, the price is $26 + (200 - 26) \times ((48-1)$ year $/90$ year$) = 117$ dollar $= 90$ euro. This is an uncertain number; however, as we shall see, my results are robust against considerable variation in this number.

Figures A3 and A4 in Durand-Lasserve et al. (2010) show respectively the future gas price (without the CO_2 cost) and the future electricity price (including the CO_2 cost). Both series are more or less constant over time; in the model they are exactly constant over time.

4.3.2 Electricity and gas from the grids

The present CO_2 emission in the Netherlands[7] associated with energy from the two grids (electricity and natural gas) is given in table 3. The prices and taxes are without VAT[8].

I assume that the production cost of grid electricity does not decrease in the future. Then it follows from the above mentioned assumption of a constant electricity price including the CO_2 cost that the gradual reduction of the CO_2 associated with grid electricity offsets the rise of the CO_2 price.

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[7] Emissions associated with the use of natural gas from the gas grid in the Netherlands are similar to Belgium; see Van der Veken et al. (2006), table 4. The same table shows that emissions associated with the production of grid electricity vary widely with the fuel mix: 0.06, 0.31, 0.59 kg/kWh in respectively France, Belgium and Germany. Compare with the Dutch 0.57.

[8] For the VAT and energy taxes, see appendix B.

Table 4: Characteristics of heating equipment in a neighbourhood with gas

		old house			new house		
	unit	HP	mCHP	CB	HP	mCHP	CB
natural gas	1000 m^3/yr	0.8	2.4	2.0	0.5	1.6	1.3
electricity	MWh/yr	2.9	−3.3		2.0	−2.2	
CO$_2$ (2010)	ton/yr	3.1	2.4	3.6	2.0	1.6	2.3
cost (2010, without CO$_2$)							
capex	1000 euro	2.6	3.4	1.0	2.6	3.4	1.0
opex	1000 euro/yr	0.41	0.52	0.60	0.27	0.35	0.39
total	1000 euro	6.4	8.2	6.5	5.1	6.6	4.5

Source physical data: Profile Generator;   total cost = capex + 9.1 year × opex.

(I will denote this as the greening of grid electricity.) Hence the CO$_2$ cost per kWh stays 1.1 eurocent over the entire model time period[9].

4.4 Heating equipment

Table 4 shows some characteristics of the three types of heating equipment in the neighbourhood with gas[10].

The heat pump (HP) used in the neighbourhood with natural gas makes the outside air colder and uses that energy to heat water ("air-to-water"). This process is driven by machinery which requires electric energy. It uses also an auxiliary gas boiler when necessary to satisfy the heat demand. Below approximately zero degrees Celsius outside air temperature, only the auxiliary gas boiler is used. Of course, in the all-electric neighbourhood heat pumps have electric auxiliary heating, as in Said (2010).

Table 4 reflects clearly the reduced heat demand in the new house: each quantity for a new house (except the capex) is smaller than the corresponding quantity for an old house. The electricity use of the conventional boiler (CB) is ignored.

The CO$_2$ emission and the opex in the table are computed from the emission factors and prices in table 3. The emission of the mCHP is the net

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[9] This is coincidental, based on the stylized fact of the electricity price including the CO$_2$ cost being constant. Of course this changes with alternative assumptions about the future CO$_2$ price.

[10] The efficiency of a micro CHP boiler can be demonstrated with the following simple and exaggerated example. Let its output be half electricity and half heat. Let this also be the case with a gas-fired power plant, where the heat is wasted. Then the mCHP saves as much gas at the power plant as it uses itself.

emission from gas (positive) and electricity (negative).

The bottom line of table 4 adds the costs (except the CO_2 cost), using the prices in table 3. The 9.1 in the note below the table is the discounted stream over the lifetime of 12 years, from table 2. In the old house the heat pump has (only just) the lowest cost, while in the new house the conventional boiler has the lowest cost.

In the future, the difference between the investment cost of the micro CHP and the conventional boiler decreases with 5% per year. This is due to technological progress, learning, etcetera. For the heat pump (which is a more mature technology) this is 3%.

In the all-electric neighbourhood, there is no choice of heating equipment: heating is done with a ground-source heat pump, with auxiliary electric heating when necessary. This creates the following problem which occurs after a power failure in the winter: then all houses run their heating facilities at maximum, at the same time. I assume that this problem will be solved, though at some cost. This cost is not relevant for the model, since the neighbourhoods are not compared with each other.

4.5   Solar PV

The model allows for the residential investment in solar PV. When and where it is used, it is included in the costs and in the extreme loads. In the latter, the intermittency of solar energy requires extremes both with and without sunshine.

The size of a solar PV panel is expressed in kWp (short for kWpeak). This gives the number of kW output in the laboratory when the radiation input is one kW/m^2 (and some other requirements are satisfied)[11].

Hence the surface (in m^2) of one kWp is the reciprocal of the efficiency in the laboratory. In the model this efficiency is at present 15% and then the surface of one kWp is $1/0.15 = 6.7$ m^2. The available roof surface is assumed to be 30 m^2, or $30/6.7 = 4.5$ kWp. The efficiency increases over time, together with the decreasing costs per kWp. See appendix C.2 for details.

A widely used value for the annual output of solar PV in the Netherlands, averaged over the past years, is 850 kWh/kWp. This applies to an optimal orientation (in fixed position) to the sun. Then the benefit from investment in solar PV, discounted to the year of investment, is:

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[11] This input is seldom attained in, say, central Europe. Hence the size of a system expressed in kWp gives more or less the maximum ("peak") output in kW.

$$850 \text{ kWh/kWp/year} \times 13.9 \text{ year} \times (5.9 + 1.1) \text{ eurocent/kWh}$$
$$= 830 \text{ euro/kWp}$$

when rounded at 10 euro/kWp. The 13.9 year is from table 2; the 5.9 and 1.1 cent are from table 3, assumed constant over time. This gives the socially break-even price of an installed solar PV system, including electronic equipment. The various price scenarios for model year 25 (=2034) range from 400 to 1200 euro/kWp; in the reference solution this is 1200 euro/kWp.

Of course, a different CO_2 cost changes the break-even price. For instance a doubling of the CO_2 cost increases the break-even price with $850 \times 13.9 \times 0.011 = 130$ euro/kWp. I ignore the CO_2 emission associated with the use of energy in the production of solar PV systems[12].

Finally, the following two related concepts are used below. First, solar PV is called *just profitable* if it is profitable but the profit per kWp is less than the cost of the related increase of the network strength. Then the optimal amount of solar PV is limited by the strength of the network (or by the roof surface, whichever comes first). In appendix C.3, the width of the range within which solar PV is just profitable is estimated to be 260 euro/kWp. Second, solar PV is called *amply profitable* if it is more than just profitable: its profit per kWp is more than the cost of the related increase of the network strength. Then the optimal amount of solar PV is equal to the available roof surface. With an unlimited surface, the optimal amount of solar PV in the model would be infinite, with an infinitely strong network.

4.6 The Profile Generator

4.6.1 Introduction

The Profile Generator is a software system which generates typical time series ("profiles") of residential energy use[13]. The seasonal variation is compressed in six days per year. From these time series, the extreme loads of the electric network are obtained. (These loads are at the moments s in the model equations in section 3.) Hence the Profile Generator is the foundation of both the economics in the model (kWh/year electricity and m^3/year gas) and the physics in the model (extreme kW electricity).

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[12] The "energy pay-back period" of solar PV is approximately five years in the Netherlands. See Koenders (2009), table 36, page 64. See also MacKay (2009), page 41/42. Do not confuse with the required "monetary" pay-back period of five years for households; see appendix A.

[13] The Profile Generator has been built mainly by Jan Willem Turkstra (kema.com).

Table 5: A selection of extreme electric loads ($\times$ 100 watt), 2010

		net load		components	
		house	house		heat-
extreme		with mCHP	with CB	mCHP	unrelated
		(1)+(2)	(2)	(1)	(2)
max	house with micro CHP	**5**	5	0	5
min	house with micro CHP	**−6**	2	−8	2
max	house with conventional boiler (CB)	−1	**8**	−9	8
min	house with conventional boiler (CB)	−5	**2**	−7	2

The Profile Generator simulates the heating and cooling down of a house, from which follows the use (or production) of heat-related electricity and natural gas as a function of the outside temperature and the thermal properties of the house and the type of heating equipment. The energy use related to the production of hot water from the tap is added. Also added is the electricity use which is heat-unrelated (tv, lighting, etcetera). Finally, a series of solar radiation is added.

The Profile Generator is designed assuming that all equipment performs according to its specifications. More details in appendix D. See also the chapter about the Profile Generator in Verbong et al. (2010).

4.6.2 A simplified example of an extremes table

Table 5 shows part of the table of extreme moments for a new house with gas. The heat pump and solar PV are omitted; showing only a part of the table makes it more easy to explain the principle.

The left-hand side of the table defines the nature of the extreme moments: whether it is a maximum or a minimum, and what heating equipment is used in the house. A minimal load is relevant with local generation, which may result in a negative net load, and hence in a reverse power flow.

The numbers in bold face, in the two "with" columns, show the value of the maximum or minimum. The numbers in ordinary print in these columns give the load in a house with the *other* heating method, at the *same* moment. For example, the second line shows that when a house with a mCHP has its (negative) minimum, a house with a conventional boiler consumes 200 watt[14].

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[14] Compare with Aalbers et al. (2011) where it is assumed that the largest generation takes place at a moment when the consumption is zero.

These numbers are based on the production of the mCHP and the heat-unrelated consumption at these moments, as shown in the last two columns ("components"). The lowest three lines are in the winter, with large production from the mCHP.

The heat-unrelated load is its value at 2010. This is assumed to grow with 0.8% per year, based on the outlook of the growth per house of heat-unrelated electricity use in the Netherlands. This implies an increase with a factor 1.45 after 47 years. See appendix D.5 about the implementation.

4.6.3 Other aspects of the extremes tables

The extremes of the old neighbourhood are different from a new neighbourhood, due to the larger heat demand. The all-electric neighbourhood has only one type of heating, a heat pump with electric auxiliary heating; by far its largest load is in the winter.

A neighbourhood with gas has also the option of a heat pump. This increases both the rows and the columns of the table. Solar PV adds several rows to the tables, for various sizes of the solar PV surface. The extremes of the school, with only conventional heating, are computed separately.

The energy for cooking is not available in the present version of the Profile Generator and hence not included in the model. This is only relevant for the network load in the all-electric neighbourhood, assuming cooking with gas in the neighbourhoods with gas.

More details and complete extremes tables are given in appendix D.

5 The results

5.1 The new neighbourhood with natural gas

5.1.1 The reference result

In the optimum, every house uses a conventional boiler (CB) in the first subperiod and a heat pump (HP) in the remaining three subperiods, with the exception of 1.7% of the houses using a mCHP in the last subperiod; the mCHP reduces the required network somewhat. The optimal network strength is 71% of the standard electric network of section 4.2.

This result is called the reference result, to be distinguished from the results of scenarios with alternative parameter assumptions, discussed below.

Table 6 shows the time pattern of the reference result, over the four subperiods in the first column. The last column shows in what subperiods the network is a binding restriction, with the electric current reaching 100%

Table 6: Time pattern in the new neighbourhood with gas, reference solution

subperiods	heating (%)			largest electric current
(years)	HP	mCHP	CB	(% of maximum permitted)
1-12			100	59
13-24	100			95
25-36	100			100
37-48	98.3	1.7		100
1-48	74.6	0.4	25	100

of its restriction $E^{\max}$ in one or more links in the network[15]. One might expect that the network is a binding restriction at the end, when the heat-unrelated load is largest. In fact, due to the mCHP, both the last subperiod and the penultimate subperiod have no slack in the network capacity left. The no-slack years are at the end of these subperiods: the years 36 and 48. In both years, the no-slack moment within the year is the moment at which the load of a house with a heat pump is at its largest[16].

As noted above, the results vary also over the location in the network. The load-reducing investments (if any) are concentrated at end-points of the network branches, as far as possible from the connection to the grid.

5.1.2   Robustness

Table 7 shows the result of the scenarios with alternative parameter values, indicating the sensitivity of the solution for changes in the parameters[17].

Large network strengths, indicated in boldface, occur with higher load (99%) and with much local generation with solar PV (199%) or solar PV and a mCHP (184%). These are percentages of the standard electric network.

The growth of the heat-unrelated electricity has little effect on the type of heating, but of course it has an effect on the size of the network. The table shows the effect of 1.3% per year (large, but without electric car) and 2.3% (with electric car)[18].

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[15] The $E^{\max}$ occurs in equations (8) and (9) in section 3.3. The voltage deviations never reach their bound. See appendix E for the principles of the network restrictions and the effect of short distances in this neighbourhood.

[16] Line 5 of extremes table 13 in Appendix D.

[17] This is the reverse of DeCarolis (2010), who searches for solutions which deviate much from the optimal solution but show only a small deterioration of the objective.

[18] The resulting 99% network suggest that the present standard network is just enough to accommodate the electric car. Of course this 2.3% growth and the extreme network

Table 7: Results of parameter scenarios for the new neighbourhood with gas

				heating (%)			solar	network
				HP	mCHP	CB	(kWp)	(%)
	reference		=	75		25		71
growth electr. use per house	0.8	%/year	1.3 =	74	1	25		78
(heat-unrelated)			2.3 =	72	3	25		**99**
electricity price	7	cent/kWh	4 =	100				71
(w/o tax, with CO_2)			16 =		75	25	6.7	**184**
gas price	30	cent/m^3	20 =	48	2	50		67
(w/o tax, w/o CO_2)			50 =	100				71
price CO_2 in year 2057	90	euro/tonCO$_2$	45 =	73	2	25		67
			135 =	reference				
			180 =	100				71
price HP	1600	euro	600 =	100				71
(compared with CB)			2600 =	reference				
price solar PV system	1200	euro/kWp	400 =	75		25	7.9	**199**
in year 2034			500 =	75		25	3.6	74
			800 =	75		25	3.5	71
			900 =	reference				

See table 1 for abbreviations. All "cent" is eurocent.
The "solar (kWp)" is the average per house in 2034.

The table shows large changes in the type of heating with alternative values about the price of electricity and the price of natural gas. The direction of the changes are as expected, noting that a heat pump uses mainly electricity and a mCHP produces electricity and uses gas. The other scenario with a large change in the type of heating is the price of the heat pump (HP), where a very low price would cause the heat pump to be used in all time periods.

Only a very large increase of the CO_2 price in 2057 has a substantial effect, increasing the use of the heat pump to 100%.

Most houses have the maximum solar PV on the roof if the electricity price would be 16 eurocent/kWh. This pushes the network to 184%. The network strength is a binding restriction only at the moment of the solar investment, in year 25: after that year the increase of heat-unrelated electricity consumption reduces the reverse power flow.

The price of solar PV has no effect on the type of heating. Furthermore, the table shows that solar PV becomes profitable between 800 and 900 euro/kWp; this is consistent with the break-even price of 830 euro/kWp computed in section 4.5. The range in which solar PV is "just profitable" is from less than 500 to more than 800 euro/kWp; this is wider than the theoretical width of 260 euro/kWp computed in appendix C.3. The difference in kWp between 500 and 800 euro/kWp is due to the spatial variation over network: the effect of a particular load change on the network depend on the location of the change; see appendix H.

Some other scenarios were computed, but their results differs little or nothing from the reference result and they are not reported in table 7. These scenarios are: increasing or decreasing the present mCHP price with 1000 euro; changing the decrease per year of this price (compared with the CB) to 2% or to 9%; changing the decrease per year of the price of a heat pump (compared with the CB) to 1% or to 8%; halving or doubling the network costs; changing the discount rate to 2.5% or to 8.5%; increasing or decreasing the 35 kW peak of the school with 20 kW.

5.1.3 The "business case" for the household

The "business case" for households is based on the household's private costs. These are derived from social costs by adding energy taxes and removing the CO_2 emission costs. Also, the discount rate is increased to 19%, based on a pay-back period of five years for heating equipment; see appendix A

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loads used in the model are not very accurate and this result must be coincidental.

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Table 8: Social costs of the "business case" optimum, new neighbourhood with gas

	heating (%)			solar PV (kWp)	social costs (euro)
	HP	mCHP	CB		
business case (private optimum)	0	50	50		+70
reference solution (social optimum)	75	0	25		(0)
business case, without energy taxes	24	0	76		+44
business case, without ignoring CO_2 cost	0	50	50		+70
business case, without increased discount rate	0	75	25	3.0	+118

See table 1 for abbreviations. The cost is per year and per house; see footnote 19.

for details. Willingness to pay for renewable energy is not included. The new neighbourhood with gas is used, with the network strength fixed at the reference outcome.

The optimum is: in the first 24 years each house uses a conventional boiler and in the remaining 24 years each house uses a mCHP; see the first line of table 8. The fixed network is never a binding restriction.

The second line shows the reference solution. The costs in the last column are the deviation from the costs in the reference solution. The difference in social costs is 70 euro per house and per year[19].

The other lines in table 8 show the business case with one component missing, ordered by cost. One might conclude from these lines that the energy taxes are the most important (compared with ignoring the CO_2 and with increasing the discount rate), since omitting them gives a result which is nearest to the reference solution; in cost and in the heating percentages. More precisely, the *ratio* between the two taxes is the problem, rather than their level: the electricity tax is per energy content six times larger than the gas tax; see table 3. Per ton CO_2 emission this ratio is two[20]. If the

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[19] More precisely, this is the "levelized" cost: the amount per year which gives the same present value as the actual time series. This is computed by dividing the present value by the number of years of the "discounted stream" in the discount table 2 on page 9. (For the new neighbourhoods of 48 years, this is 17.7 years.) Next, this is divided by the number of houses, giving the average per house. Of course all social costs are computed with the reference discount of 5.5% per year.

[20] Which pair of energy taxes (per m^3 gas and per kWh electricity) would tax CO_2 emission equally, leaving the total energy tax revenue the same? This is a taxation of 136 euro per ton CO_2, giving a gas tax of 24 cent/m^3 (an increase of 7 cent) and an electricity tax of 7.8 cent/kWh (a decrease of 3.2 cent). This follows from the present

gas tax is *increased* by a factor 3 or more then the business case optimum has the socially optimal 75% HP (or more). Note that these taxes were not distorting the choice of heating at their date of introduction, because there was no such choice at that time: all residential heating was with gas.

The table shows also that it does not matter whether or not the CO_2 cost are included. The solar PV in the bottom line is installed in model year 25 (calendar year 2034).

For the old neighbourhood, the general results of which are discussed below, the "business case" optimum is similar to table 8: in the first 12 years each house uses a conventional boiler and in the remaining 12 years each house uses a mCHP. Detailed results are available from the author upon request.

5.2   The all-electric new neighbourhood

Even without electric cooking, the reference solution of the all-electric new neighbourhood has a much stronger network than the neighbourhood with gas: 117%. This is caused by the large load of the heat pump with a ground source.

The all-electric neighbourhood has no alternative types of heating in the model. Hence parameters variations have effect only on the amount of solar PV and the network. This is shown in table 9, as follows.

With a high growth rate of the heat-unrelated electricity use, the network strength increases.

With the high electricity price of 16 eurocent/kWh, solar PV is "amply profitable" in most of the network after 25 years: nearly all houses have the maximum kWp on the roof, which pushes the network to 164%. This is similar to the new neighbourhood with gas.

The solar PV price in the "just profitable" range of $500\ldots800$ euro/kWp results in more solar PV than with the new neighbourhood with gas, because the network is stronger here. On the other hand, the solar PV price in the

Dutch residential electricity consumption (in kWh) being 2.3 times the residential gas consumption (in m³), combined with the emission factors and tax rates in table 3.

Hence using the first two lines of the heating equipment table 4 for the new neighbourhood with gas, the annual energy tax for heating changes from 305, 30 and 221 euro (for HP, mCHP and CB respectively) to 280, 220 and 320 euro (rounded to 10 euro). Note the large increase for the mCHP. With the old neighbourhood with gas this is a change from 455, 45 and 340 to 420, 330 and 490 euro. In each neighbourhood, these tax shifts move the private optimum in the direction of the social optimum but the two optima are not quite the same.

Table 9: Results of parameter scenarios for the all-electric neighbourhood

				solar (kWp)	network (%)
	reference		=		117
growth electr. use per house (heat-unrelated)	0.8	%/year	1.3 =		**132**
			2.3 =		**174**
electricity price (w/o tax, with CO_2)	7	cent/kWh	16 =	6.8	**164**
price solar PV system in year 2034	1200 euro/kWp		400 =	7.9	**195**
			500 =	5.3	117
			800 =	5.2	117
			900 =	reference	

See table 1 for abbreviations. All "cent" is eurocent.
The "solar (kWp)" is the average per house in 2034.

"amply profitable" range gives the same solar PV as the new neighbourhood with gas, because this depends only on the roof size.

Other parameter scenarios have no effect on the network or the amount of solar PV.

Finally, the purpose of this study does not include the comparison of neighbourhoods with and without gas supply. Even so, one might ask if, within the context of the model, the choice for all-electric is socially optimal? Indeed, the all-electric neighbourhood has 169 euro/year per household less cost, not counting the profit of not having a gas network. Note however that the cost of the solution to the all-electric problem after a power failure in the winter, discussed in section 4.4, must be added to the costs of all-electric. Outside the context of this model, one might wonder if there is enough experience with all-electric neighbourhoods to answer this question.

5.3   The old neighbourhood

The old neighbourhood has a fixed network, with half its lifetime left. The reference result is: all houses use a heat pump all the time. The network is not a binding restriction.

Table 10 shows the robustness of the reference result against alternative

Table 10: Results of parameter scenarios for the old neighbourhood, with fixed network

				heating (%)		
				HP	mCHP	CB
	reference			= 100	0	0
electricity price	7	cent/kWh	4 =	reference		
(w/o tax, with CO_2)			16 =	0	100	0
gas price	30	cent/m^3	20 =	50	0	50
(w/o tax, w/o CO_2)			50 =	reference		
price HP	1600	euro	600 =	reference		
(compared with CB)			2600 =	50	0	50

See table 1 for abbreviations. All "cent" is eurocent.

parameter assumptions. The same parameter scenarios were computed as with the new neighbourhood in section 5.1.2, including those mentioned as the end of that section. No scenario makes the network a binding restriction here. This includes a growth rate of 2.3% per year of the heat-unrelated load. As noted earlier, in footnote 4, here the standard network is rather strong.

Only the scenarios shown in table 10 give a result which differs from the reference result. The time pattern of the two 50/50 results is as follows: it starts with the conventional boiler, followed by the heat pump.

5.4   Solar PV and the network

The moment at which solar PV becomes socially profitable in the Netherlands is very uncertain. However, once it is profitable, the optimal *quantity* seems to be well defined, as follows.

If solar PV is "just profitable" then the amount of solar PV (in kWp/house) is 4 to 5 times the strength of the network. This is with uniformly distributed or optimally distributed solar PV, respectively. See the statistical analysis in appendix C.4; a theoretical analysis gives 3 times with uniform distribution.

As noted above, if solar PV is "amply profitable" then the amount of solar PV is given by the available roof size.

Table 11: No transition compared with the optimal transition

	heating (%)			social costs (euro/year per house)					
				heating				net-	total
	HP	mCHP	CB	elec-tricity	gas	invest-ment	CO_2	work	
New neighbourhood with gas									
optimal transition	75	0	25	57	279	49	44	−6	
no transition	0	0	100	0	393	0	96	−9	
difference				−57	+114	−49	+52	−3	+57
Old neighbourhood, fixed network									
optimal transition	100	0	0	167	238	157	33		
no transition	0	0	100	0	589	0	116		
difference				−167	+351	−157	+83		+110

See table 1 for abbreviations. The investment cost is in comparison with a CB.

5.5 How much better is optimal?

Table 11 shows the cost difference between the optimal transition (the reference solution of the model) and no transition, where the households continue to use a conventional boiler for heating. Both transitions have no solar PV. The costs in the table are the amount per year and per house as described in footnote 19 on page 20.

In the *new* neighbourhood with no transition, the network is minimized with all other control variables fixed. This gives a network strength of only 56% of the standard network for the new neighbourhood. Hence the heat pumps in the reference solution require a network increase of $71/56 - 1 = 27\%$. The 3 euro network cost in the table can be verified as $(0.71 - 0.56) \times 114000$ euro / (319 houses $\times$ 17.7 year).

The total cost difference is 57 euro per year per household. This might be called the social profit of the optimal transition.

For the *old* neighbourhood the cost difference is 110 euro per year and per house. This is much more than the new neighbourhood, because the heat demand is larger in an old house and on average the costs are in the less distant future.

In the *all-electric* neighbourhood there is no freedom to choose heating equipment.

Figure 1: Heating costs of a new house with gas (euros 2010)

(a) without CO_2 cost   (b) with CO_2 cost
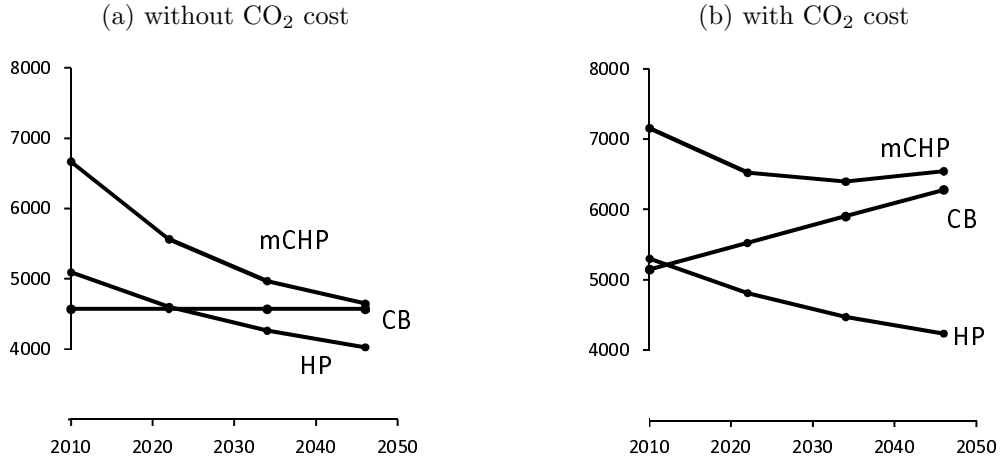
Note: present value discounted to the moment of the installation of the equipment.

5.6 The heating costs over time

As we saw above, the network costs play a minor role in the model; recall the network cost difference of 3 euro per household per year in table 11. In this section I show that the optimal result of the model follows closely the cheapest method of heating.

Figure 1 shows the heating cost over time[21] for a new house (built in 2010), based on the heating investment years 1, 13, 25 and 37 in the model, translated to calendar years (1=2010). The amounts are the present value at those investment years. This is the same definition as the bottom line of table 4 on page 12.

Graph (a) is without the CO_2 cost and hence here the lines start at the numbers in the bottom line of table 4 for the new house. (The small discrepancies are due to the rounding in the table.) The gradual cost reduction of mCHP and HP is due to the reduction of their investment costs as given in section 4.4.

Graph (b) includes the CO_2 cost. The greening of grid electricity and the rising CO_2 cost widens the distance between the HP line and the other two lines. As a result, the least cost heating method starts (only just) with the conventional boiler, followed by the heat pump during the rest of the time.

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[21] In terms of the model in section 3, this is about the cost equation (3) with $S_{nt} = 0$.

Indeed, this is nearly the same pattern as shown in table 6; the only difference is the small fraction of houses with a mCHP in the last subperiod in table 6, due to the network.

Also, the difference between the CB and HP lines in figure 1(b) agrees with the cost difference of a new house in table 11. The heating part of this cost difference is $57 + 3 = 60$ euro. This can be reproduced from the investment years 2022, 2034 and 2046 in figure 1(b) as follows:

$$1000 \text{ euro} \times \left(0.7 \times 0.53 + 1.4 \times 0.53^2 + 2.0 \times 0.53^3\right) / 17.7 = 60 \text{ euro}$$

The 0.53 and the 17.7 are from table 2.

With an old house (not shown here), the costs without CO_2 start at the numbers in the bottom line of table 4 for the old house. These costs are somewhat higher than with a new house, due to the larger heat demand of an old house. The pattern over time is similar to the pattern with a new house, though only up to the second dot in the graphs, at 2022. At both dots the least cost heating method is the heat pump, both with and without CO_2 cost. This agrees with the result of the old neighbourhood in section 5.3.

6 Conclusions

In the socially optimal new neighbourhood with natural gas, nearly all houses are going to use an air-to-water heat pump. The heat pumps require an electric network increase of 27% compared with a network with only conventional boilers. This network increase costs only 3 euro per year per household; hence the result changes very little when the network is ignored and only one house is studied. Also, the result depends very little on the future CO_2 permit price.

With a round figure, the cost reduction of the optimal transition for a new neighbourhood with natural gas, compared with the continuation of the use of conventional boilers, is 60 euro per year per household. For an old neighbourhood this is more, due to the larger heat demand and on average the less distant future.

A simple computation shows that a solar PV system price below 830 euro/kWp is required to make solar PV socially profitable. This is at present not the case.

The robustness of the result against alternative parameter values has been tested. The largest changes in the result occur with more heat-unrelated electricity use and with a higher electricity price and with a higher gas price.

The former two scenarios show a large network strength, though for opposite reasons: much load and much local generation, respectively.

The optimal amount of solar PV and the optimal type of heating are connected through the electric network. However, this connection is very weak: the optimal type of heating does not depend on the price of solar PV.

In the optimal "business case" for a household, using private costs including taxes and excluding CO_2, no heat pump is used. This causes a social cost of 70 euro per household per year. However, changing the ratio between the two taxes with a factor 3 makes the private optimal choice the same as the social optimal choice.

These results apply to the Netherlands. A generalization to other countries must take into account the specific Dutch situation, including its prices and taxes, its country-wide network for natural gas and its relatively much gas-fired grid electricity. The Netherlands has mild summers with little need for the cooling of a house. Another limitation of the study is the Profile Generator being based on the assumption that all equipment performs according to its specifications.

Finally, the following has no effect on the results: a linear Taylor approximation of the power flow model has been derived, modelling both real and reactive power. The well-known DC approximation is a simplification of the real power part of this new approximation.

A   Dating and discounting

In order to keep the model simple, an investment and its first yearly revenue are recorded in the same year. In a way, this year is treated as both a time moment and a time period. Hence the summation in table 2 starts with $t = 0$.

This error is considered less important than the simplicity of the time keeping in the model (and a low probability of mistakes). Since a yearly flow is on average halfway the year, the error is approximately equal to one half of the discount rate, which is much smaller than the percentage accuracy of most if not all amounts in the model.

The discount rate of 19% for the "business case" of households is based on a pay-back period of five years for $T = 12$ (see section 5.1.3 on page 19). Here the summation starts halfway the first year:

$$\sum_{t=0}^{11}(1 + 0.19)^{-(t+1/2)} = 5.0 \tag{11}$$

Compare with the summation formula in table 2. For the pay-back period of five years, see for example De Jong et al. (2008).

B   Taxes and social costs

As noted in section 4.1, taxes are ignored in the model since for the society as a whole, paying and receiving them cancel each other. The same holds for subsidies.

In CPB (2011) it is suggested to consider the government as an economic agent with a balanced budget. The government returns any surplus to the consumers, after multiplying it with $1+\tau$ (where τ is the VAT rate), because from any amount spent on consumption a fraction $t = \tau/(1+\tau)$ comes back to the government. In my model this leads to the same principle of cancelling taxes. This can be shown as follows.

Let there be two alternatives, A and B, indicated by index i. Let the consumer cost C_i of alternative i be:

$$C_i = (1 + \tau)\,(F_i + T_i) \tag{12}$$

where F_i is the factor cost and T_i is the energy tax or any indirect tax other than the VAT.

In my model, the utility of the alternatives is the same for the consumer: the alternatives are two ways to satisfy the energy need. This leaves only the costs to compare. For the government, the tax receipts of A minus those of B are equal to $T_A - T_B$. This is without a change in VAT receipts[22], assuming[23] that the total consumption does not depend on the choice between A and B.

As noted above, the government balances its budget by returning any surplus to the consumers, after multiplying it with $1 + \tau$. This amounts to a tax return difference:

$$R_A - R_B = (1 + \tau)(T_A - T_B) \tag{13}$$

This can be positive or negative. Then the cost for the government is zero and the social cost difference between A and B is the consumer cost difference

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[22] Indeed, simply ignoring the VAT ($\tau = 0$) gives the same result in factor prices, namely $F_A - F_B$; see the final equation (14). However, in CPB (2011) it is noted that consumer preferences between alternatives, when expressed in money, include the VAT, and hence it is suggested to include the VAT in government investment costs in social cost-benefit studies. (Be aware of the different notation in CPB (2011), using not the tax rate τ but the fraction $t = \tau/(1+\tau)$.)

[23] This assumption was suggested by Peter Zwaneveld (CPB).

minus the tax returns difference:

$$(C_A - C_B) - (R_A - R_B) = (1 + \tau)(F_A - F_B) \tag{14}$$

This is the social cost difference including VAT. In factor prices, this is simply the factor costs difference $F_A - F_B$, which is equivalent to ignoring all taxes.

C   Solar PV

C.1   Output

The output of a residential solar PV system is somewhat lower than the output of the cells in the laboratory. This is for instance caused by the transformation to AC and expressed by the performance ratio (or "conversion efficiency" or "quality factor"). In the model this quantity is set to 77.5%.

C.2   Roof surface and maximum output

The surface of a residential solar PV system is limited by the available roof surface. This can be translated to a maximum kWp using the efficiency, as was shown in section 4.5. The future development of the maximum kWp is proportional with the future development of the efficiency, which in turn is related to the price per kWp. Let part of the assumed drop of the price per kWp be caused by the increase of the efficiency. (The other part of the price drop is caused by the decrease of the production cost per m^2.) I assume a linear relation between the efficiency and the price per kWp:

$$\eta_t/\eta_1 = 2 - p_t/p_1 \tag{15}$$

where η_t is the efficiency in year t and p_t is the price per kWp in year t. We start in year 1 with the equality $1 = 2 - 1$. When p_t drops, the upper limit of the efficiency is twice its present value of $\eta_1 = 15\%$. See MacKay (2009), page 39.

C.3   The just profitable price range

In section 5.4, the width of the range where solar PV is just profitable was given. This can be computed as follows.

Consider the equation for the amount of solar PV as a function of the network strength in section 5.4, with optimally distributed solar. Solving

Table 12: Network strength and solar PV, new neighbourhoods

section	scenario (or reference)	network strength (1)	kWp/house optimal (2)	kWp/house equal (3)	ratio optimal (2)/(1)	ratio equal (3)/(1)
5.1 gas	reference	0.71	3.7	2.9	5.2	4.1
5.1 gas	electr. use 2.3%	0.99	5.2	4.1	5.3	4.1
5.1 gas	gas price 50 cent	0.71	3.7	2.9	5.2	4.1
5.2 gas	low PV price	0.725	3.55		4.9	
5.2 no gas	reference	1.17	6.0	4.7	5.1	4.0
5.2 no gas	electr. use 2.3%	1.74	8.7	6.8	5.0	3.9
5.2 no gas	low PV price	1.17	5.25		4.5	

Note: optimal = optimally distributed over the given network.

this equation for the network strength gives: network strength = 1/5 times the average kWp per house. Hence the network cost for one extra kWp per house is the 114,000 euro variable costs of the network (see section 4.2) divided by 5, which is 23,000 euro. The network cost for one extra kWp in the network (*not* per house) is 23,000 / 319 houses = 72 euro/kWp. Then the width of the just profitable price range in year 25 is 72/0.28 = 260 euro/kWp (rounded), using discounting table 2.

C.4 The network

In section 5.4 a relation between the amount of solar PV and the strength of the electrical network is given, when the former is limited by the latter (and not by the roof size). This relation is estimated using the data in table 12.

These data are based on the computations with the new neighbourhoods discussed in sections 5.1 and 5.2. The various lines of the table refer to the scenarios where the result differs from the reference solution, or to the reference solution itself. (The scenarios where solar PV is amply profitable, with solar PV covering all available roof surface and the network a dependent variable, have been excluded.)

Except with the fourth and the last line, the following procedure was used. Starting from the indicated model result, all control variables were fixed except the solar PV investment in year 25. Also the roof was set to unlimited size. Then the model was solved maximizing the total solar PV, both with and without the restriction of equal amounts on the houses. The

fourth line and the last line were copied from tables 7 and 9 respectively, including the amount of solar PV.

I conclude from the ratio columns in table 12: 5 kWp/house with optimal distributed solar PV and 4 kWp/house with the same amount of solar PV at each house.

Also a linear regression analysis has been applied to estimate a relation with a constant term: $\text{kWp} = a + b \times \text{network}$. However, the constant term a turned out to be insignificant; both in the sense of being small compared with the kWp in the table and in the sense of being small compared with its standard error.

Finally, a rough DC-like theoretical approximation of the coefficients can be obtained for the case where the load is evenly distributed among the houses in the network, as follows. As noted in section 5, the electric current is the binding restriction. The electric current per connection to the grid, during a solar radiation peak and with 1 kWp/house, is: 75 houses per connection $\times$ 1000 watt (the approximate peak) $\times$ performance ratio 0.775 / 230 volt = 253 A. The maximum current is 260 A per phase $\times$ 3 phases = 780 A. This is 3.1 times 253 A. Hence one unit of extra network strength creates room for 3.1 kWp/house extra solar, if the other load does not change. This is smaller than the last column of table 12.

D The Profile Generator

D.1 General

In section 4.6 the basics of the Profile Generator were discussed. More details are given here, including complete tables of extremes.

Since the amount of solar PV is not known beforehand, a range of zero to 50 m^2 of solar PV is simulated, with steps of 1 m^2. The extremes of the total load *including* solar PV are identified. Fortunately the various solar steps have many extreme moments in common. The number of these extreme moments does not increase if this upper bound of 50 m^2 is increased even further.

However, the tables 13 and 14 show the amount of load *without* taking into account solar PV output, since the amount of this output in a house depends on the actual surface of solar PV on that house. The model uses the radiation per m^2 on an optimally oriented (fixed) surface to compute the actual solar PV output, depending on the amount of solar PV installed in each particular house and time period. (First the kW/m^2 input in the table

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is translated to kW/kWp output using the efficiency and the performance factor.)

The Profile Generator takes into account the simultaneity of the extremes. For example, a peak in solar PV output occurs at the same time in each house. However, peaks in heat demand do not occur at the same time in each house and hence peaks in heat demand in the Profile Generator are lower than such peaks in one house. The Profile Generator uses 200 houses. There are several statistical methods for the prediction of peaks in a network, based on empirically estimated parameters. See for instance the review in Dickert and Schegner (2010), section IV, where the work of Rusck and the work of Velander and Axelsson and Strand is discussed. However, this is only about the heat-unrelated load, while I model simultaneously the coincidence of peaks of heat-unrelated load and heat-related load (possibly negative) and solar PV output.

The table for the old neighbourhood is not given below, since the network is not relevant there.

D.2   The extremes table of the new neighbourhood with gas

Table 13 shows the extremes for a new neighbourhood with natural gas. The extremes are ordered by season.

In the winter (the upper part of the table) the mCHP generates much electricity. In the first three lines, the heat pump uses the only the auxiliary gas boiler for heating: the air temperature is too low to extract heat.

Going down the table, the heating by the mCHP diminishes gradually, as can be seen from the diminishing electricity output in column (2). The heat pump in column (1) shows the largest electricity consumption halfway the table. Extremes with much heat-unrelated load and no solar radiation are in the evening.

The four lines of the small example in table 5 on page 15 are the following lines in table 13: line 12 (max mCHP) and line 2 (min mCHP) and line 3 (max CB) and line 1 (min CB). Each of these lines has no solar radiation.

D.3   The extremes table of the all-electric neighbourhood

Table 14 shows the extremes table for an all-electric new neighbourhood. There is no choice of heating equipment: heating is done with a heat pump (which is different from the heat pump in the neighbourhood with natural gas).

Table 13: Extreme load for the new neighbourhood with gas, 2010

	season	extreme (incl. solar)	load of a house with ...			components			heat-solar radiation	solar PV
			HP	mCHP	CB	HP	mCHP	heat-unrelated	solar radiation	solar PV surface
			(1)+(3)	(2)+(3)	(3)	(1)	(2)	(3)		
			× 100 watt						kW/m²	m²
1	cold	min HP and CB	2	−5	2	0	−7	2	0	
2	↑	min mCHP	2	−6	2	0	−8	2	0	
3		max CB	8	−1	8	0	−9	8	0	
4		min mCHP	8	−4	3	5	−7	3	0.40	4...7
5		max HP	12	1	6	6	−5	6	0	
6		min mCHP	8	−1	3	5	−4	3	0.77	8...10
7		min all	6	1	3	3	−2	3	0.90	5...∞
8		min CB	2	1	2	0	−1	2	0.19	1
9		min CB	4	0	2	2	−2	2	0.52	2
10		min HP	3	3	3	0	0	3	0.85	2...35
11	↓	min CB	3	2	3	0	−1	3	0.83	3...4
12	warm	max mCHP	6	5	5	1	0	5	0	

Source: Profile Generator. The watt columns at the left of the vertical line ignore solar PV output (which depends on the actual surface).

Table 14: Extreme load for the all-electric new neighbourhood

	season	extreme	total	heating	heat-unrelated	solar radiation	solar PV surface
			× 100 watt			kW/m²	m²
1	cold	max all	20	12	8	0	
2	↑	min total	7	4	3	0.90	5...∞
3		min total	4	1	3	0.85	7...∞
4	↓	min total	3	0	3	0.68	2...3
5	warm	min total	2	0	2	0	

Source: Profile Generator. The watt columns ignore solar PV output (which depends on the actual surface).

The first line is a winter evening and the last line is a summer night, both without sunlight.

D.4   The old neighbourhood

The extremes of the old neighbourhood (not shown) are similar to the new neighbourhood with gas in table 13, with somewhat larger heat-related loads (in absolute value), due to the larger heat demand; see also table 4 on page 12.

In particular, the maximum load of an old house with a heat pump is 1.4 kW, while this is 1.2 kW in line 5 of table 13. The minimum load of an old house with a mCHP is −0.7 kW, while this is −0.6 kW in line 2 of table 13.

D.5   The growth of the heat-unrelated electricity use

The above extremes tables show only the present situation. The heat-unrelated electricity consumption per house in these extremes grows with 0.8% per year. I implement this simply by adjusting the extreme values. An alternative method would be to apply this growth rate to the raw data of the Profile Generator for future years and identify from these series the extreme moments in the future. In order to keep it simple, I have not done this.

This alternative method can be explained more formally as follows. Let $s = S$ be the solution of

$$\max_s X_s + Y_s \tag{16}$$

where s is the time moment within a year, as in section 3. The Y grows gradually over the years. Let λ be the growth factor of Y from one year to the next. Then this alternative method solves repeatedly, for all years t:

$$\max_s X_s + \lambda^t Y_s \tag{17}$$

while I simply use repeatedly:

$$X_S + \lambda^t Y_S \tag{18}$$

Note that with the latter method one might define the extreme moment S as the solution of (16) in any year. Fortunately it turned out that doing this in the last year of the model does not differ much from doing this in the first year.

E   Network restrictions and cable properties

The network capacity is defined by restrictions on voltage deviations (to protect the customer's equipment) and restrictions on the electric current (to protect the cables and transformers).

E.1   Maximal voltage deviations

In the Netherlands, the percentage voltage deviation from the nominal value is regulated in the *Netcode Electriciteit*; see Energiekamer NMa (2011), section 3.2: *De kwaliteit van de transportdienst* (The quality of the transport service). For the low voltage net, the deviation is restricted to the range $\pm 10\%$ in 95% of the ten-minute average values during one week. Also, the deviation is restricted to the range $+10/-15\%$ for all ten-minute average values. In the model only the first restriction is implemented: $\pm 10\%$ at all times in the extremes of the Profile Generator. This allows for special, rare events (which are not included in the extremes of the Profile Generator) to occur in the residual 5% of the time.

It is in principle possible to prevent for instance a too low voltage by increasing the voltage at the connection with the grid (the transformer). Of course this works only in the case of a very low voltage or a very high voltage, but not both at the same time in different places. Programming this in the model is trivial (don't fix the voltage level at the grid), but it increases the computing time of finding the optimum.

In all our scenarios, the voltage does not reach any of its two limits, not even close. Hence the electric current is the binding restriction; see below.

Since the voltage drop along a line increases with the length of the line, I also simulated a less densely populated area by multiplying all distances by an integer value. I used the new neighbourhood with gas. With a multiplication of 2, the largest voltage drop anywhere anytime is 9.0%. With a multiplication of 3, the voltage drop is a binding restriction, with 10%. Apart from the school, this drop occurred in three places at three moments. The three places are endings of the network. The three moments are in the years 24, 36 and 48: final years of the heating equipment, when the heat-unrelated load is largest. The moment within the year was always when the load of a house with a heat pump is at its largest.

Table 15: Physical cable properties, per phase

neighbourhood	cross-section (mm^2)	max current (ampere)	resistance R (ohm/km)	reactance X (ohm/km)
old	95	205	0.32	0.08
new	150	260	0.21	0.07

Notes: the cables are aluminium; the reactance is at 50 Hz.

E.2   Maximal currents and other cables properties

The electric currents in the network are restricted by a maximum. Large currents can damage a cable or decrease its residual lifetime. The maximum current depends on the surface of the cross section. The maximum currents per phase are given in table 15, together with the other electric properties. The 95 and 150 mm^2 cables are applied in the old and new neighbourhood, respectively. The symbols R for resistance and X for reactance are used in the discussion of the power flow model in appendix F.

E.3   The network strength x

In practice, a stronger (or weaker) network implies a larger (smaller) number of connections with the grid, with a constant total number of houses. However, the optimization process requires a continuously variable network strength with a fixed network topology.

In order to understand how this is implemented in the model, consider a doubling of the network strength required by a doubling of the load, with the same number of houses. Each connection to the grid now becomes two connections, serving the group of houses originally served by one connection. Think of this as two parallel identical cables to those houses. Together these two parallel cables have one half of the original impedance and twice the original maximum electric current.

This is generalized to a continuously variable network strength x by making (a) the impedance of each cable in the network inversely proportional with x and (b) the maximum electric current directly proportional. The impedance depends on the resistance R and reactance X. The ratios between R and X and the maximum currents are as in table 15.

F The electric power flow model

In this appendix the engineering model of the flow of electric power through a network is discussed. This model is at the heart of the GAMS economic-technical model, since for each iteration in the solution space, during the optimization, the restrictions on electric currents and voltages are satisfied everywhere in the network and in each extreme moment in each year.

Results of this part of the GAMS model has been verified with the PowerFactory computer program from DigSILENT. The model without a shunt capacitor is used; experiments with PowerFactory show that the effect of including a shunt capacitor in our network model is negligible (with $C \approx 0.7\mu\text{F/km}$).

I start from the standard notation in the engineering literature, with complex variables. Next, the model in real variables is derived, as coded in GAMS. In this way it is possible for a mathematically skilled economist to understand the model and relate it to the engineering literature.

In section F.5 a linear approximation is derived, to start the nonlinear iterations in GAMS.

Only the steady state is modelled, where all voltages and currents are sinusoidal.

F.1 The power flow from a node

Think of a voltage or current as a complex number with constant modulus, rotating counterclockwise along a circle in the complex plane, in the frequency of the AC system. The real part shows the sine wave of the "real" AC variable[24]. A snapshot of the complex plane at an arbitrary moment shows the modulus of the variables and the phase angles between them, which is all we need.

Complex quantities are shown here in boldface: power $\mathbf{S}$, voltage $\mathbf{V}$, electric current $\mathbf{I}$ and serial impedance $\mathbf{Z}$. For ease of exposition, only one phase of the three-phase system is modelled. (The load per phase is computed by dividing the actual load by three.) Of course, the order of the two subscripts of direction-less elements such as impedance $\mathbf{Z}_{ik}$ is not relevant.

Let nodes i and k be connected by a direct network link. The complex voltage drop over the link is equal to the impedance times the current (a

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[24] The modulus of the voltage at the grid is set at 230 volt. This is also the amplitude of the real part of the rotating complex voltage, while in fact the amplitude of the voltage is $230\sqrt{2}$ volt. Of course this is the most practical approach, giving results which do not need to be divided by $\sqrt{2}$ (voltages and currents) or by 2 (power).

generalization of Ohm's Law): $\mathbf{V}_i - \mathbf{V}_k = \mathbf{I}_{ik}\mathbf{Z}_{ik}$. Hence

$$\mathbf{I}_{ik} = \frac{\mathbf{V}_i - \mathbf{V}_k}{\mathbf{Z}_{ik}} \tag{19}$$

The complex power flow from i in the direction of k equals the voltage at i times the conjugated current from i to k. (Note that the complex power does not rotate over time.) Hence the conjugate of the complex power is:

$$\mathbf{S}_{ik}^* = \mathbf{V}_i^* \mathbf{I}_{ik} = \mathbf{V}_i^* \frac{\mathbf{V}_i - \mathbf{V}_k}{\mathbf{Z}_{ik}} = \frac{|\mathbf{V}_i|^2 - \mathbf{V}_i^* \mathbf{V}_k}{\mathbf{Z}_{ik}} \tag{20}$$

In a GAMS model all quantities are real. The relations between the complex quantities and the real quantities are:

$$V_i = |\mathbf{V}_i| \tag{21}$$

$$\mathbf{Z}_{ik} = R_{ik} + \mathrm{j}X_{ik} \tag{22}$$

$$\mathbf{S}_{ik}^* = P_{ik} - \mathrm{j}Q_{ik} \tag{23}$$

$$\delta_{ik} = \sphericalangle \mathbf{V}_i \; - \; \sphericalangle \mathbf{V}_k \tag{24}$$

The R_{ik} and the X_{ik} are respectively the line resistance (causing heat) and the line reactance X_{ik} (caused by the magnetic field around the line). The imaginary unit is $\mathrm{j} \equiv \sqrt{-1}$. The P and Q are respectively the real power and the reactive power. Note that both P and Q are "real" physical phenomena; for economists, see for instance Stoft (2002), Fig. 5-2.1 on page 384. The $\sphericalangle \mathbf{V}$ indicates the voltage phase angle in the complex plane. The definition of δ_{ik} is arbitrary; it might as well be the reverse, in which case all δ_{ik} below must be negated.

In order to find the real part and the imaginary part of (20), use the rule for a ratio of complex numbers:

$$\frac{x_1 + \mathrm{j}y_1}{x_2 + \mathrm{j}y_2} = \frac{(x_1 x_2 + y_1 y_2) + \mathrm{j}(y_1 x_2 - x_1 y_2)}{x_2^2 + y_2^2} \tag{25}$$

We need the real part and the imaginary part of $\mathbf{V}_i^* \mathbf{V}_k$ in (20). This is a complex number with modulus $V_i V_k$ and angle $-\delta_{ik}$ and hence:

$$\mathbf{V}_i^* \mathbf{V}_k = V_i V_k \cos \delta_{ik} \; - \mathrm{j}V_i V_k \sin \delta_{ik} \tag{26}$$

Now the real part of equation (20) follows straightforward:

$$P_{ik} = \frac{V_i}{R_{ik}^2 + X_{ik}^2} \left(R_{ik}V_i - R_{ik}V_k \cos \delta_{ik} + X_{ik}V_k \sin \delta_{ik} \right) \tag{27}$$

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Table 16: Reactive power

		Q/P	$\cos\phi$
tv, lighting, etc.	consumption	0.2	0.98
school	consumption	0.05	0.999
mCHP	production	0.25	0.97
heat pump (any)	consumption	0.62	0.85
solar PV	production	−0.1	0.99

Note: $\tan\phi = Q/P$.

Since Q_{ik} is minus the imaginary part of $\mathbf{S}^*_{ik}$, I have

$$Q_{ik} = \frac{V_i}{R_{ik}^2 + X_{ik}^2}\left(X_{ik}V_i - X_{ik}V_k\cos\delta_{ik} - R_{ik}V_k\sin\delta_{ik}\right) \qquad (28)$$

Equations (27) and (28) are the building blocks of our power flow model. Both equations occur in Elgerd (1971), page 35, with a less pedestrian derivation. In the economic reference work Schweppe et al. (1988) the P_{ik} equation is given without derivation in equation (A.1.8) on page 272 (with the Ω_i having the incorrect sign) and repeated as equation (D.1.1) on page 313 (with correct Ω_i).

Table 16 shows the model assumptions about the relative amounts of reactive power in consumption and production of electricity. The $\cos\phi$ is called the power factor. The reactive power delivered by solar PV is controlled by power electronics.

F.2   The simultaneous model

The total power flow model basically consists of a simultaneous system of equations with two equations per node. For each node i:

$$\overline{P}_i + \sum_{k\in\ell(i)} P_{ik} \;=\; 0 \qquad (29)$$

$$\overline{Q}_i + \sum_{k\in\ell(i)} Q_{ik} \;=\; 0 \qquad (30)$$

The $\overline{P}_i$ and $\overline{Q}_i$ together indicate the net load at node i. (They are the W_{ts*} in equation (5) on page 7.) The P_{ik} and Q_{ik} are defined by the equations (27) and (28). The $\ell(i)$ is the set of nodes directly linked to node i. In words: the net power flowing out of a node is zero.

At node i are two unknowns: V_i and $\sphericalangle\mathbf{V}_i$. Together they constitute the complex voltage. However, at the connection with the grid, the complex voltage is fixed and the complex power flow from the grid into the network is an unknown; this keeps the total number of unknowns equal to the total number of equations.

F.3 The bus admittance matrix

An alternative approach in the literature is the use of the admittance, the reciprocal of the impedance. I discuss shortly the relation with the approach above.

The bus admittance matrix is a square symmetrical complex matrix. Its order is the number of nodes ("buses"). The off-diagonal element i, k contains minus the admittance between nodes i and k, if there is a direct network link between them:

$$\mathbf{Y}_{ik} = -\frac{1}{\mathbf{Z}_{ik}} = -\frac{R_{ik} - \mathrm{j}X_{ik}}{R_{ik}^2 + X_{ik}^2} = \frac{-R_{ik} + \mathrm{j}X_{ik}}{R_{ik}^2 + X_{ik}^2} \tag{31}$$

If there is no direct link then $\mathbf{Y}_{ik} = 0$, as if the modulus of the impedance $\mathbf{Z}_{ik}$ is infinite. The diagonal of the matrix is:

$$\mathbf{Y}_{ii} = -\sum_{k \neq i} \mathbf{Y}_{ik} \tag{32}$$

Then the equations for node i can be written:

$$\overline{P}_i + V_i \sum_k V_k \left(\; G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik}\right) = 0 \tag{33}$$

$$\overline{Q}_i + V_i \sum_k V_k \left(-B_{ik} \cos \delta_{ik} + G_{ik} \sin \delta_{ik}\right) = 0 \tag{34}$$

The G_{ik} and B_{ik} are the real part and the imaginary part of $\mathbf{Y}_{ik}$, respectively, for all pairs i, k.

Compare with (29) and (30), together with (27) and (28). The terms with V_i^2 in (27) and (28) come from the diagonal of the bus admittance matrix in (32); note that $\sin \delta_{ii} = 0$ and $\cos \delta_{ii} = 1$ and $G_{ii} = \sum_{k \neq i} R_{ik}$ and $B_{ii} = -\sum_{k \neq i} X_{ik}$. The set ℓ is no longer needed here, due to the zero admittance between nodes without a direct network link.

F.4   The current between two nodes

The electric current in the network is restricted by a maximum, which limits the network capacity. Hence the currents must be computed. The modulus of the complex current between nodes i and k can be computed from (19) after the power flow has been computed, including all voltages. The squared modulus is

$$|\mathbf{I}_{ik}|^2 = \frac{|\mathbf{V}_i - \mathbf{V}_k|^2}{|\mathbf{Z}_{ik}|^2} \qquad (35)$$

with $|\mathbf{Z}_{ik}|^2 = R_{ik}^2 + X_{ik}^2$. We have

$$\begin{aligned} |\mathbf{V}_i - \mathbf{V}_k|^2 &= V_i^2 + V_k^2 - 2V_iV_k\cos\delta_{ik} \\ &= (V_i - V_k)^2 + 2V_iV_k\left(1 - \cos\delta_{ik}\right) \end{aligned} \qquad (36)$$

The first line is the cosine rule for triangles. The second line is numerically more stable[25] and also more appealing as it decomposes into the effect of the modulus difference and the effect of the phase difference.

The voltages and the currents in the left-hand side of equation (5) on page 7 are modulus values.

F.5   Linearized models

Using a linear approximation of the power flow model saves computing time. If the result is not accurate enough, then such an approximation is still useful for initially setting all unknows at a value not very far from the solution: a "warm start". (With a "cold start", GAMS sets all unknowns at zero.)

First I discuss a linear Taylor approximation of the model[26]. Then the approximation is further simplified by setting all $R = 0$.

Define the ratio

$$r_i \equiv V_i/V \qquad (37)$$

where V is the nominal voltage. Define r_k similarly, and also the constant

$$\lambda_{ik} \equiv \frac{V^2}{R_{ik}^2 + X_{ik}^2} \qquad (38)$$

---

[25] Rounding errors cannot produce a negative result here. Also, consider the case of δ_{ik} being exactly zero. Let $V_i = 1$ and $V_k = V_i + \epsilon$ with ϵ somewhere in between single and double precision, say 1.E−9. Then in GAMS and Matlab and Excel, the first line of (36) produces a zero and the second line produces the correct result.

[26] This was suggested by Rob Aalbers.

The Taylor approximation is around the point defined by:

$$r_i = r_k = 1 \quad \text{and} \quad \delta_{ik} = 0 \tag{39}$$

Substitution of (38) and (39) into (27) and (28) gives the following result. For brevity, the subscripts of λ_{ik} and R_{ik} and X_{ik} are omitted:

$$[P_{ik}] \,/\, \lambda \;=\; r_i \left(Rr_i - Rr_k \cos\delta_{ik} + Xr_k \sin\delta_{ik} \right) = R - R = 0 \tag{40}$$
$$[Q_{ik}] \,/\, \lambda \;=\; r_i \left(Xr_i - Xr_k \cos\delta_{ik} - Rr_k \sin\delta_{ik} \right) = X - X = 0 \tag{41}$$

The values at the point defined by (39) are indicated with square brackets. The derivatives at this point are:

$$\left[\frac{\partial P_{ik}}{\partial r_i} \right] /\lambda \;=\; 2Rr_i - Rr_k \cos\delta_{ik} + Xr_k \sin\delta_{ik} \qquad = +R \tag{42}$$

$$\left[\frac{\partial P_{ik}}{\partial r_k} \right] /\lambda \;=\; \qquad - Rr_i \cos\delta_{ik} + Xr_i \sin\delta_{ik} \qquad = -R \tag{43}$$

$$\left[\frac{\partial P_{ik}}{\partial \delta_{ik}} \right] /\lambda \;=\; \qquad + Rr_ir_k \sin\delta_{ik} + Xr_ir_k \cos\delta_{ik} = +X \tag{44}$$

$$\left[\frac{\partial Q_{ik}}{\partial r_i} \right] /\lambda \;=\; 2Xr_i - Xr_k \cos\delta_{ik} - Rr_k \sin\delta_{ik} \qquad = +X \tag{45}$$

$$\left[\frac{\partial Q_{ik}}{\partial r_k} \right] /\lambda \;=\; \qquad - Xr_i \cos\delta_{ik} - Rr_i \sin\delta_{ik} \qquad = -X \tag{46}$$

$$\left[\frac{\partial Q_{ik}}{\partial \delta_{ik}} \right] /\lambda \;=\; \qquad + Xr_ir_k \sin\delta_{ik} - Rr_ir_k \cos\delta_{ik} = -R \tag{47}$$

Then the first-order Taylor approximation of P_{ik} around the point defined by (39) is:

$$
\begin{aligned}
P_{ik} &\approx \; \left[P_{ik} \right] + (r_i - 1) \left[\frac{\partial P_{ik}}{\partial r_i} \right] + (r_k - 1) \left[\frac{\partial P_{ik}}{\partial r_k} \right] + \delta_{ik} \left[\frac{\partial P_{ik}}{\partial \delta_{ik}} \right] \\
&= \; \lambda_{ik} \left(R_{ik}(r_i - r_k) + X_{ik}\delta_{ik} \right) \\
&= \; \frac{V}{R_{ik}^2 + X_{ik}^2} \left(R_{ik}V_i - R_{ik}V_k + X_{ik}V\delta_{ik} \right)
\end{aligned}
\tag{48}
$$

Compare with (27). In the same way I get for Q_{ik}:

$$
\begin{aligned}
Q_{ik} &\approx \; \lambda_{ik} \left(X_{ik}(r_i - r_k) - R_{ik}\delta_{ik} \right) \\
&= \; \frac{V}{R_{ik}^2 + X_{ik}^2} \left(X_{ik}V_i - X_{ik}V_k - R_{ik}V\delta_{ik} \right)
\end{aligned}
\tag{49}
$$

The result is a set of equations which is linear in all P_{ik}, Q_{ik}, V_i, V_k and δ_{ik}, given all R_{ik} and X_{ik} and the nominal voltage V. Note that $P_{ik} + P_{ki} = 0$; in this sense this is a lossless model, although the resistances are not zero[27].

This can be simplified further by setting $R_{ik} = 0$, creating a "really" lossless model:

$$P_{ik} \approx V^2 \delta_{ik}/X_{ik} \tag{50}$$
$$Q_{ik} \approx V(V_i - V_k)/X_{ik} \tag{51}$$

Equation (50) is called the DC power flow approximation. See for instance Schweppe et al. (1988), appendix D; Grainger and Stevenson (1994) formula (14.58) on page 626; Wood and Wollenberg (1996) pages 108/109 and 341; Stoft (2002), table 5-2.1 on page 387; Purchala et al. (2005). It is usually motivated by the substitution of

$$R_{ik} = 0\,;\ V_i = V_k = V\,;\ \sin\delta_{ik} = \delta_{ik}\,;\ \cos\delta_{ik} = 1 \tag{52}$$

into the exact formula (27). However, there is no such set of substitutions which produces both (50) and (51) from the two exact formulas (27) and (28).

The model consisting of both (50) and (51) will be indicated as DC$^+$. When computing the electric current with DC$^+$, substitute $|\mathbf{Z}_{ik}| = X_{ik}$ in equation (35), giving $R_{ik} = 0$.

F.6   The accuracy of the two linearized models

All results given in section 5 are based on the exact model. Some of these results have also been computed with the Taylor model of equations (48) and (49) and with the DC$^+$ model. Table 17 shows differences with the exact model in the new neighbourhood with gas. The voltages with the DC$^+$ model show large errors.

The accuracy of the Taylor approximation in the other two neighbourhoods is similar to table 17.

Table 18 shows the outcome of table 17 in a network with only two nodes, and hence one link. This simulates a section of a street in the reference situation (with mainly heat pump heating and without solar). The end of this section is at the end of the street. All quantities are per phase. A cluster of houses is located at the end (see appendix G for clustering). It

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[27] Compare with equations (16) and (17) in Singh et al. (1994). Due to their constant terms L_{ij} and N_{ij} (with $L_{ij} = L_{ji}$ and similarly for N), this is not lossless. Note that in their equations (1) and (2), the definition of δ_{ik} is the reverse of our definition.

Table 17: The accuracy of the linearized models, new neighbourhood with gas

	reference			solar 400 euro/kWp		
	exact	Taylor	DC$^+$	exact	Taylor	DC$^+$
network strength (%)	71	68	68	199	205	205
fraction mCHP (%)	0.4	0.5	0.5			
solar (kWp/house)				7.9	7.9	7.9
largest voltage drop (%)	4.3	4.4	0.5	1.6	1.5	0.3
largest voltage raise (%)				4.2	4.2	0

Exact = (27)+(28); Taylor = (48)+(49); DC$^+$ = (50)+(51). Empty cells are zero.
Some of the exact results are in tables 6 and 7.

Table 18: The accuracy of the linearized models, one link

	exact	Taylor	DC$^+$
voltage drop (V)	0.43525	0.41739	0.02609
voltage lag (radians)	2.4618E-4	2.2684E-4	5.6711E-4
current (A)	9.2	8.9	8.5

Impedance $(R, X) = (0.045, 0.015)$ ohm. Load $(P, Q) = (2000, 400)$ watt.
Nominal voltage is 230 V. Voltage at the sending end is 221 V.
For model definitions, see notes below table 17.

is easy to verify that these numbers indeed satisfy the three models. Note that the exact voltage drop is 17 times the DC$^+$ voltage drop. At the same time, the voltage phase lag is much too large. These two errors compensate each other somewhat in equation (36), giving a moderate error in the approximate current. The approximate currents being too low explains why the approximate optimal network strengths in the reference part of table 17 are too low. (However, it is not clear why the Taylor and DC$^+$ optimal networks are the same, while their currents are not.)

I conclude that there is little reason to use the DC$^+$ approximation; in particular in studies where the voltages might be a binding restriction. Hence the DC$^+$ adds little to the DC itself.

The computer program supplied with this paper can optimize either with the Taylor approximate model or with the exact model. The Taylor model can reduce the computing time by a factor of 10, compared with the exact model. If the Taylor model is not used in the optimization, it is used to create a "warm start" for the first computation of the exact model.

F.7 Electric network loss

The network loss is only known at the moments of extreme load, when the power flow is computed. Hence the network loss in kWh per year is not known and not included in the costs.

Of course, the *level* of the network loss is not relevant; only the effect of the various choices on the loss is relevant. Loosely speaking this effect is as follows. Starting with a positive electric load, adding a heat pump increases the load and hence the network loss, while a mCHP decreases the loss. A small amount of solar PV output decreases the loss but a large amount might create a large reverse power flow with a large loss. Of course, starting with a negative overall load, the above remarks must be reversed (with the exception of the large amount of solar).

It follows that the outcome of a large fraction of heat pumps in the reference solution will still hold when the network loss would be included in the costs.

G   The clustering of houses

In order to reduce the model size and the time to solve it, adjacent houses along a street are clustered, as follows. Each series of houses between two forks in the street is divided in two equal groups, each of them located near one of the two forks. For example, let such a series contain 10 houses,

numbered 1 to 10. Then the houses 2 to 5 are moved to the position of house 1 and the houses 6 to 9 are moved to the position of house 10. In fact there are now only two houses, each with a weight of 5. It turns out that this has very little effect on currents and voltages, while the number of endogenous (free) variables is reduced from 1.2 to 0.3 million and the computing time reduced with a factor 50.

In section 4.2 the neighbourhoods are described, based on the Meekspolder, which had originally 183 houses and a school. Without the street containing the school, they have four streets with together 172 houses, giving 43 houses per cable. However, the usual number of houses per cable is at present much higher for a new network, and this number is set at 75. This higher number is implemented by multiplying the above mentioned weights of the houses with 75/43.

H   The spatial distribution of the investments

In the economic optimum, the various types of heating are not evenly distributed over the neighbourhood. The same holds for the location of solar PV. As noted in section 5.1.1, the load-reducing investments (mCHP and solar PV) are concentrated at the end-points of the network.

In order to understand this subtle relation between economics and physics, a simplified network consisting of one street was modelled. For each house, consider the effect of the load at that house on the largest electric current anywhere along the street. This can be expressed as the derivative of this current with respect to the load at that house. (The unit of this quantity is V^{-1}. With a lossless DC-style approximation this is simply the reciprocal of the nominal voltage of 230 V, for each house.) It was found that this quantity varies systematically along the street. Its reciprocal is everywhere smaller than 230 V.

The derivative of the lowest *voltage* with respect to the load at a house varies much more than the derivative discussed above. (The unit is A^{-1}.) However, the voltage is never a binding restriction in my model. Of course, with a negative net load, the largest voltage is the problem, rather than the smallest.

I am not aware of any systematic study of this subject.

I Computational aspects of the model

For the new neighbourhood with gas, which is the largest application of the model, there are over 300,000 scalar variables. (This is after the clustering of houses; otherwise this number would be much higher; see appendix G.)

Most of these variables are from the engineering model indicated as the function v in equation (5) on page 7, and described in appendix F. An approximate calculation of this number of variables is as follows:

$$48 \times 12 \times (45 \times 2 + 44 \times 7) = 229248$$

with 48=years; 12=moments per year; 45=nodes; 2=voltage and phase; 44=links; 7=current and $V_i V_k$ and phase difference and P_{ik} and Q_{ik} and P_{ki} and Q_{ki}.

The number of scalars in the control variables H_{***}, S_{**}, x in (10) on page 8 is only

$$34 \times (4 \times 3 + 2) + 1 = 477$$

with 34=clusters of houses; 4=time periods heating equipment; 3=types of heating equipment; 2=time periods solar equipment; 1=network. The number of nodes (45) is larger than the number of clusters of houses (34), due to forking; there are no houses exactly at a fork in the street.

Note that even if a linearized power flow model is used, the total model is not linear. Nonlinear elements are (a) the network strength (and hence the impedances) being an unknown variable and (b) the modulus of the current being a nonlinear function of voltages in section F.4.

The model is solved numerically with the GAMS software (gams.com), using a nonlinear optimization solver. In GAMS, an equation like $Y = aX$ might also be written as $X = Y/a$ or $aX - Y = 0$, etcetera. Hence this equation does not define Y; it only gives a condition which must be true in the solution. Such equations are said to be *not normalized*. Hence the concept of control variables has a limited use in an optimizing model in GAMS; in the example X can be said to control Y, but the reverse is equally true. However, one might define a set of control variables in GAMS as any set of variables such that when all are fixed at some given value, the number of remaining variables is equal to the number of equalities and hence the model can be solved without optimization. Our control variables satisfy this criterion and they are also variables which can be controlled in the practical sense of the word.

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