CPB Netherlands Bureau for
Economic Policy Analysis

30 Years of Generational Accounting: A Critical Review

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The standard approach to assess fiscal sustainability is that of generational accounting (GA). This paper reviews GA. It argues that GA is a powerful instrument that puts the finger on the balance between generations. At the same time, GA suffers from some weaknesses that have become more significant over time. We conclude that it is time to update GA.

CPB Background Document

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Abstract

The question whether fiscal policies can be considered sustainable in the light of population ageing is old, but still relevant. Even more so, its relevance has increased recently as public debt levels have gone up dramatically as a result of the COVID-19 crisis. The follow-up question if public finances are deemed unsustainable is also still relevant: to what level should the public debt ratio be reduced in order to restore fiscal sustainability? The standard approach to assess fiscal sustainability is that of generational accounting (GA). This paper reviews GA. It argues that GA is a powerful instrument that puts the finger on the balance between generations. At the same time, GA suffers from some weaknesses that have become more significant over time. We conclude that it is time to update GA.

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1 Introduction

It was some thirty years ago that Auerbach, Gokhale and Kotlikoff introduced the methodology of generational accounting (GA) (Auerbach *et al.* 1991; 1994). Other than existing measures of the fiscal deficit and the fiscal debt, GA made it possible to assess whether fiscal policies at that time, if left unchanged, would ultimately imply an explosive public debt. As an explosive public debt is incompatible with continued access to financial markets, these policies would then indirectly pass a bill to future generations. GA made it possible to quantify this bill.

By then, the awareness was growing that populations were ageing as a result of falling fertility levels and decreasing mortality rates and that this would create financial problems for pay-as-you-go (PAYG) financed social security arrangements like pensions and health care. Moreover, population ageing was a worldwide phenomenon (albeit countries differ in the timing and intensity of the process) and PAYG-financed insitutions are common in many countries (although, again, large differences between countries exist). Hence, it is no surprise that GA became quite popular in a short amount of time, leading researchers in many countries to apply GA to assess the sustainability of fiscal policies in their countries.[1]

This paper reviews GA thirty years later.[2] It argues that GA has been very successful in pointing out how vulnerable fiscal policies are with respect to permanent demographic shocks like population ageing. The ability to express a big financial problem with many aspects into a single number probably contributed to the popularity of GA. It is difficult not to connect the widespread use of GA with pension reform that has taken place in many countries, among others in the form of increasing pension eligibility ages.

On the other hand, GA has largely ignored criticism raised fairly soon after the introduction of GA that it does not fully take into account insights about economic behaviour in response to (or in anticipaton of) policy changes and that it tends to focus on a quite narrow definition of net benefits. Similarly, GA has traditionally paid little attention to uncertainties which are generally quite huge given the long horizons involved. Further,

[1]The studies used for this paper are the following: Adema and Van Tilburg (2019) (the Netherlands), Calmfors (2020) (Nordic countries), CBO (2020) (US), EC (2019), OBR (2020) (UK), Van Ewijk *et al.* (2006) (the Netherlands), and Werding *et al.* (2020) (Germany).

[2]The issue of fiscal sustainability has been covered before in the excellent review by Debrun *et al.* (2019). Unlike the present paper, Debrun *et al.* (2019) pays little attention to generational accounting.

three decades turned out not be long enough to solve the question how to discount future primary balances that are inherently stochastic.

Some of the problems have recently become more manifest. The narrow definition of net benefits has become more problematic now that climate change policies involve huge investments, mainly for the benefit of future generations. The problem of what is an appropriate discount rate has also become more problematic, but for a different reason: GA finds it difficult to handle scenarios in which the interest rate is structurally below the rate of economic growth.

More recently, several alternative ways to look at the sustainability of fiscal policies have emerged. Reinhart and Rogoff (2010) claimed that a public debt to gdp ratio of 90 percent would be a critical value; debt ratios higher than this value would imply lower economic growth. A large literature has arisen that questions whether such a critical value indeed exists, whether it is this uniform across countries and whether it centers at a level of 90 percent or close to it. Second, following Bohn (1998), a large literature has arisen which tries to find the fiscal response functions that connect primary balances to public debt levels. Obviously, if fiscal policies respond (strongly enough) to changes in public debt, unsustainability of fiscal policies is less of an issue. These fiscal response functions are at the heart of studies that apply debt sustainabilty analysis (DSA) that focus on the likely development of public finances in the future. Finally, there is some literature that claims that fiscal reponse functions loose some of their power at high levels of the public debt (fiscal fatigue). This again puts the unsustainability of fiscal policies in the spotlights. Indeed, one can use the new insights on the fiscal response functions to quantify so-called fiscal limits above which financial problems ar such large that a restructuring of the public debt cannot be avoided.

The structure of this paper is as follows. Section 2 discusses the basic idea of GA and section 3 discusses the instrument of the sustainability gap. Section 4 reviews some salient aspects of GA, whereas section 5 lists some critical issues. Section 6 describes some recent alternative approaches to assess fiscal sustainability. Conclusions can be found in section 7.

2 The basic idea of GA

Until the beginning of the nineties of the previous century, it was common to assess public debt policies on the basis of figures of public deficit and public debt ratios. In a series of papers, Auerbach, Gokhale and Kotlikoff

(Auerbach *et al.* (1991, 1994)) made clear that these figures provide little to no information on the impact of foreseen future trends such as that of the ageing of the population. In particular, they argued that the combination of institutions that make transfers from working to retired generations with that of an ageing population implies a huge implicit debt. If fiscal policies at that time would be left unchanged, they continued, the generations at that time were passing a large unpaid bill to the then future generations.

Auerbach *et al.* (1994) applied the concept to the US and concluded that the generational imbalances at that time were huge. For a basic scenario, they calculated that newborn generations faced positive net payments, *i.e.*, their lifetime payments to the public sector exceeded the transfers obtained from the public sector. Given the outcome of their study that fiscal polcieis were unsustainable, for future generations they calculated net payments that were more than double as large as those for the newborn generations. As mentioned above, GA spread quickly all over the world. See for example Kotlikoff and Raffelhueschen (1999) for a study of fiscal sustainability in 22 countries in the world and Raffelhueschen (1999) for a similar study for 12 EU countries. These studies pointed out that countries differ a lot when it comes to the severity of their fiscal sustainability problems. They also indicate unsustainable policies were more common than sustainable policies. For example, in only 3 out of 22 countries studied in Kotlikoff and Raffelhueschen (1999) fiscal policies were assessed to be sustainable.[3]

To set the stage for our discussion, it is good to describe briefly the basic idea of GA. Generational accounting splits the primary balance into a number of spending and revenue items. For a particular spending item, an identity is specified with says that in a basic year t (mostly the most recent year for which reliable statistics are available), aggregate spending, denoted X_t, equals the sum of spending on all age groups in that year. If we write spending on each age cohort as spending per capita, denoted $x_{i,t}$, times the size of the age cohort, denoted $N_{i,t}$, the identity reads as $X_t = \sum_i x_{i,t} N_{i,t}$. This procedure can be repeated for the other spending items and for all the revenue items.

Typical for GA is the concept of intergenerational neutrality. The simulations are based on the assumption that future generations will benefit in the same way from the public sector as current generations. Hence, if we assume that all the age profiles $x_{i,t}$ will remain unchanged in the future, *i.e.*, $x_{i,\tau} = x_{i,t}$ for all $\tau \geq t$, then all future generations will face the same lifetime net benefits or the same lifetime net payments as current generations.

[3] Including borderline cases, sustainability was not a problem in 4 out of 22 countries.

The population structure is obviously not assumed constant over time: the population structure, as contained in the variables $N_{i,\tau}$ $\tau \geq t$, will develop in accordance with projections by demographic agencies. Hence, a GA simulation combines constant age profiles of primary spending per capita and revenue per capita with changing demographics. A typical outcome of such a simulation is that primary balances deteriorate over time and that ultimately the primary balance will stabilize at some negative value.[4] If, as is commonly assumed, the interest rate exceeds the rate of economic growth, then this implies an explosive public debt.

An explosive public debt is incompatible with the No-Ponzi-Game (NPG) condition. Formally, this condition states that $\lim_{T\to\infty}(1+r)^{-T}D_T = 0$, where r denotes the interest rate and D the public debt. If this condition is not met, investors know that future primary balances are insufficiently large to cover future interest payments, which will make them reluctant to hold public debt. Fiscal policies are then unsustainable.

In order to make policies sustainable, GA then requires future generations to make additional lifetime transfers to the government. These transfers must raise the primary balance such that ultimately the debt to gdp ratio will stabilize. The more unsustainable fiscal policies, the larger will be these lifetime transfers by future generations. Appendix A shows that the lifetime transfer to be paid by a representative future cohort, z_b, can be written as $\mu_b(d_{t-1} + b^c_{t-1} + b^f_{t-1})$, where $\mu_b = (1+r)(r-g)/(1+g)^2$, d refers to the statutory debt, b^c refers to the implicit debt due to current generations and b^f refers to the implicit debt due to future generations, all expressed in terms of period-t gdp .[5] This expression makes clear that it is the total of public debt, *i.e.* the statutory debt and the implicit debt that is due to existing public arrangements, that determines the extent to which fiscal policies are unsustainable.

One important point to note is that the concept of fiscal sustainability as it is used in GA studies is more than a NPG condition. The NPG condition refers to what has been called financial sustainability (Westerhout, 2021) and this reflects the relation between the public debt and primary balances, without any reference to current fiscal policies. Fiscal sustainability as it is used in GA studies relates more specifically to the combination of financial

[4]This stabilization stems from the assumption that beyond some point in the future, exogenous variables like fertility and mortality rates stabilize, so that bultimately the primary balance to gdp ratio will also stabilize. Combining the intertemporal government budget constraint with a constant primary balance to gdp ratio, one derives that the public debt to gdp ratio is a constant.

[5]See equation (6) in appendix A for expressions for B^c and B^f.

sustainability and the continuation of current fiscal policies.

The GA approach has a few salient characteristics. First, the projections are rather mechanical as, apart from population ageing, they assume many things to remain unchanged in the future. Foreseen changes in labour productivity growth, hours worked, retirement or saving behaviour of individuals, whether or not as a response to the perspective of changing demographics, are not always taken into account. The same is true for changes in economic behaviour in relation to policy reforms.

The second characteristic is particularly interesting from a generational perspective. GA attributes the whole fiscal gap to future generations. This should be taken as a warning signal to policymakers: if they would not change fiscal policies, they would implicitly pass on a bill to future generations. It is not obvious whether one can take such a calculation as a policy recommendation, however. It would be surely unrealistic from a political-economy perspective to let one group of generations pay the entire bill that is due to population ageing. In a second stage, GA studies have been extended with simulations of feasible policy reforms.[6]

The third feature is the use of a discount rate, rather than the interest rate on government bonds, to make calculations of fiscal sustainability. The motivation that Auerbach *et al.* (1994) provided was that the discount rate that they were to use should reflect that real-world government spending and revenues are inherently stochastic, even if the simulations produced are non-stochastic.

3 The sustainability gap

Around the same time when GA was introduced, Blanchard (1990) and Blanchard *et al.* (1990) proposed the sustainability gap. This gap was defined as the difference between some sustainable tax rate and the tax rate at the time of analysis. The studies stress that instead of taxes one could also think of government spending or government transfers as the policy instrument. Therefore, it seems better to refer to the indicator as a sustainability gap, which is a more policy-neutral term.

This sustainability gap is calculated as the immediate and permanent increase in the primary balance to gdp ratio that achieves sustainability.

[6] For example, in the overview studies of Kotlikoff and Raffelhueschen (1999) and Raffelhueschen (1999) for 22 countries in the world and 12 EU countries respectively, one finds calculations of not only the necessary adaptations for future generations, but also of alternative reforms that restore sustainability such as an immediate adjustment in the primary balance sheet.

Importantly, implementation of this sustainability gap spreads the burden over all generations. Newborn generations pay as much as future generations, whereas currently living generations pay as well, proportional to their remaining lifetimes. Perhaps because of this more neutral treatment of different generations, this indicator of fiscal sustainability has become the standard to calculate the unsustainability of fiscal policies.

As an illustration, figure 1 displays a case that is representative for a study on sustainability.[7] The solid curve in Panel 1A depicts the projection of the primary government balance. It starts at zero, but turns into an ever-growing deficit, which may be interpreted as the reflection of ageing. The solid curve in Panel 1B show the corresponding projection of the public debt. This debt increases over the projection period at an ever growing pace. Fiscal policies are thus clearly unsustainable. It is straightforward to calculate how large is the sustainability gap in this case. Implementation of this sustainability gap raises the primary balance curve; because the increase in the primary balance to gdp ratio is constant over time, the dotted curve is parallel with the solid curve. The impact upon the public debt ratio can be seen in panel 1B. Due to the increase in primary balances, the public debt starts to decrease immediately in period 0 (the period in which sustainable policies are implemented). As panel 1B shows, the public debt ratio stabilizes at the end of the projection period.

< Include Figure 1 >

Note that stabilization of the public debt ratio means that fiscal policies are sustainable. This can be seen by looking at the No-Ponzi-Game (NPG) condition $\lim_{T\to\infty}(1+r)^{-T}D_T = 0$. If the public debt ratio is stable, D grows at rate g and, if, as assumed, $r > g$, the NPG condition will be met.

A stable debt ratio also implies a relation between the debt ratio and the primary balance ratio. It can be easily derived from the budget accumulation equation (equation (8) in appendix B) that the steady-state debt ratio d^* and the steady-state primary balance ratio p^* are proportional to one another: $d^* = ((1+g)/(r-g))p^*$ (g denotes the rate of economic growth and an asterisk is used to refer to steady-state values). The higher the public debt ratio, the higher the primary balance must be in order to compensate for the (growth-corrected) interest payments on the debt.

[7] To produce this figure and the following figures, I constructed a very stylized model. The model cannot be used for quantitative predictions. The only purpose of this model is to illustrate some features of policies that restore fiscal sustainability.

Figure 1: Unsustainable and sustainable fiscal policies

One peculiar feature of the GA approach is that the value to which the public debt ratio converges, is endogenous. The public debt ratio may stabilize at values of 50% or 100%, but also 0% is possible, or -50%. These numbers illustrate that sustainability does not mean that the public debt should finally be paid off, but, instead, that the public debt ratio should stabilize at the end of the projection period at a level that is related to the primary balance ratio (see above). Nothing in the definition of sustainability refers to the level at which stabilization occurs.

The explanation for this peculiar feature lies in the fact that different types of policy reforms imply different time paths for the primary balance ratio. If different types of policy reform have different implications for the steady-state primary balance ratio, they will differ in the level at which the public debt ratio will stabilize.

Figures 2 and 3 illustrate this point. The curves for the primary balance ratio and debt ratio for unsustainable policies are identical to those in Figure 1. Sustainability is achieved with different types of policy reform, however. Underlying Figure 1 is a cut in spending that does not relate to age (think, for example, of spending on civil service). In Figure 2, the government adopts a reform that puts a heavy burden on old generations (think, for example, of economizing on health care). As the demographic weight of older generations increases over time, the impact on the primary balance ratio grows over time. Hence, the primary balance ratio achieves a higher value at the end of the projection period and, as a result, the debt ratio stabilizes also at a higher level than in Figure 1.

Figure 3 is the opposite of Figure 2. In case of Figure 3, we adopt a reform that affects mainly young generations (like increasing the labour income tax, for example). Since the number of younger generations decreases over time, the adjustment of the primary balance ratio decreases over time. Hence, the primary balance ratio achieves a lower value at the end of the projection period than in Figure 1 and, as a result, the debt ratio stabilizes at a lower level than in Figure 1.

< Include Figures 2, 3 >

The approach described thus far is more or less representative for the GA studies that are carried out in many countries. However, one some aspects different studies adopt different approaches. Different studies differ in the extent to which they include aspects of economic behaviour, in the way they discount future primary balances, in the number and types of scenarios calculated, to name a few examples. The next section elaborates on these

Figure 2: Sustainability policies targeted at older generations

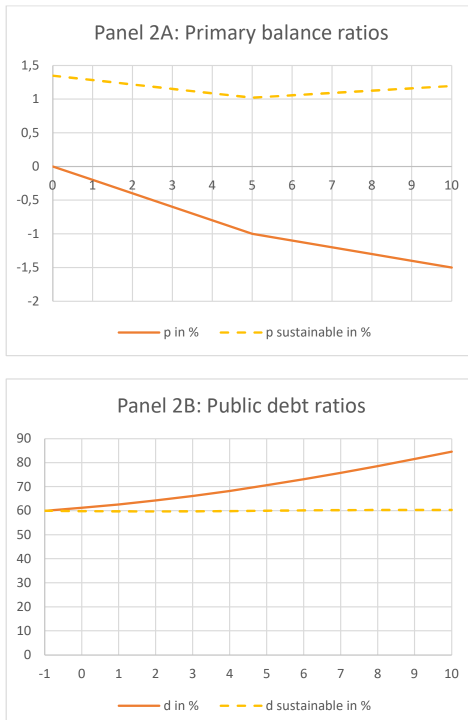
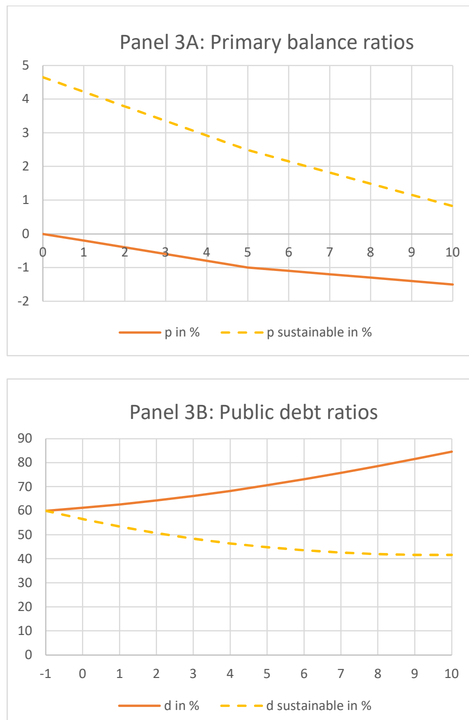


Figure 3: Sustainability policies targeted at younger generations



differences.

4 A comparison of different ageing studies

We discuss several aspects of GA studies on which they may adopt a different approach: economic behaviour, the issue of discounting, the concept of constant net benefits, handling an uncertain future, and the medium-term fiscal sustainability indicator.

4.1 Economic behaviour

As discussed above, the models that are used in GA studies often assume that the demographic structure of the population will develop in the future as foreseen by demographic or statististical agencies, whereas the age profiles of the various spending and revenue items will remain unchanged. This leaves room for different approaches with regard to changes in the economic environment or in the economic behaviour of households and firms. We distinguish three approaches.

The approach originally adopted allows for (constant) economic growth. Apart from that, it assumes that the economic environment and the economic behaviour of households and firms will remain unchanged. Obviously, the projections that result are quite mechanical in nature and have been criticized for that (Haveman, 1994; Buiter, 1997).

The second approach - let us call it the projections approach - is to allow for trends or, more generally, dynamics in the economic environment (think of, for example, changing factor productivity growth, capital deepening, changing interest rates) and in the economic behaviour of households and firms (think of, for example, changing labour market and saving behaviour, changing investment behaviour). Examples can be found in Adema and Van Tilburg (2019) and EC (2021). To the extent that these dynamics match with future dynamics, the simulation in this second approach will provide better predictions of the development of public finance variables.

The third approach - let us call it the simulations approach - is by far the most ambitious one. It sets up an overlapping-generations general-equilibrium model to derive the economic behaviour of households and firms from optimality principles (Van Ewijk *et al.*, 2006; Fehr *et al.*, 2005; DREAM, 2015; Kitao, 2015; Lassila and Valkonen, 2018). Like the second approach, this third approach may result in better predictions of the development of public finance variables than the first approach. Moreover, the general-equilibrium approach is more appropriate to calculate the impact of different policy reforms. One can think of many policy reforms that restore fiscal sustainability: raising the retirement age, increasing taxes or social security contributions, cutting public spending, reforming the health care system and so on. These policy reforms will in general have different effects on the economic behaviour of agents. The advantage of general-equilibrium models is that they account for these behavioural changes and their impact on the development of public finance variables.

That being said, it is important to note that many GA studies focus not that much on calculating the effects of different policy reforms, but mainly on assessing the fiscal impact of ageing under different assumptions about environmental and behavioural variables. In that case, the projections approach may be better suited than the simulations approach. Indeed, if one makes simulations assuming constant tax and social security contribution rates, one does not need to bother that much about the impact of tax reform on the labour market behaviour of households.

4.2 Discounting uncertain cash flows

Most GA studies have in common that they use bond interest rates in their projections of future deficits and debts. When these projections are non-stochastic, this amounts to discounting future primary balances with bond interest rates. Note that these interest rates do not need to be risk-free. Sometimes interest rates carry a country risk premium. It is common to assume that the underlying risk of default will not materialize in the projection period, however.

As noted above, the original Auerbach *et al.* (1994) study used a discount rate which was much higher than the interest rate at that time. The motivation to do so was to account for the stochastic nature of primary balances. Later studies have abandoned this approach, except for the GA studies of the Dutch CPB.[8] The latter studies follow a similar reasoning as Auerbach *et al.* (1994). In order to account for the riskiness of future primary balances, they use as discount rate the rate of return on a broad measure of financial wealth and this implies a discount rate that is higher than the risk-free interest rate.

Which of the two approaches is better? Bohn (1995) contained a powerful message. He showed that using a risk-free interest rate can produce misleading conclusions. The appropriate approach is to discount different scenarios with the corresponding stochastic discount rates. This changes

[8]These include Van Ewijk *et al.* (2006), Van der Horst *et al.* (2010), Smid *et al.* (2014), and Adema and Van Tilburg (2019).

the weights attached to different scenarios. Scenarios in which consumption is low (and the marginal utility of consumption is high) get a larger weight than scenarios in which consumption is high (and the marginal utility of consumption is low).

However, this does not necessarily prove that an approach that rests on interest rates is completely wrong. It is true that this latter approach neglects the insights from the theory of asset pricing. On the other hand, the theory of asset pricing rests on an assumption which is not that obvious. Particularly, it assumes complete markets. In our context, this assumption is quite crucial. Assuming complete markets, an investor in the Netherlands and one in Bahrein agree on the valuation of future Dutch primary balances. However, not assuming complete markets means that the Dutch and the Bahraini investor will disagree. Probably, the Dutch investor will use a high discount rate that reflects the positive correlation between consumption in the Netherlands and primary balances in the Netherlands. But the Bahraini investor will apply the risk-free interest rate for discounting as the correlation between consumption in Bahrein and primary balances in the Netherlands is close to zero. The problem is not trivial: empirical evidence suggests that financial markets are far from complete (Sørensen and Yosha, 1998). As far as I know, this problem has not been resolved in the literature.

But suppose we can get rid of this problem and can use the properties of Dutch consumption to assess the value of Dutch primary balances. Does that imply the approach adopted in Auerbach *et al.* (1994) and studies of the Dutch CPB? Not necessarily so. Bohn (1995)'s approach relies on stochastic discount rates, discount rates that differentiate between states of nature and time periods, whereas the former studies adopt discount rates that are uniform across states of nature and time periods. Instead, the ageing studies could potentially benefit from Gollier (2014), which shows that one can derive a discount rate that is uniform across states of nature and time periods if one is willing to make a few additional assumptions. Applying this approach would likely require a distinction between primary spending and revenues, as in Jiang *et al.* (2020). We cannot know as of now whether applying this approach would result in a discount rate that is close to the ones in Auerbach *et al.* (1994) and studies of the Dutch CPB. This makes it an interesting exercise for GA studies to explore this approach in future work.

4.3 Constant net benefits

As mentioned above, it is common to assume unchanged age profiles of spending and revenue items. This is often interpreted as intergenerational neutrality: different generations, including future generations, derive the same (growth-corrected) net benefits from the public sector. Andersen (2012) points out that this approach is not grounded in standard economic theory. The point is that if future generations are richer than current ones, standard economic theory states that their marginal utility of consumption will be lower than that of current generations. Equating the marginal utilities of different generations would imply that the net benefits to future generations should be lower than those of current generations. Needless to say, the criticism does not necessarily hold true once the standard utility function is extended with economic growth (such as in the case of, for example, habit formation).

Apart from that, the concept of unchanged age profiles has been criticized as it narrows public finance to direct income transfers from and to the public sector. Other spending and revenue items are often neglected or necessarily treated in a simplistic way (such as the benefits from civil service which, lacking detailed informaton about their value for different generations, are assumed to have equal value for different generations). One may question the validity of this approach when it comes to investments in the future, however. One example concerns infrastructural investment. The benefits of such investments lie in the future and, hence, will benefit future generations more than current generations, especially the older ones. As these benefits are not taken into account in the common definition of net benefits, application of the idea of constant net benefits would imply that current generations should pay as much (after correction for economic growth) as future generations. Should the benefits from investment in new infrastructure be taken into account, the burden of ageing would fall more on future generations.

There are other examples, think about investments in education and investments that combat climate change. It may be an extremely difficult problem to assess the benefits of these investments that accrue to different generations. But leaving these investments out completely as is standard practice in GA is a very simple and probably too simple answer to this problem.

4.4 Handling uncertainty about the future

Commonly, GA studies present more scenarios to indicate that the future is uncertain. Quite often, these studies calculate a sort of benchmark scenario and a set of alternative scenarios.[9] In the benchmark scenario, the exogenous variables in the model take their most likely values. The alternative scenarios change the time path of one of the exogenous variables. Combining things, one gets an indication of the distribution of the public debt in the future.

To present a set of scenarios is a very useful way to illustrate that results on fiscal sustainability are uncertain. Still, the approach fails to address future uncertainties in the best possible way. One thing is that in defining alternative scenarios, information about the variances of the exogenous variables in the model and their covariances are not taken into consideration. This makes it difficult to attach probabilities to the alternative scenarios. Another thing is that a set of alternative scenarios gives an indication of the future distribution of endogenous variables, but no more than that. By failing to derive the full distribution, the calculations cannot tell us something about the probability that some variable will exceed some threshold value at some point in the future (like the probability that the public debt will exceed the value of 100% gdp in 2060).[10]

Conceptually, it would be quite straightforward to extend calculations in this way. Studies that apply stochastic debt sustainability analysis (SDSA) - to be discussed below - are a good example. Simple fan charts show how the distribution of endogenous variables may evolve over time.

A full stochastic analysis would also be helpful in another way: it would make explicit that fiscal sustainability is actually an abuse of terminology. To explain, let us turn to figure 4. This displays three lines that depict the development of the public debt ratio over time. All three lines are drawn under the assumption that policies have been implemented that make public finances sustainable. The middle line refers to a basic scenario which is identical to the line in panel 1B of figure 1. What are the other two lines? The line with small dots assumes that the interest rate is lower than assumed in the basic scenario. Now, the public debt ratio does not stabilize as with the former scenario, but implodes. Still, the scenario is sustainable. The line with large dots shows the counterpart of the line with small dots. It

[9] Alternatively, OBR (2020) distinguishes three scenarios: a central scenario, a downside scenario and an upside scenario.

[10] Studies undertaken as part of the DEMWEL project, a consortium of European institutions, were different (see Armstrong *et al.*, 2007; 2008 and the contributions of different institutions in Alho *et al.*, 2008). The approach adopted in this project was a stochastic one, albeit that only demographic risk factors were taken into consideration.

assumes that the interest rate takes higher values than assumed in the basic scenario. Now, the public debt ratio explodes over time. Clearly, public finances are now unsustainable.

This exercise illustrates that public finances that are made sustainable conditional on the assumptions of a basic scenario, can turn out to be unsustainable if these assumptions do not materialize. Sustainability is thus a probabilistic concept, as argued by Blanchard (2022). Note that nothing suggests that this is a minor problem. In a stylized case, one can easily show that the probability that sustainable policies will result in unsustainable public finances can be as large as 50%.

< Include Figure 4 >

4.5 A medium-term fiscal sustainability indicator

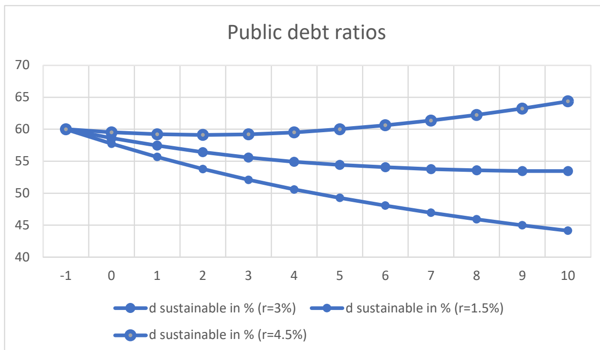
Above we noted that fiscal sustainability does not say anything about the level at which the public debt ratio will stabilize. We also noted that sustainability may even imply that the public debt ratio stabilizes at a negative level. These types of results may be difficult to understand by the general public.

An alternative indicator that does not fall prey to this issue is the so-called medium-term fiscal sustainability indicator.[11] The medium-term fiscal sustainability indicator adopts a target value for the public debt ratio in some future year and calculates how much the primary balance ratio should be adjusted once-and-for-all to achieve this target value. As with the sustainability gap, different scenarios are possible with different values for economic parameters or different types of policy reform, but, unlike the sustainability gap, the target value is the same in all scenarios. Appendix C provides the formula for the medium-term sustainability indicator.[12]

[11] EU studies refer to this medium-term fiscal sustainability indicator as the S1 indicator. The sustainability gap discussed above (a sort of long-term sustainability indicator) is referred to in EU studies as the S2 indicator.

[12] The EU applies this medium-term indicator in its sustainability studies. The 2018 Fiscal Sustainability Report (EC, 2019) calculates this indicator for EU member states based on a target of 60% for the public debt ratio, achievable over a 15-year period (2033). The Congressional Budget Office in the US chooses a longer term. CBO (2020) calculates the primary balance sheet adjustment needed to reduce debt to a level of 79% of GDP (its level in 2019) or 100% of GDP by 2050. The Office for Budget Responsibility in the UK (OBR, 2020) makes two calculations. The first concerns the question of how much fiscal adjustment is needed to stabilize the debt ratio at current levels. The second is about

Figure 4: Public debt ratios when r equals 1.5, 3 or 4.5% when sustainable policies assume r equals 3%



Different from the approach that uses the sustainability gap, the analysis itself does not say anything about what would be a reasonable target value. Hence, adopting this approach might result in unsustainable policies. Moreover, the analysis loses its power to signal the implications of changes in economic circumstances. If, for example, changes in demographic or economic factors deteriorate sustainability, this will go unnoticed if one relies exclusively on the medium-term fiscal sustainability indicator.

5 Critical issues about GA

Above, we have pointed at several issues on which GA studies have been criticized. One is that GA studies not always account for economic behaviour by households and firms for which there is ample empirical evidence. This may give biased predictions, especially when it comes to simulating the effects of policy reforms that aim at restoring fiscal sustainability. A second critical point concerns the way that future stochastic primary balances are discounted. The common approach - use bond interest rates - may give the wrong answer to the question how sustainable are fiscal policies as it neglects insights from the theory of asset pricing. GA studies are pretty unanimous when it comes to the definition of constant net benefits. However, these net benefits are narrowly defined and this may be a handicap when public policies aim at increasing sharply public investments, such as in climate change. Moreover, the treatment of future uncertainties is rather rough. Hence, GA studies are silent on the spread of the distributions of fiscal variables in the future and, therefore, on the probabilities of typical outcomes.

In this section, we discuss two additional points. The first is that GA studies do not work well if the interest rate is structurally below the rate of economic growth; the second concerns the treatment of endogenous interest rates as exogenous variables. We will discuss the two issues in turn.

5.1 What if R<G?

Globally, interest rates have been declining during the last four decades. Recently, interest rates have fallen to levels lower than rates of economic growth. This phenomenon is not unique; it happened before (Schmelzing, 2020).

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what adjustment is needed for the debt-to-GDP ratio to end at 75% of GDP by 2070. Guillemette and Turner (2021) make simulations of what policy adjustments need to be made if the public debt to gdp ratio stabilizes at its current level in 2060.

Until now, we have been assuming that the interest rate is higher than the rate of economic growth. If one instead assumes that the interest rate is structurally lower than the rate of economic growth, the analysis changes fundamentally. In particular, it is no longer possible to calculate the sustainability gap. Technically, the reason is that an infinite sum of discounted primary balance flows no longer converges to some finite value. Economically, the reason is that no adjustment in primary balances is required in order to stabilize the public debt ratio. Indeed, the debt ratio will stabilize automatically if the interest rate-growth differential is negative.

As mentioned above, most GA studies use the interest rate on government bonds to assess sustainability. Still, the R<G problem does not arise in these studies as they make the assumption that the interest rate will converge to a level higher than the rate of economic growth in some finite period of time. In CPB studies the R<G problem does not arise either. CPB studies use a discount rate to assess sustainability. This discount rate adds a risk premium to the interest rate and this risk premium is sufficiently large to ensure that the discount rate exceeds the rate of economic growth.

5.2 The fallacy of an exogenous interest rate

One more salient aspect of GA studies is that they generally assume that interest rates are exogenous. Hence, policy reforms aimed at restoring fiscal sustainability will in general not affect interest rates. This assumption seems too simple, for two reasons. The first one has to do with the nature of interest rates. As they are determined on capital markets, they are endogenous. Hence, if a large number of countries in the world would react to population ageing by decumulating the public debt (or starting to accumulate a public fund that can be used to meet future expenditures, which amounts to the same thing), world saving would increase, putting downward pressure on interest rates. On the opposite, if governments in a large number of countries would increase their debts (perhaps reasoning that interest rates will be below economic growth rates for an extended period of time), interest rates would rise.

The exogeneity assumption is also uneasy for a second reason. Particularly, if a country's public debt becomes so high that financial sustainability is at stake, investors may question the creditworthiness of the country and command a risk premium. The relationship between this risk premium and the public debt ratio may be strongly convex (Ghosh *et al.*, 2013). Hence, the argument may have little relevance for low and medium debt levels, but be very relevant for high debt levels.

6 Alternative approaches of fiscal sustainability

More recently, different approaches have emerged to assess fiscal sustainability. A popular field stems from the work of Reinhart and Rogoff (2010) and focuses on the public debt-growth nexus. Next, there is a field of literature inspired by Bohn (1998) which focusses on fiscal reaction functions - linear or nonlinear. A third new approach is that of debt sustainability analysis in general and the stochastic version of it in particular.

6.1 The public debt-economic growth nexus

The idea that a public debt ratio of 90 percent can be considered a turning point stems from empirical work by Reinhart and Rogoff (Reinhart and Rogoff, 2010). They do not find a clear relationship between the public debt ratio and the rate of economic growth for public debt ratios below 90 percent, but find a negative relationship for debt ratios higher than 90 percent.[13]

Other studies explore not so much whether public debt and economic growth are correlated, but more whether public debt and economic growth are causally related. Checherita-Westphal and Rother (2012) find a statistically significant inverted U-shape relationship between economic growth and the public debt ratio, with a peak at a debt ratio of 90-100 percent. In the same spirit, Baum *et al.* (2013) find that economic growth is increasing or flat with respect to the debt ratio below a level of 95 percent, whereas the relationship turns negative for debt ratios higher than 95 percent.

Other studies draw different conclusions, however. Eberhardt and Presbitero (2015), Woo and Kumar (2015), and Chudik *et al.* (2017) find that debt has a negative effect upon economic growth - this effect is strongest for debt ratios higher than 90 percent, but reject the hypothesis of a turning point. Some literature even calls into doubt whether debt exerts a negative impact upon economic growth at all (Kourtellos *et al.*, 2013; Panizza and Presbitero, 2014; Proaño *et al.*, 2014; Jacobs *et al.*, 2020).

Hence, we conclude that the hypothesis of a turning point for the public debt ratio is not very robust. And if there were a turning point, it is

[13]Later, it turned out some errors had been made in the empirical analysis (Herndon *et al.*, 2014). After correcting for these errors, the result on the turning point could not be reproduced for the data used in the original study. The relevance of the latter finding is not very clear, however. A study of Reinhart, Reinhart and Rogoff that focuses on 26 cases in which the public debt ratio exceeds 90 percent for a period of at least five years also concluded that there is a turning point beyond which economic growth is significantly lower (Reinhart *et al.*, 2012).

implausible that it would be the same for different countries - given the diversity of fiscal institutions across countries. This corresponds to the idea of debt tolerance, which suggests that a given level of public debt can have very different implications for countries that have a different economic history (Reinhart *et al.*, 2003). As indicated above, an increase in the public debt may adversely affect economic growth, especially at very high levels of debt (Égert, 2015). The link of the public debt-economic growth nexus to the debate on fiscal sustainability is weak, however. Although lower economic growth may endanger sustainability, it does not necessarily imply a sustainability problem.

6.2 Fiscal reaction functions, fiscal fatigue and the fiscal limit

At the end of the last century, Bohn (1998) introduced the concept of a linear fiscal reaction function. This fiscal reaction function expresses that the primary balance reacts positively to increases in the public debt ratio. Such a reaction makes it less likely that public finances are unsustainable, as adverse developments in the public debt will automatically invoke a corrective policy reaction. Indeed, Bohn finds that in case of the US in the 1916-1995 period, debt sustainability is not an issue. Bohn (2008), Mendoza and Ostry (2008), and Checherita-Westphal and Žd'árek (2017) draw similar conclusions for the US and other countries; the picture sketched by the evidence in Galí and Perotti (2003) and in Lukkezen and Rojas-Romagosa (2012) is more mixed, however.

A critical issue is whether the fiscal reaction function is linear. There does not seem to be a theoretical basis for such a linearity. Indeed, governments may be less inclined to adjust their primary balances when their debt is already quite high – for example, because it is politically difficult or economically undesirable to undertake an adjustment. In that case, sustainability may still be an issue. Bohn (1998) found this is not worrying: he concluded that the response of the primary balance to debt is *stronger* at high debt ratios. However, Mendoza and Ostry (2008) found that the impact of the debt ratio upon the primary balance ratio is smaller for countries with high debt ratios. They concluded that these countries do face a sustainability risk, despite the positive sign of the debt coefficient. Similarly, Celasun *et al.* (2007) found a kink in the fiscal reaction function that implies a weaker response of the primary balance to debt for high debt ratios. Lukkezen and Rojas-Romagosa (2012) present mixed evidence. Ghosh *et al.* (2013) also found that the fiscal reaction function is nonlinear. In particular, they introduced the idea of fiscal fatigue: at higher levels of the debt ratio it becomes increasingly difficult for governments to adjust their policies if necessary. The implication is that there is a sustainability risk, even if the coefficient of the fiscal reaction function is positive at moderate levels of the public debt ratio.

Based on their estimates of fiscal reaction functions, Ghosh *et al.* (2013) calculated fiscal limits for a number of countries, *i.e.* the levels of the public debt ratio beyond which fiscal policies in these countries are unsustainable. The fiscal limits they calculate are in the range of 150-265% gdp. Fournier and Fall (2017) perform a similar analysis for OECD countries and find fiscal limits in the range 170-335% gdp.

It is a question how robust are the findings of Ghosh *et al.* (2013), however. Although Medeiros (2012) also found fiscal fatigue in an analysis for the EU, Checherita-Westphal and Žd'árek (2017) were unable to find statistical evidence for the results in Ghosh *et al.* (2013). Furthermore, more research into the determinants of the fiscal limit seems needed. Fournier and Fall (2017) note that small changes in the interest rate or the growth rate can have a big impact upon fiscal limit estimates. In addition, there is reason to analyse more explicitly the composition of the public debt. Kim and Ostry (2020) show that a stronger reliance on GDP-indexed bonds and long-term debt can help to raise the fiscal limit and expand the fiscal space.

A slightly different approach to derive a fiscal limit can be found in Bi (2012) and in Jiang *et al.* (2022). This is based on the idea of a Laffer curve for taxes which implies a maximum for tax revenues. The public debt reaches its limit if it is so high that maximum tax revenues are insufficient to keep the debt from increasing. A second alternative approach is also elaborated in Jiang *et al.* (2022). That is that the government may have an incentive to default on its debt. It is beyond the scope of this paper to go into this literature. However, we note that this relates to the risk premia in interest rates that emerge when debt levels become too high.

6.3 Debt sustainability analysis

The tool of debt sustainability analysis (DSA) is fastly gaining popularity (see Barnhill and Kopits (2004) and Celasun *et al.* (2007) for a discussion of the methodology). DSA studies are performed for individual countries (Budina and Van Wijnbergen, 2008; Hajdenberg and Romeu, 2010; Schumacher and Weder di Mauro, 2015) or sets of countries (Celasun *et al.*, 2007;

Lukkezen and Rojas-Romagosa, 2012; Tielens *et al.*, 2014).[14] Like GA, DSA makes simulations of deficit and debt trajectories in the near and far future.

The two approaches feature some subtle differences. DSA does not put central the ageing of the population as GA does. Rather, it takes a somewhat broader view by including all trends that may be relevant for the development of public finances. In addition, DSA does not distinguish between different generations. The concept of intergenerational neutrality, *i.e.* the equality of net benefits from the government for different generations, does not play any role in DSA studies.

These are not the most important differences between GA and DSA, however. More important are, first, the inclusion of fiscal reaction functions, and, second, the form of the simulation exercise. As regards the former, the fiscal reaction functions can play an important role in governing the future of public debt dynamics. They can even make the difference between sustainable and unsustainable public finances. In particular, whereas a GA study may find current fiscal policies to be unsustainable, a DSA study based on largely the same assumptions may conclude that sustainability is not an issue. The fiscal reaction functions included in many DSA studies make fiscal policies flexible to absorb shocks in the public debt ratio. GA on the other hand assumes that current fiscal policies will be maintained forever, even if a small adjustment could prevent policies from becoming unsustainable.

If the fiscal reaction functions used in DSA express the behaviour of future governments (note that past relations may not hold in the future), the predictions in GA studies seem to look too pessimistic. But notice that the two approaches ask different questions. DSA asks what is the most likely development of public finances in the future; GA asks which development of public finances in the future is most likely if current fiscal policies are maintained.

The second major distinction between GA and DSA exercises is the form of the simulation exercise. GA studies commonly calculate a handful scenarios. In DSA and, in particular, in stochastic DSA, the number of simulations is large enough to be able to derive the distribution of public finance variables in the future. The GA approach is very informative, as it shows how robust outcomes are with respect to changes in the values of exogenous variables. But the approach is largely uninformative about the distribution of variables in the future. Tail scenarios, scenarios that are not

[14] EC (2014), Bouabdallah *et al.* (2017) and IMF (2013) present detailed overviews on how to perform DSA, but do not apply it.

that likely to occur but which may have dramatic implications for the fiscal budget, may go unnoticed. In addition, as discusssed above, it is generally impossible to assign probabilities to the different scenarios that GA studies distinguish.

In these respects, stochastic DSA is broader than GA. Stochastic DSA does inform about the distributions of variables in the future and can assess the likelihood of particular scenarios, including tail scenarios (see also Cochrane, 2021). Moreover, by basing the distributions of risk factors on a VAR analysis, any correlations between different risk factors are taken into account. As argued in Celasun *et al.* (2007), uncertainties remain, also with DSA. In particular, it is difficult to account for parameter uncertainty or structural breaks in the statistical properties of the (joint) distributions of risk factors.[15]

7 Concluding comments

What can we conclude from this review about the future of GA, thirty years after their introduction? First of all, the aim of GA has been to inform policymakers that population ageing implies big challenges and that not taking policy action would mean that an often huge burden would be transferred to future generations. This message seems to have been well taken, witness the many countries that have started to increase their pension retirement dates. The translation of a complex problem into one single measure (the sustainability gap) may have helped to get this important message understood.

There are also critical issues. Common across GA studies is the use of the concept of net benefits. These are quite narrowly defined - mainly, the concept focuses on financial transfers between the public and the private sector. In many cases, it is extremely difficult to attribute the costs and benefits of collective arrangements to different generations. But this practice is not without consequences, as the example of investments in climate change forcefully illustrates.

Discounting an uncertain future has been an issue since the start of GA. Remarkably, it has not been resolved. Common practice in GA studies is to use a bond interest rate, whereas an academic literature points to the use of stochastic discount rates. Whereas the latter approach relies on assumptions which may not always be appropriate, it has the virtue of accounting for the uncertainties that surround debt projections. Given the important role of

[15] Hajdenberg and Romeu (2010) explore the role of parameter uncertainty.

uncertainties and their implications for risk-averse individuals, one would welcome future research that could narrow the gap between prescriptions from economic theory and the practice of GA.

A further common practice in GA studies is to calculate sustainability in a handful of different scenarios. As we have argued above, this practice implies that GA can only sketch a very rough picture of the future distributions of fiscal variables, that it cannot say how likely are different scenarios and that tail scenarios - scenarios which are very unlikely to occur, but which may have severe consequences for sustainability, remain unknown. In this respect, GA could benefit from including aspects of stochastic DSA studies into their studies.

Concluding, our review finds GA to be a tool that powerfully demonstrates the fiscal implications of population ageing and the balance between generations. However, GA could benefit from exploring more the role of discount factors, from focussing on more types of net benefits than financial transfers only, and from taking more seriously the huge future uncertainties that are inherent to long-term projections.

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Appendices

A The original generational accounts

Originally, GA derived how much the lifetime net benefits of future generations should be adjusted to obey the intertemporal government budget constraint. We derive a formula for this adjustment in order to show that it is remarkably close to the well-known sustainability gap.

Define D_t as the government debt at the end of year t, r as the interest rate and $B_{t,j}$ as the net benefit in year t of the generation who is j years old. We distinguish T generations with age 0 until $T-1$. Further we assume for simplicity that the interest rate is a constant. These assumptions imply the following budget accumulation equation:

$$D_t = (1+r)D_{t-1} + \sum_{j=0}^{T-1} B_{t,j} \tag{1}$$

We will assume, as is common, that the net benefit terms are proportional to gdp, denoted Y. Further, GA studies assume that the net benefit to gdp ratios are constant through time. Hence, we write $B_{t,j}$ as $b_j Y_t$. Now define the debt ratio, d, as D/Y and assume a constant rate of economic growth, $g \equiv Y_t/Y_{t-1} - 1$. Equation (1) can then be rewritten in the following form:

$$d_t = \left(\frac{1+r}{1+g}\right) d_{t-1} + \sum_{j=0}^{T-1} b_j \tag{2}$$

Throughout this appendix, we will assume that the interest rate exceeds the rate of economic growth, $r > g$. This implies that equation (2) is an unstable first-order difference equation in the public debt ratio.

Forward substitution till the end of time gives an alternative version of this budget accumulation equation:

$$\lim_{N\to\infty} \left(\frac{1+r}{1+g}\right)^{-N} d_{t+N-1} = d_{t-1} + \sum_{i=1}^{\infty} \left(\frac{1+r}{1+g}\right)^{-i} \sum_{j=0}^{T-1} b_j \tag{3}$$

Note that the final term in equation (3) sums over years. For our purpose it is more useful to sum over generations. This allows us to split the final term in equation (3) into a part that corresponds to current generations (the second term at the RHS of (4)) and another part that corresponds to future generations (the third term at the RHS of (4)):

$$\begin{aligned}\lim_{N\to\infty}\left(\frac{1+r}{1+g}\right)^{-N} d_{t+N-1} &= d_{t-1}+\sum_{k=t-T+1}^{t}\sum_{i=1}^{k+T-t}\left(\frac{1+r}{1+g}\right)^{-i} b_{t+i-1-k}\\ &+ \sum_{k=t+1}^{\infty}\sum_{i=1}^{T}\left(\frac{1+r}{1+g}\right)^{-i-(k-t)} b_{i-1} \qquad (4)\end{aligned}$$

Now, we impose the transversality condition that reduces the term at the LHS of equation (4), the discounted value of the debt ratio at the end of time, to zero.[16] In order to remain consistent with equation (4), we add the term z_b to the lifetime net benefits of future generations. This gives us the intertemporal government budget constraint:

$$\begin{aligned}0 &= d_{t-1}+\sum_{k=t-T+1}^{t}\sum_{i=1}^{k+T-t}\left(\frac{1+r}{1+g}\right)^{-i} b_{t+i-1-k} \qquad (5)\\ &+ \sum_{k=t+1}^{\infty}\left[\sum_{i=1}^{T}\left(\frac{1+r}{1+g}\right)^{-i-(k-t)} b_{i-1} - z_b\left(\frac{1+r}{1+g}\right)^{-k+t-1}\right]\end{aligned}$$

This intertemporal government budget constraint tells us that the total public debt should be zero. If there is a positive statutory debt, there should be an equally-sized negative implicit debt. If the statutory debt is positive and the remaining lifetime net benefits for current generations are positive, the lifetime net benefits to future generations must be negative, which will be reflected in the term z_b.

This can be seen more clearly by rewriting equation (5) in terms of z_b:

$$\begin{aligned}z_b &= \frac{(1+r)(r-g)}{(1+g)^2}\left[d_{t-1}+\sum_{k=t-T+1}^{t}\sum_{i=1}^{k+T-t}\left(\frac{1+r}{1+g}\right)^{-i} b_{t-1+i-k}\right]\\ &+ \sum_{i=1}^{T}\left(\frac{1+r}{1+g}\right)^{-i+1} b_{i-1} \qquad (6)\end{aligned}$$

Equation (6) tells us that it is the total public debt that determines the size of the adjustment to the lifetime net benefits to future generations that

[16] It is common in GA studies to impose that beyond some future year the economy reaches a steady state in which the exogenous variables in the model grow at constant rates. The transversality condition then implies that the debt ratio stabilizes.

is needed to ensure sustainability. Both a large statutory public debt and large net benefits contribute to a large adjustment.

Equation 6) can be written more briefly as follows,

$$z_b = \mu_b(d_{t-1} + b^c_{t-1} + b^f_{t-1}) \tag{7}$$

where

$$\begin{aligned}
\mu_b &\equiv ((1+r)(r-g)/(1+g)^2) \\
b^c_{t-1} &\equiv \sum_{k=t-T+1}^{t} \sum_{i=1}^{k+T-t} \left(\frac{1+r}{1+g}\right)^{-i} b_{t-1+i-k} \\
b^f_{t-1} &\equiv \mu^{-1} \sum_{i=1}^{T} \left(\frac{1+r}{1+g}\right)^{-i+1} b_{i-1}
\end{aligned}$$

B The long-term fiscal sustainability indicator

Here, we derive an expression for the long-term fiscal sustainability indicator. We start again with equation (1), but now use the primary balance P rather than the sum of generational net benefits:

$$D_t = (1+r)D_{t-1} - P_t \tag{8}$$

Forward substitution yields the following budget accumulation equation:

$$\lim_{N\to\infty}(1+r)^{-N}D_{t+N-1} = D_{t-1} - \sum_{i=1}^{\infty}(1+r)^{-i}P_{t-1+i} \tag{9}$$

Equation (9) is the counterpart of equation (3). Each of the (discounted) future primary deficits in equation (9) replaces the corresponding sum of (discounted) future generational net benefits in equation (3).

Let us now define p_{t+i} as the primary balance to gdp ratio, P_{t+i}/Y_{t+i}. We can then rewrite equation (9) as follows:

$$\lim_{N\to\infty}(1+r)^{-N}D_{t+N-1} = D_{t-1} - \sum_{i=1}^{\infty}\left(\frac{1+r}{1+g}\right)^{-i} p_{t-1+i}Y_{t-1} \tag{10}$$

Like in the previous section, we now impose the transversality condition. In order to remain consistent, we now add z_p to the RHS of equation (10), the so-called sustainability gap. This gives us the following version of the intertemporal government budget constraint:

$$0 \;=\; D_{t-1} - \sum_{i=1}^{\infty} \left(\frac{1+r}{1+g}\right)^{-i} (p_{t-1+i} + z_p) Y_{t-1} \tag{11}$$

This budget constraint can be rewritten as an expression for the sustainability gap:

$$z_p \;=\; \left(\frac{r-g}{1+g}\right) \left[d_{t-1} - \sum_{i=1}^{\infty} \left(\frac{1+r}{1+g}\right)^{-i} p_{t-1+i} \right] \tag{12}$$

This expression makes clear that the sustainability gap is a fraction of the total public debt, consisting of the statutory debt and the implicit debt, which is defined as the sum of (discounted) future primary deficits. Technically, this is almost equivalent to the expression for the adjustment to lifetime net benefits for future generations that we derived above in appendix A.[17] Economically, their interpretation is very different. The adjustment to lifetime net benefits rests entirely on the new-born generations, whereas the sustainabiity gap is an adjustment to the primary balance that can be achieved in different ways, but will generally affect many if not all generations alive.

Equation (12) is also useful to discuss the double roles of the interest rate and the rate of economic growth. Let us define two extreme cases. One is a case without any population ageing, but with a positive initial debt: $p_{t-1+i} = 0$ for $i \geq 1$, $d_{t-1} > 0$. In this case, an increase in the interest rate increases the sustainability gap. Higher interest payments require that the primary balances are increased upwards. The second case is one without initial debt, but with population ageing that takes the form an increase in primary deficits over time: $p_{t-1+i} < p_{t-2+i}$ for $i \geq 1$, $p_t < 0$, $d_{t-1} = 0$. Now, the effect of a higher interest rate is negative: a higher interest rate reduces the weight of high primary deficits in the far future and thereby lowers the sustainability gap. Reality will likely fall in between these two cases. Hence, in reality the effect of the interest rate upon the sustainability gap can have either sign, depending on the size of the initial debt and the pattern of future primary deficits.

A similar comment can be made about the rate of economic growth. In the former case higher growth reduces the unsustainability of fiscal policies

[17] Almost as the coefficients corresponding to the initial public debt in equations (6) and (12) differ a factor $(1+r)/(1+g)$. This is entirely due to the fact that the adjustment of lifetime net benefits to future generations in equation (6) starts in year $t+1$, whereas the sustainability gap in equation (12) is implemented immediately in year t.

(we keep assuming that $r > g$). A smaller adjustment to primary balances is required to maintain sustainability as higher growth reduces the instability of the debt ratio. In the latter case, the opposite occurs. Higher growth increases the weight attached to high primary deficits far in the future or, stated alternatively, high growth increases deficits far in the future more than in the near future, as deficits are proportional to gdp. In the latter case, higher growth thus increases the sustainability gap. Like with the interest rate, in reality the effect of the rate of growth upon the sustainability gap can then have either sign, depending on the size of the initial debt and the pattern of future primary deficits.

C The medium-term fiscal sustainability indicator

The medium-term fiscal sustainability indicator is derived by imposing that the public debt ratio achieves some target value in some year in the future. The expression that we derive for this medium-term fiscal sustainability indicator is very similar to the one derived in appendix B for the long-term fiscal sustainability indicator.

We start by recalling the budget accumulation equation of the previous section (10):

$$\lim_{N\to\infty}(1+r)^{-N}D_{t+N-1} = D_{t-1} - \sum_{i=1}^{\infty}\left(\frac{1+r}{1+g}\right)^{-i} p_{t-1+i}Y_{t-1}$$

Rather than imposing the transversality condition, we now impose that the debt ratio after N years is equal to $\hat{d}$. Again, a sustainability indicator is added to the RHS of the budget accumulation equation in order to keep consistency:

$$\left(\frac{1+r}{1+g}\right)^{-N}\hat{d} \;=\; d_{t-1} - \sum_{i=1}^{N}\left(\frac{1+r}{1+g}\right)^{-i}(p_{t-1+i}+z_m) \qquad (13)$$

We rewrite equation (13) as an expression for the sustainability indicator:

$$\begin{aligned} z_m \;=\; & \left(\frac{r-g}{1+g}\right)\left(1-\left(\frac{1+r}{1+g}\right)^{-N}\right)^{-1} \times \\ \times \; & \left[d_{t-1} - \left(\frac{1+r}{1+g}\right)^{-N}\hat{d} - \sum_{i=1}^{N}\left(\frac{1+r}{1+g}\right)^{-i}p_{t-1+i}\right] \end{aligned} \qquad (14)$$

There is a clear correspondence between the expressions for this medium-term sustainability indicator (14) and for the sustainability gap (12). In both cases, the gap or indicator multiplies the total public debt with some proportionality factor. Further, there are three differences. First, in the case of the sustainability gap, primary deficits up to infinity are taken into account. In case of the medium-term sustainability indicator, only the primary deficits up to the target year N are taken into account. Second, the proportionality factors are different. Thirdly, the medium-term sustainability indicator relates to the target value for the debt ratio.

The impact of this target upon the debt is negative. The lower the debt target, the higher will be the sustainability gap. The size of this effect depends on the number of years that policies can take to achieve the target. The further away the target year, the lower is the impact of the target debt. Indeed, if we choose infinity for the target year, the target debt ratio does not play any role and the expression for the medium-term sustainability indicator equals that for the sustainability gap.