Bubbles and Investment Horizons*

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Abstract

The current theoretical literature makes contradicting predictions regarding the impact of an investor's horizon on his optimal trading strategy in the presence of bubbles. We analyze this relation empirically using a Regime Switching Model to identify bubbles and crashes. We base our analysis on industry returns and find high positive returns after bubbles at the one-month horizon. At intermediate horizons of 2-4 months our findings are mixed, but thereafter, for horizons up to five years, returns following a bubble are again more positive than returns in the absence of a bubble. We compare a mean-variance as well as a downside-risk averse investor's portfolio allocation in the presence and absence of a bubble. The weight allocated to the bubbly asset is higher for horizons up to 5 years. These findings suggest that even for a rather unsophisticated trader who does not follow daily market news, riding bubbles is a more profitable strategy than refraining from investing in the bubbly asset. Given the broad range of horizons during which riding bubbles is the optimal strategy, our results question the idea that bubbles are zero-sum games.

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1 Introduction

Bubbles can pose a serious risk to investors' wealth if they crash but might also offer profitable trading opportunities. As Abreu and Brunnermeier (2003) derive theoretically and Guenster et al. (2008) document empirically, actively investing in bubbly assets, or "riding the bubble", seems to be the optimal choice at very short horizons. However, it is questionable whether this conclusion is generalizable to investors with longer horizons. Empirical studies analyzing the behavior of mutual fund managers' behavior during the internet bubble, such as Greenwood and Nagel (2007) and Dass et al. (2008), provide evidence that managers who were heavily invested in bubble stocks earned high returns in the short run. However, in the long run, as the bubble started deflating, they were also the ones incurring the largest losses. Managers who invested less heavily in tech stocks kept loosing money in the short-term but outperformed as the bubble burst.

From a theoretical perspective, the relation between the investor's horizon and his optimal strategy during bubbles is not straightforward. The efficient market hypothesis predicts that investors short overpriced assets, independent of their horizon. However, in the limits-to-arbitrage literature (see, for example, De Long et al. (1990a) and Shleifer and Vishny (1997)), short horizons induce investors to refrain from trading against the bubble. In turn, De Long et al. (1990b) and Abreu and Brunnermeier (2003) suggest that investors should ride the bubble at short horizons and sell out as the risk of the crash increases. Empirically, Brunnermeier and Nagel (2004) and Temin and Voth (2004) document that this strategy is profitable for sophisticated investors who possess the ability to predict the crash.

To understand the impact of time horizons on the optimal asset allocation is not only relevant for investors but also important from a general equilibrium perspective. If bubbles do not affect the fundamental value of the asset, as assumed in the models discussed above, they must in the end be a zero-sum game. This reasoning implies that riding bubbles can only be a profitable strategy for a limited time horizon. This argument no longer holds if bubbles have an impact on the real economy. Jerzmanowski and Nabar (2008) provide evidence that the NASDAQ bubble had a positive effect on aggregate net wealth. Hirshleifer et al. (2006) present a model that includes a feedback effect from the firm's stock price to its cash-flows. In this setting, even irrational investors can earn positive abnormal returns. Their irrationally high expectations of the firm's future cash-flows drive up the price. The higher price motivates stakeholders, for example employees, to make more firm-specific investments, which in turn raises the fundamental value of the firm. In such a setting, crashes will not wipe out all gains accrued during the bubble: at least the increase in fundamental value caused by the bubble will remain. Consequently, riding bubbles can in this setting even be a profitable investment strategy for long-term investors without timing capabilities.

In this paper, we investigate how the investor's optimal strategy during bubbles is affected by his horizon. Our analysis is based on a Markov regime switching model as proposed by Hamilton (1994). This methodology allows us to replicate the uncertainty a real-world investor faces. Further, we can explicitly derive how the risk and return forecasts as well as the optimal weight develop over time. We disentangle how different effects, such as crash probability, the probability that the bubble continues and the distribution parameters of the different regimes, determine the development of the forecasts over time. Our analysis is based on industry returns. Famous historical episodes of bubbles that started in specific industries are the recent internet bubble, the electricity and the railway boom.

The investor infers from past abnormal returns whether the asset's return is currently in the bubble, normal or crash regime. To compute the abnormal returns, we use three asset pricing models: the CAPM, the Fama and French (1993)-Model (hereafter: 3-Factor Model) and the Carhart (1997)-Model (hereafter: 4-Factor Model). The bubble regime is characterized by large positive mean abnormal returns. They range from 2.46% per month for the 3-Factor Model to 2.67% per month for the CAPM. In the normal regime where the growth rate of the price should in expectation be equal to fundamental value, the mean abnormal returns are close to zero. The crash regime is characterized by large negative abnormal returns, which are around -8.5% per month. To ensure that the bubbles we detect are substantial deviations from fundamental value and to clearly distinguish them from industry momentum described by Moskowitz and Grinblatt (1999), we require bubbles to last at least one year. In our estimation, we also require that a bubble ends with a crash since historical episodes of bubbles are associated with subsequent crashes. Whether these crashes wipe out all gains, depends on the probability of a crash and its size.

We investigate the optimal choice of a mean-variance investor and a downside risk averse investor. Paying special attention to downside risk ensures that we take crash risk appropriately into account. We assume that the investor holds a zero-investment portfolio and can choose between a bubbly and a non-bubbly asset, implying that he holds a long-short position. Our evidence shows that riding bubbles, i.e., taking a long position in the bubbly asset and a short position in the non-bubbly asset, is the optimal strategy for most horizons. We observe a rather large positive weight allocated to the bubbly asset during the first month following the bubble regime. For the mean-variance investor, it ranges from 80% of every dollar invested for the CAPM to 16% for the 4-Factor Model. For the downside risk averse investor, it is somewhat lower, but still economically large. Following the first month, the optimal weight allocated to the bubbly asset declines substantially. The risk and return forecasts deteriorate due to a sharp rise in crash probability. However, this increase is only temporary and after a couple of months, we observe that the bubbly asset becomes again more and more attractive. Our results show that even for horizons up to five years, the weight allocated to the bubbly asset is rather high. Our most conservative estimates indicate an optimal weight of 0.32 for the variance risk averse investor and a weight of 0.18 for the downside risk averse investor at the 5-year horizon.

This evidence is consistent with models that propose a positive effect of bubbles on the asset's fundamental value, such as Jerzmanowski and Nabar (2008) and Hirshleifer et al. (2006). Since bubbles lead to increases in fundamental value, subsequent crashes will not wipe out all gains. Therefore, in line with our findings, riding bubbles can even be a profitable strategy in the long run and for investors who have no timing ability.

We proceed our analysis as follows. First, we discuss the theoretical and empirical literature. Then, we present our model to detect bubbles. In the third section, we derive the investors optimal asset allocation. In section 5, we start our empirical analysis. First, we estimate the regime switching model. Subsequently, we evaluate the risk and return forecasts over time and analyze the optimal weight. Section 6 concludes.

2 Literature Review

2.1 Theoretical Literature

The theoretical proposition on how the rational investor's optimal trading strategy during bubbles varies with his horizon diverge.[1] The traditional efficient market hypothesis predicts that it is optimal for any investor to short the bubbly asset (see Fama, 1965)) independent of his horizon. The fact that this literature does not explicitly address horizons is a direct consequence of its prediction that prices will immediately return to fundamental value. As Fama (1965, p. 38) puts it: "For example, if there are many sophisticated traders who are extremely good at estimating intrinsic values, they will be able to recognize situations where the price of a common stock is beginning to run up above its intrinsic value. Since they expect the price to move eventually back toward its intrinsic value, they have an incentive to sell this security or to sell it short. If there are enough of these sophisticated traders, they may tend to prevent these "bubbles" from ever occurring."

However, history suggests that this theory is incomplete as we observe many examples of prices that deviate from fundamental value.[2] As a response the limits-to-arbitrage literature has evolved, which explains why rational investors do not trade against bubbles and do not prevent their existence. In this line of literature, a crucial factor that deters investors from trading against bubbles is their short horizon. In De Long et al. (1990a) irrational noise traders push prices away from fundamental value. The rational arbitrageurs do not take offsetting positions, as they do not know whether noise trader sentiment will change during their investment horizon. Only if the arbitrageur can be sure that his horizon is longer than the noise traders' horizon, it is optimal for him to short the bubbly asset. Shleifer and Vishny (1997) extend the analysis of De Long et al. (1990a) by providing an explanation of why sophisticated investors have short horizons. They model sophisticated investors as delegated portfolio managers hired by less sophisticated individuals. These individuals do not understand the investment strategy. They observe the returns and accordingly evaluate the manager. If they observe a prolonged sequence of low or even

[1] Throughout this literature review, we will use the term "rational" investor and "informed" investor interchangeably

[2] For an overview, see Hirshleifer (2001).

negative returns (while the mispricing persists), they doubt the manager's capabilities and withdraw their funds. The manager is forced to liquidate his position, potentially at a loss, before prices return to fundamental value.

More recently, Barberis and Shleifer (2003) use the general idea of De Long et al. (1990a) to explain the performance of different investment styles over time. In their model, two groups of traders, "switchers" and "fundamental traders", can allocate their resources to two different styles. Switchers, who are similar to positive feedback traders, allocate their assets to a specific style based on the style's relative past performance. Thereby, they push up prices away from fundamental value. The "fundamental traders" do not take opposing positions due to their short horizons. The Barberis and Shleifer (2003)-model can, for example, explain the difference in returns between old and new economy stocks in the late 90s and the poor returns of value stocks despite their relatively good cashflow performance.

The model of Dow and Gorton (1994) complements the previous theoretical studies by introducing the concept of arbitrage chains. They show that it is only profitable for an informed investors with a short horizon to trade upon his information, if he can be sufficiently certain that subsequent arbitrageurs will trade into the same direction before he has to sell out. As a consequence, not the relative length in horizons between the rational and irrational traders is crucial. Instead, the important factor is the length of the informed investor's horizon relative to the speed at which other investors become informed and act. As the likelihood that other investors become informed increases as time progresses, the Dow and Gorton (1994)-model also supports the prediction that the longer the rational investor's horizon, the more likely it will be profitable for him to short the bubbly asset.

Two theoretical models predict that it can be profitable for rational investors to fuel bubbles independent of their horizon. In the first model, written by De Long et al. (1990b), the market consists of feedback traders and rational arbitrageurs. The feedback traders demand is positively correlated with past price changes. As the rational arbitrageurs anticipate their demand, they buy more than they would in the absence of feedback traders. The feedback traders observe the positive returns, start buying and thereby push prices even further away from fundamental value. As the rational arbitrageurs are aware of the mispricing and the firm's liquidation in the following period, they sell their holdings and profit at the expense of the feedback traders. The second model, proposed by Abreu and

Brunnermeier (2003), stresses the need of coordination among rational arbitrageurs. As every rational arbitrageur cannot burst the bubble by himself, his optimal strategy is to ride the bubble and profit at the expense of the noise traders. It is only profitable for him to sell his holdings if the risk of a crash outweighs the positive returns he earns from riding the bubble. Both models lead to the conclusion that rational investors actively ride bubbles independent of their horizon. However, such a strategy only seems profitable if investors are able to predict the time of the crash. Otherwise, all their gains might be lost again.

While the models discussed so far all focus on rational investors, the model of Hirshleifer et al. (2006) proposes that irrational investors can benefit from bubbles. The crucial difference between their work and the previous studies is that they introduce a feedback loop from prices to cashflows. The irrational traders drive up prices, which in turn motivates stakeholders of the firm to increase their firm-specific investment. The higher stakeholder investment raise the firm's cashflows which in turn has positive effect on prices in the following period. In this way, the bubble has a positive effect on the real value of the firm and the irrational investors potentially profit from the mispricing they originally caused. Because the price rise leads to an increase in fundamental value, investing in the bubbly asset might in this models even be a good strategy in the long run. A crash that happens subsequently should only wipe out the irrational part of the price increase but not the part that can be attributed to an increase in fundamental value.

2.2 Empirical Literature

Empirical studies commonly analyze how different types of investors behaved during historical bubble periods. Brunnermeier and Nagel (2004) analyze the behavior of hedge fund managers and find that they were riding the tech bubble. Interestingly, hedge funds had superior timing ability and were able to leave the market before the crash. Thereby, they profited greatly from less informed traders. A similar strategy is documented by Temin and Voth (2004) for a highly sophisticated investor, Hoare's Bank, during the South Sea Bubble.

However, not all investors have these timing abilities, eventually needed to make riding bubbles a profitable strategy. Greenwood and Nagel (2007) document that experienced mutual fund managers did not ride the tech bubble, but inexperienced managers were heavily invested in tech stocks. Managers who were riding the bubble earned higher returns in the short run. However, they also faced larger losses as the bubble started deflating and they were not able to time the crash. Dass et al. (2008) provide evidence that managers with higher incentive contracts invested less in bubble stocks. The high incentive managers earned about 2% lower returns per quarter during the bubble period. However, they earned 2.7% higher returns per quarter once the bubble started deflating.

A rather different approach is taken by Guenster et al. (2008). They do not analyze the behavior of a specific type of investor. Instead, they investigate the risk-return trade-off of bubbles in a large sample of industry returns. Their findings show that riding bubbles is a very risky but also profitable strategy at a one-month horizon. The additional return an investor can earn from riding bubbles ranges from 0.41% to 0.64% per month. However, the risk of a crash is also more than twice as large.

We can conclude that first investing in the bubbly asset and then shorting the bubbly asset is a profitable strategy for investors who can foresee the crash. However, we need to take into account that a lot of investors, even professional ones like mutual fund managers, do not have these capabilities. For these investors, previous theoretical and empirical research suggests that their optimal strategy changes along their horizon. It seems that the theoretical predictions made by Abreu and Brunnermeier (2003) and De Long et al. (1990b) are applicable for investors with a very short, one-month horizon. As the investor's horizon increases and simultaneously the risk that the bubble starts deflating, the propositions of the limits-to-arbitrage literature might be become increasingly relevant. Riding bubbles is dangerous due to crash risk. However, the fact that the bubble might also continue makes it risky to take an extreme short position. At intermediate horizons, it seems that sidelining, i.e., not trading upon the information might be the optimal strategy. If the investor has long horizon and can be reasonable certain that the bubble will crash before he has to close his position, going short seems to be the optimal strategy given that bubbles are zero-sum games. If, following the idea of Hirshleifer et al. (2006), bubbles are related to increases in fundamental value, this conclusion might not be valid and holding a long position might even be a good long-run strategy.

While we can draw these inferences by combining the findings of current empirical and theoretical papers, it is far from certain whether they correctly describe reality. To our knowledge, there is currently no study that investigates the profitability of different strategies in relation to the investor's horizon. We intend to fill this gap.

3 A Regime-Switching Model for Bubbles

3.1 Model Design

Our analysis is based on a regime switching model since it allows us to separately describe the price process in case a bubble continues to inflate, in case a crash possibly ends a bubble, and the base case in which no bubble is present. Evans (1991), van Norden and Schaller (1999) and Brooks and Katsaris (2005) are examples of the use of regime switching models to study asset price bubbles.

An advantage of using a regime switching model is the ease with which the actual presence of the bubble can remain latent. As in reality, the investor does not know for sure whether a bubble is present but has to make a probabilistic inference. In determining his optimal allocation he has to take into account that his inference may be wrong. This approach allows us to describe a more realistic setting than most theoretical models, where at least a fraction of investors knows with certainty that the price contains a bubble component (see among others Abreu and Brunnermeier (2003), De Long et al. (1990b), or De Long et al. (1990a)).

We let the latent process for the presence of a bubble be governed by a first order Markov chain. With a certain probability, the process can switch from one state to another and eventually to the bubble state. This switch can correspond with a displacement in a Minsky model (see Kindleberger, 2000) or "new economy thinking" as in Shiller (2000). Once the process switched to the bubble state, it can remain there for the following periods or leave it with a crash. We deviate from van Norden and Schaller (1999) and Brooks and Katsaris (2005), who do not use a Markov chain. In their studies, the latent process of a bubble evolves much more gradually and cannot accommodate the sudden switches that are considered typical characteristics of bubbles (see for example Figures 2 and 3 in Brooks and Katsaris, 2005).

Besides by a sudden change, bubbles are characterized by a price that grows faster than fundamental value. While such exuberant growth is present in all bubble models, we tie it directly to an asset pricing model like the CAPM or a multi-factor model. We do not assume that the fundamental growth rate is simply given, as is typical for the theoretical rational bubble literature (see, for example, Blanchard and Watson, 1982), nor do we tie it to dividends as many articles on testing for bubbles propose (see Flood and Hodrick, 1990, for an overview).

In our setting, structural growth beyond what can be explained from covariance with systematic risk factors (or, equivalently, the pricing kernel) is considered a bubble. We do not require that bubble growth is constant over time. Instead, we allow a stochastic growth rate which is strictly positive in expectation as in Brooks and Katsaris (2005).

Mathematically, the asset return r_t obeys

$$r_t = r_{\mathrm{f},t} + \boldsymbol{\beta}' \boldsymbol{f}_t + \sigma_t u_t(S_t), \tag{1}$$

where $r_{\mathrm{f},t}$ is the risk-free rate, $\boldsymbol{f}_t$ denotes the vector of realizations of the (traded) risk factors, $\boldsymbol{\beta}$ the vector of sensitivities to the risk factor, σ_t the idiosyncratic (deterministic) volatility of the asset, and $u_t(S_t)$ a random variable, independent from $\boldsymbol{f}_t$, and depending on a latent state variable S_t. The first two terms capture the systematic part of the asset return. The last term captures the idiosyncratic part of the asset return, which may contain a bubble depending on the realization of S_t.

The latent process S_t can be in one out of three regimes. In the normal regime N, no bubble is present, and the asset price grows at the fundamental growth rate. The expectation of u_t under this regime, denoted by μ_{N}, will be close to zero. In the bubble regime B, the expected value of u_t, denoted by μ_{B}, will be strictly larger than zero. Under both regimes u_t follows a normal distribution, with equal standard deviation ω. We explicitly incorporate that bubbles end with a crash, and therefore introduce a crash regime C. To ensure that the realization of u_t under the crash regime is always below some minimum crash value $k < 0$, u_t follows a transformed lognormal distribution, $k - \mathrm{e}^{Z}$, where Z follows a normal distribution with mean μ_{c} and standard deviation ω_{C}. This approach puts our model in line with other models that distinguish return sources for tranquil and for stressful periods, such as Das and Uppal (2004). We summarize the model for $u_t(S_t)$:

$$u_t \sim \begin{cases} N(\mu_{\mathrm{N}}, \omega^2) & \text{if } S_t = \mathrm{N} \\ N(\mu_{\mathrm{B}}, \omega^2) & \text{if } S_t = \mathrm{B} \\ k - \mathrm{e}^Z, \quad Z \sim N(\mu_{\mathrm{C}}, \omega_{\mathrm{C}}^2) & \text{if } S_t = \mathrm{C} \end{cases} \tag{2}$$

While the distributions of u_t conditional on S_t have well-defined characteristics, the unconditional distribution of u_t will be time-varying, and exhibit skewness and excess kurtosis.

The process S_t is governed by a first order Markov chain with the associated matrix of transition probabilities $\boldsymbol{P}$. We restrict the process in two ways. First, as a bubble should correspond with a prolonged period of exuberant growth, we require that it has a minimum length L. We refer to the first $L-1$ states as transitory bubble states. Transitory bubble state B_l, $1 \leq l < L$, prevails, when the process has spent l months in the bubble regime. This state can only be preceded by B_{l-1} and followed by B_{l+1}. The process can only switch to state B_1 from the normal or crash regime. The last bubble state is the non-transitory bubble state B_{nt}. From state B_{L-1}, the process must switch to this state. Contrary to the transitory bubble states, the process can remain in the non-transitory bubble state infinitely. As a second restriction we impose that the process can only leave the non-transitory bubble state B_{nt} by switching to the crash state. This restriction puts the process in line with anecdotal and empirical evidence that bubbles end with crashes.[3] These restrictions imply a probability of zero for some transitions while others are by construction equal to 1. In Table 1, we show which transition probabilities are free parameters, and which ones have a fixed value.

[Table 1 about here.]

By including these restrictions, we intend to put enough structure on our model to ensure that we indeed detect bubbles, and to preserve enough flexibility to infer from return series how bubbles actually occur. The restrictions prevent that a single-period large return is identified as a bubble. By explicitly imposing that bubbles are ended by

[3]For a discussion on the relation between bubbles and crashes, see McQueen and Thorley (1994).

one or more crashes, a prolonged adjustment of fundamental value due to a market under-reaction is not likely to be identified as a bubble. The persistence of bubbles after their first L periods, or their average growth rate are free parameters to be estimated.

3.2 Estimation, Inference and Forecasts

The investor does not know in which regime the process is at any point in time. Instead, he has to infer the current regime and form an expectation on future regimes and their risk-return tradeoff. His information set Ψ_t at time t contains the time-series of returns and risk factors from t_0, the beginning of the sample period, to t. He applies a filtering procedure to infer with which probabilities the different states currently prevail. This procedure uses the following recursive relation to construct a times series of vectors of forecast probabilities $\boldsymbol{\phi}_{\tau|\tau-1}$ and inference probabilities $\boldsymbol{\phi}_{\tau|\tau}$ for each state s (see Hamilton, 1994, Ch. 22), where τ ranges from t_0 to t:

$$\boldsymbol{\phi}_{\tau|\tau-1} = \boldsymbol{P}\boldsymbol{\phi}_{\tau-1|\tau-1} \tag{3}$$

$$\boldsymbol{\phi}_{\tau|\tau} = \frac{1}{\boldsymbol{\phi}'_{\tau|\tau-1}\boldsymbol{g}(u_\tau)}\boldsymbol{\phi}_{\tau|\tau-1} \odot \boldsymbol{g}(u_\tau), \tag{4}$$

where $\boldsymbol{g}()$ is the vector of the probability density functions of the different states, $\boldsymbol{P}$ is the transition matrix, and $\odot$ denotes the Hadamard product. The procedure starts with inference probabilities for $t_0 - 1$. The forecast probabilities give a forecast of the state process for period τ, conditional on information up to period $\tau - 1$. When the information (i.e. the returns) of period τ becomes known, a Bayesian update is applied to arrive at the inference probabilities. We estimate the distribution parameters, transition probabilities and initial regime at $t - 1$ by recursively applying the Expectation Maximization (EM) algorithm of Dempster et al. (1977) which yields maximum likelihood estimates (see also Hamilton, 1993).

The investor applies the filtering procedure to determine the probabilities with which the different states prevail at time t. He uses these inference probabilities to construct forecast for future periods m. The m-period ahead forecast probabilities can be calculated as $\boldsymbol{\phi}_{t+m|t} = \boldsymbol{P}^m\boldsymbol{\phi}_{t|t}$. Based on the forecast probabilities and the probability density functions, he constructs a prediction for the distribution of abnormal returns $g_{t+m}(u)$:

$$g_{t+m}(u) = \boldsymbol{\phi}'_{t+m|t}\boldsymbol{g}(u). \tag{5}$$

Equation (5) shows that the m-period ahead forecast of the abnormal return distribution consists of the probabilities of the different states and their respective distributions. Along the same lines, any raw moment of order n can be stated as a sum of state-specific moment weighted by the states' forecast probabilities:

$$\mathrm{E}_t\left[u^n_{t+m}\right] = \sum_{s\in\mathcal{S}} \phi_{t+m|t}(s)\,\mathrm{E}[u^n_{t+m}|S_{t+m} = s]. \tag{6}$$

4 The Investor's Asset Allocation Decision

We choose a mean-variance utility function, which we extend to incorporate downside risk following Harlow and Rao (1989) and Fishburn (1977). This functional form allows us to focus on the "usual" fluctuations around the mean and keeps our results comparable to theoretical papers that use mean variance utility functions. However, it also enables us to take a more conservative perspective and focus on downside risk. Since crashes are the most serious risk to a strategy of riding bubbles, it is particulary relevant for our research question to pay special attention to downside risk. Only taking into account aversion to variance risk might make a strategy of riding bubbles seem more attractive than it actually is.

Formally, the investor's utility function v is defined as:

$$v(r^{\mathrm{p}}) = \begin{cases} r^{\mathrm{p}} - \frac{1}{2}\gamma_1\left(r^{\mathrm{p}} - \mathrm{E}_t\left[r^{\mathrm{p}}\right]\right)^2 - \frac{1}{2}\gamma_2\left(K - r^{\mathrm{p}}\right)^{\nu} & \text{for } r^{\mathrm{p}} < K \\ r^{\mathrm{p}} - \frac{1}{2}\gamma_1\left(r^{\mathrm{p}} - \mathrm{E}_t\left[r^{\mathrm{p}}\right]\right)^2 & \text{for } r^{\mathrm{p}} \geq K \end{cases} \tag{7}$$

where r^{p} is the portfolio's returns, γ_1 is the variance risk-aversion coefficient, γ_2 is the downside risk-aversion coefficient and K is a threshold value, which is proportional to the crash threshold k. For returns below this threshold, the investor subtracts an additional discount. In line with decreasing absolute risk aversion (see Arditti, 1967) and to ensure that marginal utility is positive, we choose $\nu = 2$.

The investor makes an investment decision at time t for the following M periods. At every point in time $t + m$, $1 \leq m \leq M$, he evaluates his investment decision. This approach allows us to take intermediate fluctuations in wealth, for example a sudden crash, into account. He maximizes the sum of his expected utility over the complete investment horizon M:

$$\mathrm{E}_t\left[U(\cdot)\right] = \sum_{m=1}^{M} \left\{ \mathrm{E}_t\left[r^{\mathrm{p}}_{t+m}\right] - \frac{1}{2}\gamma_1\, \mathrm{E}_t\left[r^{\mathrm{p}}_{t+m} - \mathrm{E}_t\left[r^{\mathrm{p}}_{t+m}\right]\right]^2 - \frac{1}{2}\gamma_2\, \mathrm{E}_t\left[\left(\left(K - r^{\mathrm{p}}_{t+m}\right)^+\right)^2\right] \right\}. \quad (8)$$

The first term of this utility function captures the expected portfolio returns, while the second and third term penalize for the risk of the portfolio. The second term is a standard variance term, while the third term is a lower partial moment of order 2.

We assume that the investor has no initial wealth and present him with the choice between an asset that can experience a bubble and an asset which never encounters a bubble. The "non-bubbly" or "normal" asset is restricted to be in the normal regime or the crash regime. The Markov chain that governs the regime process of this asset is denoted by $\tilde{S}_t$. The corresponding transition matrix $\tilde{\boldsymbol{P}}$ is a reduced form of the transition matrix $\boldsymbol{P}$, excluding bubble states:

$$\tilde{\boldsymbol{P}} = \begin{bmatrix} p_{\tilde{\mathrm{N}}\tilde{\mathrm{N}}} = \dfrac{p_{\mathrm{NN}}}{1 - p_{\mathrm{NB}}} & p_{\tilde{\mathrm{C}}\tilde{\mathrm{N}}} = \dfrac{p_{\mathrm{CN}}}{1 - p_{\mathrm{CB}}} \\ p_{\tilde{\mathrm{N}}\tilde{\mathrm{C}}} = \dfrac{p_{\mathrm{NC}}}{1 - p_{\mathrm{NB}}} & p_{\tilde{\mathrm{C}}\tilde{\mathrm{C}}} = \dfrac{p_{\mathrm{CC}}}{1 - p_{\mathrm{CB}}} \end{bmatrix} \quad (9)$$

The "bubbly asset" follows a regime process S^*_t, governed by the transition matrix $\boldsymbol{P}^*$ shown in Table 2. The return process can be in the normal regime, the bubble regime or the crash regime. To avoid that bubbles which might occur infinitely far in the future affect the investor's asset allocation decision a t, we assume that the asset only experiences a bubble once. After this bubble deflates, the bubble asset becomes identical to the normal asset and enters the regime process $\tilde{S}_t$. Due to this assumption, we need to formally define two different crash states. The difference between these crash states is that the process can enter and leave the state from different regimes. The one crash state, $\mathrm{C_N}$, can occur after the normal regime and after another crash of the same type. It can be followed by the normal regime, the bubble regime or another crash regime $\mathrm{C_N}$. The other crash state, $\mathrm{C_B}$, can be entered from the bubble regime or a crash of the same type. It can be followed by the bubble regime, another crash $\mathrm{C_B}$ or the asset transforms into the non-bubbly type and follows the chain $\tilde{S}_t$. However, while we define these two crash states to be different, we assume that they are empirically identical and will refer in the estimation to both states as C.

[Table 2 about here.]

The investor can take a position w in the bubbly asset and the opposite position $-w$ in the non-bubbly asset. Since the assets only differ with respect to their idiosyncratic return, we can write the m-period ahead portfolio return as:

$$r^{\mathrm{p}}_{t+m} = w\sigma_{t+m}\left(u^*_{t+m}(S^*_{t+m}) - \tilde{u}_{t+m}(\tilde{S}_{t+m})\right), \tag{10}$$

where u^*_{t+m} is the regime-dependent idiosyncratic return for the potentially bubbly asset, and $\tilde{u}_{t+m}$ its counterpart for the non-bubbly asset. After the bubbly asset has become identical to the non-bubbly asset, the portfolio return will equal zero.

Equation (10) allows us to rewrite the cumulation in Equation (8) as a product of the allocation w and the assets' risk and return characteristics. The cumulative expected abnormal return difference is:

$$\eta_t \equiv \sum_{m=1}^{M} \hat{\sigma}_{t+m}\,\mathrm{E}_t\left[u^*_{t+m} - \tilde{u}_{t+m}\right]. \tag{11}$$

from which it follows that the expected cumulative portfolio return equals $w\eta_t$. Similarly, we define the cumulation of variances of abnormal return differences over time as:

$$\upsilon_t \equiv \sum_{m=1}^{M} \hat{\sigma}^2_{t+m}\mathrm{Var}_t\left[u^*_{t+m} - \tilde{u}_{t+m}\right], \tag{12}$$

and use it to rewrite the cumulative portfolio variance as $w^2\upsilon_t$. For the lower partial moment, we need to distinguish whether the investor takes a long position in the bubbly asset and a short position is the non-bubbly asset or vice versa. We assume that the threshold K is proportional to the portfolio volatility $\sqrt{w^2\sigma^2_{t+m}} = |w|\sigma_{t+m}$, and corresponds with the cut-off value of the crash regime, i.e., $K = |w|\sigma_{t+m}k$. For a long position (i.e., $w > 0$), it follows that $\mathrm{E}\big[\big((K - r^{\mathrm{p}})^+\big)^\nu\big] = w^\nu\sigma^\nu\,\mathrm{E}\big[\big((k - u^*_{t+m} + \tilde{u}_{t+m})^+\big)^\nu\big]$. For $w < 0$, we find $\mathrm{E}\big[\big((K - r^{\mathrm{p}})^+\big)^\nu\big] = (-w)^\nu\sigma^\nu\,\mathrm{E}\big[\big((k - \tilde{u}_{t+m} + u^*_{t+m})^+\big)^\nu\big]$. We can define the cumulation of the lower partial moments of the abnormal returns as:

$$\rho_t(k,w) \equiv \sum_{m=1}^{M} \sigma_{t+m}^2 \, \mathrm{E}_t \left[\left(\left(k - \mathrm{sgn}(w)(u_{t+m}^* - \tilde{u}_{t+m})\right)^+ \right)^2 \right], \tag{13}$$

which depends on w only with respect to its sign. The cumulation of the portfolio lower partial moments takes the form $w^2\rho_t(k,w)$.

Substituting the expressions for the mean, variance and lower-partial moment into Equation (8) and differentiating with respect to w leads to the optimal weight:

$$w^* = \frac{\eta_t}{\gamma_1 \upsilon_t + \gamma_2 \rho_t(k,\eta_t)}. \tag{14}$$

As $\upsilon_t > 0$ and $\rho_t(k,w) > 0$ by construction, the sign of w and thus whether the investor takes a long or short position depends only on the expected return η_t. Therefore, we substitute η_t for w in $\rho_t(k,w)$. Equation (14) shows that the investor's position increases if he is less risk averse, the expected return differences η_t are higher or the asset's abnormal return differences have a smaller cumulative variance υ_t or downside risk ρ_t. If the weight optimal w^* is positive for a certain horizon, the investor holds a long position in the bubbly asset and a short position in the normal asset. In this case, we conclude that riding bubbles is consistent with the propositions of Abreu and Brunnermeier (2003) and De Long et al. (1990b) the optimal strategy at this specific horizon. If w^* is negative, we conclude that shorting is optimal. If shorting the bubbly asset is optimal for a wide range of investment horizons, our findings speak in favor of the efficient market hypothesis. Alternatively, if we find that shorting is only an optimal long-run strategy, our evidence points towards the limits-to-arbitrage literature. If w^* is close to zero, we conclude that it is optimal for an investor to refrain from trading against the bubble or actively investing in it as the risk outweighs the potential returns. We call this strategy "sidelining". Especially at shorter horizons, this result would provide as well evidence in favor of the propositions of the limits-to-arbitrage literature.

To implement his asset allocation, the investor needs to obtain risk and return forecast for his investment horizon. Broadly speaking, these forecasts are based on the probabilities assigned to the different regimes and their distributional characteristics as shown in Equation (6). The exact derivation of the mean, variance and lower partial moment forecast is discussed in Appendix A.

5 Empirical Analysis

5.1 Regime-Switching Model

To analyze how the investor's horizon and his risk-aversion influence his investment strategy upon the detection of a bubble, we estimate the model empirically. We base our analysis on monthly returns of the 48 industries previously used by Fama and French (1997), which are available on French's website[4]. Our dataset starts in July 1963 when the CRSP database is extended by stocks traded on the AMEX.

To estimate the model described in the theoretical section, we follow a two-step procedure. First, we estimate the fundamental part of the asset returns pertaining to time t over the last T months:

$$r_{i,\tau} - r_{\mathrm{f},\tau} = \alpha_{i,t} + \boldsymbol{\beta}'_{i,t}\boldsymbol{f}_\tau + \varepsilon_{i,\tau},\ \mathrm{E}[\varepsilon_{i,\tau}] = 0,\ \mathrm{E}[\varepsilon^2_{i,\tau}] = \sigma^2_{i,t} \quad \tau = t - T + 1, ..., t, \tag{15}$$

where $\boldsymbol{f}_\tau$ is a vector of risk factors. We use three different sets of risk factors, the CAPM, the 3-Factor Model and the 4-Factor Model. The market portfolio, the risk-free rate, and the factor portfolios HML, SMB and MOM were taken from French's website as well. We choose T equal to 120 months.

In the second step, we estimate the parameters for the Markov regime switching model introduced in Section 3. We use the estimates of the first step to construct standardized abnormal returns for the next month:

$$u_{i,t+1} = \frac{r_{i,t+1} - r_{\mathrm{f},t+1} - \hat{\boldsymbol{\beta}}'_{i,t}\boldsymbol{f}_{t+1}}{\hat{\sigma}_{i,t}}. \tag{16}$$

Table 3, 4 and 5 show descriptive statistics of the estimation results of Equation (15) and the abnormal returns in Equation (16) for the CAPM, the 3-Factor Model and the 4-Factor Model, respectively. The coefficient of the market factor is close to one for all models. Further, as indicated by the positive coefficient on SMB, it seems that our set of industry returns is slightly tilted towards smaller firms. The aggregate value versus growth and momentum exposure is close to zero. We find that the abnormal return volatility ranges

[4]The data can be downloaded from the Kenneth French Data Library at `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`. We have used the set of industry returns constructed with the new specifications.

from 3.68 for the 4-Factor Model to 4.04 for the CAPM. The pooled mean abnormal return is slightly negative for all three asset pricing models. The standard deviation is significantly larger than one. It ranges from 1.10 for the CAPM to 1.15 for the 4-Factor Model. In line with previous findings, we also observe that returns are slightly negatively skewed.

[Table 3 about here.]

[Table 4 about here.]

[Table 5 about here.]

We estimate the regime switching model based on the standardized abnormal returns in Equation (16). To determine the maximum likelihood estimates, we use the Expectation Maximization algorithm developed by Dempster et al. (1977). We require that bubbles last at least twelve months (i.e., $L \geq 12$) to ensure that we pick up sizable deviations from fundamental value and to avoid any overlap with industry momentum documented by Moskowitz and Grinblatt (1999). The cut-off value for crashes is $k = -1$, implying that investors consider drops of one standard deviation or more as possible crashes. We also estimate the initial inference probabilities. As the investor has no information on how long a bubble has already lasted, we restrict the initial inference probabilities for the bubble states (i.e. $\mathrm{B}_L, L = 1, \ldots, 12$) to be equal.

Table 6 shows the estimation results of the regime switching model. Panel A shows the distribution parameters of the different regimes. In Panel B, these parameters are transformed into means and volatilities. The transition probabilities are reported in Panel C. The standardized abnormal returns are in the normal regime very close to zero. They range from about -0.01 for the 3-Factor Model and the 4-Factor Model to 0.02 for the CAPM. The volatilities cluster around one. The probability that the normal regime continues in the following month is between 98% and 99%. The transition probability from the normal to the crash regime is between 1% and 2%. In line with the idea that bubbles are rare events, the transition probability from the normal to the bubble regime is extremely small. It ranges from 0.04% for the CAPM to 0.11% for the 4-Factor Model. The bubble regime is characterized by large positive abnormal returns. The estimate for the expected abnormal return in case of the CAPM is 2.67% per month. For the 3-and 4-Factor Model the estimates are 2.46% and 2.63%, respectively. Although bubbles start rarely, they have a tendency to continue. The probability of staying in the bubble regime lies between 81% and 87%. The alternative is that the return process moves to the crash regime, which is characterized by a high volatility and very negative expected abnormal returns of around -8.5%. These results hint towards a trade-off as the bubble will with a probability of 13% to 19% end with a crash during the following month. If the return process stays in the bubble regime, an investor can earn high abnormal returns by actively investing in the bubbly asset. However, if the process switches to the crash regime in the following period, the losses are even larger. From the crash regime, the return process moves most frequently to the normal regime. However, we also observe that crashes last several months. The probability that a crash is followed by another crash ranges from 18% for the 4-Factor Model to 23% for the 3-Factor Model. We do not only observe that bubbles are followed by crashes but also that crashes are in about 10% of the cases followed by bubbles. This finding suggests that bubbles might be interrupted by crashes and continue thereafter, an effect described by Abreu and Brunnermeier (2003) as "temporary strengthening".

[Table 6 about here.]

In Table 7 the ergodic probabilities are shown. They present the probability that the investor assigns to the different regimes if he has no historical information. In line with our findings for the transition probabilities, the normal regime occurs most frequently. Its probability varies from 93% for the CAPM to 95% for the 3-Factor Model. 1% to 2% of the observations belong to the crash regime. We find that the non-transitory bubble state (B_{nt}) has a probability of 1% to 2%. The probabilities of the transitory bubble states B_1 to B_{12} are much smaller and only around 0.02% since these states cannot directly reoccur.

[Table 7 about here.]

Figure 1 shows the distribution of the different regimes over time. We see a pronounced increase in the number of industries experiencing a bubble at the end of the century when the internet bubble occurred. It seems that it was also a period of higher than average volatility as several industries experienced crashes during these years as well. Especially for the CAPM and the 3-Factor Model, we find, in line with the bursting of the bubble, a sharp increase in the number of industries experiencing a crash around early 2000. Consistent with historical accounts, the number of crashes also increases substantially in Fall 1987, when the famous "Black Monday" occurred. The high number of bubbles that we observe at the beginning of the sample period is largely a statistical artifact: since we only have a small number of observations, we cannot apply the 12 months rule during the first year and our bubble identification is less strict. The distribution of bubbles across industries is shown in Table 8. The main observation is that the different regimes are well distributed across the different industries. It seems that no single industry in particular is driving our findings.

[Figure 1 about here.]

[Table 8 about here.]

5.2 Risk and Return Forecasts

To determine his optimal weight, the investor evaluates the risk and return characteristics of the bubbly and normal asset during his investment horizon. Since these characteristics depend directly on the forecast probabilities of the different regimes, as pointed out in Appendix A and Equation (6), Figure 2 displays their development. The forecast probabilities of the non-bubbly asset are based on the assumption that the investor is certain that the asset experiences the normal regime at $t = 0$. These probabilities converge very quickly to the ergodic probabilities associated with the regime process $\tilde{S}$. The ergodic probability of the normal regime ($\tilde{\mathrm{N}}$) is 0.99 for the 4-Factor Model and 0.98 for the CAPM and the 3-Factor Model. The crash probabilities are 0.02 and 0.01.

The forecast probabilities associated with the bubbly asset show a much more lively and interesting pattern. These probabilities are based on the assumption that the investor is certain that the asset experiences a bubble at $t = 0$. In the short run, the investor expects the bubble to continue. The probability that the return process stays in the bubble regime for the following month, ranges from 0.87 for the CAPM to 0.81 for the 4-Factor Model.

Two months later, the probability is still between 0.77 for the CAPM and 0.68 for the 4-Factor Model. The probability that the bubble continues declines rapidly over time. After 5 to 7 months, the probability that a crash occurred and that the two assets have converged is higher than the probability that the bubble continues. Especially during the first couple of months, the probability that the process moves to the crash regime shows a sharp increase. At $t = 2$ it is around 14% for the CAPM, 17% for the 3-Factor Model and 19% for the 4-Factor Model. Thereafter, it decreases again slightly and approaches zero at around 3 years. However, it seems that at this point in time most bubbles have already burst. The probability that the return process of the bubbly asset S^* has converged to the return process of the non-bubbly asset $\tilde{S}$ is more than 95% after three years.

[Figure 2 about here.]

Combining the forecast probabilities and the expected abnormal returns of the different regimes we can derive abnormal return forecasts shown in Figure 3. In Subfigures 3(a), 3(c) and 3(e), we compare the monthly abnormal return expectations of the normal asset if the investor is 100% certain that the return process is in the normal regime (dashed line) to the abnormal returns of the bubbly asset given that the bubble regime prevails with 100% certainty (dotted line). Similar to the average abnormal returns for the complete sample (see Tables 3 to 5) the predicted abnormal returns following the normal regime are slightly negative. In line with our findings for the forecast probabilities, they deviate only slightly from their long-term average for the first few months. The return forecasts following the bubble regime show a more lively pattern. In the first month following the bubble regime, we observe for all three asset pricing models very positive expected abnormal returns. Thereafter, they decline quickly. The expected return difference, which can be interpreted as the result of a strategy which is long in the bubbly asset and short in the non-bubbly asset (see Equation (17)) is during the first month 0.22% for the 4-Factor Model, 0.48% for the 3-Factor Model and 1.03% for the CAPM. Thereafter, the difference declines sharply for all three asset pricing models due to the strong increase in crash probability showed in Figure 2. This decline is most pronounced for the 4-Factor Model, where the returns even temporarily become negative. The reason for the relatively stronger decline is twofold. First, crashes are slightly more severe and second, the probability that the bubble continues is sightly lower than for the other models. For both, the 3-Factor Model and the 4-Factor Model, returns recover after a few months. Just like the decline, the recovery is particularly strong for the 4-Factor Model. After one year, the monthly abnormal returns to the long-short strategy are 0.40% for the CAPM, 0.25% for the 3-Factor Model and 0.27% for the 4-Factor Model.

Since the optimal weight for different horizons depends on the cumulative returns to the long-short strategy (see Equation (11)), Subfigures 3(b), 3(d) and 3(f) show how the monthly abnormal returns add up over time. Except for the 4-Factor Model in month 4, the cumulative abnormal returns are consistently positive. They increase during the first couple of years and reach thereafter an almost constant level as most bubbles have burst and the bubbly asset has with a high probability converged to the normal asset. The positive cumulative return forecasts for most of the sample imply that the investor will generally assign a positive weight to the bubbly asset and a negative weight to the non-bubbly asset. How extreme his position will be depends on the variance and lower partial moment forecasts as well as his risk aversion.

[Figure 3 about here.]

The investor's abnormal return volatility forecasts are shown in Figure 4. Subfigures 4(a), 4(c) and 4(e) show the monthly forecasts of the long-short strategy. Although the relation between the volatility forecast of the long-short portfolio and the two separate assets is not straightforward, these forecasts can give some first insights. Just like above, we assume for the normal asset that the investor is certain that it is in the normal regime $\tilde{\mathrm{N}}$ at $t = 0$. Its volatility is relatively constant over time and close to 4% for all three asset pricing models. The forecasts for the bubbly asset are based on the assumption that it was in the non-transitory bubble regime B_{nt}. At very short horizons, the volatility is very high due to the high crash risk documented in Figure 2. As the horizon increases, crash risk decreases again. Overall, we can infer that the very high volatility of the long-short position at short horizons is mainly driven by the high volatility of the bubbly asset. At the one-month horizon, the standard deviation of the long-short position is between 7.5% and 8% for the different factor models. Just like the volatility of the bubbly asset, it decreases thereafter. After one year, the volatility equals 4.28% for the CAPM, 3.65% for the 3-Factor Model and 3.41% for the 4-Factor Model. After 5 years, the monthly volatility for the long-short position has become negligible due to the high probability that the assets have converged. It is about 0.5% for the CAPM and 0.30% for the other models. Subfigures 4(b), 4(d) and 4(f) show how the monthly volatility forecasts translate into the cumulative variance forecasts defined in Equation(12), which directly determine the investor's optimal weight. The cumulative variance increases steeply during the first few months but looses speed thereafter. Over time, as the probability that the two assets are identical approaches one, the monthly increase approaches zero. Comparing the evolution of the return to the volatility forecasts suggests that the additional risk during bubbles provides a clear counterweight to the additional returns that an investor can earn. As the risk seems to be mainly due to crashes, we will investigate in turn the lower partial moments (LPM).

[Figure 4 about here.]

For the LPMs we need to distinguish whether the investor holds a short position in the bubbly asset and a long position in the non-bubbly asset or vice versa. As shown in Equation (14), the sign of the weight is determined by the difference in expected abnormal returns between the two assets. As the difference in abnormal returns between the bubbly and normal asset is positive for most of our sample period, the LPMs shown in Figure 5 are usually the appropriate risk measure.[5] Subfigures 5(a), 5(c), and 5(e) document that the evolution of the monthly LPM forecast is similar to the volatility forecast. It seems to be slightly more extreme which can be explained by the fact that it focuses on crash risk. At very short horizons, the LPM is rather high. For the CAPM and the 3-Factor Model, it is during the second month even slightly higher than during the first month. This increase can be explained by the increase in crash probability documented in Figure 2. Thereafter, it decreases consistently over time. While it ranged from 12.80 for the CAPM to 17.93 for the 4-Factor Model during the second month, it is virtually equal to zero after 5 years. Subfigures 5(b), 5(d) and 5(f) show how the monthly forecasts translate into cumulative

[5] As the LPMs of the short position or the long position do by definition not relate to the partial moments of the zero investment portfolio, we deviate from our approach for the other moments and do not discuss them.

forecasts over the investor's horizon as defined in Equation (13). We observe first a steep increase which flattens over time.

[Figure 5 about here.]

Figure 6 shows the LPM of an investment portfolio that is long in the normal asset and short in the bubbly asset. As the cumulative abnormal return forecast is only negative during the second month for the 4-Factor Model, we can infer that this LPM is only the appropriate risk measure for this specific case. The LPMs of this strategy are generally somewhat smaller in magnitude, which can be explained by the absence of crash risk. Just like the variance and the other LPMs, they are higher at the beginning and decline as the investor's horizon increases. Thus, it seems that not only riding bubbles is a risky strategy but also shorting the bubbly asset. While riding the bubble, an investor faces the risk of a crash. When shorting, he may face large negative returns in case the bubble continues.

[Figure 6 about here.]

5.3 Investment Horizon and Optimal Portfolio Weight

The investor determines his optimal weight based on the risk and return forecasts and his aversion to downside or variance risk. As the current empirical literature does not provide any risk aversion parameters for the utility function proposed in Equation (7), we calibrate it to the market. The underlying idea is that an investor with an average risk-aversion would hold the market portfolio. We discuss the details of the estimation of the risk aversion coefficient in Appendix B.

Figure 7 shows the optimal weight for a mean-variance and a downside risk averse investor. As in Equation (14), the weights are defined in terms of a long position in the bubbly asset and a short position in the normal asset. A negative weight at a certain horizon therefore implies that the investors shorts the bubbly asset and takes a long position in the non-bubbly asset at this investment horizon. A positive weight means that it is optimal to ride the bubble. If the weight is close to zero, we conclude that it is optimal to sideline at this horizon. In line with our approach in the previous section, we compare the bubbly asset given that the investor is certain that the asset experienced a bubble at $t = 0$ to the non-bubbly asset, assuming that it was with certainty in the normal regime.

[Figure 7 about here.]

Although the variance-risk averse investor takes more extreme positions, we observe a similar development for both investors over time. During the first month, both the variance- as well as the downside-risk averse investor assign a relatively large positive weight to the bubbly industry and consequently a negative one to the normal asset. For the CAPM, the downside risk averse investor's optimal weight in the bubbly asset is 0.57 and the variance risk investor's optimal weight is 0.80. For the 3-Factor and 4-Factor Model, the weights are lower but still positive and economically large. The variance-risk averse investor's optimal weight is 0.38 for the 3-Factor Model and 0.16 for the 4-Factor Model. The optimal weights of the downside risk averse investor are 0.23 and 0.09 respectively. Following the first months, the weight allocated to the bubbly asset declines rapidly, reflecting the increased crash probability and consequently worse risk-return tradeoff documented above. The decline is particularly pronounced for the 4-Factor Model, where the weight becomes even slightly negative during the fourth month. For the other factor models, we also observe a steep decline, but the weight stays positive. The lowest point for the CAPM is during the 5th and 6th month, where the optimal weight for the variance risk averse investor is 0.60 and the optimal weight of the downside risk averse investor is 0.40. The lowest weight of the 3-Factor Model invested in the bubbly asset is 0.14 and 0.08 at the 5 month horizon for a variance-risk and downside-risk averse investor respectively. During the following months, the optimal weight allocated to the bubbly asset continuously increases again until it reaches a steady state level. For the 4-Factor Model, the weight allocated to the bubbly asset at the 3-year horizon is even higher than the weight allocated during the first month. It is 0.31 for the variance-risk averse investor and 0.18 for the downside-risk averse investor. The optimal weights at the 3-year horizon are higher for the other two models. The variance-risk averse investor's optimal weight allocated to the bubbly asset is 0.75 for the CAPM and 0.37 for the 3-Factor Model. The weight of the downside-risk averse investor is, as usual, somewhat lower at 0.53 for the CAPM and 0.22 for the 3-Factor Model.

Our evidence suggests that riding bubbles is a profitable strategy, particularly at a very short horizon of one month and rather long horizons of at least a year or longer. While the first result is in line with the finding of Guenster et al. (2008) and the theoretical models of Abreu and Brunnermeier (2003) and De Long et al. (1990b) the second result is at first sight quiet surprising. Traditionally, bubbles are thought of as zero-sum games and one expects crashes to wipe out all gains previously accrued. Therefore, the difference in weights should approach zero for a horizon that is sufficiently long to ensure a high probability that the bubble burst. Our finding of a positive and economically large weight allocated to the bubbly asset in the long-run points towards a close connection between bubbles and the real economy.

Anecdotal evidence suggests that bubbles are often coupled with technological innovations and "New Economy" thinking as described in Shiller (2000). As bubbles go in line with technological enhancements, it could be that the run-up in stock prices simultaneously originates from two sources. The one source is a continuous adjustment in fundamental value and the other source is the irrational price increase. If these two development occur at the same time, our methodology might not be able to disentangle them. However, this explanation does not endanger our conclusion that riding bubbles is the optimal strategy.

Another explanation for these findings goes one step further and proposes a positive causal effect of bubbles on the real economy. For example, Jerzmanowski and Nabar (2008) suggest that the internet bubble had a positive effect on aggregate GDP growth. The idea underlying their analysis is that financing constraints for R&D spending lead usually to a suboptimal level of innovation. Bubbles can ease these financing constraints as investors have higher return expectations and consequently they are willing to invest more. The increase in R&D spending brings the economy closer to an optimal innovation level, which has a positive effect on net welfare. While the model of Jerzmanowski and Nabar (2008) analyzes the effect of bubbles on welfare at the aggregate level, the model Hirshleifer et al. (2006) which we discuss in Section 2.1 analyzes feedback at the firm level. Although rather different factors play a role in the model of Hirshleifer et al. (2006) and Jerzmanowski and Nabar (2008), they both arrive at the conclusion that bubbles can have a positive effect on the real economy. This proposition can provide an explanation of our finding that the weight allocated to the bubbly asset is positive, even in the long run.

6 Conclusion

In this paper we analyze how the investor's horizon affects his optimal portfolio allocation during bubbles. We assume that he can only make a zero-cost investment and offer him two assets, one that experiences a bubble and one that can never enter the bubble regime. When confronted with this choice, an investor with a mean-variance utility function as well as an investor who is averse to downside risk actively invests in the bubbly asset for most of the investment horizons. Especially in the very short run as well as in the long run, the bubbly asset offers a substantially better risk-return trade-off than the non-bubbly asset. At intermediate horizons of 2 to 7 months, the risk-return trade-off deteriorates temporarily due to a very high crash risk.

Our results indicate that riding bubbles seems to be the optimal strategy, even in the long run, for investors who have no superior timing abilities. This finding stands in stark contrast to the predictions of the efficient market hypothesis, which states that it is optimal to short the bubbly asset at any horizon. It also contradicts the limits-to-arbitrage literature. Based on this stream of literature, we would expect that shorting the bubbly asset is risky and potentially unprofitable for short-term oriented investors. However, it should at least be the optimal strategy for investors who have a longer horizons. These investor should ultimately be able to profit of the bubble bursting. And even the theoretical models that propose riding bubbles as the optimal short-term strategy, predict that the investor takes a short position as the bubble progresses and the risk of a crash increases.

Our results are consistent with models that suggest a causal relation between bubbles and the asset's fundamental value. As the bubble leads to an increase in fundamental value, subsequent crashes should not wipe out all of the gains. Instead, the part of the price increase which is due to the bubble's impact on fundamental value should remain. Consequently, in line with our findings, riding bubbles can even be a good strategy in the long term.

A Derivation of Regime-Specific Moment Forecasts

The investor has to make a decision at point t. He has constructed inference probabilities telling him with what probability the different regimes apply to the bubbly asset, $\phi^*_{t|t}(s), s \in \mathcal{S}^*$. For the non-bubbly asset he has calculated $\tilde{\phi}_{t|t}(s), s \in \tilde{\mathcal{S}}$. His allocation decision depends on the risk and return characteristics of the difference between the idiosyncratic part of the assets, i.e. $u^*_{t+m} - \tilde{u}_{t+m}$ over the complete horizon $m = 1, 2, \ldots, M$. If the assets have converged (i.e., $S^*_{t+m} = \tilde{S}$), the two assets are identical and both the risk and return characteristics are equal to zero.

Based on Equation (6), we derive the expectation, the variance and the lower partial moment. The expected value of u for period $t + m$ follows directly:

$$\begin{aligned}
\mathrm{E}_t \left[u^*_{t+m} - \tilde{u}_{t+m}\right] &= \sum_{s \in \mathcal{S}^*} \phi^*_{t+m|t}(s)\psi(s) - \sum_{s \in \tilde{\mathcal{S}}} \tilde{\phi}_{t+m|t}(s)\psi(s) \\
&= \sum_{s \in \{\mathcal{S}^* \setminus \tilde{S}\}} \phi^*_{t+m|t}(s)\psi(s) - \left(1 - \phi^*_{t+m|t}(\tilde{S})\right) \sum_{s \in \tilde{\mathcal{S}}} \tilde{\phi}_{t+m|t}(s)\psi(s),
\end{aligned} \tag{17}$$

where the predictions for the means of the different regimes are denoted by $\psi(s)$. In the second equation, we use the fact that the two assets have equal expectation if $S^*_{t+m} = \tilde{S}$. The forecast probabilities are constructed from the time t inference probabilities and the transition matrices, $\boldsymbol{\phi}^*_{t+m|t} = (\boldsymbol{P}^*)^m \boldsymbol{\phi}^*_{t|t}$ and $\tilde{\boldsymbol{\phi}}_{t+m|t} = (\tilde{\boldsymbol{P}})^m \tilde{\boldsymbol{\phi}}_{t|t}$. The regime-specific means follow directly in case of the normal and bubble regime. For the crash regime, we use the fact that u_t follows a lognormal distribution:

$$\psi(s) \equiv \mathrm{E}[u_t | S_t = s] = \begin{cases} \mu_{\mathrm{N}} & \text{if } S_t = \mathrm{N} \\ \mu_{\mathrm{B}} & \text{if } S_t = \mathrm{B} \\ k - \mathrm{e}^{\mu_{\mathrm{C}} + \frac{1}{2}\omega^2_{\mathrm{C}}} & \text{if} S_t = \mathrm{C} \end{cases} \tag{18}$$

The variance, $\mathrm{Var}_t \left[u^*_{t+m} - \tilde{u}_{t+m}\right]$, can be calculated as $\mathrm{E}_t \left[\left(u^*_{t+m} - \tilde{u}_{t+m}\right)^2\right] - \mathrm{E}_t \left[u^*_{t+m} - \tilde{u}_{t+m}\right]^2$. We can compute the first part using Equation (6), and the second part is given in Equation (17). Summarizing the raw second moments of the different regimes in $\zeta(s)$ we can express the portfolio's raw second moment as:

$$
\begin{aligned}
\mathrm{E}_t\left[\left(u^*_{t+m}-\tilde{u}_{t+m}\right)^2\right] =& \, \mathrm{E}_t\left[(u^*_{t+m})^2 - 2u^*_{t+m}\tilde{u}_{t+m} + \tilde{u}^2_{t+m}\right] \\
=& \sum_{s\in\{\mathcal{S}^*\setminus\tilde{S}\}} \phi^*_{t+m|t}(s)\zeta(s) \\
& - 2\sum_{s\in\{\mathcal{S}^*\setminus\tilde{S}\}} \phi^*_{t+m|t}(s)\psi(s)\sum_{s\in\tilde{\mathcal{S}}}\tilde{\phi}_{t+m|t}(s)\psi(s) \\
& + \left(1-\phi^*_{t+m|t}(\tilde{S})\right)\sum_{s\in\tilde{\mathcal{S}}}\tilde{\phi}_{t+m|t}(s)\zeta(s)
\end{aligned}
\tag{19}
$$

The regime-specific second moments are consequently also a combination of the squared regime-specific first moments, given in Equation (18), and the regime-specific raw second moments, which can be stated as:

$$
\zeta_s \equiv \mathrm{E}[u_t^2|S_t=s] = \begin{cases} \mu_{\mathrm{N}}^2+\omega^2 & \text{if } S_t=\mathrm{N} \\ \mu_{\mathrm{B}}^2+\omega^2 & \text{if } S_t=\mathrm{B} \\ k^2-2k\mathrm{e}^{\mu_{\mathrm{C}}+\frac{1}{2}\omega^2_{\mathrm{C}}}+\mathrm{e}^{2\left(\mu_{\mathrm{C}}+\omega^2_{\mathrm{C}}\right)} & \text{if } S_t=\mathrm{C} \end{cases}
\tag{20}
$$

Finally, we derive the partial moments. Partial moments are raw moments by construction and consequently, we can directly apply Equation (6). To compute the partial moments, we need take into account whether the investor takes a long or a short position in the bubbly versus the normal asset shown in Equation (13). We find:

$$
\begin{aligned}
&\mathrm{E}[((k-\mathrm{sgn}(w)(u^*_{t+m}-\tilde{u}_{t+m})^+)^2] = \\
&\sum_{s^*\in\{\mathcal{S}^*\setminus\tilde{S}\}}\sum_{\tilde{s}\in\tilde{\mathcal{S}}}\phi^*_{t+m|t}(s^*)\tilde{\phi}_{t+m|t}(\tilde{s})\times \\
&\qquad\qquad \mathrm{E}\left[\left(\left(k-\mathrm{sgn}(w)(u^*_{t+m}-\tilde{u}_{t+m})\right)^+\right)^2|S^*_{t+m}=s^*,\tilde{S}_{t+m}=\tilde{s}\right],
\end{aligned}
\tag{21}
$$

which depends on the convolution of u^*_{t+m} and $\tilde{u}_{t+m}$ under the different regimes. To compute regime-specific partial moments, we need to distinguish between a short and long position in the risky asset to evaluate the expectations. Only in case both u^*_{t+m} and $\tilde{u}_{t+m}$ follow normal distributions, their convolution will also follow a normal distribution and we can calculate the lower partial moment in closed form. If one of them follows a log-normal distribution that is no longer the case, and we use numerical integration.

Consider a random variable $X \sim N(\mu, \sigma^2)$ which has pdf $g_{\mathrm{X}}(\cdot)$. Further, $g_{\mathrm{N}}(\cdot)$ represents the standard normal pdf and $G_{\mathrm{N}}(\cdot)$ the related cdf. For $w > 0$, the lower partial moment is computed as:

$$\begin{aligned}\int_{-\infty}^{k}(k-x)^2 g_X(x)\,\mathrm{d}x &= \int_{-\infty}^{\frac{k-\mu}{\sigma}}(k-\mu-\sigma z)^2 g_{\mathrm{N}}(z)\,\mathrm{d}z \\ &= \left((k-\mu)^2+\sigma^2\right) G_N\left(\frac{k-\mu}{\sigma}\right)+\sigma(k-\mu)g_{\mathrm{N}}\left(\frac{k-\mu}{\sigma}\right)\end{aligned} \tag{22}$$

and for $w < 0$, we find:

$$\begin{aligned}&\int_{-k}^{\infty}(k+x)^2 g_X(x)\,\mathrm{d}x = \int_{-\frac{(k+\mu)}{\sigma}}^{\infty}(k+\mu+\sigma z)^2 g_{\mathrm{N}}(z)\,\mathrm{d}z \\ &= \left((k+\mu)^2+\sigma^2\right)\left(1-G_{\mathrm{N}}\left(-\frac{k+\mu}{\sigma}\right)\right)+\sigma(k+\mu)g_{\mathrm{N}}\left(-\frac{k+\mu}{\sigma}\right).\end{aligned} \tag{23}$$

B Estimation of Risk Aversion Coefficients

The utility function in Equation (8) shows two risk aversion coefficients, γ_1 for variance and γ_2 for the second order lower partial moment. While the typical value for the risk aversion coefficient on variance are well-known and range between 1 and 5, for more exotic risk measures such as lower partial moments it is unclear what typical values should be. Instead of simply picking some numbers, we calibrate the parameter values to the market. We pick the values in such a way that the investor would want to exactly hold the market portfolio, so combinations of γ_1 and γ_2 should solve

$$1 = w^{\mathrm{m}} = \frac{\mathrm{E}[r_t^{\mathrm{m}}]}{\gamma_1 \mathrm{Var}[r_t^{\mathrm{m}}] + \gamma_2 \mathrm{LPM}_2[r_t^{\mathrm{m}}]}, \tag{24}$$

where r^{m} is the one-period market return. This expression is similar to the optimal portfolio in Equation (14).

We estimate the moments in Equation (24) by a Markov regime switching model, in particular to pay attention to downside risk. This regime switching model has a state variable S_t^{m} which can be in a normal regime and a crash regime, making it comparable to the regime switching model we use for the non-bubbly asset in Section 4. The model is parameterized as follows

$$r_t^{\mathrm{m}} = \sigma_t^{\mathrm{m}} u_t^{\mathrm{m}},\ u_t^{\mathrm{m}} \sim \begin{cases} N(\mu_{\mathrm{mN}}, \omega_{\mathrm{mN}}^2) & \text{if } S_t^{\mathrm{m}} = \mathrm{N}^{\mathrm{m}} \\ k - \mathrm{e}^{Z^{\mathrm{m}}}, Z^{\mathrm{m}} \sim N(\mu_{\mathrm{mC}}, \omega_{\mathrm{mC}}^2) & \text{if } S_t^{\mathrm{m}} = \mathrm{C}^{\mathrm{m}} \end{cases} \tag{25}$$

$$P^{\mathrm{m}} = \begin{bmatrix} p_{\mathrm{NN}}^{\mathrm{m}} & p_{\mathrm{NC}}^{\mathrm{m}} \\ p_{\mathrm{CN}}^{\mathrm{m}} = 1 - p_{\mathrm{NN}}^{\mathrm{m}} & p_{\mathrm{CC}}^{\mathrm{m}} = 1 - p_{\mathrm{NC}}^{\mathrm{m}} \end{bmatrix} \tag{26}$$

We do not include bubble components in this model, as the limited number of observations will complicate the identification of the bubble states.

We use the CRSP All Shares Index from July 1963 until December 2006 to estimate the parameters. We first estimate σ_t as the volatility over the prior $T = 120$ months, and construct $u_t^{\mathrm{m}} = r_t^{\mathrm{m}}/\sigma_t^{\mathrm{m}}$. This resulting series of u_t^{m} is used to estimate the regime switching model by applying the EM-algorithm. The resulting estimates are given in Table 9. For the normal regime we find an average return of 1.07% per month and a volatility of 4.01%. When an average crash occurs the market goes down with 11.49% and volatility increases to 5.15%. The normal regime is quite persistent with a probability of 0.967 to continue. However, when the market encounters a crash, the probability that another crash follows is also high at 0.283. We found similar patterns for the industry results.

[Table 9 about here.]

To calculate the values for γ_1 and γ_2 we combine the regime-specific predictions with the ergodic probabilities. As such the horizon and the information set of the investor do not matter. The ergodic probability for the normal regime equals 0.957, which leaves 0.043 for the crash regime. Consequently, the unconditionally predicted return equals 0.52% per month, the volatility equals 4.8% per month, and the root of the second order lower partial moment equal 1.92%. Substituting these numbers into Equation (24) means that the following linear combination of γ_1 and γ_2 makes sure that an investor holds the market

$$\gamma_2 = 14.19 - 6.27\gamma_1. \tag{27}$$

An investor who only cares about downside risk will have $\gamma_2 = 14.19$, while a mean-variance investor will have $\gamma_1 = 2.26$.

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Table 1: Structure of the Transition Matrix

	N	C	B_1	B_2	$\cdots$	B_j	B_{j+1}	$\cdots$	B_{L-1}	B_{nt}
N	p_{NN}	p_{CN}	0	0	$\cdots$	0	0	$\cdots$	0	0
C	p_{NC}	p_{CC}	0	0	$\cdots$	0	0	$\cdots$	0	p_{BC}
B_1	p_{NB}	p_{CB}	0	0	$\cdots$	0	0	$\cdots$	0	0
B_2	0	0	1	0	$\cdots$	0	0	$\cdots$	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
B_j	0	0	0	0	$\ddots$	0	0	$\cdots$	0	0
B_{j+1}	0	0	0	0	$\cdots$	1	0	$\cdots$	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\ddots$	$\vdots$	$\vdots$
B_{L-1}	0	0	0	0	$\cdots$	0	0	$\ddots$	0	0
B_{nt}	0	0	0	0	$\cdots$	0	0	$\cdots$	1	p_{BB}

This table shows the structure of the matrix of transition probabilities for the different regimes. N indicates the normal regime, C indicates the crash regime and B indicates a copy of the bubble regime. We restrict the bubble regime to last at least L periods. Therefore, we include L copies of the bubble regime: the transitory bubble states B_1, B_2, $\ldots, B_{L-1}$, and the non-transitory bubble state B_{nt}. We require that the process S_t can enter state $B_j, j = 2, \ldots, L$ only from state B_{j-1}.

Table 2: Structure of the Transition Matrix $\boldsymbol{P}^*$

	N	$\mathrm{C_N}$	$\mathrm{B_1}$	$\mathrm{B_2}$	$\cdots$	B_j	B_{j+1}	$\cdots$	B_{L-1}	$\mathrm{B_{nt}}$	$\mathrm{C_B}$	$\tilde{S}$
N	p_{NN}	p_{CN}	0	0	$\cdots$	0	0	$\cdots$	0	0	0	0
C	p_{NC}	p_{CC}	0	0	$\cdots$	0	0	$\cdots$	0	0	0	0
$\mathrm{B_1}$	p_{NB}	p_{CB}	0	0	$\cdots$	0	0	$\cdots$	0	0	p_{CB}	0
$\mathrm{B_2}$	0	0	1	0	$\cdots$	0	0	$\cdots$	0	0	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\ddots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
B_j	0	0	0	0	$\ddots$	0	0	$\cdots$	0	0	0	0
B_{j+1}	0	0	0	0	$\cdots$	1	0	$\cdots$	0	0	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\ddots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
B_{L-1}	0	0	0	0	$\cdots$	0	0	$\ddots$	0	0	0	0
$\mathrm{B_{nt}}$	0	0	0	0	$\cdots$	0	0	$\cdots$	1	p_{BB}	0	0
$\mathrm{C_B}$	0	0	0	0	$\cdots$	0	0	$\cdots$	0	p_{BC}	p_{CC}	0
$\tilde{S}$	0	0	0	0	$\cdots$	0	0	$\cdots$	0	0	p_{CN}	1

This table shows the structure of the matrix of transition probabilities for the Markov chain S_t^*, which is associated with a bubbly asset. N indicates the normal regime, C indicates a copy of the crash regime, B indicates a copy of the bubble regime and $\tilde{S}$ indicates the chain of the non-bubbly asset. The transitory and non-transitory bubble states are defined as explained in Table 1. The crash state C_{N} can only be entered from the normal regime. The crash state C_{B} can only be entered from the non-transitory bubble regime. From the crash state C_{B} the chain can be left to the non-bubbly chain $\tilde{S}$.

Table 3: Descriptive Statistics of CAPM Regression and Abnormal Returns

	Estimates of Equation 15			Abnormal Returns (Equation 16)					
Industry	$\bar{\alpha}$	$\bar{\beta}$	$\bar{\sigma}$	mean	stdev.	skew.	kurt.	min.	max.
Agric	−0.004	0.94	4.63	0.012	1.01	0.08	4.97	−3.43	4.84
Food	0.336	0.74	3.07	0.109	1.16*	0.24	5.82	−4.76	5.36
Soda	0.114	0.88	4.88	−0.036	1.24*	−0.22	5.73	−5.85	4.88
Beer	0.286	0.86	3.88	0.025	1.13*	−0.41	4.38	−4.28	3.27
Smoke	0.425	0.69	4.92	0.093	1.15*	−0.41	4.52	−5.53	3.63
Toys	−0.302	1.19	4.80	−0.073	1.05	−0.65	5.54	−5.92	3.25
Fun	0.181	1.29	4.61	0.021	1.02	−0.24	3.82	−3.72	2.94
Books	0.105	1.05	3.07	0.039	1.04	−0.52	6.26	−5.65	3.04
Hshld	−0.005	0.88	2.77	−0.064	1.15*	−0.68	8.02	−6.52	4.53
Clths	−0.096	1.16	4.33	−0.012	1.12*	−0.28	4.84	−4.46	3.99
Health	−0.323	1.29	6.83	−0.040	0.99	−0.80	6.65	−5.19	2.76
MedEq	0.097	0.94	3.48	−0.006	1.02	−0.19	3.94	−4.22	3.66
Drugs	0.239	0.85	3.53	0.019	1.14*	−0.42	5.57	−5.33	5.07
Chems	−0.049	1.00	2.84	−0.030	1.14*	0.13	4.94	−4.07	5.55
Rubbr	0.000	1.07	3.40	−0.011	1.10*	−0.38	5.29	−4.29	4.82
Txtls	−0.130	1.00	4.15	−0.034	1.12*	−0.38	5.09	−4.63	4.18
BldMt	−0.006	1.12	2.74	−0.005	1.11*	−0.33	4.44	−4.81	2.89
Cnstr	−0.243	1.30	4.11	−0.028	1.09*	0.14	3.47	−3.63	3.33
Steel	−0.419	1.17	4.21	−0.061	1.12*	0.58	5.36	−3.20	6.02
FabPr	−0.448	1.08	4.77	−0.100	1.07*	−0.10	4.37	−4.28	4.29
Mach	−0.185	1.17	2.74	−0.065	1.13*	0.04	3.49	−3.33	3.83
ElcEq	0.121	1.17	3.13	0.027	1.05	−0.23	3.66	−4.27	3.05
Autos	−0.164	1.00	4.15	−0.053	1.14*	−0.32	5.58	−5.99	4.10
Aero	0.063	1.16	4.35	0.035	1.09*	−0.60	5.95	−5.66	3.59
Guns	0.234	0.90	4.99	0.038	1.13*	−1.30	11.44	−8.25	3.75
Gold	−0.198	0.71	9.84	−0.022	1.10*	0.42	5.19	−3.42	6.03
Ships	−0.208	1.05	4.95	−0.039	1.10*	0.00	4.25	−4.14	4.18
Mines	−0.187	0.99	4.75	−0.011	1.12*	−0.13	3.35	−3.95	3.19
Coal	−0.114	1.08	7.01	−0.025	1.17*	0.29	4.54	−4.64	5.03
Oil	0.203	0.77	3.87	0.069	1.12*	0.28	3.55	−2.99	3.74
Util	0.173	0.51	3.10	0.074	1.11*	−0.01	3.74	−3.68	4.14
Telcm	0.131	0.73	3.08	0.045	1.08*	0.02	3.53	−3.69	3.47
PerSv	−0.375	1.19	4.13	−0.057	1.02	−0.48	4.81	−4.45	3.05
BusSv	0.000	1.34	2.96	0.031	1.06	0.17	3.87	−2.95	3.71
Comps	−0.278	1.17	4.26	−0.086	1.10*	−0.11	3.85	−4.28	3.49
Chips	−0.173	1.40	3.67	−0.029	1.16*	−0.13	4.15	−4.22	4.46
LabEq	−0.213	1.35	4.04	−0.065	1.09*	0.21	5.05	−3.69	4.85
Paper	0.113	0.96	3.28	0.009	1.07	0.55	5.22	−3.47	5.37
Boxes	−0.020	0.92	3.63	−0.006	1.15*	−0.60	4.25	−4.89	3.06
Trans	−0.094	1.09	3.26	−0.023	1.05	0.08	4.11	−3.49	4.06
Whshl	−0.027	1.11	2.59	−0.068	1.11*	−0.31	6.91	−6.37	5.16
Rtail	0.058	1.06	3.22	−0.013	1.12*	−0.19	3.57	−3.71	3.70
Meals	0.054	1.16	3.89	−0.028	1.06	−0.32	5.31	−4.80	3.79
Banks	0.176	1.03	3.32	0.037	1.05	−0.52	5.91	−4.85	4.22
Insur	0.207	0.91	3.46	0.053	1.05	−0.51	7.54	−6.24	4.88
RlEst	−0.525	1.12	4.59	−0.137*	1.06	−0.60	5.27	−4.63	3.40
Fin	0.162	1.09	2.29	0.090	1.13*	−0.32	4.56	−4.53	4.33
Other	−0.592	1.22	4.36	−0.092	1.07	−0.19	5.77	−5.29	4.31
Pooled	−0.040	1.04	4.04	−0.010	1.10*	−0.20	5.07	−8.25	6.03

The first three columns of this table report the averages of the coefficients of the rolling regressions of the market model in Equation 15 for every industry. For each regression, we construct an abnormal return for the month following the estimation window as in Equation 16. The remaining columns show the descriptive statistics of these abnormal returns. To correct for time-varying volatility, we standardize the abnormal return by a division by the residual volatility of the regression model displayed in column 3. An asterisk denotes a significant difference from zero for the mean and a significant divergence from one in case of volatility, both at the 5% significance level.

Table 4: Descriptive Statistics of 3-Factor Model Regressions and Abnormal Returns

	Estimates of Equation 15					Abnormal Returns (Equation 16)					
Industry	$\bar{\alpha}$	$\overline{\beta}_{\mathrm{m}}$	$\overline{\beta}_{\mathrm{SMB}}$	$\overline{\beta}_{\mathrm{HML}}$	$\bar{\sigma}$	mean	stdev.	skew.	kurt.	min.	max.
Agric	−0.132	0.85	0.62	0.07	4.29	−0.004	1.03	0.48	5.87	−3.28	5.48
Food	0.238	0.80	−0.17	0.08	2.90	0.066	1.19*	0.25	5.54	−4.73	5.29
Soda	0.038	1.01	−0.09	0.19	4.73	−0.039	1.25*	0.12	5.72	−5.24	5.32
Beer	0.176	0.88	−0.04	0.04	3.73	0.004	1.18*	−0.39	4.16	−4.10	3.50
Smoke	0.367	0.78	−0.24	0.06	4.79	0.060	1.17*	−0.54	5.08	−6.13	3.63
Toys	−0.327	1.06	0.55	−0.10	4.44	−0.100	1.07*	−0.48	5.19	−6.00	3.02
Fun	0.162	1.17	0.54	−0.07	4.33	−0.007	1.05	−0.11	3.65	−3.91	3.19
Books	0.010	1.01	0.26	0.06	2.90	0.025	1.07*	−0.45	6.24	−6.20	3.50
Hshld	0.064	0.91	−0.15	−0.12	2.66	−0.040	1.23*	−1.44	17.06	−10.44	5.12
Clths	−0.273	1.08	0.59	0.18	3.69	−0.096	1.23*	−0.23	5.94	−5.65	5.00
Health	−0.348	1.03	0.94	−0.29	5.95	−0.072	1.07*	−1.25	10.04	−7.23	3.25
MedEq	0.342	0.83	0.02	−0.51	3.15	0.043	1.11*	−0.27	4.40	−4.85	3.62
Drugs	0.482	0.80	−0.37	−0.49	3.12	0.096	1.20*	−0.38	4.42	−5.32	3.95
Chems	−0.171	1.10	−0.02	0.27	2.67	−0.081	1.13*	0.17	4.58	−4.13	5.23
Rubbr	−0.186	0.98	0.67	0.17	2.74	−0.081	1.19*	−0.11	4.10	−4.19	4.37
Txtls	−0.425	1.01	0.76	0.49	3.36	−0.149*	1.15*	0.02	4.09	−3.99	4.26
BldMt	−0.192	1.13	0.30	0.23	2.38	−0.084	1.13*	−0.17	3.98	−4.72	3.39
Cnstr	−0.346	1.26	0.51	0.14	3.75	−0.081	1.11*	0.07	3.32	−3.61	2.72
Steel	−0.631	1.20	0.42	0.44	3.88	−0.128*	1.15*	0.54	4.82	−3.78	5.67
FabPr	−0.508	1.03	0.63	0.08	4.35	−0.128*	1.09*	−0.07	4.10	−3.75	4.32
Mach	−0.210	1.15	0.35	0.11	2.55	−0.094	1.15*	0.09	3.96	−3.57	4.64
ElcEq	0.152	1.12	0.05	−0.14	3.03	0.052	1.07	−0.25	3.30	−3.97	2.86
Autos	−0.480	1.15	0.21	0.64	3.75	−0.155*	1.16*	−0.09	4.47	−4.93	3.65
Aero	−0.082	1.14	0.35	0.16	4.02	−0.019	1.10*	−0.45	5.86	−6.11	3.63
Guns	0.015	0.94	0.26	0.36	4.75	−0.023	1.15*	−1.05	11.10	−8.37	4.91
Gold	−0.348	0.70	0.60	0.28	9.75	−0.041	1.10*	0.54	5.96	−3.79	6.38
Ships	−0.264	1.09	0.31	0.22	4.70	−0.063	1.12*	0.25	3.92	−3.66	4.11
Mines	−0.327	1.02	0.51	0.34	4.46	−0.059	1.12*	−0.19	3.30	−4.06	3.08
Coal	−0.234	1.09	0.28	0.19	6.98	−0.043	1.18*	0.40	5.08	−4.73	5.19
Oil	0.118	0.93	−0.28	0.28	3.63	0.047	1.15*	0.23	3.50	−3.08	3.84
Util	−0.075	0.70	−0.24	0.47	2.68	−0.022	1.11*	0.04	3.87	−3.90	3.98
Telcm	0.038	0.80	−0.24	0.18	2.95	0.032	1.11*	−0.02	3.68	−3.68	3.67
PerSv	−0.455	1.09	0.55	−0.02	3.70	−0.101	1.08*	−0.68	6.14	−5.60	3.17
BusSv	0.153	1.09	0.44	−0.45	2.20	0.121*	1.12*	0.10	4.44	−3.77	4.76
Comps	0.010	0.99	0.08	−0.54	3.94	−0.019	1.09*	−0.21	3.56	−4.14	3.03
Chips	−0.012	1.20	0.39	−0.34	3.25	0.008	1.20*	−0.19	3.92	−4.28	4.03
LabEq	−0.018	1.14	0.47	−0.43	3.60	−0.027	1.11*	0.10	4.55	−4.15	4.37
Paper	−0.011	1.05	0.01	0.28	3.18	−0.047	1.08*	0.42	4.80	−3.33	5.04
Boxes	−0.011	0.99	−0.11	0.03	3.53	−0.006	1.17*	−0.67	4.52	−5.43	2.93
Trans	−0.244	1.10	0.29	0.25	3.01	−0.065	1.07*	−0.09	4.01	−4.35	3.66
Whshl	−0.099	1.06	0.40	0.07	2.30	−0.130*	1.15*	−0.33	6.31	−5.57	5.08
Rtail	0.033	1.02	0.20	−0.02	3.12	−0.017	1.17*	−0.14	3.93	−4.24	4.01
Meals	−0.024	1.09	0.42	−0.03	3.49	−0.045	1.12*	−0.25	4.65	−5.01	4.13
Banks	−0.034	1.19	0.01	0.47	2.92	−0.007	1.03	0.14	4.20	−3.64	4.15
Insur	0.052	1.00	−0.02	0.26	3.17	0.027	1.03	−0.05	4.11	−3.54	3.30
RlEst	−0.898	1.06	0.99	0.51	3.49	−0.286*	1.08*	−0.09	5.82	−4.98	5.45
Fin	0.000	1.18	0.07	0.34	2.05	0.022	1.13*	−0.12	4.24	−4.49	3.85
Other	−0.655	1.10	0.44	−0.05	4.05	−0.094	1.22*	0.28	6.55	−5.53	5.64
Pooled	−0.116	1.02	0.26	0.09	3.71	−0.039*	1.13*	−0.14	5.26	−10.44	6.38

The first five columns of this table report the averages of the coefficients of the rolling regressions of the Fama and French (1993) Model in Equation 15 for every industry. For each regression, we construct an abnormal return for the month after the estimation window as in Equation 16. The remaining columns show the descriptive statistics of these abnormal returns. To correct for time-varying volatility, we standardize the abnormal return by a division by the residual volatility of the regression model displayed in column 3. An asterisk denotes a significant difference from zero for the mean and a significant divergence from one in case of volatility, both at the 5% significance level.

Table 5: Descriptive Statistics of 4-Factor Model Regressions and Abnormal Returns

	Estimates of Equation 15						Abnormal Returns (Equation 16)					
Industry	$\bar{\alpha}$	$\overline{\beta}_{\mathrm{m}}$	$\overline{\beta}_{\mathrm{SMB}}$	$\overline{\beta}_{\mathrm{HML}}$	$\overline{\beta}_{\mathrm{UMD}}$	$\bar{\sigma}$	mean	stdev.	skew.	kurt.	min.	max.
Agric	−0.223	0.85	0.64	0.09	0.11	4.28	−0.029	1.03	0.51	5.89	−3.24	5.55
Food	0.274	0.79	−0.17	0.07	−0.02	2.84	0.099	1.21*	0.31	4.80	−3.60	5.20
Soda	0.111	1.00	−0.09	0.17	−0.05	4.68	−0.019	1.28*	0.12	5.53	−5.28	5.39
Beer	0.108	0.87	−0.02	0.06	0.10	3.69	−0.014	1.20*	−0.38	4.09	−4.20	3.75
Smoke	0.459	0.78	−0.24	0.04	−0.09	4.77	0.081	1.19*	−0.47	4.74	−6.03	3.60
Toys	−0.195	1.04	0.54	−0.13	−0.13	4.37	−0.062	1.08*	−0.38	4.57	−5.43	3.42
Fun	0.154	1.16	0.55	−0.06	0.04	4.29	−0.014	1.06	−0.08	3.69	−3.92	3.16
Books	0.049	1.02	0.25	0.05	−0.04	2.89	0.046	1.10*	−0.31	5.86	−6.18	3.68
Hshld	0.095	0.91	−0.15	−0.13	−0.02	2.64	−0.030	1.27*	−1.42	17.60	−10.90	5.27
Clths	−0.076	1.07	0.55	0.13	−0.21	3.59	−0.036	1.24*	−0.03	4.88	−5.07	5.18
Health	−0.431	1.05	0.96	−0.27	0.12	5.92	−0.084	1.10*	−1.05	8.50	−6.35	3.54
MedEq	0.345	0.84	0.02	−0.51	0.00	3.14	0.044	1.13*	−0.30	4.54	−4.68	3.78
Drugs	0.467	0.81	−0.36	−0.49	0.03	3.10	0.086	1.23*	−0.44	4.48	−5.34	4.07
Chems	−0.060	1.10	−0.03	0.24	−0.12	2.63	−0.035	1.17*	−0.08	4.66	−4.55	4.47
Rubbr	−0.167	0.98	0.68	0.17	0.00	2.73	−0.071	1.22*	−0.20	4.24	−4.32	3.99
Txtls	−0.320	1.00	0.77	0.46	−0.10	3.31	−0.124*	1.14*	0.12	3.97	−4.16	4.18
BldMt	−0.138	1.13	0.29	0.22	−0.06	2.37	−0.052	1.14*	−0.14	4.04	−4.60	3.45
Cnstr	−0.311	1.26	0.51	0.13	−0.05	3.74	−0.066	1.11*	0.10	3.20	−3.54	2.81
Steel	−0.530	1.20	0.43	0.40	−0.12	3.84	−0.106	1.16*	0.54	4.47	−3.34	5.12
FabPr	−0.459	1.02	0.65	0.05	−0.05	4.24	−0.130*	1.09*	−0.01	3.79	−4.31	3.61
Mach	−0.101	1.15	0.35	0.07	−0.12	2.49	−0.058	1.16*	0.08	3.91	−4.04	4.36
ElcEq	0.160	1.13	0.06	−0.14	−0.01	3.02	0.057	1.09*	−0.39	3.69	−4.65	2.71
Autos	−0.273	1.13	0.18	0.59	−0.23	3.66	−0.094	1.16*	−0.21	4.60	−5.28	3.45
Aero	−0.034	1.13	0.34	0.15	−0.05	4.01	−0.001	1.12*	−0.47	5.95	−5.81	3.72
Guns	0.044	0.94	0.26	0.34	−0.03	4.75	−0.013	1.16*	−0.95	10.71	−8.36	5.05
Gold	−0.493	0.72	0.65	0.30	0.16	9.71	−0.066	1.11*	0.48	5.89	−4.07	6.34
Ships	−0.278	1.09	0.32	0.22	0.01	4.67	−0.069	1.13*	0.05	4.64	−5.20	4.08
Mines	−0.224	1.03	0.52	0.30	−0.13	4.41	−0.040	1.13*	−0.22	3.38	−4.19	3.04
Coal	−0.339	1.11	0.29	0.21	0.10	6.97	−0.052	1.18*	0.37	5.06	−4.83	5.14
Oil	0.048	0.92	−0.27	0.30	0.07	3.58	0.021	1.17*	0.11	3.35	−3.09	3.82
Util	−0.084	0.69	−0.25	0.47	0.01	2.65	−0.024	1.13*	0.12	3.94	−3.92	4.01
Telcm	0.082	0.79	−0.23	0.16	−0.04	2.92	0.047	1.13*	−0.01	3.77	−3.89	3.96
PerSv	−0.402	1.09	0.54	−0.04	−0.05	3.69	−0.081	1.08*	−0.64	5.91	−5.24	3.22
BusSv	0.164	1.08	0.44	−0.45	−0.01	2.19	0.117*	1.14*	0.07	4.95	−4.79	4.99
Comps	0.121	0.99	0.07	−0.57	−0.14	3.84	0.004	1.11*	−0.11	3.28	−3.73	3.47
Chips	0.076	1.18	0.39	−0.36	−0.10	3.21	0.039	1.21*	−0.18	3.89	−4.35	4.11
LabEq	0.031	1.14	0.47	−0.44	−0.05	3.58	−0.010	1.13*	0.24	4.65	−4.03	4.64
Paper	0.066	1.06	0.01	0.26	−0.09	3.16	−0.015	1.12*	0.13	5.61	−5.69	4.26
Boxes	0.065	0.98	−0.10	0.01	−0.07	3.50	0.011	1.19*	−0.58	4.75	−5.70	4.12
Trans	−0.181	1.10	0.29	0.23	−0.07	2.99	−0.040	1.08*	−0.03	3.94	−4.41	3.64
Whshl	−0.069	1.07	0.39	0.06	−0.03	2.29	−0.108	1.17*	−0.27	6.01	−5.49	5.20
Rtail	0.194	1.02	0.17	−0.06	−0.17	3.03	0.048	1.19*	−0.10	3.72	−4.63	3.43
Meals	0.078	1.09	0.42	−0.06	−0.10	3.46	−0.020	1.13*	−0.17	4.34	−4.95	4.09
Banks	0.102	1.19	0.00	0.43	−0.15	2.87	0.043	1.04	0.10	3.67	−3.31	3.80
Insur	0.114	1.01	−0.04	0.24	−0.08	3.15	0.053	1.04	−0.03	4.10	−4.17	3.70
RlEst	−0.859	1.05	0.99	0.50	−0.04	3.49	−0.276*	1.08*	−0.07	5.71	−4.97	5.46
Fin	0.037	1.19	0.06	0.33	−0.05	2.04	0.052	1.15*	−0.12	4.18	−4.56	3.92
Other	−0.664	1.09	0.44	−0.05	0.02	4.05	−0.098	1.23*	0.23	6.34	−5.51	5.58
Pooled	−0.072	1.02	0.26	0.08	−0.04	3.68	−0.023*	1.15*	−0.13	5.20	−10.90	6.34

The first six columns of this table report the averages of the coefficients of the rolling regressions of the Carhart (1997) Model in Equation 15 for every industry. For each regression, we construct an abnormal return for the month after the estimation window as in Equation 16. The remaining columns show the descriptive statistics of these abnormal returns. To correct for time-varying volatility, we standardize the abnormal return by a division by the residual volatility of the regression model displayed in column 3. An asterisk denotes a significant difference from zero for the mean and a significant divergence from one in case of volatility, both at the 5% significance level.

Table 6: Estimation results for the regime-switching models

a: *Estimates for the distribution parameters*

	CAPM	3F	4F
μ_{N}	0.018	−0.014	−0.009
ω	0.995	1.043	1.058
μ_{C}	0.420	0.520	0.588
ω_{C}	0.588	0.552	0.530
μ_{B}	0.662	0.662	0.716

b: *Means and volatilities for the different regimes*

	CAPM	3F	4F
typical vol. (% p.m.)	4.04	3.71	3.68
mean N (% p.m.)	0.072	−0.050	−0.033
vol N (% p.m.)	4.02	3.87	3.89
mean C (% p.m.)	−8.31	−8.27	−8.62
vol C (% p.m.)	4.70	4.34	4.34
mean B (% p.m.)	2.67	2.46	2.63
vol B (% p.m.)	4.02	3.87	3.89

c: *Estimates for the transition probabilities*

	CAPM	3F	4F
p_{NN}	0.982	0.987	0.987
p_{NC}	0.017	0.013	0.012
p_{NB}	0.0004	0.0005	0.0012
p_{CN}	0.706	0.684	0.721
p_{CC}	0.202	0.226	0.178
p_{CB}	0.091	0.089	0.101
p_{BC}	0.126	0.161	0.185
p_{BB}	0.874	0.839	0.815

This table reports the estimates of the regime switching model and their implications based on the pooled set of standardized abnormal returns. For each industry we first estimate an asset pricing model with a moving window of 120 months as in Equation (15). We use the estimates to construct a series of standardized abnormal returns as in Equation (16). In the second step we use these to estimate a regime switching model for each industry under the assumption that the standardized abnormal returns are industry independent, but have the same parameters. Panel A shows the estimates for the distributions under the different regimes, i.e. normal distributions for the normal and bubble regimes, and a log-normal distribution below $k = -1$ for the crash regime, as specified in Equation (2). The volatility of the normal and bubble regimes are restricted to be equal. Panel B shows the implied means and volatilities for the different regimes for the typical industry multiplied by the idiosyncratic volatility. Panel C reports the estimates for the free transition probabilities in Table 1. We consider the CAPM, the 3-Factor Model of Fama and French (1993) and the 4-Factor Model of Carhart (1997) as asset pricing model.

Table 7: Ergodic Probabilities of Different Regimes

Regime	CAPM	3-Factor Model	4-Factor Model
N	0.93	0.95	0.94
C	0.02	0.02	0.02
B_1	2.52 E^{-3}	2.09 E^{-3}	2.78 E^{-3}
B_2	2.52 E^{-3}	2.09 E^{-3}	2.78 E^{-3}
B_3	2.52 E^{-3}	2.09 E^{-3}	2.78 E^{-3}
B_4	2.52 E^{-3}	2.09 E^{-3}	2.78 E^{-3}
B_5	2.52 E^{-3}	2.09 E^{-3}	2.78 E^{-3}
B_6	2.52 E^{-3}	2.09 E^{-3}	2.78 E^{-3}
B_7	2.52 E^{-3}	2.09 E^{-3}	2.78 E^{-3}
B_8	2.52 E^{-3}	2.09 E^{-3}	2.78 E^{-3}
B_9	2.52 E^{-3}	2.09 E^{-3}	2.78 E^{-3}
B_{10}	2.52 E^{-3}	2.09 E^{-3}	2.78 E^{-3}
B_{11}	2.52 E^{-3}	2.09 E^{-3}	2.78 E^{-3}
B_{nt}	0.02	0.01	0.02

This table presents the ergodic probabilities of the different regimes. "N" indicates the normal regime, "C" indicates the crash regime and the bubble regime is represented by "B_1" to "B_{nt}". B_1 to B_{11} are the transitory bubble regimes. B_{nt} is the non-transitory bubble regime. The probabilities shown in column 1 correspond with the abnormal returns based on the CAPM, column 2 and column 3 correspond with the Fama and French (1993)-Model and Carhart (1997)-Model.

Table 8: Identification of regimes per industry

	CAPM			3F-Model			4F-Model		
industry	normal	crash	bubble	normal	crash	bubble	normal	crash	bubble
Agric	390.4	6.2	6.4	388.2	3.8	10.9	390.2	3.1	9.7
Food	297.9	14.1	91.0	340.4	12.4	50.2	310.6	13.3	79.1
Soda	368.8	12.4	21.7	373.9	12.3	16.8	373.9	12.3	16.7
Beer	356.2	10.7	36.1	381.7	8.6	12.7	368.0	8.6	26.4
Smoke	380.7	8.8	13.5	396.4	5.1	1.5	396.5	4.3	2.2
Toys	385.8	8.5	8.7	390.5	6.8	5.7	389.7	6.4	6.9
Fun	358.7	6.7	37.7	365.0	4.4	33.6	355.8	4.6	42.7
Books	386.7	10.3	6.0	389.2	8.0	5.8	384.1	8.8	10.1
Hshld	360.9	14.6	27.5	367.9	14.9	20.2	369.3	11.6	22.1
Clths	375.9	7.6	19.5	377.9	6.6	18.6	370.2	7.6	25.2
Health	364.1	11.8	27.1	367.3	9.6	26.1	363.4	10.0	29.7
MedEq	368.8	10.4	23.8	372.3	7.7	23.1	359.2	8.9	35.0
Drugs	362.1	13.8	27.2	381.4	8.4	13.3	377.2	6.2	19.5
Chems	375.6	10.4	17.0	392.7	7.2	3.2	394.4	5.6	3.0
Rubbr	362.4	6.9	33.7	386.2	5.2	11.6	384.6	4.9	13.5
Txtls	355.0	9.3	38.8	351.1	7.9	44.0	347.1	7.2	48.7
BldMt	355.9	12.1	35.0	365.8	9.8	27.4	361.5	8.0	33.5
Cnstr	387.1	5.9	10.0	386.6	4.8	11.7	389.5	4.2	9.3
Steel	381.1	11.6	10.3	390.0	9.4	3.6	386.7	8.3	8.0
FabPr	366.0	8.0	28.9	391.9	5.9	5.2	387.2	5.8	10.0
Mach	353.8	11.0	38.2	358.7	9.4	35.0	360.9	6.8	35.3
ElcEq	353.9	10.3	38.7	377.5	7.5	18.0	376.5	7.1	19.5
Autos	330.9	12.8	59.2	333.4	10.5	59.1	338.0	9.2	55.8
Aero	363.4	7.3	32.3	369.3	5.8	27.9	366.8	6.8	29.4
Guns	373.6	8.9	20.4	392.3	4.8	5.9	396.3	5.6	1.1
Gold	367.1	9.0	26.9	378.5	6.2	18.3	367.0	6.9	29.1
Ships	378.4	5.6	19.0	372.7	6.2	24.1	372.6	6.0	24.4
Mines	368.5	9.0	25.5	390.1	6.7	6.2	368.4	5.2	29.4
Coal	326.0	11.1	65.8	312.4	10.2	80.3	322.8	9.1	71.0
Oil	359.5	11.0	32.5	365.3	8.2	29.5	370.0	7.4	25.6
Util	361.5	15.3	26.2	366.1	12.4	24.4	369.1	9.3	24.6
Telcm	380.1	8.3	14.6	379.1	6.9	17.0	384.8	4.7	13.5
PerSv	372.2	12.8	18.0	377.0	11.3	14.7	378.0	10.1	14.9
BusSv	362.6	11.3	29.1	363.2	9.8	30.0	349.2	9.2	44.6
Comps	384.3	8.0	10.7	390.3	6.9	5.9	393.5	5.2	4.3
Chips	378.2	9.7	15.1	392.5	5.3	5.2	390.5	4.4	8.1
LabEq	389.9	9.2	3.9	384.8	5.1	13.1	384.2	3.8	15.0
Paper	392.0	10.4	0.6	393.8	8.6	0.5	395.1	7.4	0.6
Boxes	340.8	14.0	48.3	358.9	9.2	34.9	356.6	8.2	38.1
Trans	384.8	9.5	8.7	366.3	8.6	28.1	362.6	10.2	30.2
Whshl	368.5	7.5	27.0	378.7	6.6	17.7	394.6	6.6	1.9
Rtail	389.7	9.1	4.2	387.0	8.7	7.3	392.5	6.3	4.2
Meals	367.9	8.9	26.2	394.7	6.1	2.2	388.2	6.1	8.7
Banks	367.8	7.8	27.3	374.2	5.0	23.8	383.6	4.4	15.0
Insur	363.3	13.2	26.5	372.0	9.5	21.4	357.8	9.8	35.4
RlEst	369.0	10.9	23.1	370.0	10.3	22.6	362.3	10.2	30.5
Fin	365.5	8.7	28.8	380.6	11.3	11.0	381.9	10.9	10.2
Other	376.8	7.0	19.2	377.1	5.7	20.2	365.5	4.4	33.1
Pooled	91.1%	2.5%	6.4%	93.1%	2.0%	4.9%	92.5%	1.8%	5.7%

This table shows the identification of the different regimes per industry. For each industry we report the sum of the smoothed inference probabilities over time for the normal regime and the crash regime. For the bubble regime we gather the smoothed inference probabilities for the transitory bubble states $B_1, \ldots, B_{11}$ and the non-transitory bubble state B_{nt}, and sum these also over time. For the pooled set of industries we do the same calculations, and report the results as a proportion of the total number of observations. We show the identification for each risk factor model: the CAPM, the 3-Factor Model of Fama and French (1993) and the 4-Factor Model of Carhart (1997).

Table 9: Parameter Estimates for a Regime Switching Model for the Market

(a) *distribution parameters*

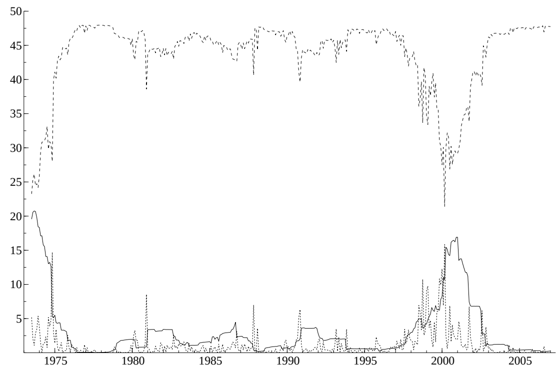
Regime	Normal	Crash
μ	0.234	0.197
σ	0.879	0.664
mean (in %)	1.07	-11.49
volatility (in %)	4.01	5.15
LPM_2	0.023	3.57

(b) *transition matrix*

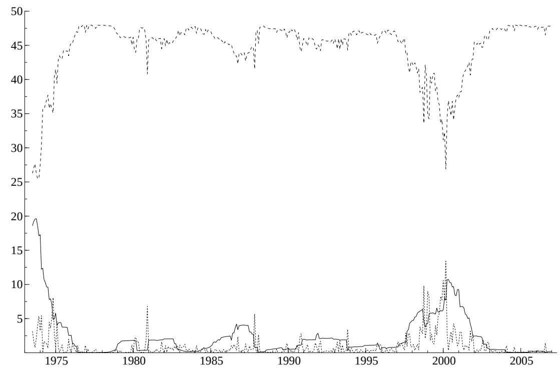
	N	C
N	0.967	0.717
C	0.033	0.283

This table reports the parameter estimates for a regime switching model for the market portfolio. The market is proxied by the CRSP All Shares Index from July 1963 until December 2006. The model specification is given by Equations (25)-(26). The series u_t^{m} is constructed by dividing each excess market return r_t^{m} by the volatility calculated over the prior 120 months. The cut-off value for the crash regime has value $k = -1$. For each regime we report the resulting mean, volatility and second order lower partial moment below -1 for the market, based on the average market volatility of 4.56% per month.

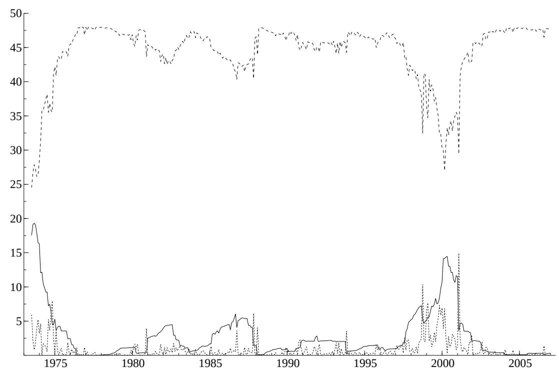
Figure 1: Identification of regimes over time



(a) CAPM



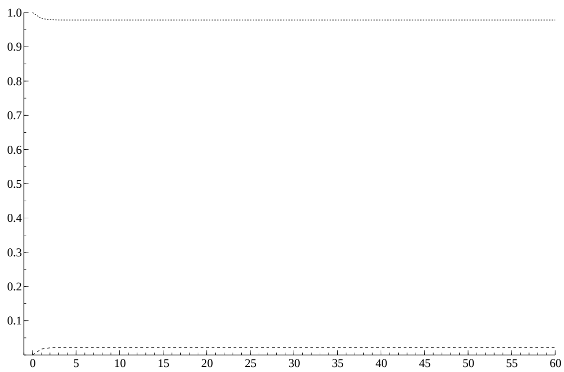
(b) 3-Factor Model



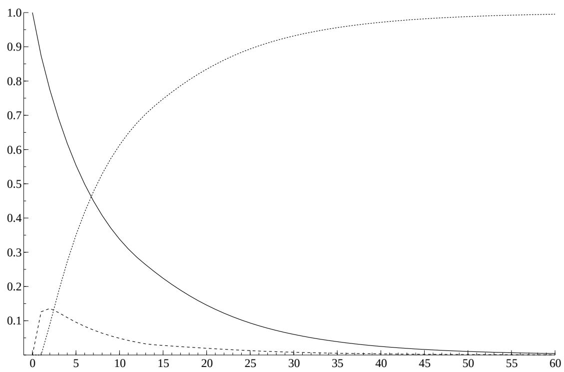
(c) 4-Factor Model

This figure shows the identification of the different regimes over time. A dashed line corresponds with the normal regime, a dotted line with the crash regime and a solid line with the bubble regime. To calculate the values for the normal regime we sum the smoothed inference probabilities at each point in time over the 48 industries. We follow the same procedure for the crash regime. For the bubble regime we also take the smoothed inference probabilities for the transitory bubble states $B_1, \ldots B_{11}$ and the non-transitory bubble state B_{nt} together. We show the identification for each risk factor models: (a): the CAPM, (b) the 3-Factor Model of Fama and French (1993) and (c) the 4-Factor Model of Carhart (1997).

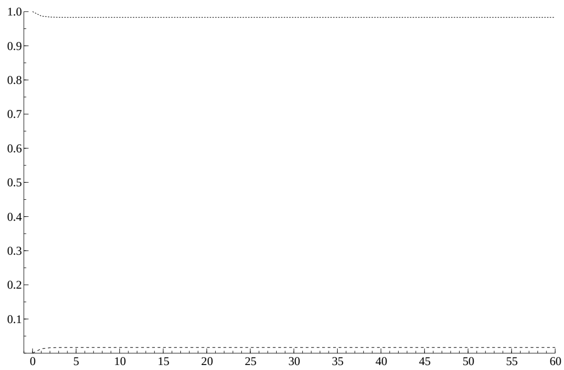
Figure 2: Forecasts of Regime Probabilities



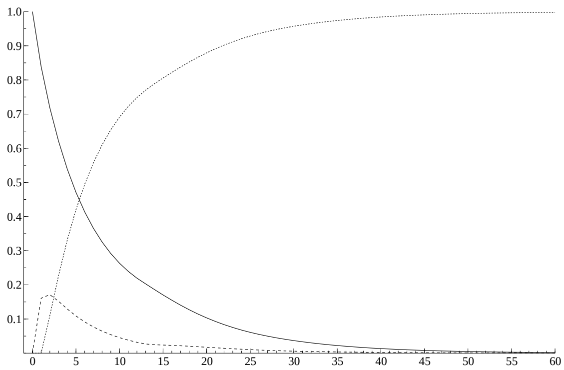
(a) non-bubbly asset, CAPM



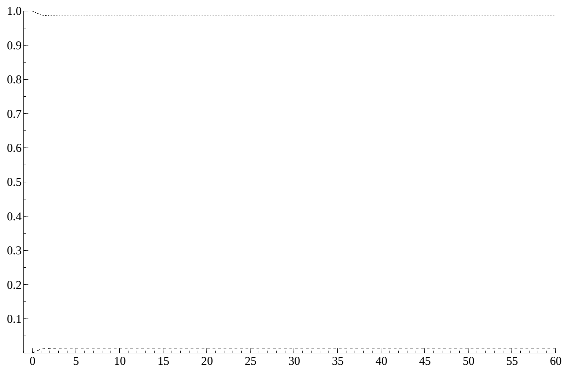
(b) bubbly asset, CAPM



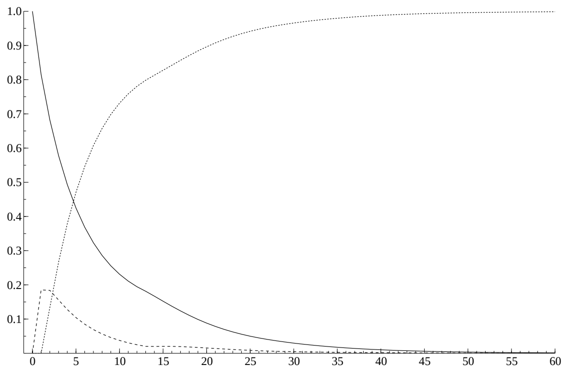
(c) non-bubbly asset, 3-Factor Model



(d) bubbly asset, 3-Factor Model



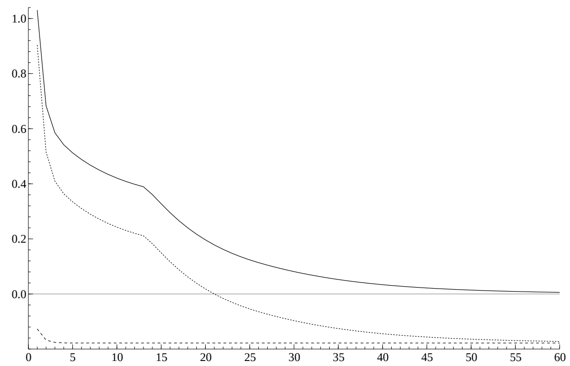
(e) non-bubbly asset, 4-Factor Model



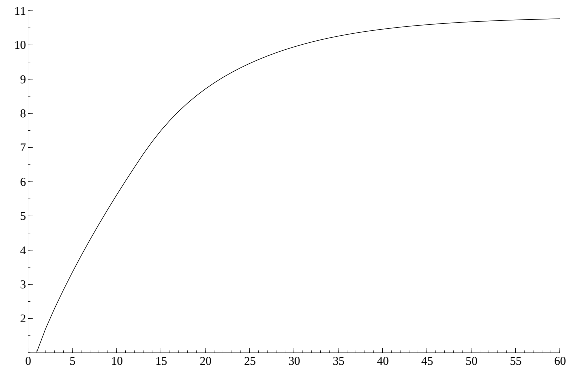
(f) bubbly asset, 4-Factor Model

This figure shows the forecasts of the probabilities with which a regime prevails for different horizons. Subfigures (a), (c) and (e) show the forecast probabilities of the crash (dashed) and normal regime (dotted) for the non-bubbly asset, assuming that the return process was at $t = 0$ in the normal regime ($\phi_{t|t}(\tilde{S}_t = \tilde{\mathrm{N}}) = 1$). In subfigures (b), (d) and (f), we plot the probabilities for regimes of the bubbly asset, which are the bubble regime (solid), the crash regime (dashed) and the regime $\tilde{S}$ (dotted). We assume that the return process was at $t = 0$ in the non-transitory bubble regime ($\phi_{t|t}(S_t^* = \mathrm{B_{nt}}) = 1$). The forecast probabilities for the bubble regime are the sum of probabilities for the transitory and non-transitory bubble states. The subfigures correspond with the different risk factor models: the CAPM, the 3-Factor Model of Fama and French (1993) and the 4-Factor Model of Carhart (1997). The transition probabilities that govern the forecast probabilities can be found in Table 6.

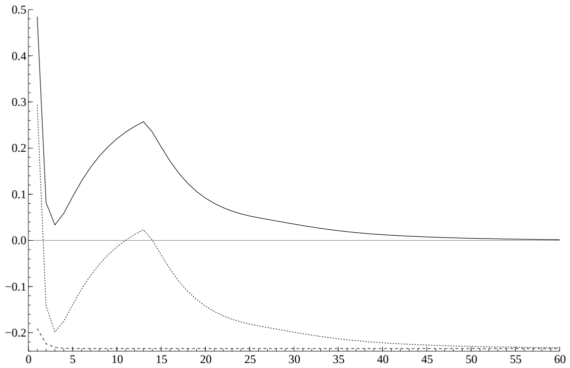
Figure 3: Abnormal Returns Forecasts



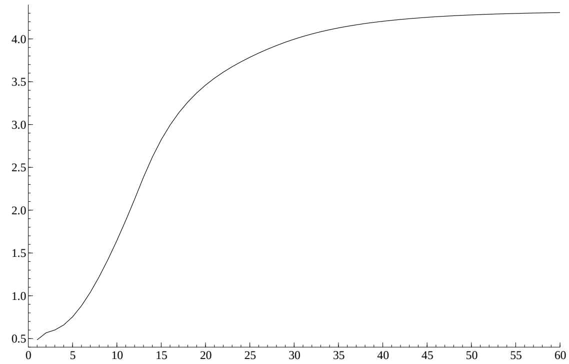
(a) CAPM, returns per month



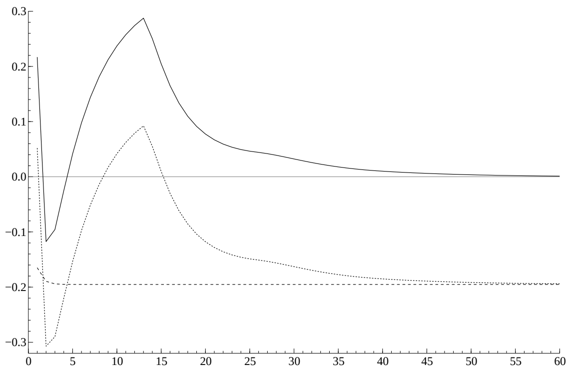
(b) CAPM, cumulative returns



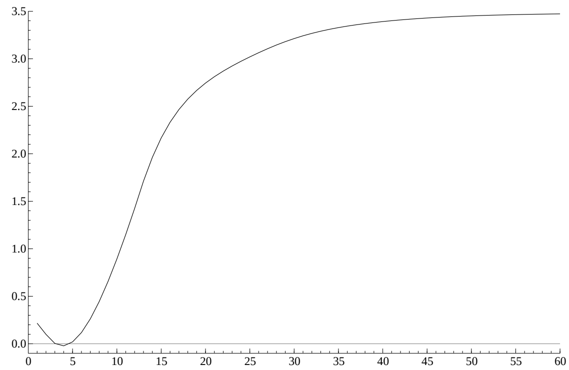
(c) 3-Factor Model, returns per month



(d) 3-Factor Model, cumulative returns



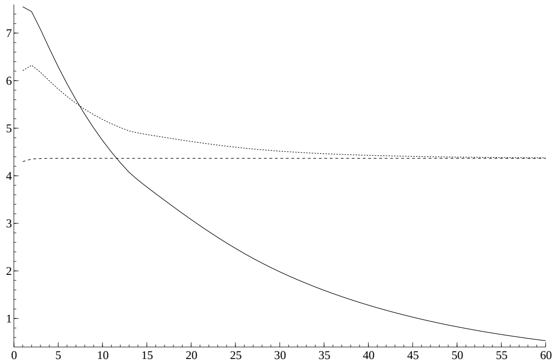
(e) 4-Factor Model, returns per month



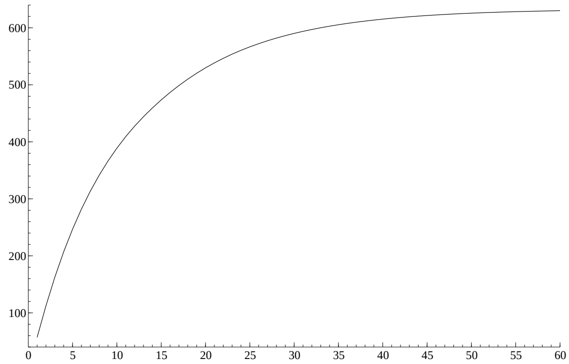
(f) 4-Factor Model, cumulative returns

This figure shows the abnormal return forecasts (in %) as a function of the forecast horizon (in months). They forecasts are based on estimates for the regime switching models in Table 6 and multiplied by their idiosyncratic return volatility. The subfigures correspond with the different risk factor models: (a,b): the CAPM, (c,d) the 3-Factor model of Fama and French (1993) and (e,f) the 4-Factor model of Carhart (1997). The dotted line shows the expected abnormal returns for the bubbly asset, when the investor is 100% certain that the typical industry is in the non-transitory bubble regime when he makes the forecast at time t (i.e., $\phi_{t|t}(S_t^* = \mathrm{B_{nt}}) = 1$). The dashed line indicates the expected returns for the non-bubbly asset, which is in the normal regime with probability 1 at time t (i.e., $\phi_{t|t}(\tilde{S}_t = \tilde{\mathrm{N}}) = 1$). The solid line gives the expected returns for a position of one unit long in the bubbly asset and one unit short in the non-bubbly asset. The subfigures on the left show the predictions per month. The subfigures on the right show the forecast of the cumulative return for the long-short position for each month.

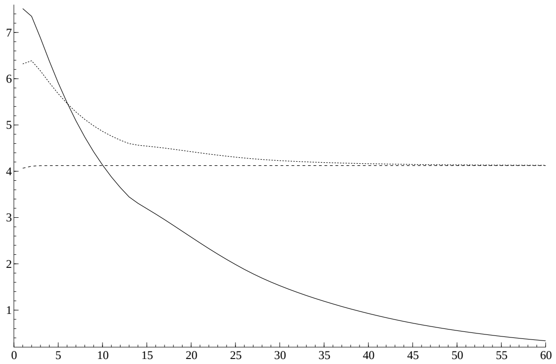
Figure 4: Forecast of Abnormal Return Volatility



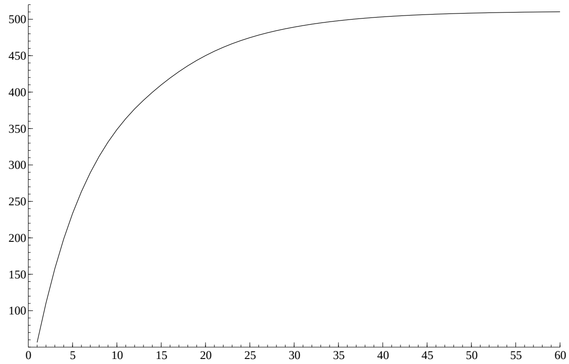
(a) CAPM, volatility per month



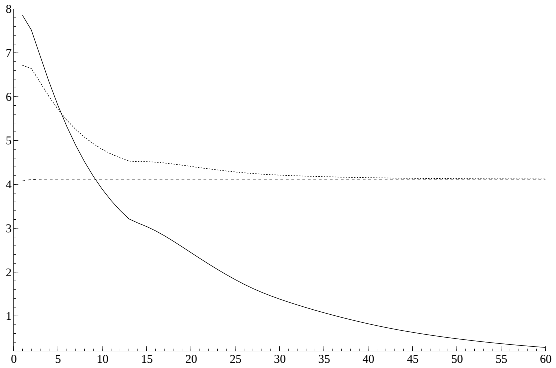
(b) CAPM, cumulative variance



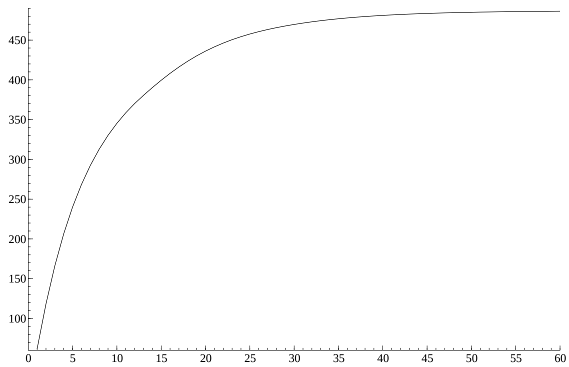
(c) 3-Factor Model, volatility per month



(d) 3-Factor Model, cumulative variance



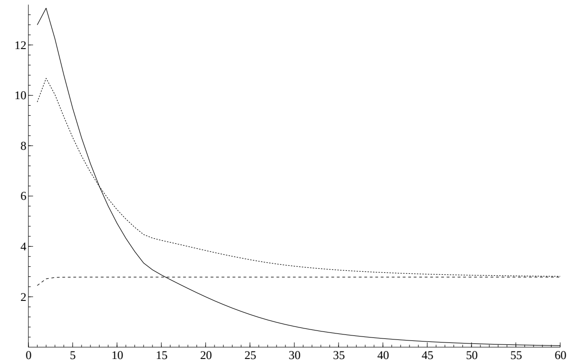
(e) 4-Factor Model, volatility per month



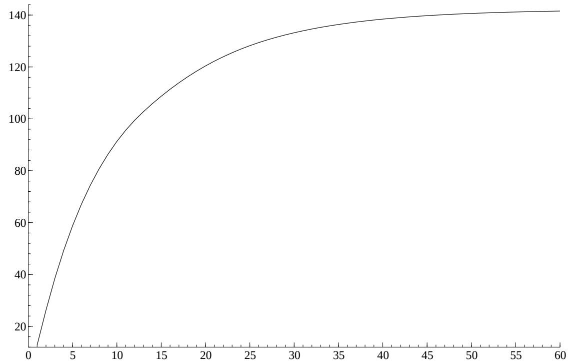
(f) 4-Factor Model, cumulative variance

This figure shows the forecasts of the volatility of abnormal returns as a function of the forecast horizon (in months). The forecasts are based on estimates for the regime switching models in Table 6 and multiplied by their idiosyncratic return volatility. The subfigures correspond with the different risk factor models: (a,b): the CAPM, (c,d) the 3-Factor Model of Fama and French (1993) and (e,f) the 4-Factor Model of Carhart (1997). The dotted lines show the forecasted volatility of the bubbly asset, when the investor is 100% certain that the typical industry is in the non-transitory bubble regime when he makes the forecast at time t (i.e., $\phi_{t|t}(S_t^* = \mathrm{B}_{\mathrm{nt}}) = 1$). The dashed line indicates the volatility for the non-bubbly asset, which is in the normal regime with probability 1 at time t (i.e., $\phi_{t|t}(\tilde{S}_t = \tilde{\mathrm{N}}) = 1$). The solid line gives the volatility forecast for a position of one unit long in the bubbly asset and one unit short in the non-bubbly asset. The subfigures on the left show the predictions per month. The subfigures on the right show the forecast of the cumulative variance for the long-short position for each month.

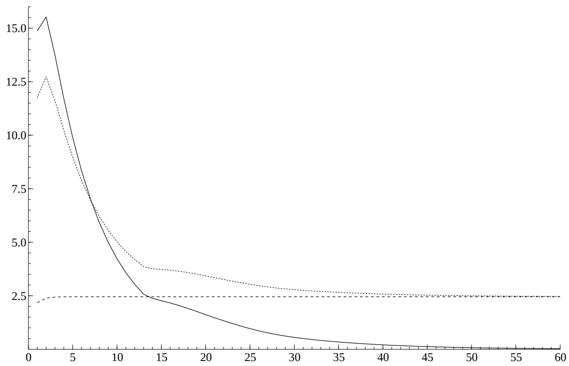
Figure 5: LPM Forecast for Positive Weight in Bubbly Asset and Negative Weight in Normal Asset



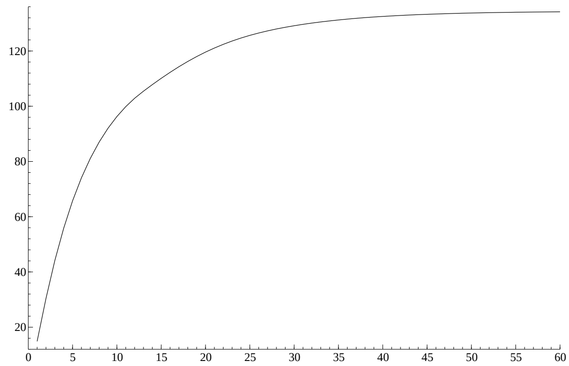
(a) CAPM, LPM per month



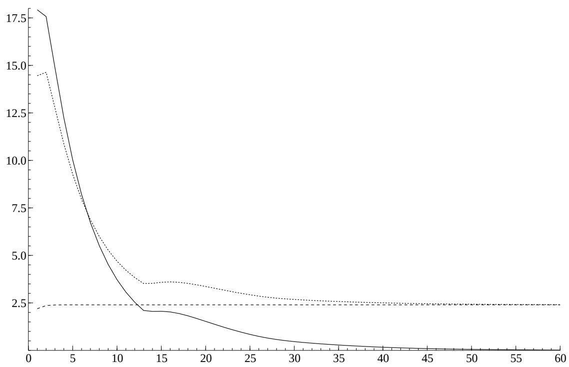
(b) CAPM, cumulative LPM



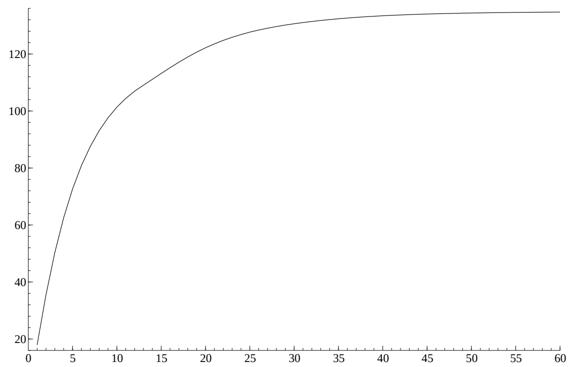
(c) 3-Factor Model, LPM per month



(d) 3-Factor Model, cumulative LPM



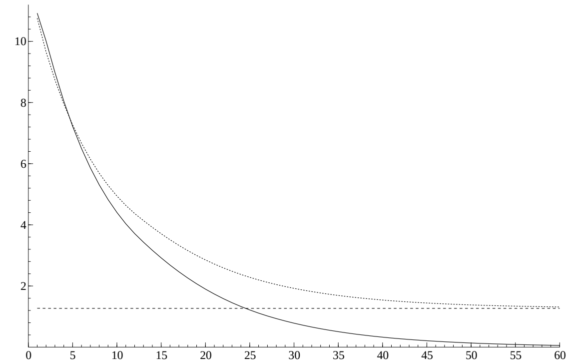
(e) 4-Factor Model, LPM per month



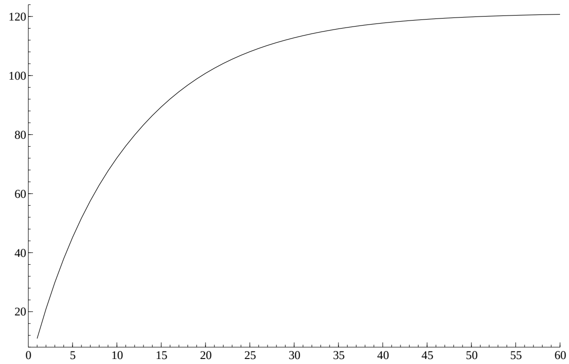
(f) 4-Factor Model, cumulative LPM

This figure shows the forecasts of the second order lower partial moment below $k = -\sigma$ of the abnormal returns (in %) as a function of the forecast horizon (in months) for a positive weight in Equation (14). The forecasts are based on estimates for the regime switching models in Table 6 and multiplied by their idiosyncratic volatility σ. They are for a portfolio that is one unit long in the bubbly asset and one unit short in the normal asset. We assume that the abnormal returns of the normal asset were in the normal regime at $t = 0$ (i.e., $\phi_{t|t}(\tilde{S}_t = \tilde{\mathrm{N}}) = 1$) and that the returns of the bubbly asset were in the non-transitory bubble state (i.e., $\phi_{t|t}(S_t^* = \mathrm{B_{nt}}) = 1$). The subfigures correspond with the different risk factor models: (a,b): the CAPM, (c,d) the 3-Factor model of Fama and French (1993) and (e,f) the 4-Factor model of Carhart (1997). The subfigures on the left show the predictions per month. The subfigures on the right show the forecast of the cumulative lower partial moments for the long-short position for each month.

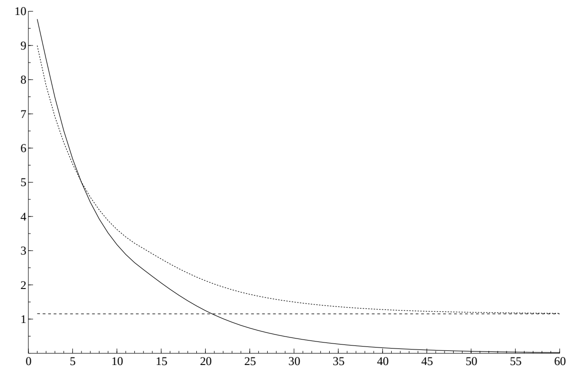
Figure 6: LPM Forecast for Positive Weight in Normal Asset and Negative Weight in Bubbly Asset



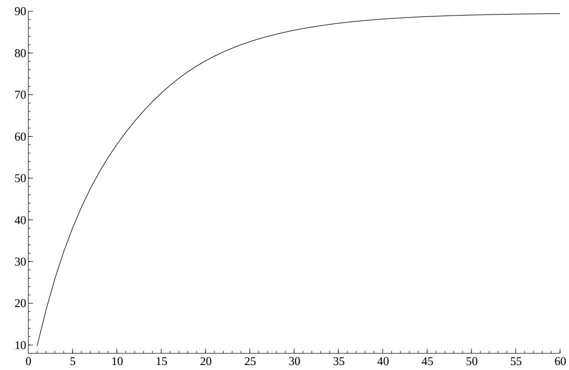
(a) CAPM, UPM per month



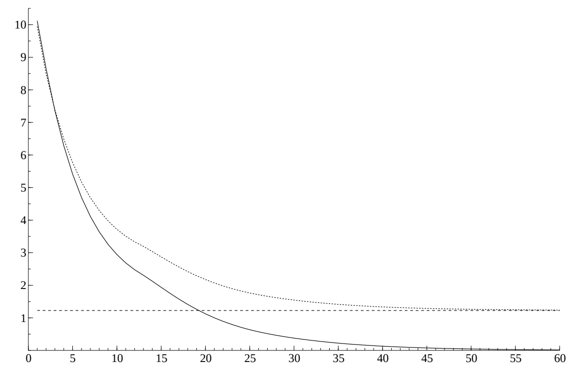
(b) CAPM, cumulative UPM



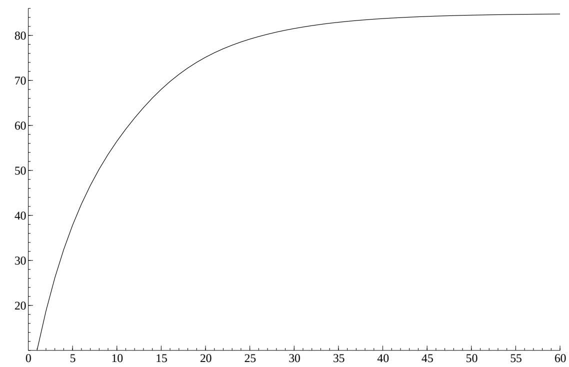
(c) 3-Factor Model, UPM per month



(d) 3-Factor Model, cumulative UPM



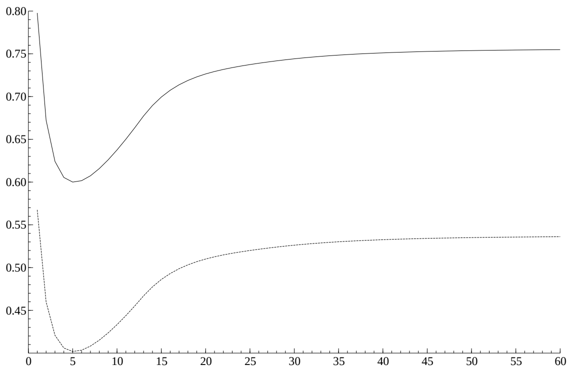
(e) 4-Factor Model, UPM per month



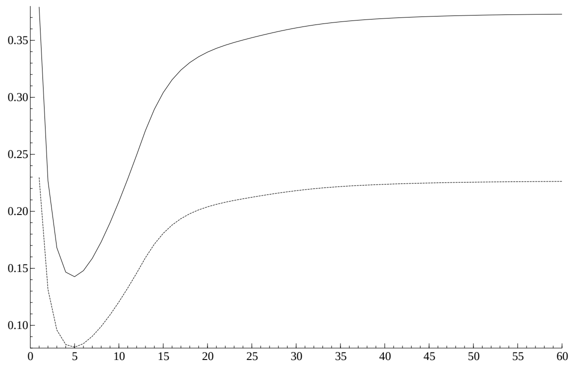
(f) 4-Factor Model, cumulative UPM

This figure shows the forecasts of the second order lower partial moment below $k = -\sigma$ of the abnormal returns (in %) as a function of the forecast horizon (in months) for a negative weight in Equation (14). The forecasts are based on estimates for the regime switching models in Table 6 and multiplied by their idiosyncratic volatility σ. They are for a portfolio that is one unit short in the bubbly asset and one unit long in the normal asset. We assume that the abnormal returns of the normal asset were in the normal regime at $t = 0$ (i.e., $\phi_{t|t}(\tilde{S}_t = \tilde{\mathrm{N}}) = 1$) and that the returns of the bubbly asset were in the non-transitory bubble state (i.e., $\phi_{t|t}(S^*_t = \mathrm{B}_{\mathrm{nt}}) = 1$). The subfigures correspond with the different risk factor models: (a,b): the CAPM, (c,d) the 3-Factor model of Fama and French (1993) and (e,f) the 4-Factor model of Carhart (1997). The subfigures on the left show the predictions per month. The subfigures on the right show the forecast of the cumulative lower partial moments for the long-short position for each month.

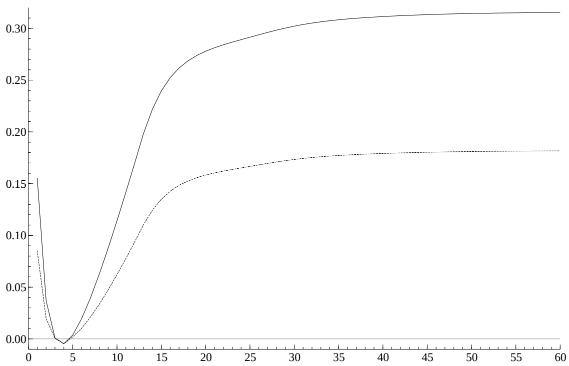
Figure 7: Optimal long-short positions



(a) CAPM



(b) 3-Factor Model



(c) 4-Factor Model

This figure shows the optimal weight invested in the bubbly asset and the non-bubbly asset as a function of the investment horizon (in months) as a fraction of wealth. The optimal portfolios are calculated as given in Equation (14). The solid line corresponds with an investor who is averse to variance risk, and the dashed line with an investor who is averse to downside risk. The risk aversion coefficients γ_1 and γ_2 are calibrated to the market (see Appendix B).