CPB Netherlands Bureau for
Economic Policy Analysis

Appendix to the Dutch publication: "Heeft de contracyclische kapitaalbuffer een effect op de kredietverlening?"

I analyze the effects of an increase in the countercyclical capital buffer on credit provision by banks and non-bank financial intermediaries in a tractable general equilibrium model of banking and macroprudential regulation. I show that an increase in the countercyclical capital buffer leads to a decrease in total credit supply because banks reduce credit supply to meet higher capital requirements.

Non-bank financial intermediaries are not affected by the countercyclical capital buffer and increase lending. Although the effect on total credit provision is negative, the reduction depends on the extent to which non-bank financial intermediaries can increase lending.

Appendix

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1 Introduction

In this document I analyze the effects of an increase in the countercyclical capital buffer on credit provision by banks and non-bank financial intermediaries. I do so in a tractable general equilibrium model of banking and macroprudential regulation. I analytically solve for the general equilibrum effect of an increase in the countercyclical capital buffer (CCyB), and show how the impact on credit provision depends on the ease with which non-bank financial intermediaries (NBFIs) can increase credit supply. My analytical results are used to provide theoretical backing and economic intuition to empirical hypotheses concerning the effect of CCyB changes on credit provision in the paper that this document accompanies.

In my model, production firms attract lending from banks and households to finance purchases of physical capital goods that are used in production. Banks are balance-sheet-constrained due to a principal agent friction between banks and their creditors, such that the size of banks' balance sheets is limited by their stock of net worth (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011). Households function as NBFIs in the model: they also extend credit to production firms. However, households face quadratic adjustment costs in increasing credit supply to capture that they are less skilled at monitoring and screening borrowers and thus intermediate credit less efficiently (Gertler and Kiyotaki, 2015; Gertler et al., 2020).[1] The two-period structure with financial frictions follows Van der Kwaak and Van Wijnbergen (2017) and Van der Kwaak (2020).

I analytically show that an increase in the CCyB leads to a reduction in total credit provision when both banks and NBFIs intermediate credit. An increase in the CCyB leads to a decrease in credit provision by banks, who are required to shrink the balance sheet after an increase in the CCyB. In response, NBFIs increase credit supply, as they are not affected by the CCyB. Hence: macroprudential policy leads to a leakage of credit from banks to NBFIs. Although total credit provision falls when both NBFIs and banks intermediate credit, the size of the total reduction in credit supply depends on how efficient or inefficient NBFIs are at lending relative to banks.

I build on the literature that investigates the effectiveness of macroprudential policy in macroeconomic models. Faria-e Castro (2021) and Darracq Paries et al. (2021) study the effectiveness of the CCyB at fostering macroeconomic and financial stability in nonlinear DSGE models, while my main focus is on the effect of the CCyB on credit supply and whether

[1] In reality, direct lending by households in Europe is limited. However, many households indirectly participate in financial markets through pension funds and investment funds. Introducing another group of agents who attract funding from households and lend to firms subject to adjustment costs would be isomorphic to the current setup. I will use 'households' and 'NBFIs' interchangeably when discussing credit provided by this particular agent.

macroprudential policy leads to a shift from bank to non-bank credit. Faria-e Castro (2021) finds that an increase in the CCyB during a boom mitigates the risk of a bust, while lowering the CCyB during a financial crisis lowers the probability of a bank run. Darracq Paries et al. (2021) find that the CCyB can prevent contractionary spillovers from monetary policy cuts when policy rates are already negative. Closer to my analysis is Fève et al. (2019): they estimate a DSGE model with both commercial banks, which face a CCyB, and shadow banks, which are unregulated, and find that CCyB activations lead to a leakage of credit towards the shadow banking sector. Hence, unregulated financial institutions reduce the effectiveness of macroprudential policy. I find similar results, but do so analytically.

This Background Document follows the following structure: in Section 2 I set up the two-period model and I present the main analytical results in Section 3. I conclude in 4.

2 Model

Time is discrete, with periods $t = 1$ and $t = 2$. Agents enter period $t = 1$ with initial assets from an exogenous period $t = 0$. In $t = 1$ the regulator can decide whether or not to change the CCyB, after which agents make their decisions. Banks issue deposits, which they combine with net worth to purchase corporate securities issued by production firms, and face an agency friction such that the size of the banking sector's balance sheet depends on its net worth (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011). In addition, banks are subject to changes in capital regulation, which further limits the size of their balance sheet. Households own all firms and banks in the economy. They consume final goods and decide on their holdings of bank deposits and corporate securities. I assume households are less efficient at intermediating credit, which I capture by introducing quadratic adjustment costs. Hence, households are the NBFIs in my model. Production firms use last period's capital stock to produce final output. After production, the capital stock depreciates completely. Firms issue corporate securities, which are a claim on their profits, to banks and households which firms use to purchase final goods to construct the capital stock. Variables that are determined in period $t = 0$ are exogenous and are not affected by changes in endogenous variables. First I set up the model without the CCyB, after which I show how the model changes when I introduce capital regulation.

Some points on notation: variables with numerical subscripts are chosen in said period, while variables with subscript 0 are exogenous and can be thought of as endowments. Finally, objects without a numerical subscript are parameters.

2.1 Households

Households derive utility from consumption c_1 and c_2, and invest in corporate securities s_1^h subject to quadratic adjustment costs and in bank deposits b_1. The severity of the quadratic adjustment cost is determined by the parameter κ_h. Corporate securities are a claim on production firms' profits, and securities bought in $t = 1$ pay a net return r_2^k in $t = 2$, while bank deposits pay an exogenous real interest rate r.[2] Households are the owners of firms and banks, and therefore receive profits Π^f from firms and terminal net worth n from banks. Households enter period 1 with asset holdings from period 0 (i.e. initial holdings of assets), which can be thought of as exogenous endowments. Households maximize the following utility function

$$\max_{b_1, c_1, c_2, s_1^h} u\left(c_1\right) + \beta \mathbb{E}_1 \left\{u\left(c_2\right)\right\},$$

β is the household's subjective discount factor, $\mathbb{E}_1$ is the conditional expectations operator, and the utility function satisfies the standard assumptions: $u'\left(c_t\right) > 0$, $u''\left(c_t\right) < 0$ for $t = 1, 2$.[3] The household maximizes its utility subject to the following budget constraints for each period:

$$c_1 + b_1 + s_1^h + \frac{\kappa_h}{2}\left(s_1^h - \hat{s}^h\right)^2 = \left(1 + r\right) b_0 + \left(1 + r_1^k\right) s_0^h + \Pi_1^f,$$
$$c_2 = \left(1 + r\right) b_1 + \left(1 + r_2^k\right) s_1^h + n_2 + \Pi_2^f.$$

I relegate most of the first order conditions to the Appendix, but present the optimality conditions for the household's portfolio choice:

$$b_1 : \mathbb{E}_1 \left\{\beta \frac{u'\left(c_2\right)}{u'\left(c_1\right)} \left(1 + r\right)\right\} = 1, \tag{1}$$

$$s_1^h : \mathbb{E}_1 \left\{\beta \frac{u'\left(c_2\right)}{u'\left(c_1\right)} \left(\frac{1 + r_2^k}{1 + \kappa_h \left(s_1^h - \hat{s}^h\right)}\right)\right\} = 1, \tag{2}$$

[2] I follow Van der Kwaak (2020) in assuming an exogenous real interest rate. This assumption greatly facilitates solving for the model's equilibrium, but otherwise does not affect my results. An exogenous, constant interest rate on bank deposits can be rationalized by postulating that the model is a small open economy and households can arbitrage returns away between bank deposits and an internationally traded asset that pays an exogenously determined world interest rate, or that this accurately captures that nominal interest rates in most advanced economies are at the zero lower bound. Derivations for the effect of an increase in the CCyB when the return on deposits is endogenous are available on request.

[3] Households form expectations in $t = 1$ over realizations of variables in $t = 2$ due to a potential activation of the CCyB in $t = 1$. After that shock is realized, all uncertainty is resolved.

where equations (1) and (2) are the first order conditions for bank deposits and corporate securities, respectively, $u'(c_t)$ is the marginal utility of consumption in periods $t = 1, 2$, and $\beta \frac{u'(c_2)}{u'(c_1)}$ is the household's stochastic discount factor. Households can essentially be thought of as a group of agents that combine both households in the traditional sense and other agents that are active on financial markets but are not balance-sheet-constrained and do not face regulatory limits. The presence of quadratic adjustment costs captures limited asset market participation vis-a-vis commercial banks (Gertler and Kiyotaki, 2015).

2.2 Financial intermediaries

Financial intermediaries are operated by bankers. In turn, each banker is a member of a household. They enter period $t = 1$ with an initial endowment of net worth n_1. They buy corporate securities s_1^b that pay a net return r_2^k in period $t = 2$. Banks fund their assets by their own net worth n_1 and by attracting deposits from households b_1 which pay a constant net return r in period $t = 2$. They face the following balance sheet constraint:

$$s_1^b = n_1 + b_1. \tag{3}$$

Bankers pay out their expected terminal net worth $\mathbb{E}_1 \{n_2\}$ in $t = 2$ to their household, which is given by the difference between the return on assets and payments on liabilities:

$$\mathbb{E}_1 \{n_2\} = \mathbb{E}_1 \left\{ \left(1 + r_2^k\right) s_1^b - (1 + r)\, b_1 \right\}. \tag{4}$$

The optimization problem of bankers consists of maximizing expected discounted terminal net worth $\mathbb{E}_1 \left\{ \beta \frac{u'(c_2)}{u'(c_1)} n_2 \right\}$ to their respective households in $t = 2$, which they discount using the household's stochastic discount factor $\beta \frac{u'(c_2)}{u'(c_1)}$. I introduce financial frictions following Gertler and Karadi (2011) by postulating that banks face an incentive compatibility constraint (ICC): bankers can divert a fraction λ_k of assets s_1^b in the transition from period $t = 1$ to $t = 2$. Depositors take this into account when deciding on how many deposits to provide: they will only fund the bank such that the gains of diverting are smaller than or equal to the bank's continuation value. The bank's ICC therefore takes the following form:

$$\mathbb{E}_1 \left\{ \beta \frac{u'(c_2)}{u'(c_1)} n_2 \right\} \geq \lambda_k s_1^b. \tag{5}$$

As such, the financial intermediary's maximization problem is given by:

$$\max_{b_1, s_1^b} \mathbb{E}_1 \left\{ \beta \frac{u'(c_2)}{u'(c_1)} n_2 \right\}, \tag{6}$$

subject to the bank's balance sheet constraint (3) and the ICC (5). I solve the bank's optimization problem in the Appendix. Rewriting the first order condition for deposits and plugging it into the first order condition for capital then yields the following optimality conditions for, respectively, capital and deposits:

$$\lambda_k \left(\frac{\mu_1}{1+\mu_1} \right) = \mathbb{E}_1 \left\{ \beta \frac{u'(c_2)}{u'(c_1)} \left(r_2^k - r \right) \right\}, \tag{7}$$

$$\frac{\chi_1}{1+\mu_1} = \mathbb{E}_1 \left\{ \beta \frac{u'(c_2)}{u'(c_1)} (1+r) \right\}, \tag{8}$$

where μ_1 and χ_1 are the Lagrange multiplier on the ICC and balance sheet constraint. These first order conditions are familiar from Gertler and Karadi (2011), where a binding ICC ($\mu_1 > 0$) drives a wedge between the return on assets and payments on liabilities. In the Appendix I show that the ICC can be rewritten in the following way:

$$(1+\mu_1) n_1 = \lambda_k s_1^b. \tag{9}$$

From this condition it follows that when the ICC is binding and banks are balance-sheet-constrained ($\mu_1 > 0$), changes in net worth n_1 directly lead to changes in lending s_1^b. Hence, the agency friction leads to creditors requiring banks to have a certain level of net worth to fund a certain amount of assets.

2.3 Non-financial firms

Production firms are owned by households and produce output in periods $t = 1$ and $t = 2$ using capital using the following Cobb-Douglas technology:

$$y_1 = k_0^\alpha, \tag{10}$$

$$y_2 = k_1^\alpha. \tag{11}$$

Since the capital stock used in production in period $t = 1$ is predetermined, output in $t = 1$ is essentially exogenous. Firms issue securities to banks and households s_1^i, where $i = h, b$ to buy capital k_1, such that $k_1 = s_1^b + s_1^h$. I assume capital depreciates completely every period. Firms credibly pledge next period's after-wage profits to the households and banks that purchase their securities, who are repaid with a return r_2^k (Gertler and Kiyotaki, 2010; Gertler and Karadi, 2011). The representative production firm's optimization problem consists of choosing capital k_1:

$$\max_{k_1}\left[y_1-\left(1+r_1^k\right)k_0+\mathbb{E}_1\left\{\beta\frac{u'\left(c_2\right)}{u'\left(c_1\right)}\left(y_2-\left(1+r_2^k\right)k_1\right)\right\}\right].$$

The first order condition for capital is then given by:

$$k_1 : r_2^k = \alpha k_1^{\alpha-1} - 1. \tag{12}$$

2.4 Market clearing

In both periods, markets clear and thus aggregate demand equals aggregate supply. In period $t = 1$ total output is exogenously given and consumed by households, used by firms to construct capital goods, and used by households when they adjust the amount of corporate securities in their portfolio:

$$y_1 = c_1 + k_1 + \frac{\kappa_h}{2}\left(s_1^h - \hat{s}^h\right)^2. \tag{13}$$

In period $t = 2$ output is consumed by households:

$$y_2 = c_2. \tag{14}$$

Finally, total lending to production firms must equal the physical capital stock:

$$s_1^h + s_1^b = k_1. \tag{15}$$

2.5 Introducing the CCyB

I follow Darracq Paries et al. (2021) in modeling the CCyB. Remember that $\left(1+\mu_1\right)n_1 = \lambda_k s_1^b$. Therefore $\lambda_k\left(1+\mu_1\right)^{-1} = n_1/s_1^b$ is the capital to assets ratio when the ICC is binding. I define $\phi_1 = n_1/s_1^b$ to be the capital to assets ratio including the CCyB shock, where

$$\phi_1 = \lambda_k\left(1+\mu_1\right)^{-1} + \varkappa_1, \tag{16}$$

and $\varkappa_1$ is a CCyB shock. Hence, an increase in the CCyB ($d\varkappa_1 > 0$) increases the amount of net worth that banks need to have to finance a certain amount of assets. As such, an increase (decrease) in the CCyB leads to a decrease (increase) in bank lending. Finally, the bank's leverage ratio is the inverse of the capital to assets ratio:

$$l_1 = s_1^b/n_1 = 1/\phi_1.$$

3 Model analysis

In this section I analyze the effect of an increase in the CCyB on total credit supply and credit supply by banks and NBFIs. I assume this increase in the CCyB is a shock that hits the economy in $t = 1$. Afterwards all uncertainty is resolved. As such I can drop the expectations operator that features in the portfolio decisions of NBFIs and banks. Extensive derivations can be found in Appendix A.8.

First, I will prove that an increase in the CCyB leads to a reduction in credit supply:

Proposition 1. *An increase in the CCyB leads to a reduction in credit supply and a fall in the physical capital stock:*

$$\frac{dk_1}{d\varkappa_1} < 0.$$

Proof. Differentiating the first order condition for bank lending (7), the household's first order condition for corporate securities (2), the definition of the capital to assets ratio (16), and the market clearing condition for capital (15) yields:

$$\frac{dk_1}{d\varkappa_1} = -\frac{s_1^b l_1}{1 - \left(\kappa_h^{-1} + s_1^b l_1\right)\Omega} < 0, \tag{17}$$

where $\Omega \equiv \beta \frac{u'(c_2)}{u'(c_1)} \frac{(\alpha-1)r_2^k}{k_1} < 0$ since $0 < \alpha < 1$. Therefore $\frac{dk_1}{d\varkappa_1} < 0$. □

Hence, an increase in the CCyB leads to a fall in total lending to production firms. Next, I investigate how the composition of credit supply changes by disentangling the effect of the CCyB increase on lending by banks and lending by NBFIs.

Proposition 2. *An increase in the CCyB leads to a reduction in bank lending:*

$$\frac{ds_1^b}{d\varkappa_1} < 0.$$

Proof. Combining the closed form solution for the change in physical capital (17) with derivative of the first order condition for bank lending (7) and the definition of the capital to assets ratio (16) yields:

$$\frac{ds_1^b}{d\varkappa_1} = \frac{\left(\kappa_h^{-1}\Omega - 1\right) s_1^b l_1}{1 - \left(\kappa_h^{-1} + s_1^b l_1\right)\Omega} < 0, \tag{18}$$

where $\Omega < 0$. Therefore $\frac{ds_1^b}{d\varkappa_1} < 0$. □

The intuition behind this result is as follows. The increase in the CCyB makes banks' balance sheet constraints more binding by increasing banks' required capital to assets ratio, see equation (16). As a result, banks have to shrink the balance sheet and reduce lending to production firms, leading to a fall in bank lending s_1^b.

Next, I characterize how the CCyB shock impacts lending by NBFIs.

Proposition 3. *An increase in the CCyB leads to an increase in NBFI lending:*

$$\frac{ds_1^h}{d\varkappa_1} > 0.$$

Proof. Combining the closed form solution for the change in physical capital (17) with the derivative of the household's first order condition for corporate securities (2) yields:

$$\frac{ds_1^h}{d\varkappa_1} = -\frac{1}{\kappa_h}\frac{s_1^b l_1 \Omega}{1 - \left(\kappa_h^{-1} + s_1^b l_1\right)\Omega} > 0, \tag{19}$$

where $\Omega < 0$. Therefore $\frac{ds_1^h}{d\varkappa_1} > 0$. □

While the *increase* in the CCyB leads to a *decrease* in bank lending, lending by NBFIs *increases.* Credit supply by NBFIs expands because they are not affected by the CCyB and do not have to shrink the balance sheet. Furthermore, the fall in bank lending increases the return on corporate securities because the demand for credit is unaffected by the CCyB. Hence, NBFIs expand lending to firms to reap the benefits of these higher returns. However, because NBFIs are less efficient than banks at providing credit to firms, net total credit supply falls after the CCyB shock.

Finally, I investigate how the ease with which NBFIs intermediate credit influences the effect of the CCyB on credit supply. I do this by taking the partial derivative of equation (17) with respect to the adjustment cost parameter κ_h, as this parameter determines how efficiently NBFIs intermediate credit.

Proposition 4. *An increase in credit market frictions faced by NBFIs leads to a larger fall in credit after an increase of the CCyB:*

$$\frac{\partial}{\partial \kappa_h}\left(\frac{dk_1}{d\varkappa_1}\right) < 0.$$

Proof. Taking the derivative of 17 with respect to κ_h yields:

$$\frac{\partial}{\partial \kappa_h}\left(\frac{dk_1}{d\varkappa_1}\right) = \frac{s_1^b l_1 \Omega}{\left[1 - \left(\kappa_h^{-1} + s_1^b l_1\right)\Omega\right]^2 \kappa_h^2} < 0, \tag{20}$$

where $\Omega < 0$. Therefore $\frac{\partial}{\partial \kappa_h}\left(\frac{dk_1}{d\varkappa_1}\right) < 0$. $\square$

Hence, an increase in the adjustment cost parameter κ_h leads to a larger fall in total credit after an increase in the CCyB due to a smaller increase in NBFI lending s_1^h compared to the case where κ_h is smaller. Intuitively, a larger κ_h implies that it is relatively more costly for NBFIs to expand lending compared to the case with a smaller κ_h because NBFIs are less efficient at credit intermediation. As such, the reduction in total credit after a CCyB shock is larger when κ_h is larger. Therefore, the total impact of the CCyB on credit supply depends on how easily NFBIs can expand lending. In the case where NBFIs barely face any lending frictions, i.e. where κ_h is close to but not equal to zero, NBFIs can almost completely make up for the fall in bank lending.[4]

As the goal of this theoretical exercise is to derive hypotheses for a separate empirical study (**INSERT REFERENCE**), I restate the most important theoretical results:

1. The activation of the CCyB *reduces bank* credit supply, as the increase in required capital buffers will lead to banks shrinking the balance sheet.

2. The activation of the CCyB *increases non-bank* credit supply, as NBFIs increase lending in response to a fall in bank lending.

3. The activation of the CCyB *reduces macro* credit supply: however, if NBFIs can easily engage in credit provision, then the gap left by the reduction in bank credit can be (partially) filled by NBFIs, mitigating the fall in total credit.

[4] Note that for $\kappa_h = 0$ NBFIs take over all credit intermediation activities from banks. This case therefore represents a corner solution. In this case the CCyB would have zero effect on credit supply, as NBFIs completely crowd out banks from financial markets. Empirically, the value for κ_h would be determined by the difference in lending efficiency between banks and NBFIs.

4 Conclusion

In this background document I write down a two-period general equilibrium model of banking to analyze the effects of an increase in the CCyB on credit supply by banks and NBFIs. I find that an increase in the CCyB leads to a fall in bank credit supply. NBFIs compensate for this fall in bank credit supply by increasing credit supply. On the macro level, the total decrease in credit therefore depends on how easily NBFIs can increase lending. These analytical results form the basis and economic intuition for the empirical hypotheses in the paper that this document accompanies.

5 References

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A Mathematical derivations

Time is discrete, with periods $t = 1$ and $t = 2$. Variables that are determined in period $t = 0$ are exogenous and not affected by changes in endogenous variables. In this section I set up the model without the CCyB. I show how the model needs to be changed to allow for capital regulation.

A.1 Households

Households invest in corporate securities s^h subject to quadratic adjustment costs and in bank deposits b. The strength of the quadratic adjustment cost is determined by the parameter κ_h. Corporate securities are a claim on production firms' profits, and securities bought in $t = 1$ pay a net return r_2^k in $t = 2$. Bank deposits pay an exogenous real interest rate r. Assuming an exogenous real interest rate greatly facilitates solving for the model's equilibrium, but is otherwise unlikely to affect our results. An exogenous, constant interest rate on bank deposits can be rationalized by postulating that the model is a small open economy and households can arbitrage returns away between bank deposits and an internationally traded asset that pays an exogenously determined world interest rate, or by arguing that this accurately captures most advanced economies being at the ZLB. Households are the owners of firms and banks, and therefore reeceive profits Π^f from firms and terminal net worth n from banks. Households enter period 1 with asset holdings from period 0 (i.e. initial holdings of assets), which can be thought of as exogenous endomwments.

$$\max_{\left\{b_1, c_1, c_2, s_1^h, s_2^h\right\}} u\left(c_1\right) + \beta \mathbb{E}_1 \left\{u\left(c_2\right)\right\},$$

subject to the following budget constraints for each period:

$$\begin{aligned} c_1 + b_1 + s_1^h + \frac{\kappa_h}{2}\left(s_1^h - \hat{s}^h\right)^2 &= \left(1+r\right) b_0 + \left(1 + r_1^k\right) s_0^h + \Pi_1^f, \\ c_2 &= \left(1+r\right) b_1 + \left(1 + r_2^k\right) s_1^h + n_2 + \Pi_2^f. \end{aligned}$$

I set up Lagrangians for both $t = 1$ and $t = 2$:

$$\begin{aligned} \mathcal{L}_1 = \mathbb{E}_1 \{ & u\left(c_1\right) + \beta u\left(c_2\right) \\ & + \lambda_1 \left(\left(1+r\right) b_0 + \left(1 + r_1^k\right) s_0^h + \Pi_1^f - c_1 - b_1 - s_1^h - \frac{\kappa_h}{2}\left(s_1^h - \hat{s}^h\right)^2\right) \\ & + \beta \lambda_2 \left(\left(1+r\right) b_1 + \left(1 + r_2^k\right) s_1^h + n_2 + \Pi_2^f - c_2\right) \}, \end{aligned}$$

and

$$\mathcal{L}_2 = u(c_2) + \lambda_2 \left((1+r) b_1 + \left(1 + r_2^k\right) s_1^h + n_2 + \Pi_2^f - c_2 \right).$$

The first order condition for $t = 2$ is given by:

$$c_2 : u'(c_2) - \lambda_2 = 0 \Rightarrow u'(c_2) = \lambda_2. \tag{A.1}$$

The first order conditions in $t = 1$ are then given by:

$$c_1 : u'(c_1) - \lambda_1 = 0 \Rightarrow u'(c_1) = \lambda_1, \tag{A.2}$$

$$b_1 : \mathbb{E}_1 \{\beta \lambda_2 (1+r)\} - \lambda_1 = 0 \Rightarrow \mathbb{E}_1 \left\{ \beta \frac{\lambda_2}{\lambda_1} (1+r) \right\} = 1, \tag{A.3}$$

$$s_1^h : \mathbb{E}_1 \left\{ \beta \lambda_2 \left(1 + r_2^k\right) \right\} - \lambda_1 \left(1 + \kappa_h \left(s_1^h - \hat{s}^h\right)\right) = 0 \Rightarrow \mathbb{E}_1 \left\{ \beta \frac{\lambda_2}{\lambda_1} \left(\frac{1 + r_2^k}{1 + \kappa_h \left(s_1^h - \hat{s}^h\right)} \right) \right\} = 1. \tag{A.4}$$

Households can essentially be thought of as a group of agents that combine both households in the traditional sense and other agents that are active on financial markets but are not balance-sheet-constrained and do not face regulatory limits. The presence of quadratic adjustment costs captures limited asset market participations vis-a-vis commercial banks, which can be due to a less efficient financial intermediation technology.

A.2 Financial intermediaries

Financial intermediaries are operated by households and enter period $t = 1$ with an initial endowment of net worth n_1. They buy assets s_1^b and are funded by their own net worth n_1 and household deposits b_1. They face the following balance sheet constraint:

$$s_1^b = n_1 + b_1. \tag{A.5}$$

Net worth in period $t = 2$ is then given by:

$$\mathbb{E}_1 \{n_2\} = \left(1 + r_2^k\right) s_1^b - (1+r) b_1. \tag{A.6}$$

Following Gertler and Karadi (2011), banks face the following ICC:

$$\mathbb{E}_1 \left\{ \beta \frac{\lambda_2}{\lambda_1} n_2 \right\} \geq \lambda_k s_1^b. \tag{A.7}$$

As such, the financial intermediary's maximization problem is given by:

$$\max_{\left\{b_1, s_1^b\right\}} \mathbb{E}_1 \left\{ \beta \frac{\lambda_2}{\lambda_1} n_2 \right\}, \tag{A.8}$$

subject to:

$$\mathbb{E}_1 \left\{ \beta \frac{\lambda_2}{\lambda_1} n_2 \right\} \geq \lambda_k s_1^b,$$
$$s_1^b = n_1 + b_1.$$

The Lagrangian is then given by:

$$\begin{aligned} \mathcal{L} = (1 + \mu_1) \mathbb{E}_1 & \left\{ \beta \frac{\lambda_2}{\lambda_1} \left[\left(1 + r_2^k\right) s_1^b - (1 + r) b_1 \right] \right\} \\ & - \mu_1 \lambda_k s_1^b + \chi_1 \left(n_1 + b_1 - s_1^b \right). \end{aligned}$$

The first order conditions are then given by:

$$s_1^b : (1 + \mu_1) \mathbb{E}_1 \left\{ \beta \frac{\lambda_2}{\lambda_1} \left(1 + r_2^k\right) \right\} - \mu_1 \lambda_k - \chi_1 = 0, \tag{A.9}$$

$$b_1 : -(1 + \mu_1) \mathbb{E}_1 \left\{ \beta \frac{\lambda_2}{\lambda_1} (1 + r) \right\} + \chi_1 = 0. \tag{A.10}$$

Rewriting the FOC for deposits and plugging it into the FOC for capital then yields the following first order conditions for, respectively, capital and deposits:

$$\lambda_k \left(\frac{\mu_1}{1 + \mu_1} \right) = \mathbb{E}_1 \left\{ \beta \frac{\lambda_2}{\lambda_1} \left(r_2^k - r \right) \right\}, \tag{A.11}$$

$$\frac{\chi_1}{1 + \mu_1} = \mathbb{E}_1 \left\{ \beta \frac{\lambda_2}{\lambda_1} (1 + r) \right\}. \tag{A.12}$$

Rewriting the law of motion for n_2 such that I get rid of b_1:

$$\mathbb{E}_1 \{n_2\} = \left(r_2^k - r \right) s_1^b + (1 + r) n_1, \tag{A.13}$$

where I used the bank's balance sheet constraint. I substitute this expression into the ICC:

$$\mathbb{E}_1\left\{\beta\frac{\lambda_2}{\lambda_1}\left(\left(r_2^k-r\right)s_1^b+\left(1+r\right)n_1\right)\right\}\geq\lambda_k s_1^b. \tag{A.14}$$

Next, I use that $\mathbb{E}_1\left\{\beta\frac{\lambda_2}{\lambda_1}\left(1+r\right)\right\}=1$ from the household's optimization problem and use that $\mathbb{E}_1\left\{\beta\frac{\lambda_2}{\lambda_1}\left(r_2^k-r\right)\right\}=\lambda_k\left(\frac{\mu_1}{1+\mu_1}\right)$:

$$\begin{aligned}\lambda_k\left(\frac{\mu_1}{1+\mu_1}\right)s_1^b+n_1\geq\lambda_k s_1^b,\Rightarrow&\\ n_1&\geq\left(1-\frac{\mu_1}{1+\mu_1}\right)\lambda_k s_1^b,\Rightarrow\\ n_1&\geq\left(\frac{1+\mu_1-\mu_1}{1+\mu_1}\right)\lambda_k s_1^b,\Rightarrow\\ n_1&\geq\left(\frac{1}{1+\mu_1}\right)\lambda_k s_1^b.\end{aligned}$$

Assuming the constraint holds with equality and rewriting yields the following equation:

$$\left(1+\mu_1\right)n_1=\lambda_k s_1^b. \tag{A.15}$$

A.3 Non-financial firms

Production firms are owned by households and produce output in periods $t=1$ and $t=2$ using capital using the following Cobb-Douglas technology:

$$y_1=k_0^\alpha, \tag{A.16}$$

$$y_2=k_1^\alpha. \tag{A.17}$$

Since the capital stock used in production in period $t=1$ is predetermined, output in $t=1$ is essentially exogenous. Firms issue securities to banks and households s_1^i, where $i=h,b$ to buy capital k_1, such that $k_1=s_1^b+s_1^h$. I assume capital depreciates completely every period. Households and banks are repaid with a return r_2^k. The representative production firm's maximization problem is given by:

$$\max_{\{k_1\}}\left[y_1-\left(1+r_1^k\right)k_0+\mathbb{E}_1\left\{\beta\frac{\lambda_2}{\lambda_1}\left(y_2-\left(1+r_2^k\right)k_1\right)\right\}\right].$$

The first order condition for capital is then given by:

$$k_1 : r_2^k = \alpha k_1^{\alpha-1} - 1. \tag{A.18}$$

A.4 Market clearing

First, asset markets must clear in both periods:

$$\begin{aligned} s_0^h + s_0^b &= k_0, \\ s_1^h + s_1^b &= k_1. \end{aligned}$$

In period $t = 1$:

$$c_1 + b_1 + s_1^h + \frac{\kappa_h}{2}\left(s_1^h - \hat{s}^h\right)^2 = (1+r)\,b_0 + \left(1 + r_1^k\right) s_0^h + \Pi_1^f.$$

Using that $b_1 = s_1^b - n_1$:

$$c_1 + s_1^b - n_1 + s_1^h + \frac{\kappa_h}{2}\left(s_1^h - \hat{s}^h\right)^2 = (1+r)\,b_0 + \left(1 + r_1^k\right) s_0^h + \Pi_1^f.$$

Next, I use that $n_1 = \left(1 + r_1^k\right) s_0^b - (1+r)\,b_0$:

$$c_1 + s_1^b - \left(1 + r_1^k\right) s_0^b + (1+r)\,b_0 + s_1^h + \frac{\kappa_h}{2}\left(s_1^h - \hat{s}^h\right)^2 = (1+r)\,b_0 + \left(1 + r_1^k\right) s_0^h + \Pi_1^f.$$

Getting rid of the identical deposit expressions and using the asset market clearing condition yields:

$$c_1 + k_1 + \frac{\kappa_h}{2}\left(s_1^h - \hat{s}^h\right)^2 = \left(1 + r_1^k\right) k_0 + \Pi_1^f.$$

Profits in period $t = 1$ are given by $\Pi_1^f = y_1 - \left(1 + r_1^k\right) k_0$. Hence:

$$c_1 + k_1 + \frac{\kappa_h}{2}\left(s_1^h - \hat{s}^h\right)^2 = \left(1 + r_1^k\right) k_0 + y_1 - \left(1 + r_1^k\right) k_0.$$

This yields the following market clearing condition:

$$y_1 = c_1 + k_1 + \frac{\kappa_h}{2}\left(s_1^h - \hat{s}^h\right)^2. \tag{A.19}$$

Now for period $t = 2$:

$$c_2 = (1+r)\,b_1 + \left(1 + r_2^k\right) s_1^h + n_2 + \Pi_2^f.$$

Using $n_2 = \left(1 + r_2^k\right) s_1^b - (1+r)\,b_1$ and the asset market clearing condition:

$$c_2 = (1+r)\,b_1 + \left(1 + r_2^k\right) k_1 - (1+r)\,b_1 + \Pi_2^f.$$

Using the definition for production firms' profits, i.e. $\Pi_2^f = y_2 - \left(1 + r_2^k\right) k_1$:

$$c_2 = (1+r)\,b_1 + \left(1 + r_2^k\right) k_1 - (1+r)\,b_1 + y_2 - \left(1 + r_2^k\right) k_1.$$

This simplifies to the following market clearing condition in period $t = 2$:

$$y_2 = c_2. \tag{A.20}$$

A.5 Equilibrium conditions

First order conditions for households are given by:

$$u'(c_1) = \lambda_1, \tag{A.21}$$

$$u'(c_2) = \lambda_2, \tag{A.22}$$

$$1 = \mathbb{E}_1 \left\{ \beta \frac{\lambda_2}{\lambda_1} (1+r) \right\}, \tag{A.23}$$

$$1 = \mathbb{E}_1 \left\{ \beta \frac{\lambda_2}{\lambda_1} \left(\frac{1 + r_2^k}{1 + \kappa_h \left(s_1^h - \hat{s}^h \right)} \right) \right\}. \tag{A.24}$$

First order conditions for production firms are given by:

$$1 + r_2^k = \alpha k_1^{\alpha - 1}, \tag{A.25}$$

$$y_1 = k_0^\alpha, \tag{A.26}$$

$$y_2 = k_1^\alpha. \tag{A.27}$$

First order conditions for financial intermediaries are given by:

$$n_1 + b_1 = s_1^b, \tag{A.28}$$

$$n_2 = \left(1 + r_2^k\right) s_1^b - (1 + r)\, b_1, \tag{A.29}$$

$$n_1 = \left(1 + r_1^k\right) s_0^b - (1 + r)\, b_0, \tag{A.30}$$

$$\frac{\lambda_k \mu_1}{1 + \mu_1} = \mathbb{E}_1 \left\{ \beta \frac{\lambda_2}{\lambda_1} \left(r_2^k - r \right) \right\}, \tag{A.31}$$

$$\chi_1 = (1 + \mu_1)\, \mathbb{E}_1 \left\{ \beta \frac{\lambda_2}{\lambda_1} (1 + r) \right\}, \tag{A.32}$$

$$(1 + \mu_1)\, n_1 = \lambda_k s_1^b. \tag{A.33}$$

Finally, goods and assets markets must clear:

$$y_1 + w_1 = c_1 + k_1 + \frac{\kappa_h}{2} \left(s_1^h - \hat{s}^h \right)^2, \tag{A.34}$$

$$y_2 = c_2, \tag{A.35}$$

$$k_0 = s_0^h + s_0^b, \tag{A.36}$$

$$k_1 = s_1^h + s_1^b. \tag{A.37}$$

A.6 Introducing CCyB

Remember that

$$(1 + \mu_1)\, n_1 = \lambda_k s_1^b.$$

Therefore $\lambda_k (1 + \mu_1)^{-1} = n_1 / s_1^b$ is the capital to assets ratio when the ICC is binding. Let us define $\phi_1 = n_1 / s_1^b$ to be the capital to assets ratio including the CCyB shock, where

$$\phi_1 = \lambda_k (1 + \mu_1)^{-1} + \varkappa_1,$$

and $\varkappa_1$ is a CCyB shock. Finally, I define the bank's leverage ratio to be the inverse of the capital to assets ratio:

$$l_1 = s_1^b / n_1 = 1/\phi_1.$$

A.7 Partial derivatives

In order to characterize the effect of a shock to the CCyB, I have to take the partial derivative of all endogenous variables with respect to the CCyB shock $\varkappa_1$. I give an overview of all relevant (and some not so relevant) partial derivatives in this section. As calculating these derivatives is relatively straightforward, I do not provide an extensive derivation.

Household:

$$\begin{aligned}
\frac{d\lambda_1}{d\varkappa_1} &= u''\left(c_1\right)\frac{dc_1}{d\varkappa_1}, \\
\frac{d\lambda_2}{d\varkappa_1} &= u''\left(c_2\right)\frac{dc_2}{d\varkappa_1}, \\
0 &= \frac{u''\left(c_2\right)}{u'\left(c_2\right)}\frac{dc_2}{d\varkappa_1} - \frac{u''\left(c_1\right)}{u'\left(c_1\right)}\frac{dc_1}{d\varkappa_1}, \\
0 &= \kappa_h\frac{ds_1^h}{d\varkappa_1} - \beta\frac{u'\left(c_2\right)}{u'\left(c_1\right)}\frac{dr_2^k}{d\varkappa_1}.
\end{aligned}$$

A quick note: from the third equation I can see how assuming an exogenous risk-free return on deposits simplifies the coming derivations considerably. Rewriting:

$$\frac{dc_2/d\varkappa_1}{dc_1/d\varkappa_1} = \frac{\frac{u''(c_2)}{u'(c_2)}}{\frac{u''(c_1)}{u'(c_1)}} > 0.$$

Hence, consumption growth from period $t = 1$ to period $t = 2$ is positive and constant.

Production:

$$\begin{aligned}
\frac{dr_2^k}{d\varkappa_1} &= \alpha\left(\alpha - 1\right)k_1^{\alpha-2}\frac{dk_1}{d\varkappa_1} = \frac{\left(\alpha - 1\right)r_2^k}{k_1}\frac{dk_1}{d\varkappa_1}, \\
\frac{dy_1}{d\varkappa_1} &= 0, \\
\frac{dy_2}{d\varkappa_1} &= \alpha k_1^{\alpha-1}\frac{dk_1}{d\varkappa_1} = r_2^k\frac{dk_1}{d\varkappa_1}.
\end{aligned}$$

Market clearing:

$$\frac{dy_1}{d\varkappa_1} = \frac{dc_1}{d\varkappa_1} + \frac{dk_1}{d\varkappa_1} + \kappa_h \left(s_1^h - \hat{s}^h\right) \frac{ds_1^h}{d\varkappa_1},$$
$$\frac{dy_2}{d\varkappa_1} = \frac{dc_1}{d\varkappa_1},$$
$$\frac{dk_0}{d\varkappa_1} = 0,$$
$$\frac{dk_1}{d\varkappa_1} = \frac{ds_1^h}{d\varkappa_1} + \frac{ds_1^b}{d\varkappa_1}.$$

Banks:

$$\frac{db_1}{d\varkappa_1} = \frac{ds_1^b}{d\varkappa_1},$$
$$\frac{dn_2}{d\varkappa_1} = \left(1 + r_2^k\right) \frac{ds_1^b}{d\varkappa_1} + s_1^b \frac{dr_2^k}{d\varkappa_1} - (1 + r) \frac{db_1}{d\varkappa_1},$$
$$\frac{dn_1}{d\varkappa_1} = 0,$$
$$\frac{d\mu_1}{d\varkappa_1} = \frac{(1 + \mu_1)^2}{\lambda_k} \beta \frac{u'(c_2)}{u'(c_1)} \frac{dr_2^k}{d\varkappa_1},$$
$$\frac{d\phi_1}{d\varkappa_1} = -\frac{n_1}{\left(s_1^b\right)^2} \frac{ds_1^b}{d\varkappa_1},$$
$$\frac{d\phi_1}{d\varkappa_1} = 1 - \frac{\lambda_k}{(1 + \mu_1)^2} \frac{d\mu_1}{d\varkappa_1}.$$

A.8 Effect of CCyB shock

Calculating the effect of an increase in the CCyB is straightforward. Combining the derivative of the multiplier on the ICC, the return on capital, and the definition of the maximum allowed capital-to-assets ratio yields:

$$\frac{d\mu_1}{d\varkappa_1} = \frac{(1 + \mu_1)^2}{\lambda_k} \beta \frac{u'(c_2)}{u'(c_1)} \frac{dr_2^k}{d\varkappa_1} = \frac{(1 + \mu_1)^2}{\lambda_k} \beta \frac{u'(c_2)}{u'(c_1)} \frac{(\alpha - 1) r_2^k}{k_1} \frac{dk_1}{d\varkappa_1}, \Rightarrow$$
$$\frac{d\phi_1}{d\varkappa_1} = 1 - \frac{\lambda_k}{(1 + \mu_1)^2} \frac{d\mu_1}{d\varkappa_1} = 1 - \frac{\lambda_k}{(1 + \mu_1)^2} \frac{(1 + \mu_1)^2}{\lambda_k} \beta \frac{u'(c_2)}{u'(c_1)} \frac{(\alpha - 1) r_2^k}{k_1} \frac{dk_1}{d\varkappa_1}, \Rightarrow$$
$$\frac{d\phi_1}{d\varkappa_1} = 1 - \beta \frac{u'(c_2)}{u'(c_1)} \frac{(\alpha - 1) r_2^k}{k_1} \frac{dk_1}{d\varkappa_1}.$$

Combining this expression with the derivative of the definition of the capital to assets ratio:

$$\frac{ds_1^b}{d\varkappa_1} = -\frac{\left(s_1^b\right)^2}{n_1}\frac{d\phi_1}{d\varkappa_1} = \frac{\left(s_1^b\right)^2}{n_1}\left[\beta\frac{u'\left(c_2\right)}{u'\left(c_1\right)}\frac{\left(\alpha-1\right)r_2^k}{k_1}\frac{dk_1}{d\varkappa_1} - 1\right].$$

Rewriting the derivative of the first order condition for household holdings of corporate securities:

$$\frac{ds_1^h}{d\varkappa_1} = \frac{1}{\kappa_h}\beta\frac{u'\left(c_2\right)}{u'\left(c_1\right)}\frac{dr_2^k}{d\varkappa_1} = \frac{1}{\kappa_h}\beta\frac{u'\left(c_2\right)}{u'\left(c_1\right)}\frac{\left(\alpha-1\right)r_2^k}{k_1}\frac{dk_1}{d\varkappa_1}.$$

Combine these expressions with the asset market clearing condition:

$$\begin{aligned}
\frac{dk_1}{d\varkappa_1} &= \frac{ds_1^h}{d\varkappa_1} + \frac{ds_1^b}{d\varkappa_1}, \Rightarrow \\
\frac{dk_1}{d\varkappa_1} &= \frac{1}{\kappa_h}\beta\frac{u'\left(c_2\right)}{u'\left(c_1\right)}\frac{\left(\alpha-1\right)r_2^k}{k_1}\frac{dk_1}{d\varkappa_1} + \frac{\left(s_1^b\right)^2}{n_1}\left[\beta\frac{u'\left(c_2\right)}{u'\left(c_1\right)}\frac{\left(\alpha-1\right)r_2^k}{k_1}\frac{dk_1}{d\varkappa_1} - 1\right].
\end{aligned}$$

Collecting terms on one side, rewriting and using the definition of the leverage ratio yields the following expression for the change in credit:

$$\frac{dk_1}{d\varkappa_1} = -\frac{s_1^b l_1}{1-\left(\kappa_h^{-1} + s_1^b l_1\right)\Omega} < 0, \tag{A.38}$$

where $\Omega \equiv \beta\frac{u'(c_2)}{u'(c_1)}\frac{(\alpha-1)r_2^k}{k_1} < 0$ since $0 < \alpha < 1$. Hence, I find that increasing the CCyB decreases credit provision if $\kappa_h > 0$. If $\kappa_h \to 0$, then $\frac{dk_1}{d\varkappa_1} = 0$.

What happens to household lending? Using the solution for the derivative of total credit provision, I find that:

$$\frac{ds_1^h}{d\varkappa_1} = -\frac{1}{\kappa_h}\frac{s_1^b l_1 \Omega}{1-\left(\kappa_h^{-1} + s_1^b l_1\right)\Omega} > 0, \tag{A.39}$$

since $\Omega < 0$. Hence, household lending increases: not just relative to bank lending, but in absolute terms.

Finally, bank lending is given by:

$$
\begin{aligned}
\frac{ds_1^b}{d\varkappa_1} &= s_1^b l_1 \left[-\frac{s_1^b l_1 \Omega}{1 - \left(\kappa_h^{-1} + s_1^b l_1\right)\Omega} - 1 \right], \Rightarrow \\
\frac{ds_1^b}{d\varkappa_1} &= s_1^b l_1 \left[\frac{-s_1^b l_1 \Omega - \left(1 - \left(\kappa_h^{-1} + s_1^b l_1\right)\Omega\right)}{1 - \left(\kappa_h^{-1} + s_1^b l_1\right)\Omega} \right], \Rightarrow \\
\frac{ds_1^b}{d\varkappa_1} &= s_1^b l_1 \left[\frac{-s_1^b l_1 \Omega - 1 + \left(\kappa_h^{-1} + s_1^b l_1\right)\Omega}{1 - \left(\kappa_h^{-1} + s_1^b l_1\right)\Omega} \right], \Rightarrow \\
\frac{ds_1^b}{d\varkappa_1} &= \frac{\left(\kappa_h^{-1}\Omega - 1\right) s_1^b l_1}{1 - \left(\kappa_h^{-1} + s_1^b l_1\right)\Omega} < 0.
\end{aligned}
$$

Hence, bank lending decreases.

I can analyze how household lending influences crowding out by inspecting the total change in credit:

$$
\frac{dk_1}{d\varkappa_1} = -\frac{s_1^b l_1}{1 - \left(\kappa_h^{-1} + s_1^b l_1\right)\Omega} < 0,
$$

I show this formally by taking the partial derivative of $dk_1/d\varkappa_1$ w.r.t. κ_h:

$$
\frac{\partial}{\partial \kappa_h}\left(\frac{dk_1}{d\varkappa_1}\right) = \frac{s_1^b l_1 \Omega}{\left[1 - \left(\kappa_h^{-1} + s_1^b l_1\right)\Omega\right]^2 \kappa_h^2} < 0.
$$

Hence, increased portfolio adjustment costs for households lead to a larger fall in credit after a CCyB shock. Mathematically, for higher κ_h you're dividing by a a smaller number and hence the decrease in total credit is going to be larger.