CPB Netherlands Bureau for Economic Policy Analysis

CRASH - A Simulation Model for Extreme Events

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In addition, we study the role of initial conditions. In particular, we explore to what extent a substantially different leverage ratio of banks, funding ratio of pension funds and loan-to-value ratio of households changes the impact of extreme external shocks.

CPB Background Document

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Abstract

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Non-technical summary

The global financial crisis, the European sovereign debt crisis and recently the Covid 19 outbreak illustrate that public finances are vulnerable to large adverse fiscal shocks. The past 5 decades counted 4 years in which the Dutch public debt to GDP ratio increased more than 5% points, with the largest increase stemming thus far from the global financial crisis (12% points).

To assist policymakers in understanding the exposure of public finances to tail risk events, fiscal stress tests can be performed. In such fiscal stress tests various risks to public finances are brought together in a combined shock scenario. The fiscal stress tests identify the channels through which such tail events could impact the public finances, indicate the potential impact of these events on the public debt and illustrate the importance of a buffer (*i.e.*, a low public debt).

Since Budget Day 2011 CPB publishes every two to three years a fiscal stress test. The past 3 times the CPB macroeconomic model SAFFIER has been used to conduct the fiscal stress tests. SAFFIER is a standard macroeconometric model, specifically developed to forecast on the short and medium term. It is, however, not specifically developed for fiscal stress testing. As a consequence certain mechanisms having little relevance in non-tail risk events but high relevance in tail risk events, are not included in SAFFIER. Ad hoc adjustments are then the practical solution for fiscal stress tests simulated by SAFFIER. Mechanisms highly relevant in tail risk events are, for instance, credit rationing in the banking sector or a drop in consumer confidence, driven by, for example, the curtailing of pensions benefits or a fall of housing prices.

To address these practical difficulties, CPB developed a model, named CRASH[1], for fiscal stress testing. To fit the purpose of fiscal stress testing, the model should in the first place include the relevant mechanisms at play in a tail risk event, *e.g.* non-linearities in various sectors of the economy. Further, since uncertainties in tail risk events are typically large, it does not make sense to capture too much detail. Rather, the model should be kept stylized so that the results can be easily interpreted in terms of the underlying mechanisms. Academically popular macroeconomic models like DSGE are typically less suitable in practice for the purpose of stress testing, which requires a combination of forecasting, policy simulation and storytelling.

Calibrating a fiscal stress test model is a challenge, since we have only little data that can be used for calibration. To account for this gap, we rely

[1] CRASH stands for Chaos and Recession After a SHock.

on several techniques: at some points, we use empirical estimates from the international literature, we sometimes replicate macroeconomic variables in 2016-2018 and we sometimes replicate the impact of the Great Recession upon macroeconomic variables. Next to that, we have made use of simulation results in SAFFIER in order to mimic SAFFIER in the case of more normal shocks. In order to get a feeling of the magnitude of the possible errors involved, we perform sensitivity analysis.

There are possibly hundreds of ways to define extreme events. It is clear that in such an event, many variables move together, but the exact combination is unknown and will generally be different from one crisis to another. For practical reasons, we therefore adopt the combination as advocated by the IMF for macroeconomic stress tests, which in turn is based on stress tests that are performed for the banking industry.

Not surprisingly, the simulations indicate that extreme events can have a huge impact upon the economy and upon public finances. A benchmark simulation calculates a loss of output of more than 9 percent after two years, relative to the level of output that would have realized if no financial crisis had occurred. The economy restores after some years such that the relative loss of output is only about 6 percent after 5 years. The public debt ratio reflects that the impact of the crisis is big and long-lasting: after 5 years, this debt ratio is about 35 percentage point higher due to the crisis.

Partly, these results can be explained by the fact that the Dutch economy is strongly connected to other countries in the world economy. In addition, the Dutch economy stands out internationally in the size of the mortgage industry, the assets of pension funds and the size of the banking sector. As a consequence, one might expect that it matters for the impact of an extreme event how sizeable are mortgages relative to housing wealth, how strong is pension wealth relative to liabilities and how sizeable is bank equity relative to banking debt. We have done some experiments to assess the role of these factors. We find that all three factors play a role. More bank equity, lower mortgages and higher pension wealth all imply that a financial crisis will hit the Dutch economy less hard.

1 Introduction

In 2008 a financial economic crisis disrupted the world economy. As a result, public deficits and public debts sky-rocketed in a number of countries. Since then, research has focused on the question why the crisis could be so deep. Which factors returned a local, financial shock into a worldwide macroeconomic recession to which we refer now as the Great Recession?

Many have argued that debt, public or private, has played a role (see, *e.g.* Koo, 2008; Hall, 2011; Mian and Sufi, 2011; Eggertsson and Krugman, 2012). Large debt positions can make it difficult to absorb shocks which could have been absorbed quite easily if debt would have been less large. Without doubt, other factors will also have played a role. One example is that of linkages, within the financial sector, between this sector and the rest of the economy, and between different countries in the world.

This document constructs a model in order to explore how external economic shocks affect Dutch public finances. The shocks typically combine a series of shocks: in equity prices, housing prices, world trade, interest rates, liquidity on financial markets.

The model we construct is quite similar to other models at CPB, like the SAFFIER model. There are three important differences, however. First, the model in this document is much more stylized than SAFFIER. Second, the model in this document includes banks and pension funds that can play an important role in changing the impact of shocks. Thirdly, the model in this document includes a number of nonlinearities, *i.e.* in the banking sector, in the pension sector and in the household sector. These nonlinearities imply that the size of a shock relates to the initial state of the economy. Hence, we are back then where we started when we argued that the levels of debt present at the onset of the Great Recession may have contributed to changing a local ripple in US housing markets into the Great Recession.

A major difference with many existing models of the macroeconomy is the focus of the current model. Here, the focus is to provide a stress test for public finances, just as banks are nowadays required to perform stress tests. The stress test as performed in this paper focuses on extreme, but unlikely events. In no way whatsoever should the simulations be interpreted as projections, but as an analysis of the vulnerability of public finances to extreme economic conditions. Such stress tests become more common internationally (see, *e.g.* Benes *et al.*, 2014a, 2014b; OBR, 2017), as fostered by the IMF (IMF, 2016).

The structure of this document is as follows. Section 2 discusses the propagation of shocks and how this relates to the initial conditions referred to above. Sections 3, 4 and 5 describe in turn the specification of shocks, the model and its calibration. Next, section 6 reports on simulation results and section 7 offers concluding remarks.

2 Three nonlinearities

We include three nonlinear elements in our model. These relate to the leverage ratio of banks, the loan-to-value ratio of households and the funding ratio of pension funds. We discuss them in turn.

The leverage of banks is important as banks finance a large part of their asset positions with debt. Hence, bank equity is (far) less than total assets. A loss on bank assets, due to an economic downturn for example, implies an equally-sized loss of bank equity if measured in euros. In relative terms however, the loss in bank equity will be (much) higher than the loss in asset value. If we use L to denote the leverage position of a bank, *i.e.* its equity relative to total assets, then a one percent change in the value of bank assets implies a $1/L$ percent change in bank equity. Hence, the lower the leverage of the bank, the bigger will be the impact of the loss of asset value. This in turn will have macroeconomic implications if banks in reaction adjust their supply of bank credits, as suggested by empirical evidence (Peek and Rosengren, 199; Adrian and Shin, 2010; Puri *et al.* 2011).

The role of private debt of households is different. If households suffer from a loss of wealth, for example due to a drop in housing prices, they may reduce their private consumption. Empirically, this effect is stronger, the higher is the amount of household debt as reflected in the loan-to-value (LTV) ratio. Indeed, Dynan (2012) finds that the marginal propensity to consume (MPC) out of housing wealth is 10.5% for a LTV ratio of 84-88%, about two times as high as the corresponding figure for a LTV ratio of 32-37%. Similarly, Mian *et al.* (2013) demonstrate that a three times higher initial LTV ratio blows up the MPC out of housing wealth with a factor three. Recently, two Dutch studies found that also in the Netherlands the MPC out of housing wealth is decreasing in housing wealth (Ji *et al.*, 2019; Zhang, 2019).

A third factor is more typical for the Netherlands. In the Netherlands, the bulk of saving is done through pension funds. These funds provide pensions that can best be characterized as collective defined contribution pensions. Hence, the level of the benefits that pension funds provide is higher, the higher is their funding ratio. Now, this function is concave, reflecting that - conditional on an adverse shock - pension funds are required to change their policies more drastically, the worser is their funding ratio. For in case of a mild change in its funding ratio, a pension fund may be required to adjust the indexation of its pensions. But in case of a huge funding deficit, supervisory rules require the fund to reduce the nominal value of its pensions. The implication is that adverse shocks will have a modest impact upon pension benefits is funding ratios are strong, but a bigger impact if funding ratios are weak.

3 Risk factors

This section documents the specifications for the risk factors in the model. This serves two purposes. First, the specifications give an idea about the propagation of shocks through time. Second, they allow us to explore shocks that are a multiple of their respective standard deviations. Knowledge of the various standard deviations is important if one wants to compare different shocks that are more or less equally likely to occur. Although we will not use these specifications in this document, they may be useful in future work.[2]

The economic environment consists of five risk factors. These are the nominal interest rate on government bonds, denoted i_b; the rate of return on foreign equity, denoted r^*; the rate of return on housing investment, denoted r_h; the level of world GDP, denoted Y^*; and the interest rate on loans between banks, denoted r_{J^*} (throughout the paper, we use asterisks to denote foreign variables). The equity return variable r^* is measured in terms of foreign goods. Relevant for agents in our economy is the same variable expressed in the domestic good, denoted as r_{F^*}. Abstracting from foreign price inflation and exchange rate variations, r_{F^*} can be expressed as $r^* - \tilde{p}$, where $\tilde{p}$ refers to the rate of domestic price inflation (throughout the paper, we use a tilde to denote the growth rate of a variable).

One may argue that not all these risk factors are truly exogenous. More specifically, housing prices are also related to local economic conditions. We do not attempt to give a full explanation of housing prices, however (or of one of the other risk factors). Our research question is more modest: what is the impact of a big, unexpected fall of housing prices that is part of an extreme, external shock?

For the interest rate, we use a specification that Chan *et al.* (1992) find to perform good in a comparison of eight alternative models. This specification is a random walk without drift and volatility that depends on

[2]One can elaborate this even more by performing a VAR analysis in order to get an understanding of the correlations between the various risk factors.

the level of the interest rate:[3]

$$i_{b,t} = i_{b,t-1} + \epsilon_{b,t} \tag{1}$$

Here, ϵ_b is an error term with zero mean and standard deviation equal to $0.34 i_{b,t-1}$.

For the rate of return on (foreign) equity, we adopt the specification in Draper *et al.* (2014).[4] This specifies the equity rate of return as a normally distributed variable with constant variance, but a mean that is dependent on the real interest rate on bonds,

$$r_t^* = (i_{b,t} - \tilde{p}_t + 0.054) + \epsilon_{F^*,t} \tag{2}$$

where the error term $\epsilon_{F^*,t}$ has zero mean and a standard deviation equal to 0.15. Note that this specification implies an equity premium equal to 5.4 percent.

For structural output growth, we adopt the AR1 process estimated by Blanchard and Simon (2001) for the period 1982-2000,[5]

$$\tilde{Y}_t^* = 0.05\tilde{Y}_{t-1}^* + \epsilon_{Y^*,t} \tag{3}$$

with $\epsilon_{Y^*,t}$ an error term with zero mean and standard deviation 0.0064. This is quite close to a random walk (Campbell and Mankiw, 1987).

We equate the rate of return on housing to the rate of real housing price inflation. For the latter we adopt an AR1 process,

$$\tilde{p}_{h,t} = 0.78\tilde{p}_{h,t-1} + \epsilon_{h,t} \tag{4}$$

where $\tilde{p}_h$ denotes the rate of real housing price inflation and $\epsilon_{h,t}$ an error term with zero mean and standard deviation 0.0134.[6]

For the interest rate on bank loans, we do not specify a stochastic equation.

4 The model

The model consists of four blocks: the public sector, the macroeconomy, banks and the market for bank credits. We review them in turn.

[3]This specification is not consistent with the secular decline of the interest rate in the last three decades or so.

[4]This specification is based on that in Koijen *et al.* (2010). Whereas the latter paper uses US data, the estimate in Draper *et al.* (2014) uses data for the Netherlands.

[5]We have converted the estimation in Blanchard and Simon (2001) on quarterly data to a specification in annual terms. Furthermore, we neglect the drift parameter in their estimated equation.

[6]Thanks to Stefan Groot for providing me this specification.

4.1 Public finances

We start with modelling the crash component of government spending. This refers to the spending that results from guarantees by the government. We distinguish between explicit guarantees and implicit guarantees. The first refers to the guarantees that are laid down in rules. The value of these guarantees is public information. With implicit guarantees we refer to any government spending which is the result of economic chaos and for which no reserves have been made.

We distinguish three types of explicit guarantees. The first relates to mortgage debts. If housing prices fall, part of the households may be unable to redeem their mortgages. Part of these mortgages is guaranteed by a national fund *Waarborgfonds Eigen Woningen* (WEW). In most cases, the buffers of this fund will be sufficiently large to absorb the shock in housing prices. Only if the shock is extremely large, may these buffers be insufficient. Only then, the government might step in and spend on mortgage guarantees. Thus, we postulate that crash spending related to mortgage debts is a function of the size of the shock in housing prices, the outstanding value of mortgage debts and a threshold indicating the reserves of the WEW.

Second, a fall in the exports of goods and services may induce additional government spending. Part of the firms that export goods and services are insured against the risk of not getting paid by their counterparties. If an adverse world trade shock occurs that is too extreme, the buffers that have been built up in the past to compensate exporters might be insufficient. This type of spending thus relates to the size of the shock in world trade and a threshold, indicating the size of the buffers.

A third type of guarantee relates to the banking sector. In case of a sufficiently large financial shock, the banking sector will face a deterioration of its equity. In case of a bank failure, the Bank Recovery and Resolution Directive (BRRD) requires burden sharing by shareholders and creditors of at least 8 per cent of the bank's liabilities through bail-in, before resolution authorities can look to access other forms of stabilisation funding such as recapitalisation with public funds. Hence, there is not an explicit role for the national government in case of financial distress. Things are different if there is an appeal on the deposit guarantee. Again, there is an institute that has accumulated reserves. But if these reserves prove to be insufficient, then the government has to step in in order to guarantee the deposits.

Moreover, in case of banks we also think of an implicit guarantee. History shows that governments may set aside rules if they deem this necessary to avoid a collapse of their economies. The measures undertaken during the

Great Recession illustrate this most clearly. In general, there is no rule saying when such an implicit guarantee will become active, so the only thing we can do is make an assumption on this as good as possible. Concretely, we will assume that government spending in case of financial distress in the banking sector relates to the size of the shock in relation to the outstanding amount of bank equity and a threshold. In the simulations we will show below, it turns out that the only relevant guarantees are implicit guarantees.

Hence, we propose the following specification for crash public spending,

$$
\begin{aligned}
H_t &= H_t^h + H_t^X + H_t^E \qquad (5)\\
H_t^h &= max(-\phi_h((1+r_{h,t})W_{W,t-1} - R_{W,t-1}) - H_{min}^h, 0)\\
H_{min}^h &> 0\\
H_t^X &= max(-\phi_X \Delta X_t - H_{min}^X, 0)\\
H_{min}^X &> 0\\
H_t^E &= \phi_H max(-\tilde{E}_{t-1} + \Lambda(R_t + A_t^*) + H_{min}^E, 0)\\
H_{min}^E &> 0
\end{aligned}
$$

where Δ is the difference operator, *i.e.* for any variable z, $\Delta z_t \equiv z_t - z_{t-1}$.

H^h relates to $((1 + r_{h,t})W_{W,t-1} - R_{W,t-1})$, which is the net housing wealth of households (W_W denotes housing wealth and R_W denotes mortgage debts). Equation (5) thus tells us that government spending occurs if net housing wealth falls below a certain threshold which refers to the reserves that been accumulated in the WEW fund. The fraction $0 < \phi_h < 1$ indicates that a shock in housing prices will present a financial problem to only part of the households (for many households, mortgages are small relative to their housing wealth or even zero) and that only part of the mortgages is guaranteed (mortgages exceeding 260.000 euro are not covered by the Nationale Hypotheek Garantie (NHG), for instance). Spending is thus only non-zero in case of an extremely large shock and occurs only for a fraction of the loss of housing wealth.

Spending H^X relates to X, which denotes the export of goods and services. The expression for H_t^X indicates that only shocks that exceed a certain threshold give rise to government spending. The fraction $0 < \phi_X < 1$ indicates that only part of the exports is covered by a government guarantee. Hence, similar to the case of housing prices, in the case of export losses government spending occurs only if export losses exceed some (large) threshold.

H^E relates to $\tilde{E}$, which will be defined below as the value of bank equity after a financial crisis has reduced it. $\Lambda(R+A^*)+H_{min}^E$ denotes the minimum amount of equity that the bank supervisor requires banks to have. If bank equity falls below this amount, the government is assume to inject money into the bank. Absent any shock or in case the shock is not too large, no government spending will occur. Note that H_t^E should be interpreted as an implicit guarantee, as discussed above.

Apart from this crash spending, primary public spending consists of public consumption (G) and income transfers (O). Both spending items relate partly to the stance of the economy. In particular, public spending consists of a structural component, denoted $\dot{G}$ and a component related to GDP, which we denote Y (throughout the paper, we denote structural variables with a dot). Income transfers consist of a structural component, $\dot{O}$, and a component related to the level of unemployment, denoted U. Hence, we have

$$G_t = \dot{G}_t + \gamma_G Y_t \qquad \gamma_G \geq 0 \tag{6}$$

$$O_t = \dot{O}_t + p^U U_t \tag{7}$$

where p^U denotes the real unemployment benefit.

Tax revenues, denoted T, consist of five items: the revenues from labour income taxation (T^l), capital income taxation (T^k), consumption (T^C), investment (T^I) and revenues that relate to the development of structural GDP (T^G).

$$\begin{aligned}
T_t &= T_t^l + T_t^k + T_t^C + T_t^I + T_t^G \\
T_t^l &= \tau^l (Y_t^l + O_t + Y_{P,t} - S_{P,t}) \\
T_t^k &= \tau^k \left(\eta_1 Y_t^k - \eta_2 Y_{t-1}^k - (\eta_1 - \eta_2 - 1) Y_{t-2}^k \right) \\
T_t^C &= \tau^C (C_t + S_{W,t}) \\
T_t^I &= \tau^I I_t \\
T_t^G &= \tau^G \dot{Y}_t
\end{aligned} \tag{8}$$

Taxes on labour income, consumption and investment are proportional. The tax base for labour income taxation adds labour income (Y_l), income transfers and pension benefits and subtracts pension contributions (S_P). This mimics Dutch institutions by including pension benefits in the income tax base and excluding pension contributions to the pension scheme from this tax base. The tax base for consumption taxation is simply the spending on consumption, C, and housing investment, S_W. The tax base for investment taxation is gross investment, I.

The capital income taxation scheme is nonlinear. Empirically, the revenues from this tax respond more than proportionally to changes in capital income, Y^k, in the short run. In order to account for that, we adopt a moving-average specification in which the weight of the contemporaneous term (η_1) is larger than one. By restricting the coefficients of the scheme, proportionality is achieved in the long run. See appendix B for further details.

Finally, part of the tax revenues is related to structural GDP.

Given the specifications for primary public spending and tax revenues, the equations for the public deficit (D) and the public debt (B) are relatively straightforward, except for the crash spending term H. We will assume that crash spending ends up fully in the public deficit and the public debt.

$$D_t = G_t + O_t - T_t + H_t + (i_{b,t}/(1 + \tilde{p}_t))B_{t-1} \tag{9}$$

$$B_t = D_t + (1/(1 + \tilde{p}_t))B_{t-1} \tag{10}$$

4.2 The macroeconomy

Next to the government, our model distinguishes consumers, firms, pension funds and banks. We first review the equations that describe their behaviour. Then, we describe the assumptions for the markets for output, labour and bank credit.

Consumers

We posit that private consumption is driven by both income and wealth. This corresponds to a large empirical literature that finds that both income and wealth are determinants of consumption spending (see, *e.g.* Campbell and Mankiw, 1990; Lusardi, 1996). In addition, we allow different components of household wealth to have different propensities to consume (Hendry and Muellbauer, 2018). Hence, we posit the following consumption equation:

$$\begin{aligned}
\hat{C}_t &= \frac{1}{1+\tau^C}\Big[\alpha_Y\left(Y_t^d + r_J J_{C,t-1} - r_{R,t} R_{C,t-1}\right) \\
&+ \alpha_W\Big(F^*_{C,t-1}(1+r_{F^*,t}) + J_{C,t-1} - (R_{C,t-1} - R_{W,t-1})\Big) \\
&+ \alpha_P V_{P,t-1} + \alpha_R\Big(W_{W,t-1}(1+r_{h,t}) - R_{W,t-1}\Big)\Big] \\
& \qquad\qquad \alpha_Y, \alpha_W, \alpha_P > 0 \qquad (11) \\
\alpha_R &= \alpha_L\Big(W_{W,t-1}(1+r_{h,t}) - R_{W,t-1}\Big)^{-\alpha_X} \qquad \alpha_L > 0, 0 < \alpha_X < 1 \\
C_t &= \lambda_C C_{t-1} + (1-\lambda_C)\hat{C}_t \qquad 0 \le \lambda_C < 1
\end{aligned}$$

Household income adds disposable labour income, Y_t^d, and capital income, combining interest earned on bank deposits and interest paid on bank loans. We split household wealth into three components, namely private financial wealth, pension wealth and net housing wealth and allow the marginal propensity to consume to differ between the three groups, reflecting differences in their liquidity.

Private wealth consists of equity and bank deposits net of bank loans. In order to avoid complex simultaneities in the model, we assume households hold only equity issued by foreign firms. Net housing wealth is defined as the difference between housing wealth and mortgage debts. We take the coefficient of net housing wealth a function of the LTV ratio, *i.e.* mortgage debt relative to housing wealth. This allows us to account for the finding that the consumption response to a change in house prices is larger for highly leveraged households (*e.g.*, Cooper, 2013; Mian *et al.*, 2013).[7] Indeed, equation (11) implies that the marginal propensity to consume out of housing net worth can be expressed as $(\alpha_L(1-\alpha_X)/(1+\tau^C))(W_{W,t-1}(1+r_{h,t})(1-\Xi_{t-1}))^{-\alpha_X}$, where Ξ_{t-1} denotes the LTV ratio, *i.e.* $R_{W,t-1}/(W_{W,t-1}(1+r_{h,t}))$. This marginal propensity to consume is increasing in the LTV ratio. Finally, we allow for a lag between preferred consumption and actual consumption, as displayed in the last line of equation (11).

[7] In a study for the Netherlands, Bijlsma and Mocking (2017) do not find that the drop in consumption is larger for more leveraged households. Still, leverage can play a role if there is a negative effect of leverage upon the decision of households to move to a different house. The finding of Morescalchi *et al.* (2017) that there is a negative effect of leverage upon job mobility suggests that this mechanism may play a role.

Disposable labour income results when we take labour income (Y^l), add income transfers (O) and pension income (Y_P), and subtract labour income tax revenues (T^l), income-independent tax revenues (T^G) and pension contributions (S_P):

$$Y_t^d = Y_t^l + (O_t + Y_{P,t}) - (T_t^l + T_t^G + S_{P,t}) \tag{12}$$

Here, pension income Y_P refers to the pension benefits that are paid out by pension funds; the income from the government-provided basic pension is included in the variable O.

As mentioned above, households invest in houses. We assume households prefer housing investment and consumption to be related. Specifically, we assume that housing investment is a fixed fraction of consumption,

$$S_{W,t} = \alpha_C C_t \qquad \alpha_C > 0 \tag{13}$$

where S_W denotes housing investment.

A fraction of $1-\sigma_W$ of this housing investment is financed out of current income; the remainder is financed out of bank mortgages:

$$\Delta R_{W,t} = \sigma_W S_{W,t} \qquad 0 < \sigma_W < 1 \tag{14}$$

where S_W denotes housing investment.

Finally, we have an accumulation equation for housing wealth,

$$W_{W,t} = W_{W,t-1}(1 + r_{h,t}) + S_{W,t} \tag{15}$$

Firms

We postulate an equation for the demand for capital, denoted $\hat{Z}$, that is poportional with output and a negative function of the cost of capital.[8] The latter equals the sum of the real interest rate and the rate of depreciation of capital, denoted δ_Z:

$$\hat{Z}_t = \gamma_Y \ddot{Y}_t (r_{R,t} + \delta_Z)^{-1/(1-\rho)} \qquad \gamma_Y > 0, 0 < \delta_Z < 1 \tag{16}$$

Here, $\ddot{Y}$ denotes medium output, defined as the geometric mean of contemporaneous output and structural output, *i.e.* $\ddot{Y}_t \equiv \sqrt{Y_t \dot{Y}_t}$, where structural output will be defined below.

[8]One can derive such an equation by assuming that firms are profit maximizers and output is generated by a CES production function.

Investment, denoted I, can be derived from the demand for capital by inverting the capital accumulation equation. We will assume that because of adjustment costs actual investment gradually adjusts to this preferred investment:

$$\begin{aligned} \hat{I}_t &= \hat{Z}_t - (1-\delta_Z)Z_{t-1} \\ I_t &= \lambda_I \hat{I}_t + (1-\lambda_I)I_{t-1} \qquad 0 < \lambda_I < 1 \end{aligned} \tag{17}$$

It is this actual investment that governs the actual stock of capital, Z:

$$Z_t = (1-\delta_Z)Z_{t-1} + I_t \tag{18}$$

Finally, we define structural output, which follows from a combination of capital and the working population, denoted N:

$$\dot{Y}_t = \left(N_t^\rho + \kappa Z_{t-1}^\rho\right)^{1/\rho} \qquad \rho < 1, \kappa > 0 \tag{19}$$

Pension funds

End-of period wealth of pension funds can be obtained through the accumulation equation,

$$W_{P,t} = W_{P,t-1}(1 + r_{P,t}) + S_{P,t} - Y_{P,t} \tag{20}$$

where the rate of return on the portfolio of pension funds averages the rates of return on bonds and on foreign equity.

$$r_{P,t} = \omega_P^b(i_{b,t} - \tilde{p}_t) + (1-\omega_P^b)r_{F^*,t} \tag{21}$$

We model the pension scheme as a collective defined contribution scheme. That means that financial market shocks are absorbed by changing benefits rather than contributions as is the case in a traditional defined benefit scheme. The scheme that is dominant in the Netherlands nowadays probably comes closer to a collective defined contribution scheme than to a defined benefit scheme (Bovenberg and Nijman, 2009). Indeed, over the last decade indexation cuts have become the most important instrument to absorb shocks, whereas the role of pension contributions has diminished (Parlevliet and Kooiman, 2015).

As a consequence, pension contributions are defined as the product of a constant pension contribution rate (σ_P) and labour income:

$$S_{P,t} = \sigma_P Y_t^l \tag{22}$$

The equation for (the change in) pension benefits is more complex. This is due to the fact that the development of pension benefits depends on the funding ratio and the fact that different pension funds have different funding ratios.[9] The behaviour of aggregate pension benefits thus depends on the development of benefits in different regimes and the distribution of funds over the different regimes.

In particular, the rate of growth of nominal pension benefits for an individual fund i, $y_{P,i}$, relates to the fund's initial funding ratio, denoted $K_{i,t-1}$:

$$\begin{aligned}
\tilde{y}_{P,i,t} &= \Upsilon_1(K_{i,t-1}) = 0.1(K_{i,t-1} - K_{max}) \qquad K_{i,t-1} < K_{min} \\
&= \Upsilon_2(K_{i,t-1}) = \Big(\frac{K_{i,t-1} - K_{min}}{K_{max} - K_{min}}\Big)\tilde{p}_t \quad K_{min} < K_{i,t-1} < K_{max} \\
&= \tilde{p}_t \qquad\qquad\qquad\qquad\qquad K_{i,t-1} > K_{max} \qquad (23)
\end{aligned}$$

Equation (23) reflects that there are three options for the change in the benefit level. Pension fund i provides full indexation if the funding ratio is sufficiently high ($K_i > K_{max}$) and partial indexation for a medium funding ratio ($K_{min} < K_i < K_{max}$). As an implication, the fund does not index pensions if $K_{i,t-1} = K_{min}$. The values of the critical funding ratios K_{max} and K_{min} depend on the indexation policy of a pension fund. In reality, these are different for different funds. The values that we work with are consistent with the average for the pension sector (Willis Towers Watson, 2018).

If the funding ratio is sufficiently low ($K_i < K_{min}$), the pension fund applies negative indexation (a nominal pension cut). This reflects supervisory policies that require pension funds to apply negative indexation if, without this measure, they are expected not to be able to achieve some target funding ratio within a period of 10 years.[10] In reality, the value of the funding ratio at which this policy rule becomes active, is different for different funds. The same holds true for the target funding ratio that the pension

[9]This heterogeneity may reflect that pension funds are heterogeneous in terms of their wealth, in terms of their pension liabilities, or both.

[10]The Pension Law requires pension funds to apply negative indexation also if their funding ratio has been below some minimum required funding ratio for five years. We assumes that this rule will be overruled by the requirement of recovery within 10 years time.

fund should be able to achieve within 10 years time. To avoid the model becoming unnecessarily complex, we equate these two critical funding ratios to K_{min} and K_{max} respectively (see equation (23)). This is not particularly relevant as other specifications will imply a similarly shaped equation for the growth of pension benefits.

The initial funding ratio thus determines how the level of benefits provided by a specific fund responds to a decline of the funding ratio. In general, equation (23) implies a kinked concave relation between the growth of benefits and the initial funding ratio. Formally, the derivative of $\tilde{y}_{P,i,t}$ with respect to $K_{i,t-1}$ equals 0.1, $\tilde{p}_t/(K_{max} - K_{min})$ and zero for the three respective regimes.

The rate of growth of aggregate nominal pension benefits is the weighted average of the changes at the microeconomic level as expressed in equation (23),

$$\tilde{Y}_{P,t} = \upsilon_{1,t-1}\Upsilon_1(\bar{K}_{1,t-1}) + \upsilon_{2,t-1}\Upsilon_2(\bar{K}_{2,t-1})\tilde{p}_t + \upsilon_{3,t-1}\tilde{p}_t \tag{24}$$

where the weights υ_i $i = 1, 2, 3$ refer to the probability distribution of the funding ratio. υ_1 is the mass of pension funds that have a funding ratio below K_{min}: $\upsilon_1 \equiv P(K_i < K_{min})$. Similarly, υ_2 is the mass of pension funds that have a funding ratio higher than K_{min} but lower than K_{max}: $\upsilon_2 \equiv P(K_{min} < K_i < K_{max})$. Finally, υ_3 is the mass of pension funds that have a funding ratio higher than K_{max}: $\upsilon_3 \equiv P(K_i > K_{max})$. $\bar{K}_1$ and $\bar{K}_2$ in equation (24) are the corresponding average funding ratios.

Pension liabilities equal the future stream of pension benefits discounted with the interest rate. The latter reflects that in the Netherlands pension funds are required to use the risk-free interest rate to discount future liabilities, calculated under the assumption of zero future indexation. One approach to calculate these pension liabilities would be to take sums over benefits over very long periods (up to 80 years). We adopt a more direct approach which uses only current variables. In particular, we split the growth of pension liabilities into the growth which is due to the indexation of pension benefits and that which is due to a change of the interest rate. In real terms, the former can be written as $(1 + \tilde{Y}_{P,t})/(1 + \tilde{p}_t)$. The latter can be specified as $(1 - \delta_P(i_{b,t} - i_{b,t-1}))$, where δ_P is the modified duration of the pension liabilities. Hence, we have the following expression for pension liabilities:

$$V_{P,t} = V_{P,t-1}\left(\frac{1 + \tilde{Y}_{P,t}}{1 + \tilde{p}_t}\right)\Big(1 - \delta_P(i_{b,t} - i_{b,t-1})\Big) \qquad \delta_P > 0 \tag{25}$$

By definition, the average funding ratio equals the ratio of pension wealth to pension liabilities:

$$\bar{K}_t = \frac{W_{P,t}}{V_{P,t}} \tag{26}$$

Equation (26) describes how the average funding ratio changes over time on account of changes in pension wealth and pension liabilities. We assume the cross-sectional distribution of the funding ratio to be lognormal, *i.e.* $ln(K_t) \sim N(\mu_{K,t}, \sigma_K)$. Further, we assume σ_K is a constant (the appendix explains how we calibrate this parameter). This implies that μ_K changes over time on account of changes in the average funding ratio, $\bar{K}$. Indeed, lognormality implies that $\mu_{K,t}$ equals $ln(\bar{K}_t) - 0.5\sigma_K^2$. Subsequently, we can use equations (39) and (40) in the appendix to calculate how $\upsilon_{i,t}$ $i = 1, 2, 3$ and $\bar{K}_{1,t}$ and $\bar{K}_{2,t}$ change over time.

Banks

To describe banks, it is useful to first consider their balance sheet. On the asset side, we distinguish three items: domestic loans, denoted R, cash reserves, denoted $CASH$, and loans to non-residents, denoted A^*. Domestic loans consist of mortgages and credits to households, denoted R_C, and loans to firms, denoted R_F (so that $R \equiv R_C + R_F$).

On the liability side figure deposits of households, J_C, and loans provided by foreign banks, denoted J^*. The balance sheet is closed by defining bank equity, denoted E, as the total of assets minus debt.

As to rates of return, we assume the following. Domestic loans earn a rate of return r_R and foreign loans a rate of return r_{A^*}. For domestic deposits we have a return variable r_J and for foreign bank loans a variable r_{J^*}. The rate of return on domestic loans is endogenous. As we will discuss below, it follows from imposing equilibrium on the market for domestic bank credit. The rate of interest on domestic deposits responds 1-to-1 to a change in the interest rate on bonds. The rate of return on foreign bank loans can vary over time. As we will discuss below, we will model a shock in this rate of return if we want to simulate a financial crisis in which the market for interbank loans is freezed. For cash reserves we assume a zero interest rate.

This structure implies that bank equity can be written as follows:

$$\begin{aligned} E_t &= (1 + r_{R,t})R_t + CASH_t + (1 + r_{A^*,t})A_t^* \\ &- (1 + r_{J,t})J_{C,t} - (1 + r_{J^*,t})J_t^* - r_{E,t}E_{t-1} \end{aligned} \tag{27}$$

Equation (27) makes clear that bank equity changes upon a liquidity crunch in which r_{J^*} jumps upward to a higher value. Important is the implication of such a shock for the leverage ratio. We define the bank's leverage ratio as $L \equiv E/(R + CASH + A^*)$ or, equivalently, $E/(E + J_C + J^*)$. As these expressions indicate, the leverage ratio drops in case of a loss of bank equity: $dL/dE = L/E > 0$. Furthermore, elaborating $d^2L/(dE)^2$ as $-L/E^2 < 0$ shows that the impact of a decline of bank equity upon the leverage ratio is larger, the lower is initial bank equity.

To model the relation between a loss of bank equity and the supply of bank credit, we adhere to earlier literature that stresses the role of leverage (Milne, 2002; Adrian and Shin, 2010; Schoenmaker and Wierts, 2015; Berben *et al.*, 2018). We assume banks choose their leverage. They choose the leverage such that this balances the benefits from further credit expansion against the costs associated with supervisory policies of doing so. In general, banks can use credits to domestic customers and loans to foreigners in order to reach this goal (De Haas and Van Horen, 2013; De Haas and Lelyveld, 2014). For empirical evidence that backs this assumption see *e.g.* Peek and Rosengren (1997), Adrian and Shin (2010) and Puri *et al.* (2011). Here, we assume that in case of a change in bank equity banks reduce foreign and domestic loans equiproportionally.

We now set up a nonlinear optimization problem for banks. Appendix D elaborates this optimization problem. The first-order condition that describes optimal banking policies can be rewritten as an equation for the preferred leverage ratio:

$$\hat{l}_t = \lambda - \left(\frac{(r_{R,t} - r_{J,t}) + \omega_B \left(r_{A^*,t} - r_{J^*,t}\right)}{\zeta\zeta_b(1 + \omega_B)} \right)^{\dfrac{-1}{1+\zeta_b}} \tag{28}$$

Here, $\hat{l}$ refers to the long-run value of l with l defined as the inverse of the leverage ratio, $l \equiv L^{-1}$. Furthermore, λ denotes the inverse minimum leverage requirement, defined as Λ^{-1}, where Λ is the minimum leverage ratio as required by the supervisor. Equation (28) implies that the long-run actual leverage ratio is always larger than the minimum leverage requirement, which corresponds with the data (Milne, 2002). It also implies that a change in capital requirements changes the leverage ratio in the same direction, which is also supported by empirical evidence. Finally, equation (28) has the appealing implication that more favourable rate of return differentials induce banks to choose lower leverage ratios.

Given the target leverage ratio as specified in equation (28), we can derive the following expression for the supply of credit to domestic households and firms,

$$\hat{R}_t = \frac{1}{1+\omega_B}\left[\hat{l}_t \tilde{E}_{t-1} - CASH_t - (A^*_{t-1} - \omega_B R_{t-1})\right] \tag{29}$$

where $\tilde{E}_{t-1}$ stands for E_t after the occurrence of period-t shocks and before the formulation of optimal credit policies.

Equation (49) shows that a loss of bank equity, for whatever reason, reduces the supply of domestic credit. The laxer the supervisory policies in place, the bigger will be this effect (use equation (28) to derive that $d\hat{l}_t/d\lambda > 0$). The effect of a change in equity upon the supply of credit will also be bigger, the less banks rely on credits to foreigners to achieve their preferred leverage. In case a loss of bank equity is due to a liquidity crunch (an increase in $r_{J^*,t}$), the effect will further amplified (use equation (28) to derive that $d\hat{l}_t/dr_{J^*,t} < 0$).

We assume that the supply of credits is governed by a partial adjustment process.

$$R_t = \lambda_R R_{t-1} + (1-\lambda_R)\hat{R}_t \qquad\qquad 0 \leq \lambda_R < 1 \tag{30}$$

One issue that we did not discuss thus far is the $CASH$ variable. Should an extreme adverse shock in equity prices occur, the government may be induced to inject capital into banks, as described above in equation (5). This is modelled as an increase in the cash position of banks, *i.e.* $CASH_t = CASH_{t-1} + H^E_t$. As follows from equation (27), this will increase bank equity to the same extent.

The market for bank credit

The demand side of the market for bank credit consists of two parts. The flow of credit demand by consumers, denoted ΔR_C, can be derived from their budget constraint. This says that any difference between consumption and housing investment on the one hand and their disposable income on the other hand implies an equally-sized increase in bank credits and mortgages,

$$\Delta R_{C,t} = (1+\tau^C)(C_t + S_{W,t}) - Y^d_t \tag{31}$$

Credit demand by firms follows from their net investment. Depreciation investment is financed out of their profits. Hence, we have:

$$\Delta R_{F,t} = (1+\tau^I)(I_t - \delta_Z Z_{t-1}) \tag{32}$$

Equating demand and supply of bank credits solves for the interest rate on bank credits, r_R.

The output and labour market

We define price competitiveness as the ratio between the domestic price, p, and the foreign price. Abstracting from foreign price inflation, we normalize the foreign price at one. Subsequently, we relate exports, denoted X, to price competitiveness and foreign GDP. The latter serves as an index of world trade.

$$X_t = p_t^{-\eta_X} Y_t^* \qquad \mu_X, \eta_X > 0 \tag{33}$$

Imports, denoted M, relate to aggregate sales, $C + S_W + I + G + X$. As the relation between imports and exports is much tighter than that between imports and other sales items (in the Netherlands, a relatively large share of imports leaves the country as exports), we distinguish between these two groups:

$$M_t = \mu_{MX} X_t + \mu_{MM} p_t^{\eta_M} (C_t + S_{W,t} + I_t + G_t) \qquad \mu_{MX}, \mu_{MM}, \eta_M > 0 \tag{34}$$

Output equals the sum of private consumption, housing investment, other investment, public consumption and exports minus imports,

$$Y_t = C_t + S_{W,t} + I_t + G_t + X_t - M_t \tag{35}$$

We take the share of labour income in GDP a constant. This is consistent with the CES production function specified above if the real wage is assumed constant. Hence, we have,

$$Y_t^l = Q Y_t \tag{36}$$

where Y^l is labour income and Q is the share of labour income in GDP.

Unemployment, denoted U, relates to GDP according to Okun's law. This means that unemployment will settle on its natural rate ($\dot{U}$) in case of a zero output gap and that it will be higher than the natural rate if output is below its structural level.

$$\begin{array}{rcll} \hat{U}_t & = & \dot{U}_t - \beta_U (Y_t/\dot{Y}_t - 1) & \qquad \beta_U > 0 \\ U_t & = & \lambda_U U_{t-1} + (1 - \lambda_U) \hat{U}_t & \qquad 0 \leq \lambda_U < 1 \end{array} \tag{37}$$

In the short run, output is driven by the demand side as specified in equation (35). In the long run, price adjustment ensures that output is determined by structural supply. To describe this price adjustment, we use the following version of the Phillips curve,

$$\tilde{p}_t = \epsilon_\lambda \tilde{p}_{t-1} + \beta_p(Y_t/\dot{Y}_t - 1) \qquad \beta_p > 0 \tag{38}$$

where the first term is an indicator of the expected rate of inflation.

5 Calibration of the model

In order to be able to perform simulations, we need to specify values for the coefficients of the model, for the exogenous variables and for some of the endogenous variables.

Tables 1 and 3 summarize the values of the parameters of the model. Some parameter values are data driven (D): their values are obtained by referring to external empirical information. Below, we report per parameter how the value relates to empirical estimates. The values of other parameters are obtained from calibration (C) of the model. The tables report per parameter which parameter value ensures that which variable is calibrated at which value. For calibration we have used data from the *Centraal Economisch Plan*, except for the calibration of the banking sector, for which we used data from DNB (the Statistieken part of the website and Berben *et al.*, 2018).

We adopt a zero value for γ_G. Government consumption thus does not change upon an external shock or a change of GDP. This is in line with Dutch fiscal policies.

The marginal propensity to consume out of current income (α_Y) follows from the requirement that non-housing consumption as predicted by the model equal aggregate consumption in the calibration year 2017. The value of 0.81 is close to the corresponding value in CPB (2010). For the marginal propensity to consume out of private savings (α_W), we adopt a value of 0.05. For the marginal propensity to consume out of pension wealth (α_P), we adopt a value half as large, reflecting the low liquidity of pension wealth. α_L and α_X follow from a two-step calibration procedure. In the first step, the value of α_X is calibrated using estimates on the relationship between the marginal propensity to consume and the loan-to-value ratio as in Mian *et al.* (2013). In the second step, α_L is calculated by equating the marginal propensity to consume out of housing wealth to 2 percent, the coefficient for housing wealth in Verstegen (2019) and Zhang (2019).

As regards pension funds, ω_P^b measures the part of the asset portfolio of pension funds invested in bonds. In 2017, this equals 0.52 (DNB, 2018).

Table 1: Parameter values

Equation	Parameter		Calibrated on
Crash spending (5)	ϕ_H	D	
	2		
	H^E_{min}	D	
	70		
Public consumption (6)	γ_G	C	$G - \dot{G}$
	0.0		0
Capital income tax revenues (8)	η_1	D	
	3.37		
	η_2	D	
	0.53		
Consumption (11)	α_Y	C	C
	0.81		310
	α_W	D	
	0.05		
	α_P	D	
	0.025		
	α_L	D	
	1.6776		
	α_X	D	
	0.56		
	λ_C	D	
	0.5		
Housing investment (13)	α_C	C	S_W
	0.10		30
	σ_W	D	
	0.95		
Investment (16)	γ_Y	C	I
	0.61		85
Capital accumulation (18)	δ_Z	D	
	0.05		
Production function (19)	κ	D	
	0.57		
	ρ	D	
	-1		

Table 2: Parameter values; continued

Equation	Parameter		Calibrated on
Rate of return pension funds (21)	ω_P^b 0.52	D	
Pension contribution rate (22)	σ_P 0.07	C	S_P 35
Individual pension benefiit (23)	K_{min} 1.05	D	
	K_{max} 1.30	D	
Aggregate pension benefit (24)	μ_K 0.06	C	$\nu_{1,0}$, $\nu_{2,0}$ 47%, 51%
	σ_K 0.11	C	
Average funding ratio (26)	δ_P 12	D	
Credit supply (30)	ζ 2.31	C	r_R 0.01
	λ 33.3	D	
	ζ_b 1.0	D	
	λ_R 0.25	D	
Exports (33)	Y^* 560	C	X 560
	η_X 3.3	D	

Table 3: Parameter values; continued

Equation	Parameter		Calibrated on
Imports (34)	μ_{MX}	D	
	0.60		
	μ_{MM}	C	M
	0.24		485
	η_M	D	
	1.2		
Labour income share (36)	Q	D	
	0.77		
Unemployment (37)	β_U	D	
	0.4		
	λ_U	D	
	0.5		
Phillips curve (38)	ϵ_λ	D	
	0.25		
	β_p	D	
	0.02		

δ_P is fixed at 12, which measures the sensitivity of pension liabilities to the interest rate if the duration of pension liabilities equals 20 years and the pension fund hedges against 40 percent of its interest payments (Lever and Loois, 2016).

As regards the banking sector, the value for the interest rate is chosen such that the simulated effect of a change in the leverage ratio upon credit supply is close to that in Aiyar *et al.* (2016).

We choose 3.3 as value for η_X, the price elasticity of exports. This value is based on estimates of the price elasticities of final goods and services as reported in CPB (2010), multiplied by their respective shares (we have corrected for the export of oil and gas and of imported goods that carry low value added).

As regards η_M, the price elasticity of imports, we adopt a similar procedure. Averaging the price elasticities of the imports of final goods and of intermediates as reported in CPB (2010) yields a value of 1.2.

We fix β_U, the coefficient in Okun's law equation, at a value of 0.4. This estimate of Okun's law is in between the estimate in Ball *et al.* (2013) for the Netherlands for the period 1980-2011 (0.511) and that for the more recent period 1995-2011 (0.336).

A regards the Phillips curve, we fix β_p at a value of 0.02, based on the estimates in DiNardo and Moore (1999) for the Netherlands.

For the labour's income share, Q, we adopt a value of 0.765. This is the average in the period 2010-2015, according to the latest definition of the labour's income share (CBS, 2016). For the elasticity of substitution between labour and capital we take a value of 0.5 (in line with Berben *et al.*, 2018). The capital stock is calibrated such that, given a depreciation rate of 5 percent, net investment is zero. Employment is then given that value that ensures that actual output and structural output coincide initially.

The capital income tax system is defined by the following parameter values: $\tau^k = 0.30$, $\eta_1 = 3.37$ and $\eta_2 = 0.53$. The appendix explains how this relates to the equation we have estimated for the revenues from capital income taxation.

In order to identify a proxy for the impulse in the interest rate on foreign loans, we calculated Great Recession effects on bank equity. We calculated these effects as the change in bank equity that occurred in the 2007-2012 period relative to that in the 2002-2007 period. The assumption on which this calculation is based is thus that, if no Great Recession had occurred, the change in the leverage ratio in 2007-2012 would have been identical to that in 2002-2007. The result is a decline in the leverage ratio of 15 percent. The impulse in the interest rate on foreign loans in the financial crisis scenario is then taken such that it generates a drop in the leverage ratio of banks of 15 percent after two years.

In order to validate the model, we adopt the same procedure to calculate Great Recession effects on consumption, investment, and bank credit to households and firms. A comparison of these Great Recession effects with the model predictions yields that in general the model mimics the Great Recession effects relatively well.

Table 4 summarizes the balance sheets of the various actors in the model, *i.e.* banks, households and pension funds. These balance sheets reflect the initial values of three different ratios. The ratio of equity to debt of banks is 6 percent (150/2500), the loan-to-value ratio of households 60 percent (650/1100) and the funding ratio of pension funds 110 percent (1250/1136).

Table 4: Balance sheets

Banks				
Assets	R_C (R_W)	R_F	$CASH$	A^*
	760 (650)	220	0	1520
Liabilities	J_C	J^*	E	
	300	2050	150	
Households				
Assets	W_W	J_C	F_C^*	V_P
	1100	300	340	1136
Liabilities	R_C			
	760			
Pension funds				
Assets	W_P			
	1250			
Liabilities	V_P			
	1136			

6 Simulations

We perform three types of simulations. First, we simulate the effects of a financial crisis based on current values for institutional variables like the leverage ratio of banks, the LTV ratio of households and the average funding ratio of pension funds. Second, we perform sensitivity analysis. Particularly, we take consumption to be more responsive to a change in disposable income in order to reflect that in case of an extreme event, normal relationships may no longer apply. In the same vein, we run a simulation in which private consumption is more responsive to a decline of housing prices than in the benchmark simulation. In the third group of simulations, we explore the impact of the nonlinearities in the model. What would be the impact of financial crisis or an external debt crisis if household leverage was a lot worse than assumed till now? What would change if the funding ratios of pension funds would be a lot weaker than assumed thus far? What is the role of leverage in the banking sector?

In reporting on the simulations, we focus upon macroeconomic variables like output and consumption and fiscal variables like tax revenues, public spending, the budget deficit ratio and the public debt ratio.

6.1 A financial crisis

We document the effects of a financial crisis upon the macroeconomy and the government budget. The setup is based on the stress tests that have been performed earlier.

In particular, the financial crisis is composed as follows:

– a drop in world trade of 11.5 percent in year 1

– a fall of equity prices of 40 percent in year 1

– a fall in housing prices of 10 percent in year 1

– an increase of the interest rate of 100 basis points in years 1 to 5

– an increase of the interest rate on interbank loans of 80 basis points in years 1 and 2.

Table 5 reports the effects. The crisis hits the economy quite immediately; the biggest impact occurs in the second year. Output drops about 8 percent on average in the first and second year, reflecting the fall of exports, consumption and investment. Investment fluctuates much more than output, with a peak effect occurring in the second year of more than 30 percent. Partly, this reflects a mechanism that we also observe during the business cycle. Partly, it is due to a crunch of credit supply by banks that suffer from a worsening of bank equity. In comparison, the decline of consumption is rather modest: about 6 percent in the first two years. The public deficit ratio immediately jumps upwards, with 7.6 percentage points after two years.

This is fully due to worsening economic conditions. The debt ratio increases even faster, fueled by a denominator effect.

After two years, the economy starts to recover. One reason is the external position. The slack on output and labour markets decreases price inflation, which in turn increases exports and decreases imports. Another reason is that we assume the liquidity crisis lasts two years. Output, consumption and investment thus start to return to pre-crisis levels. The recovery of consumption is somewhat more slowly than that of output. This is due among other things to the fact that the reduction of pension benefits as a response to funding deficits takes place quite slowly.

Unemployment responds more slowly than output, a feature we know from the literature on business cycles. It starts to recover after 3 years. The deficit ratio also returns to its pre-crisis level, but only very gradually. Hence, after 5 years, the debt ratio is still increasing. Then, it is about 35 percentage points higher than if no financial crisis had occurred. There are three reasons for the increase of the public debt ratio. The first is that tax revenues drop, whereas unemployment benefits increase. The second is that gdp decreases, which blows up the debt ratio through a denominator effect. The third reason is that the government bails out banks for an amount of 20 billion euro in year 1 of the simulation.

Table 5: A financial crisis

Cumulative deviation from base projection	After .. years	1	2	3	4	5
Y	%	-6.5	-9.4	-8.7	-7.4	-5.7
C	%	-4.2	-7.9	-7.7	-7.0	-6.1
I	%	-14.4	-30.7	-26.9	-18.6	-8.2
U	%point	1.4	2.6	3.0	2.9	2.4
T	%	-5.9	-10.1	-8.7	-6.6	-4.3
DY	%point	7.1	7.6	7.3	6.4	5.1
BY	%point	9.9	18.9	25.9	31.5	35.8

6.2 Sensitivity analysis

To assess the robustness of this result, we perform sensitivity analysis. In two simulations, we adopt different values for a crucial parameter in the model. The first of these simulations takes a 4 times as high value for the average marginal propensity to consume out of net housing wealth. This simulation thus gives insight into the effects of a financial crisis if consumption is much more sensitive to a fall of housing prices. Table 6 reports the results.

Not surprisingly, the biggest change with the previous simulation concerns the effect on private consumption. Other variables are affected as well, since the extra drop in consumption lowers gdp and thus investment. As a result, tax revenues drop and there is a bigger increase in the public debt ratio. Interestingly, the difference with the previous simulation grows over time. This can be explained from the fact that both private consumption and investment adjust partially to their optimal values.

Overall, the picture does not change dramatically, however. The peak effect on gdp is now 9.5 percent, only slightly higher as in the previous simulation. The increase in unemployment after 5 years is now 2.6 percentage points, which is in the same order as before. And the increase after 5 years in the public debt ratio is now 37.3 percentage points, whereas it was 35.8 percentage points in the benchmark simulation.

Table 6: The effect of a higher consumption-housing wealth sensitivity

Cumulative deviation from base projection	After .. years	1	2	3	4	5
Y	%	-6.7	-9.5	-8.9	-7.7	-6.1
C	%	-4.6	-8.2	-8.4	-8.0	-7.3
I	%	-14.6	-30.6	-26.4	-17.9	-7.4
U	%point	1.4	2.6	3.1	3.0	2.6
T	%	-6.1	-10.2	-8.9	-6.9	-4.6
DY	%point	7.4	7.8	7.6	6.7	5.4
BY	%point	10.3	19.5	26.8	32.7	37.3

The second of the sensitivity simulations adopts a higher value for the marginal propensity to consume out of disposable income. In particular, here this parameter adopts a value of 0.95 rather than 0.8 in the benchmark simulation. This simulation thus gives insight into the effects of a financial crisis if consumption is much more sensitive to a drop in disposbale income. Table 7 reports the results.

What we see is that the impact on the results is very modest. The change in parameter value (from 0.8 to 0.95) is too small to exert significant effects. The biggest effect after 5 years occurs for priivate investment. This investment is now 11.7 percent below its baseline value; the corresponding figure is 8.2 percent in the benchmark simulation. The upsurge in the public debt ratio after 5 years is now 36.0 percentage point; the corresponding figure in the benchmark simulation is 35.8.

Table 7: The effect of a higher propensity to consume out of disposable income

Cumulative deviation from base projection	After .. years	1	2	3	4	5
Y	%	-6.4	-9.6	-9.1	-7.7	-6.0
C	%	-4.3	-8.5	-8.3	-7.5	-6.3
I	%	-13.6	-31.8	-30.1	-22.8	-11.7
U	%point	1.4	2.8	3.3	3.1	2.6
T	%	-5.7	-10.4	-9.3	-7.2	-4.8
DY	%point	6.6	7.3	47.3	6.4	5.2
BY	%point	9.1	17.7	24.9	31.0	36.0

6.3 The role of initial conditions

As discussed above, a financial crisis might hit the economy harder if initial debt levels are higher due to nonlinearities that relate to debt levels. The question is how big is the role of these initial conditions? This section explores the role of initial conditions, starting with the leverage ratio of banks.

Before doing so, we make two caveats. First, it is a little difficult to assume that in an alternative simulation one asset or debt position changes and the rest of the economy is left unchanged. It does help to shed light on the role of a particular mechanism, however, and that is what we use this simulation for. Second, the different alternative scenarios show the role of different mechanisms. One can compare them to get an idea of their relative importance. It is difficult to translate these findings into policy implications however as there will be (different) transition paths involved.

Weaker banks

Concretely, we assume that bank equity is initially 25 percent lower than in the benchmark simulation. Hence, we assume that bank equity equals 4.5 percent of total bank assets, rather than 6 percent as assumed until now. Table 8 reports the results. On the whole, the picture is darker than before. In particular, the responses of investment and consumption are stronger than before. This is due to the stronger credit rationing by banks which in the present simulation have less buffers to curb the liquidity shock. Output peaks now at -10.6 percent, which is also stronger than before (-9.4 percent). As a consequence, the public debt ratio is now 43.2 percentage point higher in year 5, again a bigger effect than in the benchmark simulation (35.8 percentage point).

Table 8: Weaker banks

Cumulative deviation from base projection	After .. years	1	2	3	4	5
Y	%	-6.1	-10.6	-10.3	-8.8	-6.7
C	%	-3.7	-9.3	-9.3	-8.3	-6.9
I	%	-16.1	-61.6	-54.8	-35.4	-16.8
U	%point	1.2	2.7	3.3	3.3	2.8
T	%	-5.1	-11.9	-11.2	-8.7	-5.8
DY	%point	7.3	9.5	9.9	8.5	6.7
BY	%point	10.4	22.9	32.4	39.0	43.2

A higher level of mortgages

As a second experiment, we explore the role of the loan-to-value ratio of households. We assume that mortgage debt is now 25 percent larger than the value assumed in the benchmark simulation of a financial crisis. As this decreases net housing wealth of households, the marginal propensity to consume out of housing wealth will be higher. We expect thus that the consumption response to a fall of housing prices will be larger.

Table 9 reports the results. These are very similar to those in the benchmark simulation. Only the effects are somehwat larger than before, the rest of the outcomes are generally unchanged. Obviously, this relates to the relatively small effect of net housing wealth upon aggregate private consumption. On the microeconomic level, effects can be much larger.

Table 9: Higher mortgages

Cumulative deviation from base projection	After .. years	1	2	3	4	5
Y	%	-6.6	-9.5	-8.8	-7.4	-5.8
C	%	-4.3	-8.0	-7.9	-7.2	-6.2
I	%	-14.3	-30.7	-27.0	-18.8	-8.2
U	%point	1.4	2.7	3.1	2.9	2.5
T	%	-5.9	-10.2	-8.8	-6.7	-4.4
DY	%point	7.1	7.6	7.4	6.4	5.1
BY	%point	9.9	18.9	25.9	31.7	36.0

Lower pension wealth

Our third experiment is to assume a weaker financial position of pension funds. Concretely, we assume pension wealth is 25 percent lower than before. Table 10 reports the effects. In general, our comments on the previous simulation apply here as well. Only the effects upon private consumption are somewhat stronger than in the benchmark simulation, the rest of the variables are very similar to those in the benchmark simulation. Typical for the present simulation is that the stronger drop in consumption occurs only after 2 years. The worser financial position of pension funds implies that pension benefits will decrease relative to the benchmark simulation. Given that pension funds have ample time to restore their financial position, these effects occur only in the medium term.

Table 10: Lower pension wealth

Cumulative deviation from base projection	After .. years	1	2	3	4	5
Y	%	-6.5	-9.4	-8.8	-7.4	-5.8
C	%	-4.1	-7.9	-7.8	-7.1	-6.2
I	%	-14.3	-30.8	-27.3	-19.1	-8.9
U	%point	1.4	2.6	3.1	2.9	2.5
T	%	-5.8	-10.1	-8.8	-6.7	-4.4
DY	%point	7.0	7.5	7.4	6.4	5.2
BY	%point	9.8	18.8	25.8	31.6	36.0

7 Concluding comments

In this document we have described CRASH. CRASH is meant to be a model for simulating the impact of exterme external events upon the economy and public finances. As such, it should include elements that may have small relevance in normal economic times, but possibly high relevance in crisis situations. Further, it should be stylized in order to make experiments possible that help to highlight the role of structural characteristics of institutions.

The simulations on which we reported indicate that it is indeed important to include a financial sector and a pension sector. The quality of institutions plays a role in determining the impact of a financial crisis, although, as we stressed in the beginning of this document, it is impossible to give a precise estimate of the impact of the next crisis, even if we would know the shape it will have. The strength of the model is not to make predictions, but to give a better understanding of the implications of alternative assumptions on institutions and characteristics of the economy.

We do not consider this project as finished. The labour market is changing (witness the increasing role of self-employed persons) and the pension sector is changing (we expect to have a pension reform in the coming years). Meanwhile, the world economy has become the victim of a corona crisis that may fundamentally change global trade and finance patterns. It is a challenging task to gain more insight in what these and other developments may imply for the impact of future extreme external events.

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Appendix A: The distribution of the funding ratio of pension funds

As described in the main text, equation (26) is used to calculate the average funding ratio and expression $\mu_{K,t} = ln(\bar{K}_t) - 0.5\sigma_K^2$ to derive the corresponding value of $\mu_{K,t}$. Given that the distribution of the funding ratio over pension funds is lognormal, we can subsequently derive the mass of pension funds in the three groups as distinguished in the main text. We therefore apply the following expressions:

$$\upsilon_{1,t} = P(K_t < 1.05) = \Phi\Big(\frac{ln(1.05) - \mu_{K,t}}{\sigma_K}\Big)$$
$$\upsilon_{1,t} + \upsilon_{2,t} = P(K_t < 1.30) = \Phi\Big(\frac{ln(1.30) - \mu_{K,t}}{\sigma_K}\Big) \tag{39}$$

where Φ denotes the cumulative standard normal distribution function.

Similarly, we derive the average funding ratios in groups 1 and 2, $\bar{K}_1$ and $\bar{K}_2$, by applying the formulas for the conditional expectation of a lognormally distributed variable:

$$\bar{K}_{1,t} = \frac{\Big[\Phi\Big(\frac{ln(K_{min})-\mu_{K,t}}{\sigma_K} - \sigma_K\Big)\Big]}{\Big[\Phi\Big(\frac{ln(K_{min})-\mu_{K,t}}{\sigma_K}\Big)\Big]}\bar{K}_t$$
$$\bar{K}_{2,t} = \frac{\Big[\Phi\Big(\frac{ln(K_{max})-\mu_{K,t}}{\sigma_K} - \sigma_K\Big) - \Phi\Big(\frac{ln(K_{min})-\mu_{K,t}}{\sigma_K} - \sigma_K\Big)\Big]}{\Big[\Phi\Big(\frac{ln(K_{max})-\mu_{K,t}}{\sigma_K}\Big) - \Phi\Big(\frac{ln(K_{min})-\mu_{K,t}}{\sigma_K}\Big)\Big]}\bar{K}_t \tag{40}$$

In order to calibrate the model, we combine equation (39) with data on the mass of pension funds that have a funding ratio below 105 percent and a funding ratio between 105 and 130 percent. For the third quarter of 2017, these two variables equal 61.7% and 36.9% respectively (DNB, 2018). Hence, we derive values for μ_K and σ_K by combining the following two equations:

$$0.617 = \Phi\Big(\frac{ln(1.05) - \mu_{K,t}}{\sigma_K}\Big)$$
$$0.986 = \Phi\Big(\frac{ln(1.30) - \mu_{K,t}}{\sigma_K}\Big) \tag{41}$$

Elaborating these two conditions gives $\mu_K = 0.0135$ and $\sigma_K = 0.1131$. The former is an initial value, whereas the latter is a constant by assumption.

Appendix B: Capital income tax revenues

In order to obtain an equation for capital income tax revenues, we have estimated a time-series equation with capital income as explanatory variable.[11] Problematic may be to rely on capital income only as a measure of the tax base for the capital income tax. Therefore, we add gdp as explanatory variable. The preferred estimation contains the current and one-year lagged value of capital income and the current, one-year lagged and two-year lagged value of gdp. It reads as follows:

$$T_t^k = C_0 + C_1 Y_t^k - C_2 Y_{t-1}^k + C_3 Y_t - C_4 Y_{t-1} - C_5 Y_{t-2}$$

Table 11 gives the estimated coefficient values and corresponding t-statistics and a number of other statistics.

Table 11: Capital income tax revenues, 2000-2016

Variable	Coefficient	t-Statistic	Prob.
C_0	-5.522	-0.883	0.396
C_1	0.035	0.389	0.705
C_2	-0.157	-1.536	0.153
C_3	0.187	2.878	0.015
C_4	-0.002	-0.020	0.985
C_5	-0.134	-2.438	0.033

R-squared: 0.731
Adjusted R-squared: 0.609
Log likelihood: -148.302

Alternative specifications with only capital income as explanatory variable or with less lags had less explanatory power. Alternative specifications with more lags did not add much to the specification shown; in order to have a specification with not too much variables, we preferred the specification given above. We also estimated this specification in first differences; this did not change the properties of the equation substantially.

How can we change this specification in a model equation? First of all, note that we regard gdp as an alternative measure of capital income. In

[11] Thanks to Maurits van Kempen for assistance with estimating the equation for corporate tax revenues.

particular, by dividing gdp with the share of capital income in gdp, we get a second measure of capital income. Upon combining the coefficients of the two capital income measures, we get the following specification:

$$T_t^k = -5.522 + 1.045\tilde{Y}_t^k - 0.165\tilde{Y}_{t-1}^k - 0.570\tilde{Y}_{t-2}^k$$

Next, we write the last three terms at the RHS of the equation as the product of an effective tax rate and a tax base that is a moving average of current capital income, one-year lagged capital income and two-year lagged capital income. After some rounding this yields the following:

$$T_t^k = 0.3\Big[3.37\tilde{Y}_t^k - 0.53\tilde{Y}_{t-1}^k - 1.84\tilde{Y}_{t-2}^k\Big]$$

Note that, by construction, the weights of the moving-average term for the capital income tax base add up to one. Note also that the weight of the contemporaneous term is much larger than one, whereas the weights of the one-year lagged term and the two-year lagged term are both negative. Our specification thus meets the two conditions we want to impose to the equation for capital income tax revenues: overshooting of tax revenues after a shock to gdp and a long-term elasticity of tax revenues with respect to gdp that is equal to unity.

Appendix C: Calibration of the consumption/wealth ratio

In the main text we derived the following expression for the marginal propensity to consume out of net housing wealth: $(\alpha_L(1-\alpha_X/(1+\tau^C))(W_{W,t-1}(1+r_{h,t})(1-\Xi_{t-1}))^{-\alpha_X}$, where Ξ_{t-1} denotes the loan-to-value ratio. Mian *et al.* (2013) have estimated the marginal propensity to consume for a household with a loan-to-value ratio of 90 percent to be three times as large as the marginal propensity to consume of a household with a loan-to-value ratio of 30 percent. This gives the following calibration equation:

$$\frac{\dfrac{\alpha_L(1-\alpha_X)}{1+\tau_C}(W_{W,t-1}(1+r_{h,t}))^{-\alpha_X}(1-0.9)^{-\alpha_X}}{\dfrac{\alpha_L(1-\alpha_X)}{1+\tau_C}(W_{W,t-1}(1+r_{h,t}))^{-\alpha_X}(1-0.3)^{-\alpha_X}} = 3 \tag{42}$$

Solving the equation for α_X gives a value of 0.56.

In turn, we can use this result and calibrate α_L by equating the marginal propensity to consume out of net housing wealth for the average household to 0.02, the recently estimated marginal propensity to consume out of housing wealth in Zhang (2019): $(\alpha_L(1-\alpha_X)/(1+\tau^C))(W_{W,t-1}(1+r_{h,t})(1-\Xi_{t-1}))^{-0.56} = 0.02$. Solving this for the calibration year gives $\alpha_L = 1.6776$.

Appendix D: The optimization problem for banks

We can write down the following accumulation equation for bank capital:

$$\begin{aligned} E_t &= (1+r_{R,t})R_t + CASH_t + (1+r_{A^*,t})A_t^* \\ &- (1+r_{J_C,t})J_{C,t} - (1+r_{J^*,t})J_t^* - r_E E_{t-1} \end{aligned} \tag{43}$$

As regards timing, we assume that in any period t, first, one or more shocks may occur and, second, banks, who are able to observe these shocks, may change their credit policies such as to re-optimize their utility function. To elaborate this, we define bank capital after the occurence of shocks, $\tilde{E}_{t-1}$, as $\tilde{R}_{t-1} + \tilde{A}^*_{t-1} - \tilde{J}_{C,t-1} - \tilde{J}^*_{t-1}$, where for $X = R, A^*, J_C, J^*$ we define $\tilde{X}_{t-1}$ as $(1+r_{X,t})X_{t-1}$. Using ΔX_t to refer to $X_t - X_{t-1}$, we rewrite equation (43) as follows:

$$\begin{aligned} E_t &= \tilde{E}_{t-1} + (1+r_{R,t})\Delta R_t + CASH_t + (1+r_{A^*,t})\Delta A_t^* \\ &- (1+r_{J_C,t})\Delta J_{C,t} - (1+r_{J^*,t})\Delta J_t^* - r_E E_{t-1} \end{aligned} \tag{44}$$

Now, we posit the utility function for the bank. We assume utility depends on two factors. The first factor is the growth of bank capital. The other factor refers to the costs, broadly defined, that are associated with the mismatch between the bank's leverage ratio and the minimum ratio as required by the supervisor:

$$V_t = \left(\frac{E_t - \tilde{E}_{t-1}}{\tilde{E}_{t-1}}\right) - \zeta(\lambda - l_t)^{-\zeta_b} \qquad \zeta, \zeta_b > 0 \tag{45}$$

In order to understand the second term at the RHS of equation (45), define the bank's leverage ratio as bank capital over total assets, $L_t \equiv \tilde{E}_{t-1}/(R_t + CASH_t + A_t^*)$. l_t in equation (45) is the reciprocal of this leverage ratio, *i.e.* $l_t \equiv 1/L_t$. Similarly, we define Λ as the minimum leverage ratio as required by the supervisor and λ in equation (45) as its reciprocal, *i.e.* $\lambda \equiv 1/\Lambda$. Hence, $\lambda - l_t$ in equation (45) relates to the gap between the actual and the minimally required leverage ratio for banks and will be positive if the actual leverage ratio exceeds the minimum ratio.

Furthermore, we can use equation (45) to derive that the second term at the RHS of equation (45) is an increasing and concave function of the actual leverage ratio[12]. Hence, utility declines if the leverage ratio moves towards its minimum requirement and the more so the closer is the actual leverage ratio to the minimum requirement (below, we will derive that the actual leverage ratio always exceeds the minimum requirement). The idea behind this term is as in Milne (2002): a decline in the leverage ratio presses the supervisor to closely monitor the financial policies of the bank. The bank generally perceives this as a utility cost, as it has to spend more time and energy on discussions and negotiations with the banking supervisor.

The problem of the bank is now to maximize its utility, given the rates of return on the assets and liabilities on its balance sheet. The instruments that the banks in our model use to achieve maximum utility are the loans supplied domestically and abroad. In choosing its supply of loans, the bank faces a constraint: any change in assets implies an equally-sized change in liabilities.

There is a large literature that points out that banks that operate in more than one country can use both domestic and foreign loans to improve their leverage. In many instances, the two types of loans are not used symmetrically. Indeed, in times of crisis, banks have reduced credits more in countries that are further away from their base country in business terms. Our model takes this evidence into account. It treats both domestic and foreign loans as policy instruments and bases the weight placed on the two types of loans on the empirical evidence.

Formally, we elaborate the first-order condition of the maximization problem $\partial V_t/\partial R_t = 0$, taking into account the definitions of l_t and L_t given above and the constraints $dJ_{C,t} = dR_t$, $dJ_t^* = dA_t^*$ and $dA_t^* = \omega_B dR_t$. The first two constraints reflect that making a loan creates an asset as well as a liability of equal magnitude. The third constraint reflects our assumption that the change in domestic loans and that in foreign loans are proportional to one another. Hence, the first-order condition reads as follows:

$$\begin{aligned}\partial V_t/\partial R_t &= \frac{1}{\tilde{E}_{t-1}}\left((r_{R,t} - r_{J_C,t}) + \omega_B(r_{A^*,t} - r_{J^*,t})\right) \\ &- \zeta\zeta_b(\lambda - l_t)^{-\zeta_b - 1}\frac{(1+\omega_B)}{\tilde{E}_{t-1}} = 0 \end{aligned} \tag{46}$$

[12]Use equation (45) to derive that $dV_2/dL = \zeta\zeta_b(\lambda - l)^{-(\zeta_b+1)}/L^2 > 0$ and that $d^2V_2/(dL)^2 = -\zeta\zeta_b(\lambda - l)^{-(\zeta_b+2)}/L^4((\zeta_b + 1) + 2(\lambda - l)L) < 0$, where V_2 refers to the second term at the RHS of equation (45) and where it is required that $\lambda > l$, which we will show below.

This can be converted into an expression for the leverage ratio:

$$l_t = \lambda - \left(\frac{(r_{R,t} - r_{J_C,t}) + \omega_B (r_{A^*,t} - r_{J^*,t})}{\zeta \zeta_b (1 + \omega_B)} \right)^{-\frac{1}{1+\zeta_b}} \tag{47}$$

What does this imply for the supply of domestic loans? To see this, rewrite the balance sheet of banks:

$$R_t + A_t^* = l_t \tilde{E}_{t-1} - CASH_t \tag{48}$$

Further, note that $R_t + A_t^* = R_{t-1} + A_{t-1}^* + (1 + \omega_B)\Delta R_t$. Hence, we can write R_t as $R_{t-1} + ((R_t + A_t^*) - (R_{t-1} + A_{t-1}^*))/(1 + \omega_B)$. Substituting this into equation (48), we obtain the equation for the supply of domestic loans:

$$\hat{R}_t = \frac{1}{1 + \omega_B} \left[l_t \tilde{E}_{t-1} - CASH_t - (A_{t-1}^* - \omega_B R_{t-1}) \right] \tag{49}$$