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Introduction of Adjustment Costs in the Gamma Model

The modelling of the firm sector in the Gamma model is described in Draper and Armstrong (2007). This memorandum discusses the extensions in the 2010-version which is used to assess the sustainability of Dutch government finances (van der Horst et al. (2010)).

Contents

1 Introduction

The GAMMA model is used to assess the sustainability of Dutch government finances (van der Horst et al. (2010)). Adjustment costs lead to more realistic policy analyses: wages, poduction and investments will change more gradually than without such costs. These more realistic features make the introduction of adjustment costs attractive.

Private sector production in GAMMA is characterized by a simple neo-classical model for a representative firm.[1] The model's perspective is the long run of a small open economy and so it is reasonable to assume perfect competition in the goods, labour and capital markets. Hence there is one price p for goods, which is established on the world market. Perfect competition on the labour market gives the wage rate of an efficiency unit of labour[2] p_{le} which is determined by the marginal productivity of labour. Perfect competition on the capital market implies that the rate of return r is also determined on the world market.[3]
GAMMA assumes that the firm maximizes its value given a budget restriction, technology constraints and the capital accumulation equation. We will now present the details of this budget restriction, the determinants of firm's value and its production technology. Subsequently the implied capital demand and wage equations will be presented. The derivations are relegated to appendix A.

2 The budget, value and technology of the firm

The value of the representative firm is determined by a budget restriction indicating how much it can pay out in dividends each year and an arbitrage equation which values this stream of dividends on the capital market. The budget restriction of the firm can be written as:

$$D_{iv}(t)+p(t)i_e(t)=[p(t)y_{ge}(t)-p_{le}(t)l_e(t)-p(t)\Gamma(t)]-T_p(t)-G_{cb}(t)-G_{pg}(t) \tag{2.1}$$

Dividend payments D_{iv} and investment expenditures pi_e can be financed out of profits (the term between brackets) net of taxes T_p, net of central bank profits paid to the government G_{cb} and net of government income through leasing of land G_{pg}. Profits equal revenue py_{ge} minus the wage bill $p_{le}l_e$ and adjustment costs $p\Gamma$. Employment l_e is measured in efficiency units as is the wage

[1] The model follows in broad lines Draper and Huizinga (2001).

[2] Individual productivity (labour efficiency) changes over the life cycle and the productivity at the aggregated level grows over time. For these reasons we measure labour in standard efficiency units.

[3] The model does not take into account risk. So the model can not explain the risk premium which is a compensation for the disutility of risk. To prevent erroneous interpretations, the model uses one uniform market rate of return. This market rate of return is also used as discount rate.

rate p_{le}. The adjustment cost function is specified as

$$\Gamma(t) = \frac{1}{2}\gamma_{e1}\left[\frac{i_e(t)}{k_e^s(t-1)} - \gamma_{e0}\right]^2 k_e^s(t-1) \tag{2.2}$$

with i_e investment and k_e^s the capital stock. Taxes consist of the corporate tax rate τ_p times taxable revenue:

$$T_p(t) = \tau_p(t)\left[p(t)y_{ge}(t) - p_{le}(t)l_e(t) - p(t)\Gamma(t) - \rho_2\ (r-\pi)\ p(t)k_e^s(t-1) - G_{pg}(t) - A_f(t)\right] \tag{2.3}$$

with A_f the fiscal depreciation allowance and $\rho_2\ (r-\pi)\ p(t)k_e^s(t-1)$ the fiscal deductibility of the real finance costs (r is the nominal return and π the inflation rate). Fiscal depreciation is geometric with a fiscal depreciation rate v.[4] In period t the firm is allowed to deduct $v(1-v)^{i-1}p(t-i)i_e(t-i)$ for the investment purchased in period $t-i$, for all $i \geq 1$ according to this depreciation rule. Equations (2.1) and (2.3) determine the firm's budget.

The value of the firm is determined by this budget equation and an arbitrage equation that indicates how this dividend stream is valued on the capital market. The firm is valued such that the return $r\,W_e^s(t-1)$ from having invested $W_e^s(t-1)$ in alternative assets equals the return on owning the firm which consists of a capital gain of ΔW_e^s and a dividend D_{iv}:

$$r\,W_e^s(t-1) = \Delta W_e^s(t) + D_{iv}(t). \tag{2.4}$$

In this equation r is the nominal return and $W_e^s(t)$is the value of the firm at the end of period t. Forward solution of this equation, imposing the transversality condition ($\lim_{i\to\infty}(1+r)^{-i}W_e^s(t+i) = 0$), results in an explicit expression for the value of the firm.

The firm produces with capital and labour. Output is produced according to a CES production function with labour and capital as production factors

$$y_{ge}(t) = \left(\kappa k_e^s(t-1)^{\frac{\sigma-1}{\sigma}} + \theta l_e(t)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{2.5}$$

with $k_e^s(t-1)$ the relevant capital stock and σ the substitution elasticity between capital and labour. Investments are necessary for capital growth and for replacement of scrapped capital

$$i_e(t) = \Delta k_e^s(t) + \phi\, k_e^s(t-1) \tag{2.6}$$

with ϕ the technical rate of deterioration. Note that technical depreciation (ϕ) and fiscal depreciation (v) do not necessarily coincide. This completes the description of the basic specifications.

[4] Fiscal depreciation may be linear or degressive. Fiscal depreciation equal to a fixed percentage of the book value is allowed if the original investment becomes less productive with age. Since we assume that physical depreciation is exponential, a degressive fiscal depreciation scheme indeed seems most appropriate.

3 Factor demand and factor prices

The firm maximizes its value W_e^s, subject to the budget constraints and the available technology. Its instruments are investment and employment.

The value of the firm reaches its maximum in case all available labour supply is employed. Given this labour supply and the available capital stock, production is given according to (2.5). The wage rate and the compensation for capital equal their marginal products

$$p_{k_e}(t) = p(t)\kappa \left(\frac{y(t)}{k_e(t-1)}\right)^{1/\sigma} \tag{3.1}$$

$$p_{le}(t) = p(t)\theta \left(\frac{y(t)}{l_e(t)}\right)^{1/\sigma} \tag{3.2}$$

Different age cohorts j have different productivity levels, which is represented by their productivity profile $e_f(j,t)$. This assumption links age j's wages $p_l(t)$ to the macro wage in efficiency units p_{le}

$$p_l(j,t) = p_{le}(t)e_f(t)e_f(j,t) \tag{3.3}$$

with e_f the general productivity index, growing at the rate of technical progress. Employment in efficiency units, l_e, is the aggregate over the different cohorts

$$l_e(t) = \sum_j l_e(j,t) = \sum_j l_d(j,t)e_f(j,t)e_f(t) \tag{3.4}$$

with $l_d(j,t)$ employment of age j in period t.

Investment is a function of the marginal contribution of capital to the value of the firm (Tobin's q). In the steady state the marginal value of capital equals the tax-adjusted replacement value; i.e. the term between brackets (in 3.5) equals zero. The assumption that adjustment costs are zero in the steady state fixes the parameter γ_{e0} at the steady state investment ratio. The investment ratio exceeds its steady state level when q is larger than the replacement value. For a larger adjustment parameter γ_{e1}, investment is lower for a given q.

$$\frac{i_e(s)}{k_e^s(s-1)} = \gamma_{e0} + \frac{1}{\gamma_{e1}(1-\tau_p)}\left[\frac{q(s)}{p(s)} - 1 + \frac{\tau_p \nu}{r+\nu-\pi}\right] \tag{3.5}$$

The marginal value of capital q is derived as follows:

$$\begin{aligned} q(s)(1+r) = (1-\tau_p)&\left\{p_{k_e}(s+1) + p(s+1)\frac{\gamma_{e1}}{2}\left[(i_e(s+1)/k_e^s(s))^2 - \gamma_{e0}^2\right]\right\} \\ &+ \tau_p\rho_2\ (r-\pi)\ p(s+1) + q(s+1)(1-\phi) \end{aligned} \tag{3.6}$$

It consists of four parts. The first component equals the extra output from an additional unit of capital. The second part reflects that future adjustment costs are smaller for a larger capital stock. The third part equals lower corporate taxes following the deduction of real interest payments. The last part gives the remaining value of capital after depreciation.

4 Parametrization of the adjustment cost function

In the GAMMA model $\gamma_{e1} = 2$ while γ_{e0} is fixed at the steady state investment share. Hassett and Hubbard (2002) provide an excellent overview of the empirical work on adjustment costs. Early studies found estimated values of γ_{e1} ranging from 20 to 100, implying large marginal adjustment costs between one and five dollars per dollar of investment. Subsequent research has corrected for two problems: (1) measurement error in fundamental variables and (2) misspecification of convex adjustment costs. The application of improved methods yielded much lower values for γ_{e1} of 2 or lower, implying more plausible marginal adjustment costs in the range of 10 cents per dollar of additional investment. In a more recent study, Hall (2004) reports that adjustment cost parameters are not much above zero for most industries.

Table 4.1 gives an overview of the parameters chosen in other applied general equilibrium models.[5] This table indicates that rather large adjustment costs are specified in simulation models.

A final consideration in our parametrization of adjustment costs is the resulting adjustment speed of the capital stock. For this exercise we develop a partial equilibrium model consisting only of the equations relevant for the firm's decisions. The balanced growth path is constructed as the base path (i.e. all exogenous variables grow at the steady state rate). As a consequence, the real capital stock on the base path grows at a constant rate (the sum of population growth and technological progress). Employment is given by the exogenous labor supply, meaning that the wage rate adjusts to clear the labor market. We next simulate two shocks: (1) a reduction in the corporate tax rate (by 5%) and (2) an increase in the real rate of return (by 1%). We use

Table 4.1 Adjustments costs in AGE models

	γ_{e0}	γ_{e1}
Goulder and Summers (1987)	0.076	19.6
Altig et al. (1997)	0	0 (and 10)
Broer (1999)	0	10
Fehr et al. (2004)	0	10

[5] In the benchmark of Goulder and Summers (1987), total adjustment costs are 0.6% of (gross) output ($= \Gamma/(X+\Gamma)$; see Table 3).

different values for γ_{e1}, while the parameter γ_{e0} is set equal to the investment ratio in the steady state. Notice that the starting and the final capital stock is the same in all the scenarios. We first calculate the %-change of the capital stock relative to the base path, Next, we express this % relative to the final effect in the long-run (This latter effect equals 1.74% for the CIT-shock and -0.92% for the interest rate shock). Figures 4.1 and 4.2 show the fraction of the final change in the capital stock that is attained in every year. In the absence of adjustment costs ($\gamma_{e1} = 0$), capital jumps immediately to the new balanced growth path. However, the incorporation of even small adjustment costs leads to only small changes in the relative capital stock in the first year. The speed of adjustment clearly falls for larger values of the adjustment cost parameter. The length of the adjustment period becomes implausibly long in scenarios with large adjustment costs. These figures motivate the choice of a small adjustment cost parameter such that the major part of the adjustment should occur within 10 years.

Figure 4.1 Adjustment speed of the capital stock after a reduction of the CIT-rate

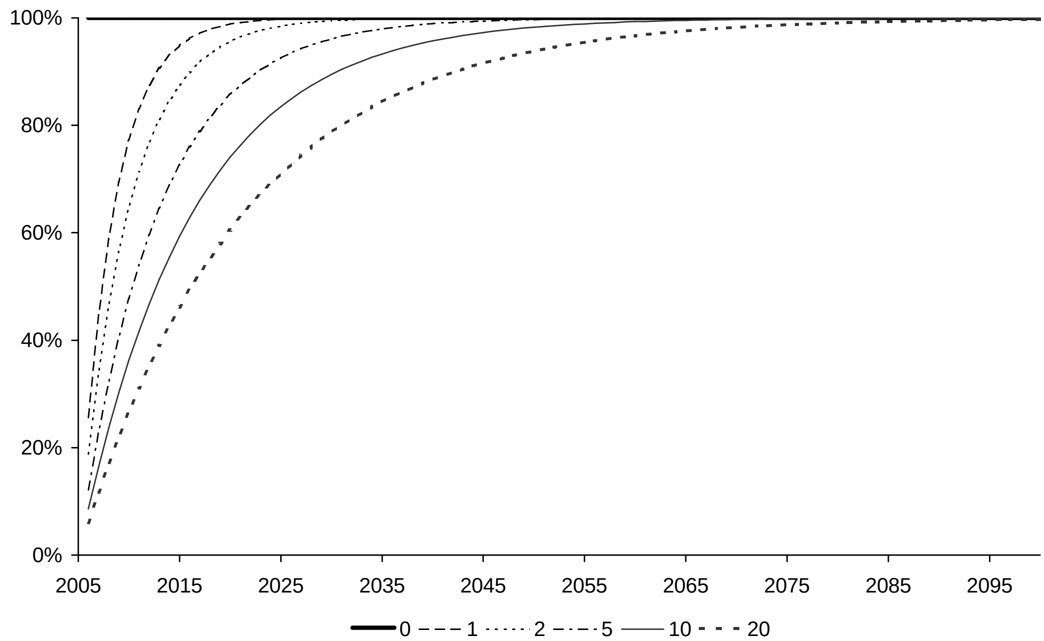
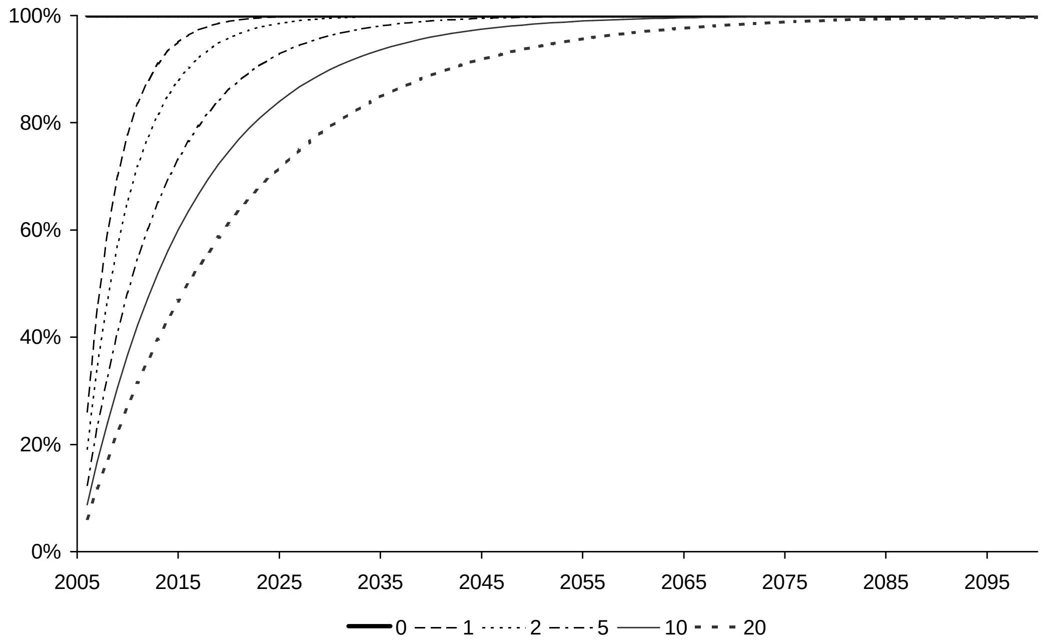


Figure 4.2 Adjustment speed of the capital stock after an increase in the real interest rate



Appendix A Derivations

The appendix in Draper and Armstrong (2007) is modified for (1) adjustment costs; (2) a single asset or a single rate of return (with a limited deductibility of interest payments) and (3) neutrality for inflation. The budget equation (2.1) in the main text can be written (after substitution of (2.3)) as

$$\begin{aligned} D_{iv}(t) &= (1-\tau_p)\left[p(t)y(t) - p_{le}(t)l_e(t) - p(t)\Gamma(t)\right] - p(t)i_e(t) \\ &\quad + \tau_p A_f(t) - G_{nt}(t) + \tau_p\rho_2\ (r-\pi)\ p(t)k_e^s(t-1) \end{aligned} \tag{A.1}$$

where r denotes the (single) nominal interest rate; ρ_2 the fraction of the interest costs that can be deducted from the corporate income tax base. To ensure inflation neutrality, interest costs are defined as the real interest rate $(r-\pi)$ times the replacement value of the capital stock. The variable G_{nt} includes payments to the government other than taxes and dividends.

$$G_{nt}(t) = G_{cb}(t) + (1-\tau_p)G_{pg}(t) \tag{A.2}$$

The adjustment cost function is specified as

$$\Gamma(t) = \frac{1}{2}\gamma_{e1}\left[\frac{i_e(t)}{k_e^s(t-1)} - \gamma_{e0}\right]^2 k_e^s(t-1) \tag{A.3}$$

where γ_{e0} equals the investment ratio in the steady state. When the growth rate of the capital stock along a balanced growth path is denoted as $\bar{g}_k$, this investment rate equals $\phi + \bar{g}_k$. Notice that the following derivatives hold:

$$\begin{aligned} \Gamma_{i_e}(t) &= \gamma_{e1}\left[\frac{i_e(t)}{k_e^s(t-1)} - \gamma_{e0}\right] \\ \Gamma_{k_e}(t+1) &= -\frac{1}{2}\gamma_{e1}\left[\left(\frac{i_e(t+1)}{k_e^s(t)}\right)^2 - \gamma_{e0}^2\right] \end{aligned}$$

Forward expanding of the capital market arbitrage equation (2.4) results in

$$W_e^s(t-1) = \sum_{j=0}^{\infty} D_{iv}(t+j)\,(1+r)^{-j}\,. \tag{A.4}$$

The discounted value of the fiscal depreciation can be split up into depreciation on the existing capital stock at time t, $AF(t)$, and the discounted value of depreciation on new investments (To make corporate taxation inflation neutral, the depreciation allowances are indexed at the inflation rate π):

$$\sum_{j=0}^{\infty}\left(\frac{1}{1+r}\right)^j A_f(t+j) = \sum_{j=0}^{\infty}\frac{\nu}{r+\nu-\pi}i_e(t+j)p(t+j)\,(1+r)^{-j} + AF(t) \tag{A.5}$$

where $AF(t)$ equals the depreciation allowance on investments installed up to time t:

$$AF(t) = \sum_{j=1}^{\infty} [(1-\nu)(1+\pi)]^j \frac{\nu}{r+\nu-\pi} i_e(t-j)p(t-j). \tag{A.6}$$

The value of $AF(t)$ is given and therefore does not affect the optimization problem.

The firm maximizes its own value (A.4) given the capital accumulation equation (2.6) and the production function (2.5). This leads to the Lagrangian

$$\begin{aligned} L = \sum_{j=0}^{\infty} \Bigg(& (1-\tau_p)p(t+j)\left[F\left\{k_e^s(t+j-1), l_e(t+j)\right\} - \frac{p_{le}(t+j)}{p(t+j)} l_e(t+j) - \Gamma(k_e^s(t+j-1), i(t+j))\right] \\ & - (1-\tau_p \frac{\nu}{r+\nu-\pi}) i_e(t+j)p(t+j) - G_{nt}(t+j) + \tau_p \rho_2 \; (r-\pi) \; p(t+j)k_e^s(t+j-1) \\ & - q(t+j)[k_e^s(t+j) - (1-\phi)k_e^s(t+j-1) - i_e(t+j)]\Bigg)(1+r)^{-j} \\ & + \tau_p AF(t) \end{aligned} \tag{A.7}$$

with F the production function (2.5). First order conditions for an optimum are :

$$\imath.\ L_{l_e} = 0\ ;\ \imath\imath.\ \ L_{k_e} = 0 \quad ; \quad \imath\imath\imath.\ \ L_{i_e} = 0 \tag{A.8}$$

$$\imath.\ \ F_{l_e}(s) = \frac{p_{le}(s)}{p(s)} \tag{A.9}$$

$$\imath\imath.\ \ q(s) = \frac{(1-\tau_p)\,p(s+1)\,[F_{k_e}(s+1) - \Gamma_{k_e}(s+1)] + \tau_p\rho_2\ (r-\pi)\ p(s+1) + q(s+1)(1-\phi)}{1+r} \tag{A.10}$$

$$\imath\imath\imath.\ \ q(s) = \left(1 - \frac{\tau_p \nu}{r+\nu-\pi} + (1-\tau_p)\,\Gamma_{i_e}(s)\right) p(s) \tag{A.11}$$

The marginal cost of capital (A.11) can be rewritten as an explicit investment relation:

$$\frac{i_e(s)}{k_e^s(s-1)} = \gamma_{e0} + \frac{1}{\gamma_{e1}(1-\tau_p)} \left\{ \frac{q(s)}{p(s)} - 1 + \frac{\tau_p \nu}{r+\nu-\pi} \right\} \tag{A.12}$$

The first-order condition for the marginal return of capital (A.10) gives the dynamic equation for q

$$q(s) = \frac{(1-\tau_p)\left\{p_{k_e}(s+1) + p(s+1)\frac{\gamma_e}{2}\left[(i_e(s+1)/k_e^s(s))^2 - \gamma_{e0}^2\right]\right\} + \tau_p\rho_2\ (r-\pi)\ p(s+1) + q(s+1)(1-\phi)}{(1+r)} \tag{A.13}$$

where the user cost is defined as

$$p_{k_e}(t) = p(t)F_{k_{e(t-1)}}(t) \tag{A.14}$$

Inspection of (A.12) and (A.13) confirms that, for a constant real interest rate $(r-\pi)$, a change in inflation does not affect the real q/p, nor the investment rate i/k. In view of the CES-production function

$$y(t)=\left(\kappa k_e^s(t-1)^{\frac{\sigma-1}{\sigma}}+\theta l_e(t)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{A.15}$$

the user cost of capital and the wage rate can be calculated as

$$p_{k_e}(t)=p(t)\kappa\left(\frac{y(t)}{k_e^s(t-1)}\right)^{1/\sigma} \tag{A.16}$$

$$p_{le}(t)=p(t)\theta\left(\frac{y(t)}{l_e(t)}\right)^{1/\sigma} \tag{A.17}$$

We now derive a relation between the capital stock and the value of the firm. We make use of the homogeneity of the production function and of the adjustment cost function (i.e. $\Gamma(t)=\Gamma_{i_e}(t)i_e(t)+\Gamma_{k_e}(t)k_e^s(t-1)$). Multiplying (A.10) by k_e^s and substituting (2.6) give

$$\begin{aligned}(1+r)q(t)k_e^s(t) &= (1-\tau_p)\,p(t+1)\,[F_{k_e}(t+1)k_e^s(t)-\Gamma_{k_e}(t+1)k_e^s(t)]\\ &+\tau_p\rho_2\;(r-\pi)\;p(t+1)k_e^s(t)+q(t+1)(1-\phi)k_e^s(t)\\ &=(1-\tau_p)\,p(t+1)\left[y(t+1)-\frac{p_{le}(t+1)}{p(t+1)}l_e(t+1)-\Gamma(t+1)+\Gamma_{i_e}(t+1)i_e(t+1)\right]\\ &+\tau_p\rho_2\;(r-\pi)\;p(t+1)k_e^s(t)+q(t+1)(1-\phi)k_e^s(t)\\ &=(1-\tau_p)\,p(t+1)\left[y(t+1)-\frac{p_{le}(t+1)}{p(t+1)}l_e(t+1)-\Gamma(t+1)\right]\\ &+q(t+1)i_e(t+1)-\left(1-\frac{\tau_p\nu}{r+\nu-\pi}\right)p(t+1)i_e(t+1)\\ &+\tau_p\rho_2\;(r-\pi)\;p(t+1)k_e^s(t)+q(t+1)(1-\phi)k_e^s(t)\\ &=D_{iv}(t+1)-\tau_p\left(A_f(t+1)-\frac{\nu}{r+\nu-\pi}p(t+1)i_e(t+1)\right)+G_{nt}(t+1)\\ &+q(t+1)k_e^s(t+1)\end{aligned}$$

Forward solution leads to the conclusion that the value of the capital stock equals the discounted value of the dividend payments (the value of the firm) minus the depreciation allowance on investments installed up to time t and an arbitrary constant

$$q(t)k_e^s(t)=W_e^s(t)-\tau_pAF(t)+\sum_{j=1}^{\infty}G_{nt}(t+j)\,(1+r)^{-j} \tag{A.18}$$

So for the value of the firm we have the value of the capital stock valued at effective prices plus the value of the depreciation allowance on investments installed up to time t minus the claims of the government on the firm other than taxes:

$$W_e^s(t) = q(t)k_e^s(t) + \tau_p AF(t) - W_{cg}^s(t) \tag{A.19}$$

with $W_{cg}^s(t) = \sum_{j=1}^{\infty} G_{nt}(t+j)(1+r)^{-j}$ the net present value of non-tax government claims on firms.

Define $\tilde{W}_e^s(t)$ as the value of the firm at the beginning of period t, or

$$\tilde{W}_e^s(t) = W_e^s(t-1)$$

Equivalent expressions for the value of the firm are:

$$\tilde{W}_e^s(t) = W_e^s(t-1) \tag{A.20}$$
$$= \frac{W_e^s(t) + D_{iv}(t)}{(1+r)} \tag{A.21}$$
$$= \frac{q(t)k_e^s(t) + \tau_p AF(t) - W_{cg}^s(t) + D_{iv}(t)}{(1+r)} \tag{A.22}$$

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