CPB Netherlands Bureau for Economic Policy Analysis

Structural causes of low interest rates in the euro area

We evaluate the impact of a number of structural factors on the real interest rate using the neo-classical overlapping generations model of Eggertsson et al. (2019) calibrated for the euro area economy.

This model allows us to quantify the impact of higher life-expectancy and lower fertility on the supply and demand for savings and, hence, on the real interest rate.

We show that the most important factor leading to the decline in the real rate of interest since the 1970s was the increase in life-expectancy and the decline of productivity growth in the euro area.

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Structural causes of low interest rates in the euro area

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Abstract

Based on the neo-classical overlapping generations model of Eggertsson et al. (2019) calibrated for the euro area economy, we evaluate the impact of a number of structural factors on the real interest rate. This model allows us to quantify the impact of higher life-expectancy and lower fertility on the supply and demand for savings and, hence, on the real interest rate. It also incorporates other structural determinants of the real interest rate such as productivity growth, borrowing constraints, the relative price of capital to consumption goods, government debt and the market power of firms. The equilibrium interest rate that results can be thought of as the natural rate of interest, because the model abstracts from business cycle shocks. We use the model to study the past impact of the different structural factors on the natural rate of interest as well as to forecast its future.

1 Introduction

In this paper we present a quantitative evaluation of the impact of a number of structural factors on the real interest rate for the euro area. We perform this assessment using the neo-classical overlapping generations model of Eggertsson et al. (2019) calibrated for the euro area economy. By incorporating a realistic number of overlapping generations, this model allows us to quantify the impact of higher life-expectancy and lower fertility on the supply and demand for savings and, hence, on the real interest rate in the euro area. The model incorporates other important structural determinants of the real interest rate such as productivity growth, borrowing constraints, the relative price of capital to consumption goods, government debt and the market power of firms. Since we abstract from business cycle shocks, the equilibrium interest rate that results from this model is the real interest rate that neither stimulates nor depresses the business cycle in the absence of business cycle shocks and rigidities: *"the natural* *rate of interest"*.[1] We use the model to illustrate the impact of the different structural factors on the natural rate of interest in the euro area in the past and to forecast its future.

The natural rate of interest is an important anchor for monetary policy, but it is an unobservable variable. It is the interest rate that the monetary authority should set in order to ensure that inflation remains stable. As with all estimations, the natural rate of interest is subject to model and parameter uncertainty. Hence the interest rate set by monetary policy can be different from the natural rate of interest.

Other papers have estimated the natural rate of interest for the euro area. Brand et al. (2018) provides an overview of all the econometric and structural estimates of the natural rate of interest for the euro area. Bielecki et al. (2020) and Papetti (2021) use overlapping generations models similar to the model we use in this paper to determine the natural rate of interest for the euro area. However, our model has a richer set of fundamental factors including borrowing constraints, the relative price of capital to consumption goods and firms' market power. The results we obtain with our model are able to replicate the decline in the real natural interest rate that took place between the years 1970 and 2020. Based on publicly available forecasts of the fundamental variables, the model predicts a slight rebound of the natural rate over the coming decades. This is mainly due to higher contributions to the pay-as-you-go pension system necessary to restore the balance of these pension systems in the euro area. Most of the structural factors have either a neutral impact on the future values of the natural rate of interest or continue to put downward pressure.

Our results indicate that the most important driving factors of the downward trend in the interest rate over the previous decades were higher life-expectancy and the decline in the growth rate of productivity. Fertility has had a lagged impact on the natural rate of interest, having a significant downward impact since 2000, as well as in the future. Looser borrowing constraints after the year 1970 and an increase in the level of government debt between 1970 and 1995 put upward pressure on the natural rate of interest, but these effects were quantitatively modest.

We show that the results are qualitatively robust to alternative parameter values. They are also quantitatively robust to alternative parameter values with the exception of the rate of time preference and the elasticity of intertemporal substitution. Changing the calibrated value of these two parameters strongly influences the level and path of the natural rate of interest. Our analysis therefore indicates the need for further research into the calibration of these two variables in order to obtain more reliable estimates of the natural rate.

This paper is organised as follows. Section 2 presents the model and discusses the structural factors influencing the natural real interest rate. Section 3 presents the data used to calibrate the model for the euro area economy and the calibration strategy, after which the results are presented in section 4. Section 5 shows the robustness of the results to different

[1] This definition of the natural rate of interest is in line with Eggertsson et al. (2019) and Holston et al. (2017).

types of calibration. In section 6 we draw some conclusions.

2 The model

In this section we present the model that we use to quantify the impact of the structural factors on the medium and long run real rate of interest, i.e. the natural rate of interest. We only present the elements of the model that are important in determining the supply and demand for savings. We also explain the mechanism through which each structural factor influences the natural rate of interest. We relegate the rest of the equations of the model to the appendix.

2.1 Households

The economy is populated with overlapping generations of households. Each household enters the labour market at the age of 26 years, has at this age a number Γ of children and lives up to a maximum of J years. If they reach age J, they leave bequests equal to x. Households have a probability to survive until age j equal to s^j. They choose consumption (c) and bequests[2] (x) at each age in order to maximize their welfare:

$$U_t = \sum_{j=26}^{J} s^j \beta^{j-1} u(c_{j,t+j-1}) + s^J \beta^{J-1} \mu v(x_{J,t+J-1}) \tag{1}$$

where j is the age of the household, t is the year, β is the time preference parameter and μ is the strength of the bequest motive.

The income of the households (y) is comprised of labour income during the working life and of the pension coming from the pay-as-you-go pension system after retirement, which takes place at age ra. The pension benefit (pb) is a replacement rate (ρ) times the average wage of the household over her working career. Households also receive the pure profits of the firms (Π). These are distribuited proportionally according to the income of the household. Households insure themselves against longevity risk. The assets that they hold are denoted by a. The per-period budget constraint is given by

[2]Bequests left intentionally are zero at all times, except at the final age J. Consequently, bequest received are also zero at all times except at age J-24.

$$c_{j,t} + \xi_t a_{j+1,t+1} + \Gamma_{26,t-j+26} \cdot x_{j,t} = y_{j,t} + \Pi_{j,t} + [r_t^k + (1-\delta)\xi_t] \left(a_{j,t} + q_{j,t} + \frac{1-s_j}{s_j} a_{j,t} \right) \quad (2)$$

$$y_{j,t} = \begin{cases} (1-\tau^w-\tau^p) w_t hc_j, \text{ if } j < ra \\ pb_t, \text{ if } j \geq ra \end{cases} \quad (3)$$

$$pb_t = \rho \frac{\sum_{k=t-(ra-26)}^{ra} w_k hc_{j+k}}{ra-26} \quad (4)$$

where r^k is the rental rate of capital, ξ the exogeneous relative price of capital in terms of the consumption good, w is the average wage and hc is labour productivity over the household's life-cycle, δ capital depreciation, q bequests received, τ^w is the income tax and τ^p is the contribution to the pay-as-you-go pension system.

Households' preferences are described by a Constant Relative Risk Aversion function:

$$U(C) = \begin{cases} \frac{C^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}}, if \gamma \neq 1 \\ log(C), if \gamma = 1 \end{cases} \quad (5)$$

where γ is the intertemporal elasticity of substitution of consumption. A positive value of γ implies that the household prefers to adjust consumption with less than the change in income that a household experiences between two different periods of time. This is possible by borrowing when income is low and saving when income is high in order to keep consumption relatively even over the life-cycle. The higher the value of γ is, the lower is the impact of income changes on consumption and, hence, the higher is the amount of savings/borrowing of the household.

Labor income $w_t hc_j$ increases over a person's career and then declines. The increase is due to experience and tenure while the decline is due to early retirement. We model this through a hump shaped profile of labor productivity that is estimated from the data and summarised in hc. As households prefer a relatively even consumption path over their life-cycle (when $\gamma > 0$), a labour income profile that increases with age means that households would like to borrow when they are young and their income is low. They repay the debt later in their career when their income is higher. Households face borrowing constraints, they cannot borrow a sum larger than D at any given time:

$$a_{j,t} \geq -\frac{D_t}{1+r_t}. \quad (6)$$

A lower D implies stricter borrowing constraints and a lower demand for savings. This pushes the interest rate down.

Depending on how high the replacement rate is in the pay-as-you-go system, households may also want to save in the second part of their career in order to prevent a too high decline in consumption during retirement. A higher life-expectancy implies that households must save more to finance a longer period of retirement. Hence, when life expectancy increases, household savings also go up. Households can also choose to finance retirement by working for a longer period of their life. This would reduce the impact of a higher life-expectancy on household savings (see Bielecki et al. (2020)).

2.2 Production of the final good

There exists a continuum of monopolistically competitive final goods firms of measure one that costlessly differentiate an intermediate good and resell it to the households. Each final goods producer i uses $y^m(i)_t$ of intermediate goods to produce output, according to a linear production function $y^f(i) = y^m(i)$. The final good composite Y is the CES aggregate of these differentiated final goods:

$$Y_t = \left(\int_0^1 y_t^m(i)^{\frac{\theta_t - 1}{\theta_t}} di \right)^{\frac{\theta_t}{\theta_t - 1}}. \tag{7}$$

The parameter θ_t is a time-varying shock to the market power of the firm. A decline in θ_t increases the firm's market power and increases the equilibrium markup defined as:

$$\text{markup}_t = \frac{\theta_t}{\theta_t - 1}. \tag{8}$$

2.3 Intermediate goods firms

There exists a perfectly competitive intermediate goods sector that sells its production to the final goods sector. These firms produce using capital and labour embedded in a CES production function with an elasticity of substitution between capital and labour of σ and a share of capital in production equal to ϵ:

$$y_t^m = \left(\epsilon \left(K_t\right)^{\frac{\sigma-1}{\sigma}} + (1-\epsilon)\left(A_{l,t} L_t\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \tag{9}$$

The variable $A_{l,t}$ denotes labour productivity. This grows at rate g_t.

Firms hire labor at price w and rent capital at price r^k. They maximise their profits, which gives the following equation for the demand for capital:

$$r_t^k = \frac{\theta_t - 1}{\theta_t} \epsilon \left(\frac{Y_t}{K_t} \right)^{\frac{1}{\sigma}}. \tag{10}$$

We note here that an increase in the market power of firms (lower θ) implies a lower demand for capital coming from firms. This pushes the rental rate of capital down.

The interest rate r_t is related to the rental rate of capital by a standard no-arbitrage condition:

$$1 + r_t = \frac{r_t^k + (1-\delta)\xi_t}{\xi_{t-1}}. \tag{11}$$

A decline in the price of capital goods with respect to consumption goods ξ_t reduces the interest rate. The relative price of capital goods is exogeneous in the model.

2.4 Government

The government spends an exogenous amount G_t, collects taxes on labour income equal to the amount T_t and may accumulate debt $b_{g,t}$. The budget constraint is given by:

$$b_{g,t+1} = (1 + r_t)b_{g,t} + G_t - T_t \tag{12}$$

$$T_t = \tau_t^w \sum_{j=26}^{ra} hc_j N_{j,t} \tag{13}$$

where $N_{j,t}$ represents the number of people aged j alive at time t in the economy. A lower fertility rate implies that cohorts become smaller and hence government revenue declines. In order to keep the debt level to GDP constant, an increase in the tax rate τ^w is necessary. A higher tax rate has implications for household finances and the supply of savings: it lowers net income and hence the savings made by households. Through this channel, a lower fertility rate leads to a lower amount of savings in the economy.

We assume that the pay-as-you-go pension system is in equilibrium every period:

$$\sum_{j=ra}^{J} N_{j,t} pb_j = \tau_t^p \sum_{j=26}^{ra} hc_j N_{j,t}. \tag{14}$$

Aging has important consequences for the budget of the pay-as-you-go pension system. As life expectancy increases and fertility rates decline, more households receive pension benefits (left side of the above equation) and less people pay contributions (right side of the above equation). In order to keep the pension system balanced, the government will increase the contribution rate to the pension system τ^P. This reduces net income and hence decreases the savings that younger cohorts make.

2.5 General equilibrium

In the general equilibrium of the economy, households maximise their present value utility subject to budget and borrowing constraints, firms maximise profits subject to their production constraints, the government budget and the pension system budget balances and the markets clear:

- Labour market: The labour supplied by households is equal to the labour demanded by firms L^d

$$L_t^d = \sum_{j=26}^{J} N_{j,t} hc_j. \tag{15}$$

- Asset market: the supply of assets provided by households is equal to the demand for assets. The latter is the sum of capital demanded by firms, government debt and capital demanded/supplied by the rest of the world (the net foreign assets of the economy, nfa):

$$\sum_{j=1}^{T} N_{j,t} \xi_t a_{j,t} = \xi_t K_t + b_t + \text{nfa}_t. \tag{16}$$

The rental rate of capital r^k and the wage w are the result of market clearing.

Population aging has an impact through general equilibrium effects as well. Firstly, a lower fertility rate means a lower supply of labour. Due to the assumption that production is given by a constant returns to scale production function, a lower supply of labour means that firms also demand less capital. This pushes the equilibrium real interest rate down. Secondly, a higher life expectancy and a lower fertility rate means that the composition of the population shifts toward older households. These save more than younger households, hence the overall supply of savings increases.

Asset market equilibrium also means that, at a constant level of supply of household savings, a higher level of government debt implies a higher demand for savings and, hence, a higher interest rate.

3 Model calibration

3.1 Data

This section presents the sources and the construction of the data series that we use for calibrating and simulating the model. In order to remove the effects of temporary shocks, all the data series used in the simulation are smoothed using a HP filter with a smoothing parameter equal to 100.

3.1.1 Demographics

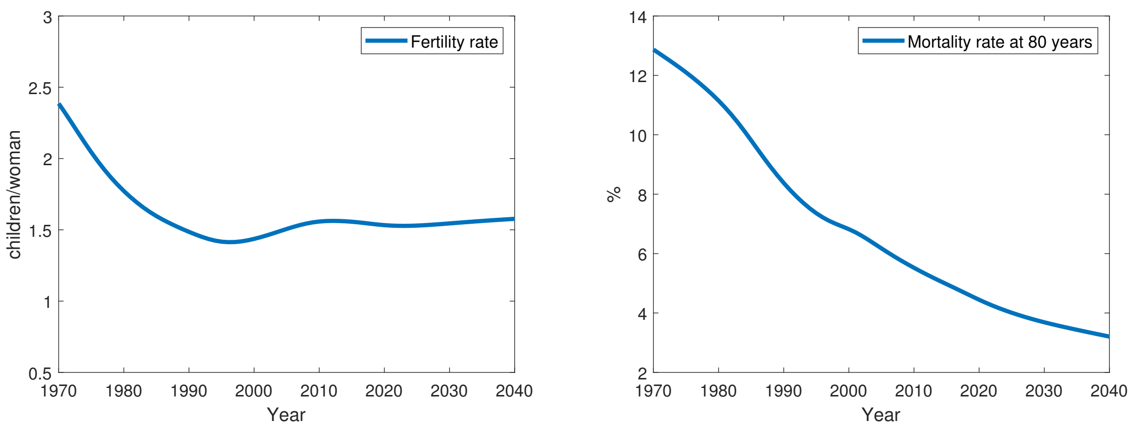
We obtain data on the total and total projected population in the euro area from Eurostat, creating a series ranging from 1960-2100. We also use Eurostat data to obtain both historical data and projections for the number of people in each one-year age category. Missings in the earlier years are either filled using the first known observation of that country or, in case there are gaps in the data, by linear interpolation. We end up with the population per age category from 1970-2100 for 101 different age categories.

To construct a fertility rate series we use both historical and projected data from Eurostat, resulting in a series ranging from 1960-2100. We use fertility data from the World Bank to fill missings in the earlier years. Next we create a weighted average for the euro area for these years, using populations as weights.

We combine data from different sources to construct historical and projected mortality rates at each age from 0 to 101 years for the period 1970-2100. Our main reference is the historical and projected population data from Eurostat. However this data has missing values. We fill in the missing data in the beginning of our sample using the UN central death rate. This is only available aggregated over five years and averaged over five age categories, thus we linearly interpolate them. We transform the central death rate of the UN into number of deaths and then mortality rates using the population by age and by country. The projected mortality rates are only reported per sex. We construct per country and per age category a weighted average of male and female mortality rates using data on population by age, sex and country from Eurostat. Finally, we construct a weighted average historical and projected mortality rate over all euro area countries using historical and projected population data by age and country.

Figure 1a presents the fertility rate included in our analysis. Figure 1b illustrates the historical and predicted decline of the mortality rates at age 80. The decline in mortality rates at all ages has led to an increase in life-expectancy in the euro area.

Figure 1: Historical and predicted demographic developments



(a) HP-filtered fertility rate

(b) HP-filtered mortality rate at 80 years

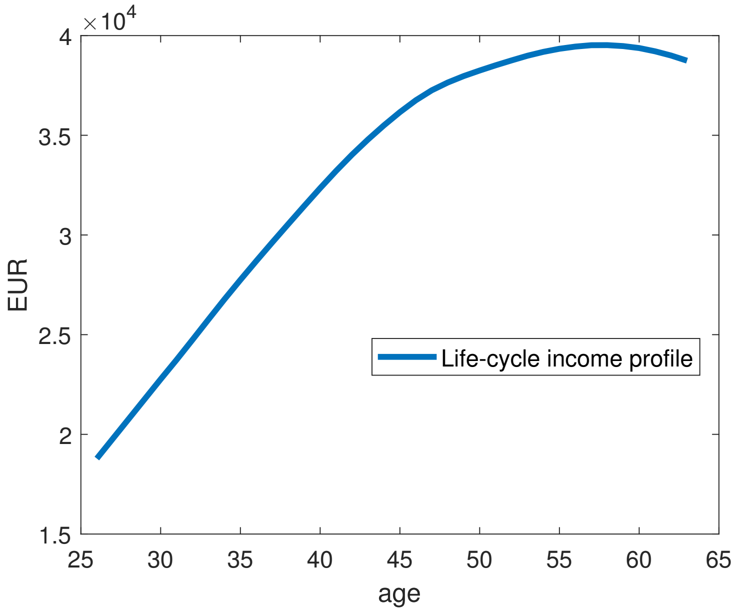
3.1.2 Other household data

In order to calibrate the strength of the bequest motive we use the calculations from Tiefensee and Westermeier (2016) based on the Household Finance and Consumption Survey (HFCS). They report median bequests in 2010 for 8 euro area countries. To calculate a euro area average we use GDP as weights.

In order to calibrate the borrowing constraint, we use consumer debt to output data from Fondeville et al. (2010) for all euro area countries except Greece. We construct a weighted euro area average using GDP as weights for the years 2000-2009.

We take the life-cycle labour productivity profile hc from Bielecki et al. (2020), see figure 2. This is constructed based on data from the HFCS.

Figure 2: Life-cycle income profile in the EA



3.1.3 Production and interest rates

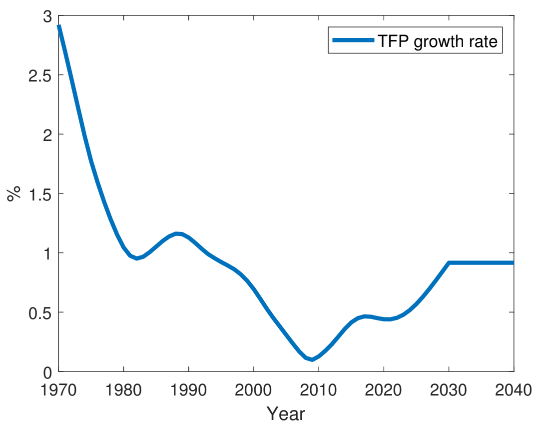
We construct output for the period 1960 to 2022 using annual GDP data in current prices from AMECO. Missing German GDP data in the period 1970-1990 is filled using data from the FRED. The remaining missing data, German GDP before 1970 and GDP of all Eastern-European euro area countries before 1991, is corrected for by rescaling euro area GDP by the average ratio of total euro area GDP over aggregate GDP for all non-missing countries in the period there is no missing data. This constructed GDP-series is also used for creating weighted averages of, for example, bequests.

The real interest rate series for the period 1970 to 2020 is created using the GDP weighted nominal short term interest rate of the euro area from AMECO. We splice the euro area-12 to the complete euro area interest rate for the period of 1980-2020. To construct the real short term interest rate we subtract euro area inflation obtained from FRED. As a proxy for the real short term euro area interest rate in the period 1970-1979 we use the GDP weighted geometric average of the real short term interest rate of Germany and France. The nominal short term interest rate and inflation for Germany and France are all obtained from the FRED.

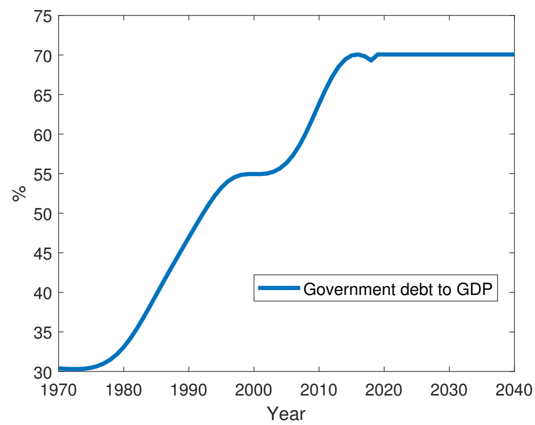
Total factor productivity (TFP) for the period 1960-2019 is taken from AMECO. We splice the euro area-12 average to the complete euro area TFP, such that we obtain a TFP series from 1960-2022. From the last year of historical data onwards, we assume that TFP growth rate increases to 1% until 2030 and remains at this value until the end of the simulation horizon. Figure 3a presents the resulting TFP growth rate series.

To construct a proxy for the average labour share of the eurozone for the period 1960-2019, we use FRED data on the share of labour compensation in GDP in current prices of Spain, Germany, Italy and France. We use GDP as weights to construct a weighted euro area average. To calibrate investment to output we use the investment-to-output ratio from Eurostat.

Figure 3: TFP and government debt



(a) HP-filtered TFP growth rate



(b) HP-filtered government debt to GDP

3.1.4 Government and rest of the world

We construct general government net debt for the period 1970 to 2019. For the period 1991-2019 we use the data from the IMF World Economic Outlook database. The earlier years are taken from Bielecki et al. (2020). After the year 2019, we assume that the ratio of government debt to GDP remains constant at the level of 70% of GDP. Figure 3b presents the resulting government debt series.

To determine the net international investment position of the euro area as a percentage of GDP in equilibrium we use quarterly net foreign assets data in millions from Eurostat for the period 1999 to 2020. Some years in the early 2000s that are missing are filled by using the earliest non-missing observation. We aggregate over all euro area countries and divide by GDP.

3.2 Calibration strategy

We set the parameters of the model following the calibration strategy from Eggertsson et al. (2019). The values of some parameters are set by minimising the distance between the values obtained in the model for some variables (see table 1 and 2) and the corresponding actual values of these variables in the data. We set the retailer elasticity of substitution in year 1970, the beginning year of our simulation, by matching the labour share in that year. We set the time preference, share of capital in production and the strength of the bequest motive, the borrowing constraint in the year 2005 and the retailer elasticity of substitution in the year 2005 by matching the model with the data for the following variables: real interest rate, investment to output, labour share and the ratio of bequests left on average by a household to the average income of a household. We target the values from 2005 when, according to ECB estimates, the output gap was around 0. As such, the actual interest rate in 2005 should have been close to the long-run neutral level. We also perform a robustness check in which we target the actual data from 1999, another year in which the output gap was 0 (section 5).

Table 1: Targeted parameters year 2005

Targeted parameter	Value	Target	Target value
Time preference (β)	1.004	Real interest rate	-0.3%
Share of capital in production (ϵ)	0.38	Investment to output	22%
Borrowing constraint (D)	0.85	Consumer debt to output	12%
Retailer elasticity of substitution (θ)	5.2	Labour share	59%
Strength of bequest motive (μ)	15.3	Bequests to income	4.3

The borrowing constraint is computed as the ratio of debt to yearly income.

Table 2: Targeted parameters year 1970

Targeted parameter	Value	Target	Target value
Borrowing constraint(D)	0.1	Consumer debt to output	1%
Retailer elasticity of substitution (θ)	8	Labour share	65%

We set another group of parameters to values from the literature (table 3). We check the robustness of the results with respect to these parameters in section 5.

Table 3: Parameters from the literature

Parameter	Value	Reference
Intertemporal elasticity of substitution (γ)	0.75	Eggertsson et al. (2019)
Capital depreciation rate (δ)	0.14	ECB (2006)
Elasticity of substitution between capital and labour (σ)	0.65	McAdam and Willman (2008)
Pension system replacement rate (ρ)	44%	Bielecki et al. (2020)
Government expenditure/GDP (G)	20%	Bielecki et al. (2020)

4 Results

We simulate the model presented in section 2 using the calibrated parameters from section 3 between the years 1970 and 2120. We assume that the euro area economy is in steady state in 1970 and 2120. The path of the natural rate of interest obtained with the model is presented in figure 4, left panel. This is declining since 1970 under the influence of the structural factors. These keep the natural rate of interest low until around the year 2030 after which the model predicts a small rebound. According to the population projections that we use as input, the fertility rate will increase slightly in the future, while death rates will continue to decline leading to further increases in the life-expectancy. As a result aging will continue to contribute to a decline in the trend of the real interest rate. The small increase in the natural rate of interest predicted by the model is due to the higher contribution rate required in the future by governments in order to rebalance the pay-as-you-go pension systems. A higher contribution paid by working cohorts into the pay-as-you-go pension system implies that they will have a lower net income and hence will save less.

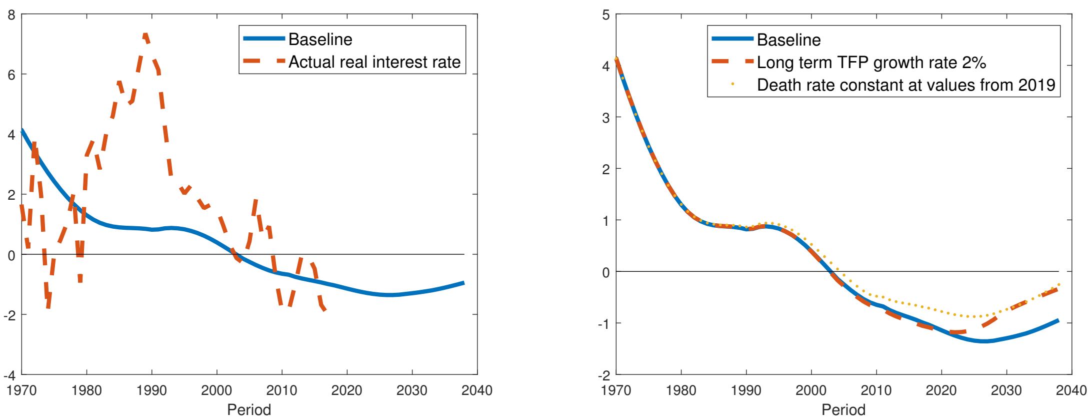


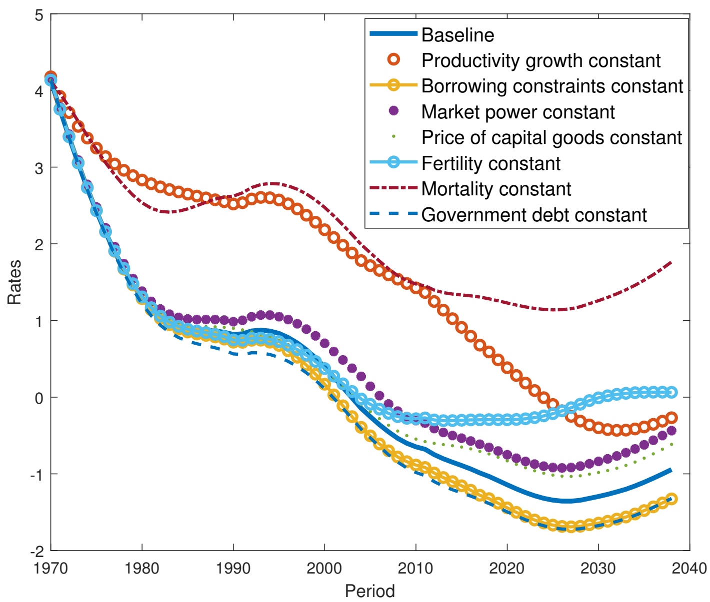
Figure 4: Results of model simulation

Our analysis indicates that the natural rate of interest is currently negative but higher than -2%. Since the nominal rate is the real interest rate plus inflation, a 2% inflation target means that, in the absence of business cycle shocks, the nominal rate of interest is just above 0%, which leaves monetary authorities limited room to intervene if a negative shock occurs.

If we assume that life-expectancy will remain constant at the level prevailing in 2019, then this results in a higher natural rate of interest (Figure 4, right panel). In general we assume that the productivity growth rate returns to 1% per year by 2030 and remains constant thereafter. The natural rate of interest will also be higher if productivity growth is higher than our baseline calibration. If productivity growth is 2% instead of 1% from 2030, the natural rate will be about 70 basis points higher (Figure 4, right panel).

Quantitatively, the most important structural factors of that pushed the real rate of interest down in the euro area in the past is the decline in the mortality rate, which increased life expectancy, and the decline in the productivity growth rate. Figure 5 presents the path of the natural rate of interest resulting from the model when we keep one structural factor at a time equal to the value prevailing in the year 1970. If the mortality rate remained at the higher level of the year 1970 throughout the simulation period, the natural rate of interest would have still declined, but much less than in the baseline scenario, reaching a level slightly below 2% in 2020. The same is the case for the productivity growth rate. However, this had more of a downward impact after 2010 when the productivity growth rate declined substantially.

Figure 5: Impact of structural factors on the natural rate of interest



The decline in the fertility rate had a delayed impact, putting downward pressure on the natural rate of interest after 2000. The rest of the structural factors considered in the model had a quantitatively small impact on the natural rate of interest. The loosening of the borrowing constraints since the 1970s and the higher level of government debt exerted an upward pressure on the natural rate of interest.

5 Robustness checks

In this section we present the results obtained with different calibrations of the parameters of the model. We do this in two steps. We first change, in turn, the calibrated value of some parameters in the model. Table 4 contains an overview of parameters we change and to which values. Second, we change the rest of the parameters from table 1 in order to meet their corresponding targets again.

Table 4: Robustness checks

Parameter	Baseline value	Alternative value
Time preference (β)	interest rate	interest year
	target year 2005	target year 1999
Intertemporal elasticity of substitution (γ)	0.75	0.5
		1
Capital depreciation rate (δ)	0.14	0.1
Elasticity of substitution	0.65	0.75
between capital and labour (σ)		
Consumer debt to output	12%	8%
Bequests to income	4.3	3

In the left panel of figure 6 we present the resulting natural rate of interest obtained by changing the depreciation rate, the substitution between capital and labour, the ratio of bequests to income and the strength of the borrowing constraints. The quantitative results are robust to changes in these parameters.

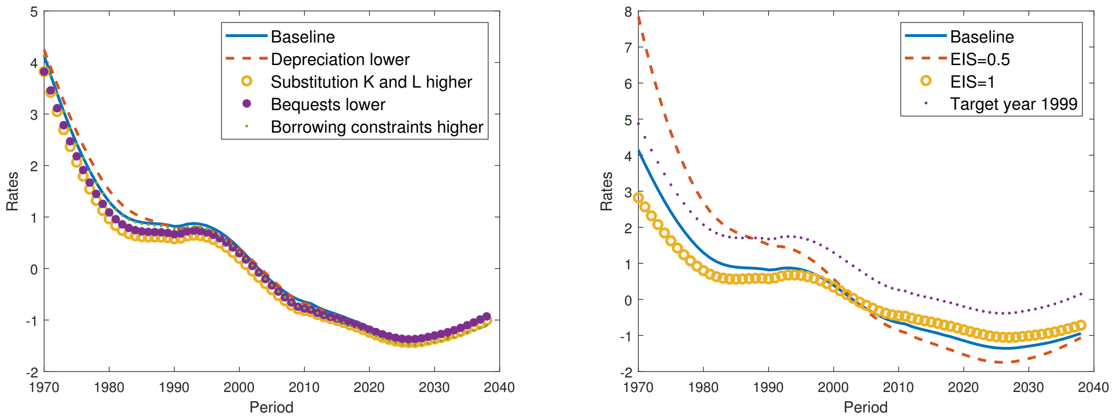


Figure 6: Robustness checks

In the right panel of figure 6 we present the resulting natural rate of interest obtained by changing the elasticity of intertemporal substitution and the time preference parameters. There is no consensus in the literature about how these variables should be calibrated. Most of the macro-literature assumes a value for the elasticity of intertemporal substitution between 1 and 2 (see Papetti (2021) for more details). Given the value calibrated for the elasticity of intertemporal substitution, time preference is set by targeting the capital to income ratio in one of the steady states of the model (eg. Papetti (2021)) or the average real interest rate over a horizon (eg. Bielecki et al. (2020)) or a year with output gap 0 (eg. Eggertsson et al. (2019)). As we illustrate in the right panel of figure 6, the choice of these parameters has important implications for the exact value of the natural rate of interest.

Time preference determines the level of the natural rate of interest resulting from our model, but the shape remains roughly the same. Specifically, calibrating the value of time preference by targeting the real interest rate in the year 1999 instead of the year 2005 yields a natural rate of interest that lies above 0 in the year 2020 instead of at -1.2%. However, the decline in the natural rate of interest between the years 1970 and 2020 is similar: 5 pp when targeting the year 1999 instead of 5.2 pp when targeting the year 2005.

The intertemporal elasticity of substitution determines the shape of the path followed by the natural rate of interest. We simulate the model with elasticities of intertemporal substitution of 0.5 and 1 instead of 0.75. The value of 0.5 was found in the meta-analysis of Havranek et al. (2015) to be the median of the estimated values for the elasticity of intertemporal substitution, while 1 is the value customarily used in macroeconomic models (see also Bielecki et al. (2020)). While the point estimates for 2020 are similar under the three calibrations, the decline in the natural rate of interest is much higher when the elasticity of intertemporal substitution is lower.

It is worth noting, however, that the qualitative results remain the same under all calibrations. While the precise estimate of the natural rate of interest is sensitive to the calibration of these two parameters, its significant decline in the past 40 years and the relatively small increase predicted by the model for the future remain a robust result.

6 Conclusions

This paper quantifies the impact of a number of structural factors on the natural real rate of interest in the euro area. We show that the most important factor leading to the decline in the real rate of interest since the 1970s was the increase in life-expectancy and the decline of productivity growth in the euro area. For the future, current projections of the fundamental factors indicate that the natural rate of interest will rebound slightly mainly due to the consolidation of the pay-as-you go pension system required by the aging of the population. Due to higher contribution rates, the future young generations will have a lower disposable income and, hence, will save less.

We also point in this paper to the importance of two parameters for the precise value of the natural rate of interest: the time preference parameter and the intertemporal elasticity of substitution. Considering the importance that the precise value of the natural rate of interest has for the conduct of monetary policy, future research should investigate the best way of calibrating or estimating these parameters such that they produce plausible macroeconomic results.

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Appendix

A Additional model equations

A.1 Bequests

Bequests received are equal to bequests given by the parents that survive until the age J when intentional bequests are distributed:

$$q_{J-24,t} = \frac{N_{J,t-1} x_{J,t-1} \Gamma_{26,t-J+26}}{N_{J-24,t}}. \tag{17}$$

A.2 Production

Final good producers are monopolistically competitive. They face a demand curve that takes the following form:

$$y_t^f(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta_t}. \tag{18}$$

The final goods firm chooses real prices $\frac{p_t(i)}{P_t}$ and production $y_t^f(i)$ to maximise real profits, subject to the production constraint:

$$\max \frac{p_t(i)}{P_t} y_t^f(i) - \frac{p_t^{int}(i)}{P_t} y_t^f(i) \tag{19}$$

$$y_t^f(i) = Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta_t} \tag{20}$$

where $\frac{p_t^{int}(i)}{P_t}$ is the price of intermediate goods taken as given by the final goods producer.

The optimality condition for the real price of the firm's good is a time-varying markup over the price of the intermediate good:

$$\frac{p_t(i)}{P_t} = \frac{\theta_t}{\theta_t - 1} \frac{p_t^{int}(i)}{P_t}. \tag{21}$$

The nominal price index implies the following expression for the price of intermediate goods:

$$P_t = \left(\int_0^1 p_t(i)^{1-\theta_t} \right)^{\frac{1}{1-\theta_t}}. \tag{22}$$

The last two equations lead to the following expression of the markup:

$$\frac{p_t^{int}(i)}{P_t} = \frac{\theta_t - 1}{\theta_t}. \tag{23}$$

Profits from monopolistically competitive firms are distributed according to wage income.

In equilibrium, the total distributed profit must equal total profits:

$$\frac{Y_t}{\theta} = \sum_{j=26}^{J} N_{j,t}\Pi_{j,t}. \tag{24}$$

With a retail elasticity of substitution of θ_t, aggregate profits that are returned to households are equal in equilibrium to:

$$\Pi_t = \frac{Y_t}{\theta_t}. \tag{25}$$

A.3 Intermediate goods firms

The optimisation problem of the representative intermediate goods firms is:

$$\max \frac{p_t^{int}(i)}{P_t} y_t^m - w_t L_t - r_t^k K_t \tag{26}$$

$$y_t^m = \left(\epsilon \left(A_{k,t} K_t\right)^{\frac{\sigma-1}{\sigma}} + (1-\epsilon)\left(A_{l,t} L_t\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}. \tag{27}$$

The first order conditions that determine the demand of capital and labour are:

$$w_t = \frac{p_t^{int}(i)}{P_t}(1-\epsilon)A_{l,t}\left(\frac{Y_t}{L_t}\right)^{\frac{1}{\sigma}} \tag{28}$$

$$r_t^k = \frac{p_t^{int}(i)}{P_t}\epsilon A_{k,t}\left(\frac{Y_t}{K_t}\right)^{\frac{1}{\sigma}}. \tag{29}$$

A.4 Population structure

The size of the population entering the labour market:

$$N_{26,t} = N_{26,t-25}\Gamma_{26,t-25}. \tag{30}$$

Population by age group:

$$N_{j+1,t+1} = s_{j,t} N_{j,t}. \tag{31}$$

Total population:

$$N_t = \sum_{j=1}^{J} N_{j,t}. \tag{32}$$