



Measuring Credit Rationing of Dutch Firms

This background document to the Policy Brief ‘Dutch SME-bank financing in a European perspective’ reports on a firmlevel microsimulation of credit rationing due to bankruptcy costs.

We estimate the amount of credit rationing that Dutch firms potentially faced in 2007-2016 based on CBS firmlevel data and relying on many simplifying assumptions.

Further analysis is required to put our results into perspective.

We obtain that the smallest 20% of firms might have experienced more credit rationing than medium-sized firms.

Our simulation study is novel, and shows that researching credit rationing with microdata is feasible and policy-relevant.

1 Introduction

This background document is associated to the CPB Policy brief on Dutch SME-bank financing in a European perspective. It explains in detail what we do in a microsimulation model that is described in a box on credit rationing on page 14.

It goes without saying that there are many factors that potentially play a big role when it comes to financing existing and new enterprises. However, a comprehensive empirical overview of the importance of various factors is lacking. In this background document we focus on one particular factor: credit rationing due to bankruptcy costs, which features prominently in the academic literature (e.g., the respective theory is commonly used to explain the popularity of simple debt contracts). We make first steps in assessing, using company-level data from CBS, whether this kind of credit rationing is economically significant in the Netherlands and whether there is a difference between smaller and larger firms in this regard. In doing so, we start with one piece of the puzzle and further analysis is required to put our results into perspective. The analysis is done without prejudice to other factors. While we assume, for example, that fixed costs of issuing credit and common macroeconomic risks are negligible—which allows us to focus on the credit rationing issue—we do not say that those factors are unimportant in practice.

A creditor charges a higher rate on a riskier loan as a compensation for the extra risk. As the risk increases, so does the interest rate, but a higher interest also makes it more likely that the debtor falls short on his obligations and declares bankruptcy. While a creditor normally recovers a part of his loan in case of bankruptcy, there are direct and indirect bankruptcy costs involved, e.g. losses due to asset specificity. In the theoretical literature, bankruptcy costs are viewed as monitoring costs that arise due to asymmetric information. In case of bankruptcy, the firm has better information about the remaining value of its assets than the bank. If the bank wants to assess that value, monitoring costs need to be incurred. Due to such monitoring or bankruptcy costs expected profits initially increase with higher interest rate but then begin to fall as higher interest rate increases the chance of bankruptcy and the associated losses to the bank. In general, if the risks are large enough, there is no interest rate that a creditor can charge and still obtain a profit. If that is the case, no credit is extended, while the net present value of the project may well be positive (i.e., the project may well be socially desirable). This phenomenon is what we refer to as credit rationing in this background document.

To compute the amount of credit rationing we need, firstly, to estimate risks on a firm-by-firm basis, and, secondly, to estimate the bankruptcy costs. We estimate risks using a simple non-parametric approach. We split firms in clusters of similar size and age, and assume that firms within the same cluster face similar risks. We can then use the observed distribution of returns for firms in a given cluster to assess the risks. The risks assessment model can be made more elaborate, obviously, but in our opinion the current approach is a good enough first approximation. No estimates of bankruptcy costs are readily available. Therefore, we simply compute credit rationing for three bankruptcy scenarios: 1) up to 100% of initial assets are recoverable and can be used to pay back the principal and the interest; 2) up to 50% of initial assets are recoverable; 3) nothing is recoverable. Some of the results are sensitive to scenarios, so there is clear scope for improvement regarding our modelling of bankruptcy costs. Unfortunately, very limited data regarding indirect bankruptcy costs is available.

Under all scenarios our simulation suggests that the smallest 20% of Dutch firms (we exclude sole proprietorships) experienced larger credit rationing in 2016 than they did in 2007, while medium-sized and large firms experienced similar or possibly lower level of credit rationing in 2016 than they did in 2007. Seemingly, the financial crisis had a disproportionate impact on smaller firms, because lower interest rates

did not mitigate the impact of the crisis. Importantly, we observe similar patterns for the middle 20% of firms, when ordered by size, and the top 20%. It is the bottom 20% that stands out. This result highlights substantial heterogeneity within the SME sector.

To the best of our knowledge, ours is a first simulation study that assesses credit rationing on a firm level. (Macro-level studies include Carlstrom and Fuerst 1997, 1998.) Many aspects of our study can be improved, so as to deliver higher precision estimates of credit rationing. In particular, the absolute level of credit rationing remains unclear without further research on bankruptcy costs. Additionally, the underlying dataset can be expanded to cover additional countries besides the Netherlands. However, our study highlights that estimating credit rationing is feasible, and that the amount of credit rationing that firms experience is potentially significant.

2 Simulation

2.1 Data

The following CBS datasets are used: NFO (balance sheet and profit and loss data on a firm-year basis), ABR (administrative firm-level data).

Table 2.1 lists CBS variables that are used in the simulation, along with their corresponding names that are used in this documentation.

Table 2.1

Local name	CBS Code	Label	Dataset.Table
A	B37	Total assets	NFO
E^e	B51_B57	Equity	NFO
E^p	B63	Provisions	NFO
L^l	B65_B73	Long-term loans	NFO
L^i	C53, F9, F10	Internal LT loans	8358SFGO*, 8358SFKO*
L^s	B75_B87	Short-term loans	NFO
R	R12	Interest expense	NFO
P	R20	Net profit	NFO
τ	VEP_DATUMONTSTAANTOEPASSING	Founding date	ABR.CBS_persoon

Let S_i^t denote the age of firm i at time t . We compute $S_i^t = t - \tau_i$, where τ_i is the founding date of firm i . The founding date comes from table 'ABR.CBS_persoon', which is joined with NFO on 'Fiscaal Identificatienummer' for small companies (SFKO) and on 'Ondernemingsidentificatie-nummer' for large companies (SFGO); the latter join is done via 'ABR.OG_persoon'. There has been a break in the ABR series in 2005, so that the ABR data beginning in 2006 gives $\max\{\tau, 2005\}$ as the founding date. To correct for this censoring, we use founding dates from the 2005 ABR data whenever available, and from the current ABR data otherwise.

Firms with different Chamber of Commerce (KVK) numbers can belong to a single business (single ownership structure). CBS identifies such firm groups with OND_ID (business id). Prior to any analysis, we aggregate the data by OND_ID. The aggregation is done for two reasons: 1) some administrative data can be joined only on OND_ID level, 2) it is reasonable to assume a bank looks at the financial situation of a business as a whole when considering a loan application, rather than at the situation of the specific legal entity that has applied for the loan. If founding dates vary across aggregated firms, then the earliest date is chosen as the business founding date.

For large firms, CBS collects consolidated financial data via a questionnaire, but the consolidation is done at the country level. The balance sheets of parent or daughter firms located outside the Netherlands do not get consolidated. Regarding our data, this rule implies that both long-term and short-term loans can include intra-business loans from related firms abroad. We have obtained custom data from CBS on the amount of intra-business loans, for large as well as for small companies. Let us denote these loans as L^i . These loans are small for all but very large companies, however for the latter group they can be substantial and we make use of this additional information in the simulation (we come back to this point later in the text).

Provisions constitute a part of equity reserved for future (certain) losses. Ideally, we would a) count provisions as equity but, at the same time, we would b) add provisions to future profit risks. However, the maturity breakdown of provisions is not available, i.e. it is not clear whether the losses will need to be expended in 1 year or, say, 5 years. It is also not clear if losses due to interest payments, which we model explicitly and separately, are already in the provisions. Hence, to avoid possible double counting, and also because it is not clear how to split provisions over time, we omit b) and only do a), that is we count provisions together with equity for the purpose of the current simulation. Namely, let equity

$$E = E^e + E^p.$$

(Whether provisions are treated as equity or not has some but limited impact on the results.)

NFO does not provide a breakdown of interest expenses into those associated with long-term loans, short-term loans, and accounts payable. We assume accounts payable accrue no interest, and, in first approximation, split the rest proportionally. Namely, let the interest expense on long-term loans be

$$R^l = R \frac{L^l}{L^l + L^s}.$$

(In the aggregate data, interest on long-term loans is higher than interest on short-term loans. However, the difference between the rates is only substantial for loans above 1 million euros. A more accurate split, based on the aggregate numbers, is possible but it should not affect most companies.)

2.2 Risk Modelling

To assess whether any given firm can obtain a loan at a bank, we need to assess its profit risks. The simplest yet robust way of doing so is to cluster firms into risk groups, and then take the cross-section distribution of profits within each group as a non-parametric risk estimator. The risk assessment can be made increasingly more sophisticated, and such possible extensions are something to keep in mind.

We define risk groups by firm size and age. Firstly, consider firm size. Let $i = 0, \dots, N - 1$ denote a particular firm, let M be the desired number of size groups ($M = 5$ at the moment), and let the firms be sorted by total assets so that $A_i < A_j$ whenever $i < j$, then

$$g_i^a = \lfloor i/(N/M) \rfloor.$$

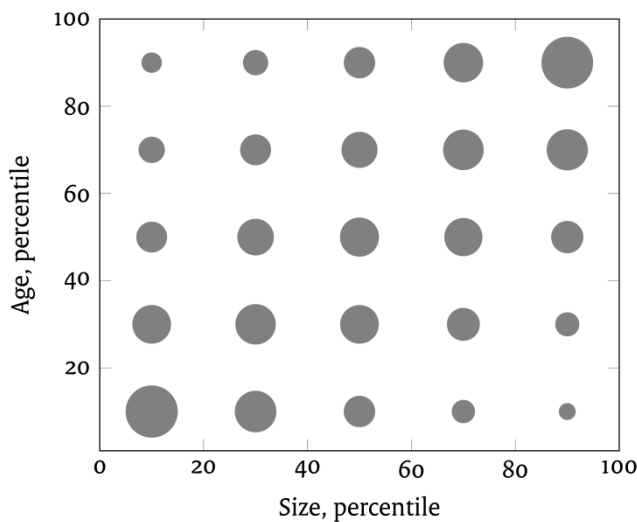
That is, with $M = 5$, smallest 20% of firms fall in group 0, second smallest 20% fall in group 1, and so on. Analogously, age groups g_i^s are constructed. Then combined risk groups are given by the intersection of size and age groups:

$$g_i = M g_i^a + g_i^s.$$

Note that the population is split into groups not on the basis of the absolute values of size and age, but on the basis of their percentiles. This manner of grouping avoids dealing with particular distributions of size and age within the population. An alternative would be a local simulation (akin to a local regression), performed for every single firm. This alternative approach can be more robust, but it is also more complex and, importantly, the running time of the simulation is likely to increase substantially.

The size of the resulting groups is presented in Figure 2.1 (an actual scatter plot cannot be shown due to CBS export limitations). While the clusters, as defined, vary substantially in size, the smallest clusters still have enough observations for simulating the amount of credit rationing in those clusters.

Figure 2.1 Risk Clusters, 2016



Notes: the area of each circle is proportional to the number of firms in the respective risk cluster; the smallest cluster has 1,913 firms, the largest—20,609.

Later on, we will be simulating the dynamics of firms' balance sheets. It is important to point out that, irrespective of whether a firm grows or shrinks, and irrespective of it becoming older, we say that the firm stays within the same risk group. This is a simplifying assumption. Relaxing this assumption would mean that every new period the firm might end up in a new risk group.

The simulation focuses on the long-term debt of firms, and the respective interest payments will be modelled explicitly. Other interest payments as well as taxes are assumed to be proportional to total assets (balance sheet dynamics can be modelled more accurately, but potentially at a big computational cost). Accordingly, we define return on assets as

$$\pi = \frac{P + R^l}{A},$$

where $P + R^l$ is earnings after short-term interest and taxes, but before long-term interest.

Let π_i^t be the ROA of firm i in period t , where $t = 0$ denotes the current period. We assume that π_i^t is a random variable whose distribution depends only on the risk group g_i . The value of π_i^0 is known from the data and so, if there are sufficiently many firms in group g_i , we can estimate the corresponding distribution. We do

so non-parametrically, using sampling with replacement to evaluate the expectations under these distributions.

Say, we want to estimate $\mathbb{E}(f(\pi_i^1, \dots, \pi_i^T))$, for some $T > 0$. We use

$$\mathbb{E}(f(\pi_i^1, \dots, \pi_i^T)) = \frac{1}{K} \sum_{k=0}^{K-1} f(\pi_{\eta(k,i,1)}^0, \dots, \pi_{\eta(k,i,T)}^0),$$

where each $\eta(k, i, t)$ is an independent draw from a random variable with discrete uniform distribution over the set $\{j: g_i = g_j\}$. That is, the current distribution of relative profits is used to forecast future growth (or decline). We use $K = 10000$ (with $K = 1000$ the results vary too much from run to run).

2.3 Loan Modelling

We ask the following question: can a firm receive an additional marginal credit from a bank? We assume there are no transaction costs associated with issuing a loan, that is, even a one euro loan will be issued if the repayment chances are good enough. This scenario is conservative: if we document the portion of smaller firms having difficulties obtaining credit under this scenario, then in practice this portion is likely to be only larger due to (i) fixed costs of issuing a loan, and due to (ii) the size of the new loan likely being larger than marginal.

If we want to move away from the conservative scenario towards a more realistic scenario, we need figures on the sizes of new loans that firms typically need, as well as on the fixed costs of issuing a loan.

A firm can receive an extra loan if it is likely to repay that extra loan as well as any outstanding loans it already has. Therefore, to assess whether a firm can receive an extra marginal loan, we essentially need to check whether the firm can refinance (roll over) its existing loans. This modelling assumption follows Gale and Hellwig (1985). Furthermore, we focus exclusively on long-term loans. We assume that short-term loans grow or shrink together with the size of the business, and that the firms do not experience any credit rationing in obtaining necessary short-term loans.

The credit rationing estimation that we make is based on the presence of asymmetric information between the creditor and the debtor, namely we assume that the creditor cannot costlessly observe the actual profits of the debtor. We discuss this point in more detail a few paragraphs later. What we would like to do now, is to draw a distinction between external and internal creditors. Arguably, internal (intra-business) credits are subject to much lower if not completely negligible information frictions, and thus should not contribute to credit rationing. We therefore exclude internal loans from total long-term loans when assessing whether a firm can refinance its outstanding long-term obligations. That is, we focus on $L = L^l - L^i$.

So, suppose a firm wants to refinance its existing long-term loans, excluding internal loans, for the next T years. (There is some evidence that long-term loans to firms are, on median, 7 years long, see Davydenko and Franks 2008. We therefore set $T = 7$.) Let r be the firm-specific interest rate that a bank charges on refinancing. We assume the interest is paid only once, at maturity. (The interest is computed implicitly, so it will be larger with longer maturity.) This assumption simplifies the analysis and presentation. Also, if we assume instead that the interest is paid annually, we need to make further deliberations about what happens if the firm runs into negative equity after paying the interest. Assume further that total assets increase or decrease by the amount of net profits the firm earns. That is, new earnings are not leveraged. Then

$$A^t = A^{t-1} + \pi^t A^{t-1}. \quad (\text{Error! Bookmark not defined.1})$$

If $A^t \leq 0$, then the firm goes bankrupt. In principle, we allow the firm to be insolvent before loan maturity. (This assumption is uncommon in the theoretical literature. However, assuming otherwise contradicts the data, as there are many firms in our sample that continue to operate while being insolvent as they have positive cash flow and are expected to be solvent in the future due to growth.) At maturity, however, the firm needs to be solvent:

$$A^T \geq A^0 - E^0 + rL.$$

That is, period T assets of the firm, A^T , should be large enough to cover period T liabilities, which are composed of the original liabilities, $A^0 - E^0$, plus the accrued interest on the long-term loan, rL . Short term interest, as well as tax expenses, enter the equation implicitly through A^T : these expenses decrease return on assets π , thus causing slower growth and smaller final assets A^T . If the firm is solvent, we assume it is also liquid enough to be able to pay back the loan. If the firm is insolvent at maturity, we assume it declares bankruptcy.

In assuming that a firm can be insolvent before maturity, and in assuming the firm is always liquid enough to be able to pay back the loan, we stay on the conservative side. That is, any credit rationing that we find is likely to be larger in practice, as banks will also assess the liquidity position of a firm before issuing a loan.

2.4 Bankruptcy Costs

We assume banks face monitoring costs when assessing ex-post performance of firms. Monitoring costs is a common framework for explaining and analyzing debt contracts, some of the earliest literature include Townsend (1979); Gale and Hellwig (1985); Williamson (1986, 1987). Recent discussions and overview of the later literature can be found in, e.g., Monnet and Quintin (2005); Antinolfi and Carli (2015); Kjenstad et al. (2015). To the best of our knowledge, our microsimulation is the first microsimulation that estimates potential credit rationing on a per-firm basis using a structural model. In contrast, credit rationing models have been applied to aggregate data. Calibrated macroeconomic models that allow for proportional agency costs with regard to debt contracts are closely related to our analysis. See Carlstrom and Fuerst (1997, 1998).

Within the costly monitoring framework, if a firm is able to pay back the loan, it does so, and no monitoring needs to occur. If a firm cannot pay back the loan, it declares bankruptcy, and the bank performs monitoring to assess how much the firm has earned and how much of the loan can be recovered. We use monitoring costs and bankruptcy costs as synonyms.

Bankruptcy costs can be split into direct costs (trustee's fees, etc.) and indirect costs (lower resale value of assets in comparison to their book value, for instance due to asset specificity). The banks in the Netherlands are well positioned in terms of creditors' priority. We therefore make a conservative assumption that the banks can recoup their loans before the direct costs are expensed, and so we focus only on the indirect costs.

Limited data is available on the indirect bankruptcy costs for the Netherlands. Recently, Couwenberg and De Jong (2009), and Van Elswijk et al. (2016) have studied Dutch bankruptcy cases, using samples of, respectively, 137 bankruptcies and 2,139 bankruptcies. However, neither paper reports sufficient statistics for us to be able to accurately assess the indirect bankruptcy costs. There is some international research on bankruptcy costs, e.g. some macroeconomic papers use proportional bankruptcy costs in the range 15%–25% (Carlstrom and Fuerst, 1997, 1998). However, these estimates are based mostly on corporate bankruptcies in the U.S. and cannot be extrapolated to the Netherlands.

Having limited information on bankruptcy costs, and given that the absolute amount of credit rationing is sensitive to the level of bankruptcy costs, we choose to make no exact assumptions and consider instead three

scenarios. Namely, we say that if a firm goes bankrupt, then a bank can recoup up to αA_0 , where $\alpha = 0$, or $\alpha = 0.5$, or $\alpha = 1$. The coefficient $1 - \alpha$ can be interpreted as indirect bankruptcy costs. Alternatively, α can be interpreted as the relative amount of collateral used to secure the loan, coupled with the assumption that a bank cannot recoup more than this initial collateral. Dutch banks are often first in line when it comes to repaying debts at bankruptcy, therefore total initial assets A_0 are used as the maximum possible collateral instead of the assets corresponding to the initial long-term debt L_0^l . It is important to note that $\alpha = 1$ does not imply that the bank gets everything back in a bankruptcy, it can still be the case that the interest on the loan is not fully paid back. Of course, were we to assume monitoring costs completely away, there would not have been any credit rationing. In this respect the assumption of positive monitoring costs is crucial to the simulation.

2.5 Credit Rationing

A bank grants a loan if there is an interest rate r such that the expected profit of the bank is positive. In assuming that a bank considers *expected* profits, we effectively assume that the bank can perfectly diversify its loan portfolio. This assumption is conservative. If, in practice, a bank is not able to fully diversify its loan portfolio, then the credit rationing is likely to be larger, both for smaller and larger firms. It will be larger for smaller firms due to higher volatility of their returns, and it will be larger for larger firms due to, presumably, fewer possibilities to diversify big loans.

Let ρ be the cost of funds. We use bank rates on deposits from households with an agreed maturity of over two years, new business, as a proxy for the cost of funds. The data is from the ECB.¹ Define $\delta = 1/(1 + \rho)$. Then, having bankruptcy costs as described earlier, the economic profit of a bank on a given loan is

$$\pi(r) = -L + \delta^T \begin{cases} (1+r)L & \text{if } A^T \geq A^0 - E^0 + rL, \\ \min((1+r)L, \alpha A^0) & \text{if } A^T < A^0 - E^0 + rL. \end{cases}$$

A bank issues a loan as long as

$$\max_r \mathbb{E}(\pi(r)) \geq 0.$$

Expanding, we obtain

$$\delta^T \max_r \left((1+r)L \cdot \mathbb{P}(A^T \geq A^0 - E^0 + rL) + \min((1+r)L, \alpha A^0) \mathbb{P}(A^T < A^0 - E^0 + rL) \right) - L \geq 0.$$

Note, if a bank issues a loan, then the particular r that the bank asks will depend on the level of competition in the banking sector, it need not be the r that maximizes expected profits.

From (1), by recursion, we have:

$$A^T = A^0 X, \quad X = \prod_{t=1}^T (1 + \pi^t).$$

Let $x_{k,i}$ denote a draw of X_i , using random sampling with replacement as described earlier. That is,

$$x_{k,i} = \prod_{t=1}^T (1 + \pi_{\eta(k,i,t)}^0).$$

Let

$$z_{k,i} = \frac{A_i^0 (x_{k,i} - 1) + E_i^0}{L_i}.$$

Then the probability that firm i is solvent can be estimated as follows:

¹ Source: [link](#)

$$\mathbb{P}(A_i^T \geq A_i^0 - E_i^0 + rL_i) = \mathbb{P}\left(r \leq \frac{A_i^0(X_i - 1) + E_i^0}{L_i}\right) = \frac{1}{K} \sum_{k=0}^{K-1} \left[r \leq \frac{A_i^0(x_{k,i} - 1) + E_i^0}{L_i} \right] = \frac{1}{K} \sum_{k=0}^{K-1} [r \leq z_{k,i}],$$

where $[P] = 1$ if P is true, and $[P] = 0$ otherwise (the Iverson bracket).

So, a bank will refinance the existing long-term loan of firm i as long as

$$\delta^T \max_r \left((1+r)L \cdot \frac{1}{K} \sum_{k=0}^{K-1} [r \leq z_{k,i}] + \min((1+r)L, \alpha A^0) \cdot \frac{1}{K} \sum_{k=0}^{K-1} [r > z_{k,i}] \right) - L \geq 0. \quad (2)$$

This optimization problem can be solved efficiently. Firstly, we can sort $z_{k,i}$ without loss of generality. Then, if $z_{k,i} < z_{l,i}$ whenever $k < l$, Eq. (2) can be equivalently written as

$$\delta^T \max_k \left((1+z_{k,i}) \cdot \frac{K-k}{K} + \min((1+z_{k,i})L, \alpha A^0) \cdot \frac{k}{K} \right) - L \geq 0. \quad (3)$$

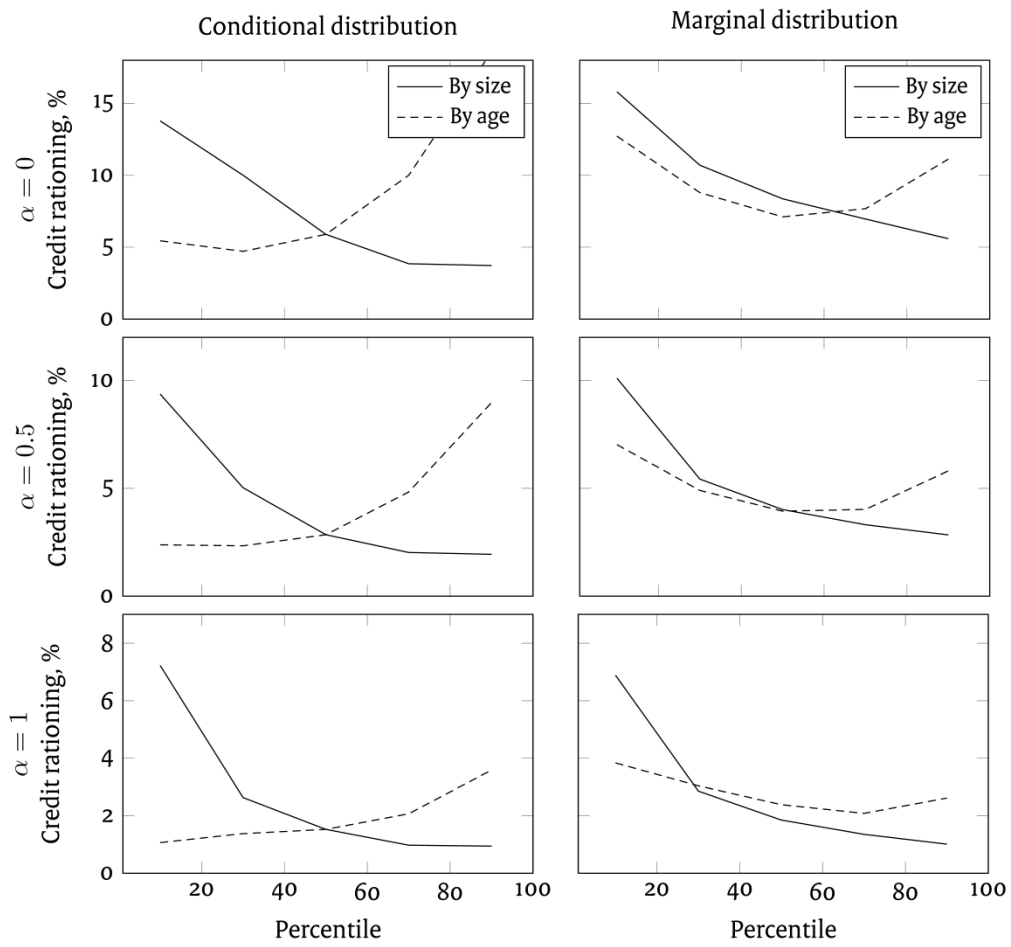
To further speed up the computation we use the same draws of X for the firms from the same risk group.

3 Results

Figure 3.1 shows marginal and conditional distributions of the number of credit rationed firms, as percentage of the total number of firms, per age and size percentile in 2016 according to the simulation. Figure 3.2 shows the time trend for the smallest, median, and largest size groups.

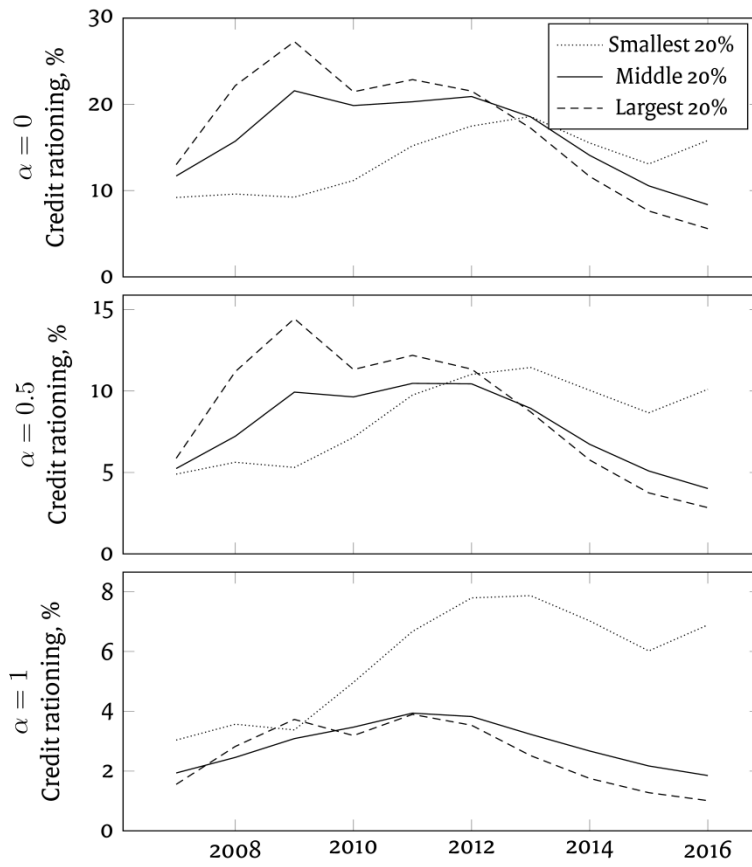
A number of conclusions can be drawn from the graphs: 1) in 2016, small firms potentially experienced substantially larger credit rationing than big firms, whereas before the crisis of 2008–2009 there was no clear difference between small and big firms, see Figure 3.2; 2) conditional on size, older age is correlated with higher potential credit rationing, see Figure 3.1, left panel; 3) big firms were more affected during the crisis than small firms, but are currently back at the pre-crisis levels of credit rationing, while small firms are experiencing a potentially worse situation than before the crisis, see Figure 3.2. These conclusions are approximate. On the one hand, we have made a number of conservative assumptions and the actual amount of credit rationing is likely to be larger. On the other hand, we neglected alternative credit channels such as market financing, which implies that for the biggest firms the problem can be smaller than shown, or even non-existent.

Figure 3.1 Credit Rationing in 2016



Notes: the size conditional distribution is conditional on the median age ($g_i^s = 2$), analogously the age conditional distribution is conditional on the median size ($g_i^a = 2$).

Figure 3.2 Credit Rationing Trend



Notes: the time-series are based on *marginal* size figures, i.e. all age groups are included.

4 References

- Antinolfi, G. and Carli, F. (2015). Costly monitoring, dynamic incentives, and default. *Journal of Economic Theory*, 159:105–119.
- Carlstrom, C. T. and Fuerst, T. S. (1997). Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis. *The American Economic Review*, 87:893–910.
- Carlstrom, C. T. and Fuerst, T. S. (1998). Agency costs and business cycles. *Economic Theory*, 12:583–597.
- Couwenberg, O. and de Jong, A. (2008). Costs and recovery rates in the dutch liquidation-based bankruptcy system. *European Journal of Law and Economics*, 26:105–127.
- Davydenko, S. A. and Franks, J. R. (2008). Do Bankruptcy Codes Matter? A Study of Defaults in France, Germany, and the U.K. *The Journal of Finance*, 63:565–608.
- Gale, D. and Hellwig, M. (1985). Incentive-compatible debt contracts: The one-period problem. *The Review of Economic Studies*, 52:647–663.
- Kjenstad, E. C., Su, X., and Zhang, L. (2015). Credit rationing by loan size: A synthesized model. *The Quarterly Review of Economics and Finance*, 55:20–27.
- Monnet, C. and Quintin, E. (2005). Optimal contracts in a dynamic costly state verification model. *Economic Theory*, 26:867–885.
- Townsend, R. M. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory*, 21:265–293.
- Van Elswijk, D. A., Jansen, J. P., Duijkers, R. P. R., and Moerman, M. P. (2016). Faillissementen oorzaken en schulden 2015. Een rapport van het Centraal Bureau voor de Statistiek.
- Williamson, S. D. (1986). Costly monitoring, financial intermediation, and equilibrium credit rationing. *Journal of Monetary Economics*, 18:159–179.
- Williamson, S. D. (1987). Costly monitoring, loan contracts, and equilibrium credit rationing. *The Quarterly Journal of Economics*, 102:135–146.