

CPB Netherlands Bureau for Economic Policy Analysis

Estimation of the Financial Cycle with a Rank-Reduced Multivariate State-Space Model

We estimate the financial cycle based on a rank-restricted multivariate state-space model. The financial cycle dynamics are captured by an unobserved trigonometric cycle component. We identify a single financial cycle from the multiple time series by imposing rank reduction on this cycle component. The rank reduction can be justified based on a principal components argument. The model includes unobserved components to capture the business cycle, time-varying seasonality, trends, and growth rates in the data. We conclude that credit and house prices are sufficient to estimate the financial cycle.

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Abstract

We propose a model-based method to estimate a unique financial cycle based on a rankrestricted multivariate state-space model. This permits us to use mixed-frequency data, allowing for longer sample periods. In our model the financial cycle dynamics are captured by an unobserved trigonometric cycle component. We identify a single financial cycle from the multiple time series by imposing rank reduction on this cycle component. The rank reduction can be justified based on a principal components argument. The model also includes unobserved components to capture the business cycle, time-varying seasonality, trends, and growth rates in the data. In this way we can control for these effects when estimating the financial cycle. We apply our model to US and Dutch data and conclude that a bivariate model of credit and house prices is sufficient to estimate the financial cycle.

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1 Introduction

In this article we estimate a single financial cycle based on a multivariate state-space model of financial and macroeconomic variables. The financial cycle is represented by a trigonometric cycle unobserved component. In order to identify a single underlying financial cycle we impose rank reduction on the financial cycle unobserved component. By restricting the rank of the covariance matrices of the error vector and initial value vector of these financial cycle unobserved components to be 1, we ensure that they will be identical up to a scaling factor.¹ Our use of rank reduction to estimate a unique financial cycle for a country is as far as we know new to the literature.

We apply our model to mixed-frequency data for the US and the Netherlands to obtain estimates of the financial cycles for both countries. The advantage of working with mixedfrequency data is that we obtain a longer sample period which helps us to identify the financial cycle with its relatively long periodicity. It also involves the introduction of missing observations into the analysis. One of the advantages of state-space models, however, is that their estimation in the presence of missing observations is straight forward.²

The rank reduction we impose to identify a single financial cycle can be justified by a principal components argument: the largest eigenvalue of the unrestricted covariance matrix of the disturbance vector driving the financial cycle components typically represents more than 90% of the sum of the eigenvalues. This suggests that the covariance of rank one is sufficient to capture the most important aspects of both cycles. We note that while this rank reduction is not supported by a model test based on the Bayes factor, the outcomes of such tests are heavily dependent on and even sometimes dominated by the priors.

We perform our estimation using Bayesian methods based on Markov Chain Monte Carlo, or MCMC simulation. A Bayesian approach has the advantage that we can include prior information in our estimation to help identify the model. Given that our model also includes unobserved components to capture the business cycle, time-varying seasonality, trends and growth rates, our priors therefore presume that the financial cycle has a longer period than the business cycle, and that the underlying growth rate only gradually changes over time.

All versions of our model include quarterly credit and housing price data. This is because these two financial series are generally seen as the principal series behind the financial cycle, see for example de Winter *et al.* (2017) and Rünstler & Vlekke (2018). Our results also lend support to this idea. We also produced estimates based on a version of the model which also includes quarterly GDP, industrial production, the S&P 500 price to earnings ratio (PE), and

¹This scaling factor is determined by the variances of the disturbance terms driving the financial cycle UCs implied by the disturbance vector covariance matrix.

 $^{^2 \}mathrm{See}$ for example Koopman et~al.~(1999) for details.

interest rate spreads. We conclude, however, that the bivariate model of credit and house prices is sufficient to obtain reliable estimates of the financial cycle.

While we opt for the model-based estimation of the financial cycle, other researchers employ filter-based methods. For example, Jordà *et al.* (2018) propose identifying financial cycles through the use of a bandpass filter using the same long-period annual data we use. Schüler *et al.* (2015) base their estimates of the financial cycle for European countries on a frequency domain based approach. Their data set begins in 1970. Rozite *et al.* (2016) propose a method of estimating a financial cycle for the US based on principal component analysis for data from 1973 to 2014. The Bank of International Settlements, or BIS, publishes estimates of their financial cycle index based on Drehmann *et al.* (2012). These estimates involve the use of filtering as well as turning points.

We argue, however, that a model-based approach to the estimation of the financial cycle has a number of advantages. In addition to the business and financial cycle unobserved components, our model also includes unobserved components to capture time-varying seasonality, trends and growth rates. By explicitly modeling these underlying processes, we can control for their effects when estimating the financial cycle.³ This model-based approach also allows us to include prior information about the unobserved components in the model and to produce model consistent forecasts. These benefits are either lacking or difficult to realize with filter-based methods.

We note that there are a number of articles in the existing literature in which the financial cycle is modeled as an unobserved trigonometric cycle component. Galati *et al.* (2016) and Rünstler & Vlekke (2018) estimate financial cycles from univariate models of a number of series. In Koopman & Lucas (2005) and de Winter *et al.* (2017) the authors also propose univariate models but with both business and financial cycles modeled as unobserved trigonometric cycle components. The ECB's Working Group on Econometric Modelling (2018) estimate financial cycles based on a multivariate state-space model. Their model, however, does not include a business cycle unobserved component, nor do they impose rank reduction on the financial cycle component to obtain a unique financial cycle for each country in their study.

Our state-space model differs in number of ways from the ones used in these articles. For one, the other models are more restricted in the stochastic processes governing the trend and drift components. Secondly, we include seasonal components in our model, which allows us to base our estimates on seasonally unadjusted data. There have been a number of articles published in which the authors argue that estimates based on seasonally adjusted data are to be preferred. The problem with seasonally adjusted data is that it tends to introduce spurious cyclicality in the data, see for example Luginbuhl & Vos (2003) and Harvey *et al.* (2007). Most

³Note that this type of state-space model is also referred to as an unobserved component time series model. We refer the reader to Harvey (1991) for further details. Further information about state-space models can be found in Durbin & Koopman (2001).

importantly, however, none of the cited articles produce estimates of an unique financial cycle for each country, as we do here.

In the following section we formulate the state-space model, after which we describe the Bayesian estimation method we employ in Section 3. Section 3 also includes a discussion of how we impose the rank reduction we need to identify a unique financial cycle. In Section 4 we specify the priors, followed by a discussion of the data in Section 5 and the results in Section 6. In Section 7 we present our conclusions.

2 The state-space model

State-space models are specified via the state space form, which consists of two equations: the measurement and state equations. The measurement equation specifies how the unobserved components and measurement error combine to produce the data. We use the logarithm of the observed series. The data is assumed to follow a long run trend. This trend is in turn influenced by a growth rate that slowly varies over time. The business cycle and financial cycles cause longer frequency fluctuations around this slowly moving trend. Therefore when the financial cycle is larger than zero, financial market conditions are above their long-term trend. As a result the cycle components are assumed to produce no permanent changes to the level of the series, only temporary ones. Our model also includes seasonal factors to capture the seasonal pattern in the data.

Formally we assume that the model involves n time series, which at period t are denoted by y_{it} for i = 1, ..., n. Therefore, we specify a measurement equation for the observed data vector $\vec{y_t} = (y_{1t}, ..., y_{nt})$. Each series is assumed to consist of a growing trend, μ_{it} , a business cycle component, ψ_{it}^B , financial cycle component, ψ_{it}^F , a set of seasonal components, $\gamma_{i,j,t}$ and a measurement error, ε_{it} . Adopting the same vector notation convention for the unobserved components and the measurement error that we use for $\vec{y_t}$ enables us to formulate the measurement equation as follows.

$$\vec{y}_t = \vec{\mu}_t + \vec{\psi}_t^F + \vec{\psi}_t^B + \sum_{j=1}^{[s/2]} \vec{\gamma}_{jt} + \vec{\varepsilon}_t, \quad \vec{\varepsilon}_t \sim N\left(0, \Omega_{\varepsilon, t}\right)$$
(1)

Note that the measurement error covariance $\Omega_{\varepsilon,t}$ is assumed to be time-varying. This is to allow us to correct for the fact that at least some of the series consist of yearly data of lower quality at the start of the sample period, while in the latter part of the sample period they consist of quarterly data. This leads us to specify $\Omega_{\varepsilon,t}$ as a diagonal matrix, with zeros as off-diagonal elements, indicating that the measurement errors between series are uncorrelated, where the elements on the main diagonal are given by

diagonal
$$(\Omega_{\varepsilon,t}) = (\sigma_{\varepsilon,1} \mathbf{1} (t \ge T_1^*) + \sigma_{h,1} \mathbf{1} (t < T_1^*), \dots, \sigma_{\varepsilon,n} \mathbf{1} (t \ge T_n^*) + \sigma_{h,n} \mathbf{1} (t < T_n^*))$$
 (2)

We denote the indicator function here by $\mathbf{1}(\cdot)$, therefore the date T_i^* is the date of the first quarter of the sample period with the lower variance $\sigma_{\varepsilon,i}$ for series *i*. The initial period, $t < T_i^*$, is assumed to have the higher variance $\sigma_{h,i}$. This point is discussed below in more detail in Section 5.⁴ Values for T_i^* are listed in Table B.2 in Appendix B.

The state equation of the state space form specifies the dynamics of the unobserved components in the model. If we stack the unobserved components into a state vector \vec{a}_t and the unobserved component disturbances into the vector $\vec{\xi}_t$, then the general expression of the state equation is given by

$$\vec{a}_{t+1} = T_t \vec{a}_t + \vec{\xi}_t,\tag{3}$$

where the matrix T_t is known as the transition matrix. This matrix is a sparse, block diagonal matrix. This enables us to unpack the state equation into a number of separate equations, each one governing the evolution of an unobserved component. We can therefore implicitly define T_t by specifying these separate equations of each unobserved component. We begin with the specification of the trend component $\vec{\mu}_t$.

The unobserved component $\vec{\mu}_t$ in (1) represents a type of time-varying trend called a local linear trend:

$$\vec{\mu}_{t} = \vec{\mu}_{t-1} + \vec{\beta}_{t-1} + \vec{\eta}_{t}, \quad \vec{\eta}_{t} \sim N(0, \Omega_{\eta}).$$
(4)

Note that the covariance matrix Ω_{η} is restricted to be diagonal to achieve a more parsimonious model. The $\vec{\beta}_t$ is an unobserved component that represents the time-varying growth rate of the trend. It evolves as a random walk:

$$\vec{\beta}_t = \vec{\beta}_{t-1} + \vec{\zeta}_t, \quad \vec{\zeta}_t \sim N\left(0, \Omega_\zeta\right) \tag{5}$$

The two components of the trend $\vec{\mu}_t$ and $\vec{\beta}_t$ together are responsible for the slowly changing, growing trend in the data.

Both unobserved components $\vec{\psi}_t^F$ and $\vec{\psi}_t^B$ in (1) are cyclical components. In general a cyclical component $\vec{\psi}_t^C$ (where C = F indicates a financial cycle, and C = B a business cycle) evolves

 $^{^{4}}$ An alternative formulation could involve allowing for this type of time-varying change in the covariance matrices of the other unobserved components in the model. Experimenting with a model version in which we impose the time-varying structure in (2) on the trend disturbance covariance instead of the measurement disturbance covariance makes no difference to the estimates we obtain for the rest of the model.

as follows.

$$\begin{pmatrix} \vec{\psi}_t^C \\ \vec{\psi}_t^{C*} \end{pmatrix} = \rho^C \begin{bmatrix} \cos\frac{2\pi}{\lambda^C} & \sin\frac{2\pi}{\lambda^C} \\ -\sin\frac{2\pi}{\lambda^C} & \cos\frac{2\pi}{\lambda^C} \end{bmatrix} \otimes I_n \begin{pmatrix} \vec{\psi}_{t-1}^C \\ \vec{\psi}_{t-1}^C \end{pmatrix} + \begin{pmatrix} \vec{\kappa}_t^C \\ \vec{\kappa}_t^{C*} \end{pmatrix}$$
(6)

Note that we adopt the notation I_n to indicate the $n \times n$ identity matrix. Further we have that $\vec{\kappa}_t^C \sim N\left(0, \Omega_{\kappa}^C\right)$ and $\vec{\kappa}_t^{C*} \sim N\left(0, \Omega_{\kappa}^C\right)$. Also note that the covariance matrices of both disturbance vectors $\vec{\kappa}_t^C$ and $\vec{\kappa}_t^{C*}$ are restricted to be equal, with $\vec{\kappa}_t^C$ and $\vec{\kappa}_t^{C*}$ taken to be uncorrelated: $Cov\left(\vec{\kappa}_t^C, \vec{\kappa}_t^{C*}\right) = 0$. These restrictions are standard, see Harvey (1991) for details.

The dampening parameter $0 < \rho^C < 1$ determines the persistence of the cycle $\vec{\psi}_t^C$, and λ^C represents the period of the cycle.⁵ We note that the unobserved component $\vec{\psi}_t^{C*}$ is only required for the construction of the cycle component $\vec{\psi}_t^C$. The specification is stationary and ensures that when included in the measurement equation that the changes it induces in the data are temporary.

The unobserved seasonal components $\vec{\gamma}_{jt}$ are also cyclical unobserved components with periods $\lambda_j = \frac{2\pi j}{4}$ and are constructed together with $\vec{\gamma}_{jt}^*$ components in the same manner as in (6). Note that $j = 1, \ldots, 2$ in the case of quarterly data. Furthermore, for seasonal components it is standard to impose the restriction that the dampening coefficient $\rho_j = 1$. The seasonal component $\vec{\gamma}_{jt}$ is then given by the following.

$$\begin{pmatrix} \vec{\gamma}_{jt} \\ \vec{\gamma}_{jt}^* \end{pmatrix} = \begin{bmatrix} \cos \lambda_j & \sin \lambda_j \\ -\sin \lambda_j & \cos \lambda_j \end{bmatrix} \otimes I_n \begin{pmatrix} \vec{\gamma}_{j,t-1} \\ \vec{\gamma}_{j,t-1}^* \end{pmatrix} + \begin{pmatrix} \vec{\omega}_{jt} \\ \vec{\omega}_{jt}^* \end{pmatrix}$$
(7)

Note that $\vec{\omega}_{jt} \sim N(0, \Omega_{\omega})$ and $\vec{\omega}_{jt}^* \sim N(0, \Omega_{\omega})$, where we impose the standard restriction that the covariance matrices of $\vec{\omega}_{jt}$ and $\vec{\omega}_{jt}^*$ for $j = 1, \ldots, 2$ are diagonal and equal. The reader is referred to Harvey (1991) for further details.

2.1 Rank reduction

As currently specified, the model allows for a different financial cycle for each series. We are however interested in the question of whether there is a single underlying financial cycle. In an attempt to answer this question, we take the approach of imposing a single underlying financial cycle in our model. We achieve this by restricting the rank of the covariance matrix of the financial cycle components Ω_{κ}^{F} to 1 instead of the full-rank value of n. In this manner the financial cycles for the series in the model are assumed to be driven by the same underlying

⁵The period of the cycle is given by $2\pi/\lambda^C$. We assume a common dampening coefficients ρ^C and cycle period λ^C . We also estimate model variants in which there is a dampening coefficient and cycle period for each series: ρ_i^C and λ_i^C . The restriction of common cycle parameters across series is supported by tests based on Bayes factors when we test whether the business and financial cycles each have their two own cycle parameters ρ^B , λ^B , ρ^F and λ^F that are shared across the series in the model.

stochastic process.

This rank reduction ensures that the financial cycles $\vec{\psi}_t^F$ will be asymptotically the same up to a scale factor determined by the size of the variances from the main diagonal of Ω_{κ}^F . This is due to the fact that the presence of the dampening coefficient ρ^F in (6) ensures that the effect of the starting values $\vec{\psi}_1^F$ and $\vec{\psi}_1^{F*}$ becomes negligible as $t \to \infty$.

In order to ensure that we obtain estimates of the financial cycle $\vec{\psi}_t^F$ (and of $\vec{\psi}_1^{F*}$) that are the same up to a scale factor from the beginning of the sample period $t \ge 1$, we impose a prior distribution of the starting values $\vec{\psi}_1^F$ and $\vec{\psi}_1^{F*}$ in which the rank of their covariance matrices are also reduced to 1. We obtain these priors from the steady state of the stationary process defined in (6). This results in the following prior specification for the starting values:

$$\vec{\psi}_1^F \sim N\left(0, \left(1-\rho^{F^2}\right)^{-1}\Omega_\kappa^F\right) \quad \text{and} \quad \vec{\psi}_1^{F*} \sim N\left(0, \left(1-\rho^{F^2}\right)^{-1}\Omega_\kappa^F\right)$$
(8)

We note that the prior specification for the starting values of the business cycle components, $\vec{\psi}_1^B$ (and of $\vec{\psi}_1^{B*}$) is similar:

$$\vec{\psi}_1^B \sim N\left(0, \left(1-\rho^{B^2}\right)^{-1}\Omega_\kappa^B\right) \quad \text{and} \quad \vec{\psi}_1^{B*} \sim N\left(0, \left(1-\rho^{B^2}\right)^{-1}\Omega_\kappa^B\right).$$
 (9)

Although we do not impose rank reduction on Ω^B_{κ} , because here we are primarily concerned with the estimation of the financial cycle, in future research we intend on exploring this possibility as well.

In addition to the rank restriction on Ω_{κ}^{F} , we also impose restrictions on both covariance matrices Ω_{κ}^{B} and Ω_{κ}^{F} to require that their implied correlation between credit and the housing price index be positive. In other words we assume that shocks to the financial cycle for credit and the housing price index produce movement in the same direction for both cycles. Economically this seems reasonable. In a financial boom, we would expect both credit and housing prices to increase. It seems reasonably to assume that this should also hold for the business cycle. In model versions which include the price to earnings ratio of the S&P 500, we also require that the correlation with both credit and the housing price index be negative in the case of the financial cycle, but positive for the business cycle. We note that these restrictions seem to have little to no affect on our estimates.

An alternative, but equivalent approach to modeling a common trigonometric cycle component is given in Koopman & Lucas (2005) and de Winter *et al.* (2017). The main difference with our approach here is due to how the cycle components are formulated. In our model the measurement equation (1) includes cycle components that are specified with correlated disturbance terms. In the alternative model by comparison, there are n underlying cycle components which by construction are independent. It is also possible to formulate an equivalent single financial cycle in this alternative model. This would be based on the idea that the same underlying financial cycle affects all n series in the model. This point is discussed in more detail in Appendix A.

3 Estimation

Some of our data series consist of a combination of yearly and quarterly data. As a result, our estimation procedure must be able to accommodate missing observations in the first three quarters of each year in which we use annual data. We obtain our estimates of the financial cycles using Bayesian MCMC simulation methods. Fortunately the estimation of state space models with MCMC simulation methods in the presence of missing observations is possible and is now standard, see for example Koopman *et al.* (1999).⁶ We wrote our own code to perform the MCMC estimation in the matrix programming language OX, see Doornik & Ooms (2007). MCMC simulation techniques are now standard, and we therefore do not discuss these sampling methods in detail. We refer the reader instead to any textbook on Bayesian statistics, such as Koop *et al.* (2007).

For most parameters it is possible to perform the simulation via the Gibbs sampler, or GS. The simulation of the cycle component dampening parameters ρ^B and ρ^F and period parameters λ^B and λ^F is not possible via the GS. In order to simulate these parameters we used the Metropolis-Hastings algorithm, or MH algorithm. The imposition of rank reduction on the co-variance matrix Ω_{κ}^F also introduces an additional degree of complexity to the MCMC simulation. This involves both extra steps in the GS, as well as the use of the MH algorithm. We describe these steps in We first briefly describe how the GS works with our model, and then discuss our implementation of the MH algorithm. We then describe how we tackle the problems introduced by the rank reduction in Ω_{κ}^F .

3.1 Gibbs Sampling

As is commonly done with state-space models, we augment the set of model parameters to simulate in the GS with the disturbance terms from our model. Given values for the model parameters, we can simulate the disturbances terms in our model using the disturbance smoother as implemented in SsfPack, see Koopman *et al.* (1999) for details.⁷ Once we have simulated the disturbance terms we then simulate new values of the covariance matrices of our model from

⁶We have encountered stability issues with the Kalman filter and related algorithms in certain areas of the parameter space of our model, introduced by the presence of missing observation at the beginning of the sample period. However, in the relevant region of the parameter space for our estimation the Kalman filter-based algorithms remained well behaved.

⁷More computationally efficient sampling is possible, see Durbin & Koopman (2002).

their posterior distributions conditional on the drawn values of the disturbance terms. Given the assumed normality of the disturbance terms in the model and the conjugate inverse Wishart priors we specify on the covariance matrices of our model, the conditional posteriors from which we draw the new covariance values also follow an inverse Wishart distribution: $\mathcal{W}^{-1}(\nu, S)$. In this standard case, we have that the posterior degrees of freedom ν is given by the sum of the prior degrees of freedom ν_p and the number of observations, $T: \nu = T + \nu_p$. We also have that the posterior parameter matrix S is equal to the sum of the prior matrix parameter S_p and the sum of outer product of the residual vector $R: S = S_p + R$.

In general the GS works by repeatedly cycling through the two simulation blocks of drawing the disturbances and drawing the covariances. Asymptotically, by repeatedly re-simulating all the values, we obtain drawings from the unconditional joint posterior of the model parameters and disturbances.⁸ This is however only true if we can also include a method to obtain updated drawings for ρ^B , ρ^F , λ^B and λ^F , as well as for the reduced rank covariance matrix Ω^F_{κ} . Drawing ρ^B , ρ^F , λ^B and λ^F is not feasible in the GS as we do not know any easily derived conditional posterior from which we could draw new values. Instead we use the MH algorithm.

 $\theta_{\setminus i}$

3.2 Metropolis-Hastings Algorithm

We use the MH Algorithm when we are unable to draw new parameter values directly from the appropriate conditional posterior required by the GS. Instead we draw a new parameter value from a candidate distribution. We either accept this new draw, or reject it and keep the original value from the previous draw. The decision to reject or accept the candidate drawing is based on the value of δ_c :

$$\delta_{c} = \frac{P\left(\theta_{i}^{*}\right) L\left(Y|\theta_{i}^{*}, \theta_{\backslash i}^{(m-1)}\right) f_{c}\left(\theta_{i}^{(m-1)}|\theta_{i}^{*}\right)}{P\left(\theta_{i}^{(m-1)}\right) L\left(Y|\theta_{i}^{(m-1)}, \theta_{\backslash i}^{(m-1)}\right) f_{c}\left(\theta_{i}^{*}|\theta_{i}^{(m-1)}\right)}.$$
(10)

When $\delta_c \geq 1$ we automatically accept the candidate value. When $\delta_c < 1$ we accept the candidate value with probability δ_c . Note that in (10) $P(\theta_i^*)$ represents the prior density of the parameter θ_i at the value given by the candidate drawing θ_i^* at step m of the MCMC algorithm. The value of the previous draw is denoted by $\theta_i^{(m-1)}$. The value of the likelihood given the candidate parameter value θ_i^* and the other model parameters values in the MCMC algorithm $\theta_{\backslash i}^{(m-1)}$ is denote by $L\left(Y|\theta_i^*, \theta_{\backslash i}^{(m-1)}\right)$. The density of the parameter value θ_i^* obtained from the candidate

⁸Via the disturbances we can also obtain drawings of the state vector: the trend, growth rate, cycles and seasonal components. The reader is referred to Koopman *et al.* (1999) for details.

density function is then given by $f_c\left(\theta_i^*|\theta_i^{(m-1)}\right)$. Note that the form of the candidate density can depend on the previously drawn parameter value $\theta_i^{(m-1)}$. In our implementation this is the case.

For the cycle period parameters λ^B and λ^F we draw candidate values from the gamma distribution with an expected value equal to the previously drawn period value. Similarly for the dampening coefficients ρ^B and ρ^F we draw candidate values from the beta distribution also with an expected value equal to the previously drawn dampening coefficient value.⁹ We obtain the required values of the likelihood from the diffuse Kalman Filter based on the prediction error decomposition of the likelihood. In our program we perform one Metropolis-Hastings rejection step for the four cycle parameters jointly.¹⁰

3.3 Sampling Ω_{κ}^{F} with Rank Reduction

In the presence of rank reduction, such as we impose on Ω_{κ}^{F} , drawing a new value for the covariance matrix is more complicated. Part of the covariance matrix can be simulated via the GS. The rest we draw using the MH algorithm. To see how we use the GS here, let us consider the general case of the covariance matrix Ω^{C} which has the reduced rank of r < n. We begin by first drawing a new value for Ω^{C} given the current simulated values of the associated disturbances $\vec{\kappa}_{t}^{C}$, $t = 1, \ldots, T$. Given the newly simulated value of Ω^{C} we then draw new values of the disturbances $\vec{\kappa}_{t}^{C}$, $t = 1, \ldots, T$ to complete the required GS steps.

We begin with the GS draw of Ω^C , and denote the conditional posterior of Ω^C in the GS by $\mathcal{W}^{-1}(\nu^C, S^C)$. Now consider the eigenvalue decomposition of the $n \times n$ parameter matrix, $S^C = E\Lambda E'$, where the matrix of orthonormal eigenvectors E is given by $E = [\vec{e}_1, \ldots, \vec{e}_n]$ such that E'E = the $n \times n$ identity matrix I_n , and Λ is a diagonal matrix with the eigenvalues λ_{Si} , $i = 1, \ldots, n$ along its diagonal. S^C has the reduced rank of r < n. If we order the eigenvalues from largest to smallest, then we have that $\lambda_{S,n-r+1} = \ldots \lambda_{Sn} = 0$. We can then denote the $n \times r$ matrix of r eigenvectors corresponding to the r non-zero eigenvalues as $E_r = [\vec{e}_1, \ldots, \vec{e}_r]$, and in the same manner the $r \times r$ diagonal matrix of non-zero eigenvalues as Λ_r . We can now re-write S^C as follows.

$$S^C = E_r \Lambda_r E_r' \tag{11}$$

To obtain a draw for the reduced rank covariance matrix Ω^C from the inverse Wishart distribu-

 $^{^{9}}$ This leaves an additional distribution parameter to be fixed, both in the case of the gamma and of the beta candidate distributions. We tune this value to ensure a rejection rate of between 20% and 50%.

¹⁰We repeat these joint MH drawings eight times in each cycle through the GS. The number 8 was arbitrarily chosen to produce more draws than for the parameters drawn from Gibbs sampling, because we assume that these draws require more replications to achieve convergence.

tion $\mathcal{W}^{-1}(\nu^C, S^C)$, we define the matrix Σ^C :

$$\Sigma^C = E_r \Lambda_r^{\frac{1}{2}}.$$
(12)

Then we draw the $r \times r$ full rank matrix \hat{X} from the standard Wishart distribution: $X \sim \mathcal{W}(\nu^C, I_r)$ and obtain

$$\hat{Q} = \Sigma^C \hat{X} \Sigma^{C'}.$$
(13)

We now perform the eigenvalue decomposition of \hat{Q} , which is $n \times n$ and of rank r, so that $\hat{Q} = E_{Qr} \Lambda_{Qr} E'_{Qr}$ as in (11). The reduced rank drawing $\hat{\Omega}^C$ for the covariance Ω^C is then given by

$$\hat{\Omega}^C = E_{Qr} \Lambda_{Qr}^{-1} E'_{Qr}.$$
⁽¹⁴⁾

To complete the required steps of the GS for our model, we must now draw new values of for the disturbances $\vec{\kappa}_t^F$, t = 1, ..., T. However, this is also more complicated than for the other disturbances associated with the unrestricted covariances in the model. The reduced rank of Ω_{κ}^F causes statistical degeneracy in the joint distribution of the disturbances $\vec{\kappa}_t^F$, t = 1, ..., T. For this reason in our model we can only draw r of the n vectors $\vec{\kappa}_t^F$, t = 1, ..., T in the disturbance smoother, see Koopman *et al.* (1999) for a detailed discussion.

To draw the $n \times 1$ disturbance vectors $\hat{\kappa}_t^C$, $t = 1, \ldots, T$ given the newly drawn covariance matrix $\hat{\Omega}^C$ with rank r < n, we assume that we have ordered the disturbance vectors $\vec{\kappa}_t^C$ and $\hat{\Omega}^C$ so that we have

$$\vec{\kappa}_t^C = \begin{pmatrix} \vec{\kappa}_{at}^C \\ \vec{\kappa}_{bt}^C \end{pmatrix},\tag{15}$$

where $\vec{\kappa}_{at}^C$ represents the r elements of $\vec{\kappa}_t^C$ that we can simulate with the disturbance smoother, and $\vec{\kappa}_{bt}^C$ represents the n-r remaining disturbances that we cannot obtain from the disturbance smoother due to the problem of statistic degeneracy caused by the rank reduction.¹¹ Similarly to (12), from the eigenvalue decomposition of $\hat{\Omega}^C$ in (14), we then define

$$\hat{\Sigma} = E_{Qr} \Lambda_{Qr}^{-\frac{1}{2}}.$$
(16)

As a result, $\hat{\Omega}^C = \hat{\Sigma}\hat{\Sigma}'$. Therefore, we have that the newly simulated values $\hat{\kappa}_{at}^C$ of the distur-

¹¹The disturbance smoother in SsfPack requires the specification of the diagonal selection matrix Γ which is the same dimension as the state vector with either ones on the diagonal or zeros for the corresponding stochastically degenerate elements of the state. Therefore, in our estimation procedure, Γ specifies the r elements of $\vec{\kappa}_{at}^{C}$, see Koopman *et al.* (1999) for details. We adjust the value of Γ so as to select the r series with the strongest cycle estimates, because we believe this may aid convergence.

bances $\vec{\kappa}_{at}^C$, $t = 1, \ldots, T$ must satisfy the following.

$$\hat{\kappa}_t^C = \begin{pmatrix} \hat{\kappa}_{at}^C \\ \hat{\kappa}_{bt}^C \end{pmatrix} = \hat{\Sigma}\hat{\epsilon}_t = \begin{bmatrix} \hat{\Sigma}_a \\ \hat{\Sigma}_b \end{bmatrix}\hat{\epsilon}_t, \qquad (17)$$

where $\vec{\epsilon}_t \sim N(0, I_r)$ is an unknown $r \times 1$ vector of disturbances. Furthermore $\hat{\Sigma}_a$ is $r \times r$ and $\hat{\Sigma}_b$ is $(n-r) \times r$, both sub-matrices of $\hat{\Sigma}$, such that

$$\hat{\Omega}^{C} = \begin{bmatrix} \hat{\Sigma}_{a} \hat{\Sigma}_{a'} & \hat{\Sigma}_{a} \hat{\Sigma}_{b'} \\ \hat{\Sigma}_{b} \hat{\Sigma}_{a'} & \hat{\Sigma}_{b} \hat{\Sigma}_{b'} \end{bmatrix}.$$
(18)

Given the simulated values $\hat{\kappa}_{at}^C$ from the disturbance smoother, we can solve (17) to obtain the following.

$$\hat{\epsilon}_t = \hat{\Sigma}_a^{-1} \hat{\kappa}_{at}^C, \tag{19}$$

Note that $\hat{\Sigma}_a^{-1}$ exists because the $r \times r$ sub-matrix $\hat{\Sigma}_a \hat{\Sigma}_a'$ from the top left corner of Ω^C in (18) has full rank by construction.¹² By combining the results from (17) and (19), we can see that we can recover $\hat{\kappa}_{bt}^C$ from the following.

$$\hat{\kappa}_{bt}^C = \hat{\Sigma}_b \hat{\Sigma}_a^{-1} \hat{\kappa}_{at}^C \tag{20}$$

We have now obtained the simulated disturbances $\hat{\kappa}_t^C$, $t = 1, \ldots, T$, which, together with the simulated covariance matrix $\hat{\Omega}^C$ completes the required steps of the GS. This leaves only the steps of the MH algorithm to ensure that Ω_{κ}^F is correctly simulated.

To see why we still require additional sampling, consider the rank reduction on Ω_{κ}^{F} where r = 1, In (13) the draw \hat{X} is a scalar, whereas the complete draw $\hat{\Omega}_{\kappa}^{F}$ requires *n* parameters: one for each of the *n* variances, with the covariance being determined by the perfect correlation implied by the rank reduction. Clearly these GS steps only manage to simulate one of the required parameters in Ω_{κ}^{F} . An additional set of steps using the MH algorithm is required to ensure that we fully sample a new value for Ω^{F} .

In the general case outlined above, the simulated value \hat{X} in (13) is an $r \times r$ symmetric matrix, and therefore is implicitly only defined by r(r+1)/2 univariate elements. In general the $n \times n$ covariance matrix Ω_{κ}^{F} of rank r < n is defined by

$$\frac{n(n+1) - (n-r)(n-r+1)}{2} > \frac{r(r+1)}{2}$$
(21)

univariate elements.

¹²This is due to the assumed ordering of the disturbance vector $\hat{\kappa}_t^C$ in (15).

Similarly, if we examine (17), we can see that the disturbance smoother only implicitly simulates the $r \times 1$ vector $\hat{\epsilon}_t$. Because $\hat{\kappa}_{at}^C = \hat{\Sigma}_{a\kappa} \hat{\epsilon}_t$, there is new information in the conditional posterior distribution of Ω_{κ}^F to define a new drawing of $\hat{\Sigma}_{a\kappa}$. We can also see, however, from (19) and (20), that the information in the $r \times 1$ drawing $\hat{\epsilon}_t$ is recycled to obtain the (n-r) vector $\hat{\kappa}_{bt}$. There are therefore no new stochastic univariate elements used to construct the $(n-r) \times r$ matrix $\hat{\Sigma}_{b\kappa}$, which defines part of the conditional posterior of Ω_{κ}^C in the Gibbs sampling draw discussed above.

We have observed in practice that the term $\hat{\Sigma}_b \hat{\Sigma}_a^{-1}$ in (20) remains constant in our applications when r = 1. In general we denote this $(n - r) \times r$ matrix as B:

$$B = \hat{\Sigma}_b \hat{\Sigma}_a^{-1}.$$
 (22)

We vectorize the elements of B and draw them as B^* from a multivariate normal candidate distribution, $N(B^{(m-1)}, S_B)$, where S_B is a diagonal matrix of variances for the vectorized elements of B, and $B^{(m-1)}$ is the previous draw of the elements of B. The variances in S_B must be set to be able to perform this application of the Metropolis-Hastings step.¹³

We note that to obtain a complete simulation of the financial cycle vector $\vec{\psi}_t^C$ for $t = 1, \ldots, T$ we require the simulated starting values $\hat{\psi}_0^C$, which we can straight-forwardly obtain from the simulation smoother. Draws for the other set of cycle disturbance vectors $\vec{\kappa}_t^{C*}$, as well as the cycle components $\vec{\psi}_t^{C*}$ for $t = 1, \ldots, T$ can be obtained in the same manner as outlined above. Once the MCMC algorithm has converged we continue to run the simulation steps to obtain a sample from the joint posterior distribution. We can then base our inference on this sample. Standard diagnostics can be used to check for the convergence of the MCMC algorithm.

Most of our results are based on a total of 200,000 replications from 4 parallel chains for each country model, where we throw away the first half of the replications from each chain as burn-in to ensure that we only sample from the MCMC algorithm once convergence has been achieved. Convergence diagnostics indicate that our MCMC algorithm has converged, the details of which are available on request. The exact number of replications for each model is listed in Appendix C in Table C.1.

4 Priors

The model we propose has a fair number of parameters, making the model quite flexible. There are therefore parameter regions that we would prefer to rule out. We achieve this using somewhat informative prior on some of the parameters. We also specify weakly informative priors to help

 $^{^{13}}$ Through experimentation we tune these variances to produce a rejection rate of between 20% to 50% for the joint test of the elements of B.

achieve our business and financial cycle decompositions with cycle periods for the business cycle that are relatively short and for the financial cycle that are relatively long. For the other priors we specify a small number of degrees of freedom and select the prior scaling factor centered around the main posterior density mass. In this way we specify fairly uninformative empirical Bayes priors. We discuss the various prior specifications we use for each unobserved component.

4.1 Cycles

Both cycle components require priors for the dampening coefficients ρ^C , the cycle periods λ^C , and the disturbance covariance matrices Ω_{κ}^C , for C = B and F, see (6) above. Given that the dampening coefficients $0 < \rho^C < 1$, we specify a beta distribution for these priors. Note that apriori we want $\rho^C < 1$ to ensure that the cycle components are stationary and that the cycle disturbances have no permanent effects on the long run level of the series. The priors for the cycle periods $\lambda^C \in (4, \infty)$ for quarterly data, follow gamma distributions. The priors for the covariance matrices Ω_{κ}^C are inverse Wishart distributions.

The beta priors are parameterized as $Beta(\alpha_p^C, \beta_p^C)$, for C = B and F.¹⁴ For the business cycle component, ρ^B , we set $\alpha_p^B = 55.88$ and $\beta_p^B = 1.925$. This implies a prior mean of 0.967, with a standard deviation of 0.0234. This prior relatively diffuse and has little impact on the posteriors. The prior parameters for the financial cycle components' parameter ρ^F , are give by $\alpha_p^F = 321.3$ and $\beta_p^B = 4.617$. These parameters imply a posterior mean of 0.986 and standard deviation of 0.0065. Although this prior is more spread out than the posteriors, the posteriors tend to lie slightly above the prior. This prior is therefore somewhat informative in that it tends to pull the posterior away from the value of 1. Experimenting with differing prior parameters suggests that our results are not very sensitive to this prior.

The prior gamma distribution for the λ^{C} is denoted by $Gamma(a^{C}, b^{C})$, for C = B and F.¹⁵ These priors are formulated using a Bayesian highest density region, or HDR. In the case of the business cycle, we make the prior assumption that the probability that the business cycle period is between five to ten years is 99%: $P(20 \text{ quarters} < \lambda^{B} \le 40 \text{ quarters}) = 99\%$. This results in the prior parameter values of $a^{B} = 55.88$ and $b^{B} = 4.617$ for the gamma prior of λ^{B} . We formulate our prior for the financial cycle period λ^{F} in a similar fashion. Here we employ the 99% prior HDR of between 15 to 20 years: $P(60 \text{ quarters} < \lambda^{F} \le 80 \text{ quarters}) = 99\%$. This implies the prior parameter values of $a^{F} = 321.3$ and $b^{F} = 1.925$ for the gamma prior of λ^{F} . Alternative priors based on the same HDR intervals, but with lower probabilities, such as 95% or 90%, result in similar estimates. If, however, we increase these intervals to encompass longer periods, then this can alter our estimates. For example an HDR for λ^{F} based on the interval

¹⁴The density function of $Beta(\alpha_p, \beta_p)$ is then given by $f(x) = x^{\alpha_p - 1} (1 - x)^{\beta_p - 1} / B(\alpha_p, \beta_p)$.

¹⁵The density function of Gamma (a, b) is then given by $f(x) = \frac{b^a}{\Gamma(a)} x^{a-1} \exp^{-bx}$.

from 20 to 25 years tends to result in somewhat different financial cycle estimates. On the whole, however, we believe that our priors for the cycle periods represent the values most cited in the literature, see for example Drehmann *et al.* (2012) and Borio (2014). Although somewhat informative, these priors still allow the posteriors to be largely determined by the data.

We denote the prior inverse Wishart distribution for Ω_{κ}^{C} by $\mathcal{W}^{-1}(\nu^{C}, S^{C})$, for C = B and F.¹⁶ The prior parameter ν^{C} represents the number of degrees of freedom. For both the business and financial cycle we set $\nu^{B} = \nu^{F} = 10 + n$. We then select the positive (semi) definite matrix S to ensure that the mean of the posterior is unaffected by the prior. These values for S^{C} for C = B and F are listed in Table B.1 in Appendix B for our preferred bivariate model variant. The prior parameters used in other model variants are available on request.

4.2 Trend & Growth Rates

The two trend components $\mu_{i,t}$ in (4) and the two growth rates $\beta_{i,t}$ in (5) follow random walks. They are therefore non-stationary. As a result we assume diffuse priors for their initial values.

The inverse Wishart prior degrees of freedom for the disturbance covariance matrices Ω_{η} for the trend component and Ω_{ζ} for the growth rate component are $\nu_{\eta} = 10 + n$ and $\nu_{\zeta} = 200 + n$, respectively. The values for the prior parameter matrices S_{η} and S_{ζ} are listed in Table B.2 of Appendix B.

In general 10+n degrees of freedom for the inverse Wishart distribution produces a prior that is relatively uninformative. We select the values for S_{η} to ensure that the highest prior density region corresponds to that of the posterior.¹⁷ In this way the priors for Ω_{η} are selected to have minimal impact on the form of the posteriors. This essentially an empirical Bayes approach.

Our prior specification for the Ω_{ζ} are more informative. We interpret the drift components $\vec{\beta}_t$ as representing the underlying growth rates. As such we believe *apriori* that these rates will only change gradually over time. It is however common in SSM's of macroeconomic time series with a local linear trend, such as we have specified here, that the likelihood tends to favor larger values for the variance of the disturbance of the drift component. These larger values for the variance imply a relatively quickly changing growth rate. In the case of our model we believe that these changes ought to be captured by the cycles in the model. For this reason we specify the larger prior parameter value of $\nu_{\zeta} = 200 + n$ in model variants with a longer sample period, and 80 + n otherwise for Ω_{ζ} of the growth rate component. This then represents

$$f(X) = \frac{|S|^{\nu/2}}{2^{\nu}\Gamma_2\left(\frac{\nu}{2}\right)} |X|^{-(\nu+3)/2} e^{-\frac{1}{2}tr\left(SX^{-1}\right)}$$

¹⁶The density function of $\mathcal{W}^{-1}(\nu^C, S^C)$ is then given by

¹⁷The off-diagonal elements of S_{η} are zero, because Ω_{η} is diagonal. These priors are therefore equivalent to inverse-gamma priors with the inverse gamma distribution parameters $\alpha_{\eta i} = \nu_{\eta}/2$ and $\beta_{\eta i} = s_{\eta i}/2$, i = 1, ..., n.

a more informative prior. Compared with the information in a sample period of more than 200 observations, this number of degrees of freedom is still fairly modest. We specify diagonal elements of the prior parameter matrices S_{ζ} which correspond to modest changes over time in the growth rates $\beta_{i,t}$. The off-diagonal elements are assumed to be zero indicating a prior of no correlation between the *n* growth rates.

In those instances where the marginal posterior variance for ζ_{it} was lower than our initial prior specification would suggest¹⁸, we lowered the corresponding value in S_{ζ} to match the posterior.

4.3 Seasonal Components

The covariance matrices Ω_{ω_1} and Ω_{ω_2} in (7) are assumed to be diagonal. Therefore the prior parameter matrices S_{ω_1} and S_{ω_2} are as well. In all cases we set the number of degrees of freedom of these inverse Wishart priors to $\nu_{\omega_1} = \nu_{\omega_2} = 10 + n$ and

$$S_{\omega_1} = S_{\omega_2} = \begin{bmatrix} c & 0 \\ & \ddots & \\ 0 & c \end{bmatrix}, \qquad (23)$$

where usually c = 0.0002. With the exception of the US industrial production series, all series exhibit only a slight degree of seasonality.¹⁹ We specify diffuse priors on the initial values $\gamma_{i,j,0}$ and $\gamma_{i,j,0}^*$, because these components are non-stationary.

4.4 Measurement Error Covariance

To specify a prior on the covariance matrix $\Omega_{\varepsilon,t}$ of the measurement error as given in (2), we need to specify priors on the diagonal matrices Ω_{ε} and Ω_h where their main diagonals are given by the following vectors.

diagonal
$$(\Omega_{\varepsilon}) = (\sigma_{\varepsilon_1}, \dots, \sigma_{\varepsilon_n}), \quad \text{diagonal} (\Omega_h) = (\sigma_{h_1}, \dots, \sigma_{h_n}).$$
 (24)

We use inverse Wishart priors: $P(\Omega_{\varepsilon}) \sim \mathcal{W}^{-1}(\nu_{\varepsilon}, S_{\epsilon})$ and $P(\Omega_h) \sim \mathcal{W}^{-1}(\nu_h, S_h)$. We can define the diagonal matrices S_{ε} and S_h as follows.

diagonal
$$(S_{\epsilon}) = (s_{\epsilon_1}, \dots, s_{\epsilon_n})$$
 diagonal $(S_h) = (s_{h_1}, \dots, s_{h_n})$ (25)

In Table B.3 of Appendix B we list the elements of the prior parameter matrices S_{ε} and

¹⁸We initially specify a prior on Ω_{ζ} that implies an expected value of 0.08 for each σ_{ζ_i} , $i = 1, \ldots, n$.

¹⁹For this reason we set C = 0.015 for US IP.

 S_h for our preferred bivariate model.²⁰ We also list in this table the dates T_i^* when our model transitions to the lower measurement error variance, see (2). We set the degrees of freedom $\nu_{\varepsilon} = \nu_h = 40$. We adjust the non-zero values in S_{ε} and S_h until the posterior is centered around the prior. The exception to this is Dutch industrial production, where we held the prior mode below the posterior to ensure the stability of the Kalman Filter in the estimation procedure. The prior values we specify for the other model variants are available on request.

5 The data

We include up to six data series in our multivariate models of the US and of the Netherlands: credit, a house price index, GDP, industrial production, and two other indices which we construct, one based on the the S&P500 PE ratio, and the other based on interest rate spreads. Plots of these data series are given in Appendix C together with their estimated trend components from various SSM variants in the case of the credit, housing price index, industrial production and GDP. The plots of the first difference of the cumulated S&P 500 PE and spreads data are shown together with their estimated drift components from various SSM variants. In the case of these latter two series, the first difference corresponds to the original data before cumulation, and these plots show the data more clearly than the trend does.

We discuss here the sources and definitions of the data and describe how we transform the data for the model. All series are analyzed on a quarterly basis²¹, with missing values for the missing quarters for the periods when we only have annual data available. When the orginal data series are monthly, we use the index value from the end of the month of the last month in each quarter as the quarterly value.

5.1 Credit & House Prices

The credit series is for total credit to the private non-financial sector, measured as the stock of outstanding credit at the end of the quarter. This credit series and the housing price index are both published by the Bank of International Settlements, or BIS on a quarterly basis. For earlier values, when no quarterly values are available, we rely on the yearly credit data published in Jordà *et al.* (2017) and the yearly housing price indices published in Knoll *et al.* (2017). In

²⁰Given that the measurement errors between the *n* series are uncorrelated, these priors are equivalent to inverse-gamma priors on σ_{ε_i} and σ_{h_i} with the inverse-gamma distribution parameters $\alpha_{\varepsilon_i} = \nu_{\varepsilon}/2$, $\alpha_{h_i} = \nu_h/2$, $\beta_{\varepsilon_i} = s_{\varepsilon_i}/2$ and $\beta_{h_i} = s_{h_i}/2$, where $i = 1, \ldots, n$.

 $^{^{21}}$ We have also produced estimates based on monthly observations of industrial production, spreads, and the S&P500. the results we obtain are similar. The estimation based on monthly data however involves the introduction of a substantial number of missing values, because credit, housing prices and GDP are quarterly series. This slows down the estimation and in some cases causes numerical instability in the Kalman Filter which we use in our estimation procedure.

this case the annual data represents a fourth quarter measurement, and the first three quarters of the year are missing.

Both the credit series and housing price indices are deflated using consumer price indices. In the case of the US data, we obtained monthly CPI data from Schiller (2015). The Dutch CPI data came from the US Federal Reserve's Federal Reserve's FRED Economic Data,²² but the source of the data is the OECD's "Main Economic Indicators - complete database".

Inspection of the data indicates that the earlier yearly data is more volatile. This motivated our decision to use the split measurement error variance in (2). We identify the transition dates T_i^* in (2) when the data transitions to a lower level of variability for the credit and housing price indices by determining when the data comes from a more reliable source. We were able to determine this based on the information in the documentation of the data series given in Jordà *et al.* (2017) for the credit data and in Knoll *et al.* (2017) for the housing price indices. These dates are listed in Table B.3 of Appendix B.

5.2 S&P 500

We construct a earnings to price ratio index from the S&P 500 stock price index, earnings, and the US CPI data. The data are available from Schiller (2015). Stock prices are the real total return price. Earnings are given by the real total return of the scaled earnings.²³

The Dutch stock index the AEX is only available starting in 1983, which does not represent a sufficiently long sample for this study. We therefore rely on the correlation between the US and Dutch economy to justify the use of the S&P 500 data in the Dutch model as well.

We transform the earnings E_t and price P_t data to obtain a growing index that we can model using the local linear trend specification in (4.) We start with the earnings to price ratio, $\beta_t^{ep} = \frac{E_t}{P_t}$ which we can think of as a rate of return. We therefore aggregate these growth rates into an index, starting at the arbitrary value of 100. We then transform this index using the logarithm to obtain the series lep_t , which is then given by²⁴

$$lep_t = 100 \log\left(100 \prod_{j=1}^t \left(e^{\beta_j^{ep}}\right)^{\frac{1}{12}}\right) = lep_0 + \frac{100}{12} \sum_{j=1}^t \beta_j^{ep}$$
(26)

To see that this definition of the index lep_t results in a series much like the credit or housing price index, first consider a level series Y_t , such as credit or the house price index, with growth

 $^{^{22}\}mathrm{Available}$ from their website <code>https://fred.stlouisfed.org</code>

 $^{^{23}}$ The data appears to be dated as if it is measured at the first of each month. However, comparison with the on-line data for the S&P 500 suggests that the data is from the end of the month. We date the data in our model as being measured at the end of the month.

 $^{^{24}}$ We take the twelfth root to reduce the dramatic growth of the index over the sample period. This simply represents multiplication of the logged index by a constant.

rate β_t^y . We have then that

$$Y_t = Y_0 \prod_{j=1}^t (1 + \beta_j^y) \approx Y_0 \prod_{j=1}^t e^{\beta_j^y} = Y_0 \exp\left(\sum_{j=1}^t \beta_j^y\right).$$
 (27)

Transforming the data series using logarithms then leads to the following expression.

$$y_t \equiv 100 \log (Y_t) = y_0 + 100 \sum_{j=1}^t \beta_j^y$$
(28)

We can see that this expression has essentially the same structure as lep_t in (26).

5.3 Interest Rate Spreads

We opt to include the spreads, s_t in our model also by transforming the series into an index in much the same manner as we do with the S&P 500 data. It seems reasonable to consider the relevant data as being the difference between the price level of the risky and safe bonds, or of short and long run bonds. As we show below, this turns out to be the same as the aggregated spread series. As we argue above for the S&P 500 data, this fits best with how we model for the level UC in (4).

To see how we construct an index based on the interest rate spreads, consider the bond price of a safe bond, B_t^l and the price of a risky bond B_t^k . The relevant series which best matches the other level series in the model is given by the ratio between the prices of the risky and safe bonds from the spread:

$$\frac{B_t^k}{B_t^l} = \frac{B_0^k}{B_0^l} \prod_{j=1}^t \left(\frac{e^{r_j^k}}{e^{r_j^l}}\right)^{\frac{1}{12}} = B_0^{kl} \prod_{j=1}^t \left(e^{r_j^k - r_j^l}\right)^{\frac{1}{12}} = B_0^{ks} \prod_{j=1}^t e^{\frac{1}{12}s_t}.$$
(29)

Here the interest rate spread s_t is the difference between the annual risky interest rate, r_t^k and the annual safe interest rate, r_t^l : $s_t = r_t^k - r_t^l$. Note that (29) is similar to (26) for the level transformation lep_t based on the S&P 500 data, and to (28) for Y_t .

We can now take the logarithm of this bond price ratio series. This results in the cumulative sum of the interest rate spread:

$$b_t^{ls} \equiv 100 \log \left(B_t^{ls} \right) = b_0^{ls} + \frac{100}{12} \sum_{j=1}^t s_j.$$
(30)

For the US, we construct an index based on Moody's seasoned AAA corporate bond minus the federal funds rate. This series is available on a monthly basis and is not seasonally adjusted. The sample period begins in July of 1954, and is available until January 2019. This series represents the best compromise between a good measure of the interest rate risk spread and a series of sufficient length to study the financial cycle.

No spreads of sufficient length are available for the Netherlands. Instead we use the difference between the ten year interest rates on government bonds and the three month interbank rate for Germany. This data is available from the US Federal Reserve's Fred Economic Data.²⁵ This data goes back to January 1960 on a monthly basis. We assume that the strong correlation between the German and Dutch economy is sufficient to justify using this German series together in a Dutch model.

5.4 Industrial Production & GDP

A seasonally-unadjusted monthly index series of US industrial production (IP) is available starting in January of 1919 from the US Federal Reserve's FRED Economic data. We also obtained Dutch IP data from the FRED. The Dutch data is seasonally adjusted, monthly data. This series starts in January of 1960.

Data for the gross domestic product (GDP) of the US and the Netherlands is only available on a quarterly basis. The US no longer publishes seasonally-unadjusted data for US GDP. We therefore use real US seasonally-adjusted quarterly GDP starting in the first quarter of 1947. This data is published by the US department of Commerce's Bureau of Economic Analysis.²⁶ The Dutch CBS does not publish real quarterly GDP for the Netherlands before 1995.However, real seasonally-adjusted quarterly GDP figures for the Netherlands are available starting in the first quarter of 1960 from the US Federal Reserve's FRED Economic Data.

6 The results

In this section we present the financial cycle estimates. In particular we want to gauge how successful the rank reduction on Ω_{κ}^{F} is as a method of identifying a unique financial cycle. We also want to determine which series are most informative about the financial cycle, and which model variants are best suited to the estimation of the financial cycle.

We estimate a number of multivariate model variants for the US and the Netherlands, as well as a univariate variant for each series. We begin with a bivariate model based on credit and the housing price index. The motivation for a model based on credit and the housing price index is that, as our results below show, the financial cycle in the US and the Netherlands is primarily determined by these two series. An additional advantage of modeling these two series

²⁵Available from their website https://fred.stlouisfed.org.

²⁶The data is available from their website https://www.bea.gov/iTable/index_nipa.cfm.

together is that the resulting sample period is long. In the case of the US, the sample period runs from fourth quarter of 1914 (following the creation of the US Federal Reserve system) until the second quarter of 2018. For the Dutch data the sample period begins in the fourth quarter of 1900 and runs until the second quarter of 2018.

We also estimate six-variable model variants for the US and the Netherlands based on all the available series. In the case of the US, the sample period runs from the fourth quarter of 1954 until the fourth quarter of 2018.²⁷ The six-variable Dutch variant has a sample period that spans the period of the first quarter of 1960 until the first quarter of 2019.²⁸ For the Dutch data only credit and the housing price index are available before 1960. The six-variable Dutch model includes the US S&P 500 series lep_t , even though this series is for the US. The spreads series we use in this case is also for Germany, not the Netherlands, but we suppose a high degree of integration between the US, German and Dutch financial markets and for this reason estimate this six-variable Dutch model with the rank of Ω_{κ}^{F} restricted to be 1 and 2. Alternatively, we also estimate a Dutch five-variable model variant which excludes the S&P 500 data.

Finally we also estimate four-variable model variants. In the case of the US data, this model includes credit, the housing price index, the S&P 500 series and IP. This combination of series still allows us to use a long sample period that runs from the fourth quarter of 1919 until the final quarter of 2018. The Dutch four-variable variant also includes credit, the housing price index and IP, but swaps the the S&P 500 series our for the Germany spreads. The sample period for this model runs from the first quarter of 1960 until the first quarter of 2019.

6.1 The Financial Cycle with rank $\Omega_{\kappa}^{F} = 1$

In this section we compare the estimates of the financial cycles we obtain from the model variants in which we restrict the rank of Ω_{κ}^{F} to be 1 to ensure a single underlying financial cycle. The figures are based on the posterior median of the financial cycle and the 68% and 90% credible regions. They also include estimates of the financial cycle produced by the BIS which start in 1970 (see Drehmann *et al.* (2012)).

We note that Appendix C includes additional figures showing plots of the financial cycles based on the other series for selected model variants. These figures also include plots of the business cycles, trends and drifts. The plots of the estimated trends also include the observed series, or in the case of the S&P 500 PE and Spreads cumulated series, the plots of the estimated drifts include the first difference of the series.²⁹

Here in Figure 1 we show the estimates we obtain based on the US credit and housing price

²⁷The US spreads are not available before 1954.

 $^{^{28}}$ Note that some series in these models having missing observations at the end of the sample period.

²⁹Plots for the other model variants as well as of the estimated seasonal components are available on request.

index for all four model variants. The match with cycle based on the univariate model of the housing price index with the multivariate estimates is close. The univariate model of credit on the other hand produces a somewhat different cycle. This suggests that the multivariate cycle estimates are more strongly determined by the housing price index than credit. The three multivariate variants produce very similar financial cycle estimates. The bivariate variant has the additional benefit of being based on a longer sample period and so producing a financial cycle estimate covering a longer period.

In Figure 2 we can compare the estimated financial cycles based on credit and the housing price index for the Netherlands. This figure excludes the estimates from the six-variable model variant, which we discuss below. As is the case with the US results, these three multivariate variants also produce similar financial cycle estimates, with the bivariate variant benefiting from a longer sample period. The estimates based on the credit series, however, now seem somewhat weaker. This is an indication that the Dutch financial cycle is dominated by the housing price index.

We can gauge the plausibility of our financial cycle estimates based on known historical events such as the Great Depression as well as the recovery led by World War II and its aftermath, the US savings and loans crisis of 1986, and the Great Recession³⁰ In the case of the Netherlands we can also see the effects of the housing boom from 1976-78 and the crash that followed from 1979-1983. We can see these events reflected in the estimates by a drop in the financial cycle during weaker periods and an increase during stronger periods. We also note that our estimates also show a substantial level of agreement with those of the BIS.

More generally, the mean posterior values for the period of the financial cycles, λ^F of the model variants were between 67 to 76 quarters, or roughly 17 to 19 years. Values for the posterior means from the bivariate model variants are given in Table E.3 in Appendix E. For the business cycle we obtained posterior mean values for λ^B of between 8 to 11 years. These values are also listed in Table E.3 for the bivariate models. They are in close agreement with standard values for the business cycle in the literature. Appendix E also lists the posterior means and standard deviations for the other parameters for the bivariate models, see Tables E.1, E.2 and E.3.

The concordance index between cycles provides us with an additional measure of the extent of agreement between the various financial cycle estimates. A value approaching 1 indicates a nearly perfect agreement between the cycles, whereas a value approaching 0 indicates a nearly perfect counter-cyclical relationship. An expected value assuming no relationship can be calculated for comparison. This value will typically be near 0.5. In Table 1 we list the concordance index between the estimated US financial cycles for each model variant, including the financial cycle estimate of the BIS. Table 2 provides the same information for the Dutch estimates. Tables of

³⁰See Reinhart & Rogoff (2009) and Laeven & Valencia (2012) for further details on systemic banking crises.



Figure 1: Financial cycle estimates for US credit & housing price index



















(g) six-variable



(b) Univariate











(f) four-variable

(h) six-variable





All inner credible bands are one standard deviation, outer bands represent a 90% credible band.





















Financial Cycle – HP

Financial Cycle – HP









(f) four-variable





(g) five-variable



All inner credible bands are one standard deviation, outer bands represent a 90% credible band.

Figure 2: Financial cycle estimates for NL credit & housing price index

the expected values assuming no relationship can be found in Appendix D.

In the case of Table 1 for the US, we can see that the estimates from the univariate models of credit and the housing price index show a high degree of concordance with the multivariate estimates and with the BIS. This is also the case for the bivariate model based on credit and the housing price index. This leads us to conclude that these two series are the most important determinants of the financial cycle. Relative to the expected values listed in Table D.1, the other series seem to have a weaker relationship with the US financial cycle.

In general the concordance values between the univariate estimates based on the S&P 500 and spreads and the other model variants suggests that these two series have a counter-cyclical relationship with the financial cycle, albeit a weak one.³¹ This can also be seen in the plots of the financial cycle based on PE and the spreads shown in Figure C.2a. The values of the concordance between the IP estimates and the other model variants shows no consistent pattern, indicating that IP is not strongly influenced by the financial cycle. GDP seems to have a positive relationship with the financial cycle, perhaps due to the contribution of financial sector. However this relationship also seems to be weak.

Table 2 for the Netherlands indicates broadly the same patterns. Credit, however, seems to produce a counter-cyclical financial cycle from its univariate model, although inspection of the plot of the estimated cycle in 2a suggests that this cycle is weak. This suggests that the Dutch financial cycle is largely driven by the housing price index. We note that the concordance between the financial cycle from the bivariate model and the cycles from the other multivariate models and the BIS is even larger than is the case for the US results.

| $ \begin{array}{c c c c c c c c c c c c c c c c c c c $ | | | | | | | | | | | |
|--|---------|----------|------|---------------|-------|-------|---------|-----------|--------|--------|------|
| Variable HP PE IP GDP Spreads Univariate 0.66 0.45 0.48 0.82 0.19 0.70 0.70 0.66 0.80 HP 0.39 0.40 0.63 0.39 0.94 0.86 0.82 0.77 PE 0.54 0.50 0.41 0.39 0.26 0.23 0.23 IP 0.61 0.43 0.39 0.51 0.58 0.61 GDP 0.61 0.43 0.39 0.51 0.58 0.61 GDP 0.61 0.43 0.39 0.64 0.65 0.70 Spreads 0.14 0.63 0.64 0.65 0.70 Spreads 0.38 0.47 0.42 0.33 Bivariate 0.84 0.82 0.81 | Model | | | | Univa | riate | | Bivariate | 4-vars | 6-vars | BIS |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | | Variable | HP | \mathbf{PE} | IP | GDP | Spreads | | | | |
| $\begin{array}{c ccccccccccccccccccccccccccccccccccc$ | Univar | iate | | | | | | | | | |
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | | Credit | 0.66 | 0.45 | 0.48 | 0.82 | 0.19 | 0.70 | 0.70 | 0.66 | 0.80 |
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$ | | HP | | 0.39 | 0.40 | 0.63 | 0.39 | 0.94 | 0.86 | 0.82 | 0.77 |
| IP 0.61 0.43 0.39 0.51 0.58 0.61 GDP 0.14 0.63 0.64 0.65 0.70 Spreads 0.38 0.47 0.42 0.33 Bivariate 0.84 0.82 0.82 4-var 0.93 0.81 | | PE | | | 0.54 | 0.50 | 0.41 | 0.39 | 0.26 | 0.23 | 0.23 |
| GDP 0.14 0.63 0.64 0.65 0.70 Spreads 0.38 0.47 0.42 0.33 Bivariate 0.84 0.82 0.82 4-var 0.93 0.81 | | IP | | | | 0.61 | 0.43 | 0.39 | 0.51 | 0.58 | 0.61 |
| Spreads 0.38 0.47 0.42 0.33 Bivariate 0.84 0.82 0.82 4-var 0.93 0.81 | | GDP | | | | | 0.14 | 0.63 | 0.64 | 0.65 | 0.70 |
| Bivariate 0.84 0.82 0.82 4-var 0.93 0.81 | | Spreads | | | | | | 0.38 | 0.47 | 0.42 | 0.33 |
| 4-var 0.93 0.81 | Bivaria | ate | | | | | | | 0.84 | 0.82 | 0.82 |
| 1 101 | 4-var | | | | | | | | | 0.93 | 0.81 |
| 6-var 0.81 | 6-var | | | | | | | | | | 0.81 |

Table 1: Concordance between US financial cycle estimates

³¹This lends credence to the negative sign restrictions we impose in the covariance matrix Ω_{κ}^{F} between PE and credit and the housing price index.

| Model | | | Univar | riate | | Bivariate | 4-var | 5-var | 6-var | BIS |
|------------|----------|------|---------|-------|------|-----------|-------|-------|-------|------|
| | Variable | HP | Spreads | IP | GDP | | | | | |
| Univariate | 9 | | | | | | | | | |
| | Credit | 0.38 | 0.43 | 0.83 | 0.68 | 0.35 | 0.40 | 0.42 | 0.36 | 0.28 |
| | HP | | 0.42 | 0.47 | 0.61 | 0.94 | 0.97 | 0.96 | 0.97 | 0.90 |
| | Spreads | | | 0.59 | 0.73 | 0.41 | 0.43 | 0.45 | 0.42 | 0.48 |
| | IP | | | | 0.80 | 0.45 | 0.48 | 0.51 | 0.45 | 0.38 |
| | GDP | | | | | 0.59 | 0.63 | 0.65 | 0.60 | 0.59 |
| Bivariate | | | | | | | 0.97 | 0.94 | 0.96 | 0.92 |
| 4-var | | | | | | | | 0.96 | 0.96 | 0.91 |
| 5-var | | | | | | | | | 0.93 | 0.89 |
| 6-var | | | | | | | | | | 0.93 |

Table 2: Concordance between Dutch financial cycle estimates

The concordance index can also help us to gauge the degree to which we have succeeded in isolating a unique financial cycle based on the restriction that the rank of $\Omega_{\kappa}^{F} = 1$. Accordingly we calculated the concordance indices between the financial cycle estimates of the series within a model variant for the US and Dutch models. For both countries the index values are generally either 0 or 1, demonstrating the perfect concordance as a result of the rank reduction. Only in the cases of IP and the spreads do we obtain values that slightly deviate from 0 or 1. This is due to the fact that we do not impose any sign restrictions on the elements of the covariance matrix Ω_{κ}^{F} associated with IP or the spreads. As a result these covariances are able to switch sign during the MCMC simulation. This can occur because these variable are not strongly influenced by the financial cycle. The resulting estimated financial cycles are then a mixture of simulations with at times positive and at other times negative correlations in Ω_{κ}^{F} . The S&P 500 PE series is also weakly influenced by the financial cycle. However, for this series the concordance index with credit, the housing price index and GDP is 0. This is ensured by the sign restriction for the elements related to PE in Ω_{κ}^{F} .

6.2 The Financial Cycle with rank $\Omega_{\kappa}^{F} > 1$

We attempt to judge the validity of the rank reduction we impose on Ω_{κ}^{F} in a number of ways. Qualitatively we can compare the financial cycle estimates we obtain with less restrictive rank reductions on Ω_{κ}^{F} with those in Figures 1 and 2 where the rank was restricted to be 1. Figure 3 contains two plots of financial cycle estimates. The left-hand side plots the financial cycle estimate based on US credit from the six-variable model with no rank reduction imposed on Ω_{κ}^{F} . The right-hand side plots the estimate based on Dutch credit from the six-variable model with the rank on Ω_{κ}^{F} reduced to 2 instead of $1.^{32}$ Both cycles are largely unchanged relative to their restricted counterparts. This suggests that the relationships implied by the rank reduction are already largely present in the data.



Figure 3: Financial cycle estimates for Credit with rank $\Omega_{\kappa}^{F} > 1$

All inner credible bands are one standard deviation, outer bands represent a 90% credible band.

Using the concordance index we can demonstrate this quantitatively. Table 3 lists the concordance indices between the six estimated financial cycles from the US six-variable model variant in which the rank of Ω_{κ}^{F} is unrestricted, as well as with the six estimates from the same US model only with the rank of Ω_{κ}^{F} restricted to be 1. The index values are close to either 0 or 1, and the agreement with the BIS estimates is also strong. The results for the Netherlands for the comparison between the Dutch six-variable model variant based on a rank of Ω_{κ}^{F} of 1 and 2 are broadly similar. These values can be found in Appendix D together with the tables of the expected values for the US and Dutch models.

Finally, we test the validity of the rank restriction on Ω_{κ}^{F} in two additional ways. In Table 4 we report on the first test based on the re-estimation of each model variant with the unrestricted Ω_{κ}^{F} covariance matrix. For this test we take the value of the largest of the eigenvalues of the unrestricted Ω_{κ}^{F} as a percentage of the eigenvalue sum. This value is shown under column denoted by $100 \frac{\lambda_{1}}{\Sigma \lambda_{i}}$. These values are similar to those used in principal components analysis. The percentages shown indicate that most of the variability in the estimated covariance matrices Ω_{κ}^{F} is due to the largest eigenvalue. Most values are above 0.90, with the lowest value for the US four-variable model still equal to 0.54. These results suggest that the restricted model is picking up most of the variability in the data related to the financial cycle.

 $^{^{32}{\}rm This}$ is arguably justified due to the presence of the S&P 500 PE series in this model variant, which is a US variable.

| $rank\left(\Omega_{\kappa}^{F}\right)$ | | | | ranl | x 6 | | | |
|--|----------|--------|------|---------|------|------|---------|------|
| | Variable | Credit | HP | Spreads | GDP | PE | Spreads | BIS |
| rank 1 | | | | | | | | |
| | Credit | 0.96 | 0.98 | 0.90 | 0.94 | 0.06 | 0.06 | 0.81 |
| | HP | 0.96 | 0.98 | 0.90 | 0.94 | 0.06 | 0.06 | 0.81 |
| | IP | 0.96 | 0.98 | 0.90 | 0.94 | 0.06 | 0.06 | 0.81 |
| | GDP | 0.96 | 0.98 | 0.90 | 0.94 | 0.06 | 0.06 | 0.81 |
| | PE | 0.04 | 0.02 | 0.10 | 0.06 | 0.94 | 0.94 | 0.19 |
| | Spreads | 0.04 | 0.04 | 0.10 | 0.06 | 0.96 | 0.95 | 0.17 |
| rank 6 | | | | | | | | |
| | Credit | | 0.97 | 0.91 | 0.96 | 0.05 | 0.03 | 0.82 |
| | HP | | | 0.91 | 0.94 | 0.06 | 0.05 | 0.82 |
| | IP | | | | 0.93 | 0.14 | 0.08 | 0.82 |
| | GDP | | | | | 0.09 | 0.01 | 0.82 |
| | PE | | | | | | 0.93 | 0.18 |
| | Spreads | | | | | | | 0.17 |

Table 3: Concordance US financial cycles, 6 variable model with $rank\left(\Omega_{\kappa}^{F}\right) = 1$ and 6

The second test we report on in Table 4 is based on Bayes Factors. For this test, we first calculate the log of the posterior data density both with and without the rank restriction on Ω_{κ}^{F} . We use the same priors for the unrestricted model as we selected for the restricted model. This should tend to favor the restricted model, given that some of these priors are selected using empirical Bayesian priors. The column denoted by BF, for Bayes Factor, shows the difference between the two log posterior data densities: the restricted value minus the unrestricted value. Positive values indicate support for the restricted model, while negative ones indicate support for the unrestricted model. The results show, however, that we are generally unable to justify the rank reduction on Ω_{κ}^{F} based on this test.

7 Conclusion

We propose a rank-reduced multivariate state-space model to estimate the financial cycle for the US and the Netherlands. In all multivariate model variants we include total credit to the private non-financial sector and the housing price index. These two series are generally regarded as the principal determinants of the financial cycle, something our results corroborate. In some model variants we also include industrial production, GDP, interest rate spreads, and a cumulative index based on the earnings to price ratio from the S&P 500.

Our model is comprised of unobserved components which capture the salient features in the data. In particular we specify the financial cycle as an unobserved trigonometric cycle component, with rank reduction imposed on the covariance matrix of this component's disturbance

| Country | model variant | $100 \frac{\lambda_1}{\Sigma \lambda_i}$ | BF |
|-------------|---------------|--|------|
| US | | | |
| | Bivariate | 90% | -16 |
| | 4-var | 54% | -79 |
| | 6-var | 98% | -313 |
| Netherlands | | | |
| | Bivariate | 99.5% | -29 |
| | 4-var | 99.4% | -159 |
| | 5-var | 98% | -135 |
| | 6-var | 91% | 21 |

Table 4: Tests of Rank Reduction on Ω_{κ}^{F}

The column header "BF" refers to the Bayes factor for the model with $r(\Omega_{\kappa}^{F}) = 1$ vs. the model with Ω_{κ}^{F} of full rank (except for the NL six-variable, for which the restricted ranks of 1 and 2 are compared).

vector and initial values to produce a single underlying financial cycle estimate. This use of rank reduction to identify a country's financial cycle is new to the literature.

The rank reduction we impose on the covariance of the financial cycle components' disturbance terms can be justified in a manner that is similar to principal components analysis: the largest eigenvalue of the covariance matrix Ω_{κ}^{F} is typically greater than 90% of the sum of the eigenvalues for the unrestricted SSM variants we estimate. The reduction does not however seem to be supported by Bayesian model testing based on the Bayes Factor.

The financial cycle estimates have periods lasting roughly 18 to 21 years. Financial events such as the Great Depression, the US savings and loans crisis, the Dutch housing boom and bust from 1976 to 1983, and the Great Recession are all reflected in the financial cycle estimates. The multivariate variants produce financial cycle estimates with a high degree of cyclical concordance, and also largely follow the financial cycle estimates produced by the BIS for the period since 1970.

In general our estimates indicate that the financial cycle is largely determined by credit and the housing price index. The bivariate model of credit and the housing price index is small and relatively parsimonious, and yet produces financial cycle estimates that closely mirror those we obtain from the larger multivariate models. The bivariate model has the additional benefit of allowing for a longer sample period as both series are available starting around 1900. For these reasons we conclude that the bivariate model is best suited for the estimation of the financial cycle. This conclusion is reflected in the related research of Soederhuizen *et al.* (2019) in which the authors use financial cycle estimates from the bivariate model to explore the possibility that the financial cycle influences the fiscal multiplier.

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A Alternative Specification of the Financial Cycle

An alternative formulation of our state-space model can be expressed by reformulating the measurement equation (1) as follows.

$$\vec{y}_t = \vec{\mu}_t + A \, \vec{\psi}_t^F + B \, \vec{\psi}_t^B + \sum_{j=1}^{[s/2]} \vec{\gamma}_{j,t} + \vec{\varepsilon}_t \tag{A.1}$$

Here the matrices A and B are both lower triangular matrices with unity along the main diagonal. The matrix A then is a loading-matrix that determines how much each of the cycles $\psi_{i,t}^F$ contributes to the data series $y_{i,t}$. The same is true for the loading-matrix B, which determines the weighted contribution of the cycles $\psi_{i,t}^B$ to the data series $y_{i,t}$. In order to ensure that this model is identified, we must also restrict the covariance matrices Ω_{κ}^C for C = B and F to be diagonal matrices. In other words, the underlying cycle components must be independent.

This second model specification is otherwise based on the same equations for the unobserved components shown above in (4) - (7). This state-space model is similar to the models proposed in Koopman & Lucas (2005) and de Winter *et al.* (2017). The difference between our model and theirs is due to how the cycle components are formulated. In our model formulation the measurement equation (1) includes cycle components with correlated disturbance terms. Unless our prior on the starting values for the cycles are assumed to be perfectly correlated, this has the effect that the cyclical components can differ in their amplitude and phase even when the cycle component disturbances are perfectly correlated. The second model in (A.1) by comparison will be based on underlying cycle components which can only differ in their amplitude when each series selects the same underlying cycle component in the measurement equation (A.1).

It is of course our goal in this research to identify a single financial cycle. There are two modeling options we can follow to achieve this. One option is based on (A.1) with the restriction that the A matrix select the same underlying financial cycle for the series in the model. The second alternative is based on (1) and requires the imposition of the restriction that the rank of the covariance matrix of the financial cycle component Ω_{κ}^{f} be reduced to one. In this latter case we must also impose the restriction that the starting values of the cycles are also perfectly correlated, as is done in (8) where Ω_{κ}^{F} has rank 1. These two modeling approaches are then observationally equivalent.

We note that Luginbuhl *et al.* (2019) base their estimates of the financial cycle on the same state-space model we use here in equations (1) - (7), only the authors use the standard prior on the starting values of the financial cycles given in (8) where Ω_{κ}^{F} is replaced with the diagonalized covariance matrix with zeros on the off-diagonal elements to impose independence between the starting values. Asymptotically the financial series are identical, because the authors

restrict the rank of the disurbance covariance Ω_{κ}^{F} to be 1. However, given the independent prior of the starting values of the financial cycles, the model still allows for differing phase shifts in the financial cycle of each series. While slightly less restrictive, the disadvantage of this approach is that the authors then must chose which of the estimated financial cycles represents the underlying financial cycle, because initially at least, they are not the same.

B Prior Parameters

We list here the prior parameters for the bivariate model variants. To avoid producing too many tables, we do not report on the priors for the other model variants. They are, however, available on request.

The inverse Wishart prior parameters S^B_{κ} and S^F_{κ} for the covariance matrices Ω^B_{κ} and Ω^F_{κ} , respectively, are listed in Table B.1. Note that $\nu^B = \nu^F = 13$ with the exception of Korea, where $\nu^F = 6$. The parameters $s^B_{\kappa_1}$ and $s^F_{\kappa_1}$ pertain to the credit series, $s^B_{\kappa_2}$ and $s^F_{\kappa_2}$ to the housing price index, and $s^F_{\kappa_{12}}$ is the scale factor for the covariance between the financial cycle disturbances for credit and the housing price index.³³ The comparable prior scale factor for the covariance of the business cycle disturbance is set to zero. We have therefore that

$$S_{\kappa}^{B} = \begin{bmatrix} s_{\kappa_{1}}^{B} & 0\\ 0 & s_{\kappa_{2}}^{B} \end{bmatrix} \quad S_{\kappa}^{F} = \begin{bmatrix} s_{\kappa_{1}}^{F} & s_{\kappa_{12}}^{F}\\ s_{\kappa_{12}}^{F} & s_{\kappa_{2}}^{F} \end{bmatrix}$$
(B.1)

| | S | S^B_{κ} | | | S^F_κ | | | | |
|-------------|------------------|------------------|--|------------------|------------------|---------------------|--|--|--|
| Country | $s^B_{\kappa_1}$ | $s^B_{\kappa_2}$ | | $s^F_{\kappa_1}$ | $s^F_{\kappa_2}$ | $s^F_{\kappa_{12}}$ | | | |
| Netherlands | 40.0 | 60.0 | | 0.1 | 25.0 | 1.581 | | | |
| US | 9.0 | 9.0 | | 1.2 | 10.0 | 3.464 | | | |

 Table B.1: Cycle Disturbance Covariance Priors

³³The latter value is set to exactly ensure that the rank of $S^F = 1$.

| | S_{η} | | S | S_{ζ} | | begin |
|-------------|--------------|--------------|---------------|---------------|--------------|--------------------|
| Country | s_{η_1} | s_{η_2} | s_{ζ_1} | s_{ζ_2} | $ u_{\zeta}$ | sample period |
| Netherlands | 0.100 | 0.100 | 2.0 | 1.28 | 203 | 1900 Q4 |
| US | 0.016 | 0.060 | 0.512 | 0.200 | 83 | $1914~\mathrm{Q4}$ |

Table B.2: Trend & Drift disturbance Covariance Priors & Sample Starting Date

Table B.3: Measurement Error Covariance Priors

| | | S_{ε} | | S_h | | | | | | |
|-------------|-------------------|-------------------|-----------|---------------------|-----------|--------------------|--|--|--|--|
| Country | $s_{arepsilon_1}$ | $s_{arepsilon_2}$ | s_{h_1} | T_1^* | s_{h_2} | T_2^* | | | | |
| Netherlands | 0.234 | 0.273 | 9.750 | $1961 \mathrm{Q4}$ | 8.190 | $1970~\mathrm{Q4}$ | | | | |
| US | 0.033 | 0.090 | 7.800 | $1952~\mathrm{Q4}$ | 2.340 | $1954~\mathrm{Q4}$ | | | | |

C Estimated Unobserved Components

The Figures in this appendix show plots of the medians³⁴ of the posterior distributions of the unobserved components in the model: the financial cycles, the business cycles, the trends and the drifts. The plots also show the Bayesian credible interval of 68%, or in other words \pm one standard deviation of the posteriors, as well as the 90% credible interval. For comparision, the plots of the financial cycle estimates also include the BIS estimates of the financial cycle.

³⁴Given the assumed normality of the disturbances, the median and mean of the posteriors will be equal.

| country | model | rank Ω_{κ}^{F} | total | burn-in |
|---------|-----------|----------------------------|---------|---------|
| US | | | | |
| | Credit | 1 | 200000 | 25000 |
| | HP | 1 | 200000 | 25000 |
| | PE | 1 | 200000 | 25000 |
| | Spreads | 1 | 200000 | 25000 |
| | IP | 1 | 200000 | 25000 |
| | GDP | 1 | 200000 | 25000 |
| | bivariate | 1 | 200000 | 25000 |
| | 4-var | 1 | 200000 | 25000 |
| | 6-var | 1 | 100000 | 12500 |
| | 6-var | 6 | 40000 | 5000 |
| NL | | | | |
| | Credit | 1 | 120000 | 15000 |
| | HP | 1 | 400000 | 50000 |
| | Spreads | 1 | 200000 | 25000 |
| | IP | 1 | 400000 | 50000 |
| | GDP | 1 | 1000000 | 125000 |
| | bivariate | 1 | 400000 | 50000 |
| | 4-var | 1 | 40000 | 5000 |
| | 5-var | 1 | 20000 | 2500 |
| | 6-var | 1 | 20000 | 2500 |
| | 6-var | 2 | 20000 | 2500 |

Table C.1: MCMC replications



(a) lefthand side: credit bivariate model

righthand side: HP bivariate model



All inner credible bands are one standard deviation, outer bands represent a 90% credible band.

Figure C.2: Drift estimates for US S&P 500 PE ratio and Spreads

(a) lefthand side: PE four-variable model



Business Cycle – PE



Trend – PE



Growth Rate - PE



righthand side: Spreads six-variable model

Financial Cycle – Spreads



Business Cycle – Spreads



Trend – Spreads



Growth Rate – Spreads



All inner credible bands are one standard deviation, outer bands represent a 90% credible band.



(a) lefthand side: IP four-variable model





Business Cycle – IP

Trend – IP

500

450

400 350

300

250 200

150 100

1900

Growth Rate - IP

- median

2,5

2,0

1,5

1,0

0,5

0,0

-0,5 -1,0

1900

- median

1920

1920 1940

log data

1960

1940 1960 1980 2000 2020

1980 2000 2020



righthand side: GDP six-variable model

Financial Cycle – GDP







Trend – GDP



Growth Rate – GDP







(a) lefthand side: credit bivariate model

righthand side: HP bivariate model



All inner credible bands are one standard deviation, outer bands represent a 90% credible band.

Figure C.5: Estimates for Dutch GDP & Industrial Production (IP)

(a) lefthand side: GDP five-variable model

righthand side: IP five-variable model























Financial Cycle – IP



Trend – IP



Growth Rate - IP



All inner credible bands are one standard deviation, outer bands represent a 90% credible band.

Figure C.6: Estimates for German Spreads & S&P 500 PE

(a) lefthand side: Spreads five-variable model

righthand side: PE six-variable model



Financial Cycle – PE



Business Cycle – Spreads



Business Cycle – PE



Trend – Spreads



Growth Rate – Spreads





Growth Rate – PE

Trend – PE



All inner credible bands are one standard deviation, outer bands represent a 90% credible band.

D Expected concordance values

| Model | s | | | Univa | riate | | Bivariate | 4-var | 6-var | BIS |
|--------|---------------|------|---------------|-------|-------|---------|-----------|-------|-------|------|
| | Variable | HP | \mathbf{PE} | IP | GDP | Spreads | | | | |
| Univa | riate | | | | | | | | | |
| | Credit | 0.51 | 0.51 | 0.52 | 0.54 | 0.54 | 0.51 | 0.51 | 0.55 | 0.55 |
| | HP | | 0.50 | 0.51 | 0.51 | 0.52 | 0.50 | 0.50 | 0.52 | 0.52 |
| | \mathbf{PE} | | | 0.52 | 0.53 | 0.53 | 0.51 | 0.51 | 0.54 | 0.54 |
| | IP | | | | 0.55 | 0.55 | 0.51 | 0.51 | 0.56 | 0.56 |
| | GDP | | | | | 0.59 | 0.52 | 0.52 | 0.61 | 0.61 |
| | Spreads | | | | | | 0.52 | 0.52 | 0.61 | 0.62 |
| Bivari | ate | | | | | | | 0.51 | 0.53 | 0.53 |
| 4-var | | | | | | | | | 0.53 | 0.53 |
| 6-var | | | | | | | | | | 0.61 |

Table D.1: Expected concordance values between US financial cycle estimates

Table D.2: Expected concordance between Dutch financial cycle estimates

| Model | | | Univar | iate | | Bivariate | 4-var | 5-var | 6-var | BIS |
|------------|----------|------|---------|------|------|-----------|-------|-------|-------|------|
| | Variable | HP | Spreads | IP | GDP | | | | | |
| Univariate | ; | | | | | | | | | |
| | Credit | 0.50 | 0.49 | 0.49 | 0.49 | 0.50 | 0.49 | 0.49 | 0.49 | 0.49 |
| | HP | | 0.51 | 0.51 | 0.51 | 0.50 | 0.52 | 0.52 | 0.51 | 0.52 |
| | Spreads | | | 0.65 | 0.65 | 0.51 | 0.66 | 0.66 | 0.65 | 0.69 |
| | IP | | | | 0.65 | 0.51 | 0.66 | 0.66 | 0.65 | 0.69 |
| | GDP | | | | | 0.51 | 0.66 | 0.66 | 0.65 | 0.69 |
| Bivariate | | | | | | | 0.51 | 0.51 | 0.51 | 0.51 |
| 4-var | | | | | | | | 0.67 | 0.67 | 0.70 |
| 5-var | | | | | | | | | 0.66 | 0.70 |
| 6-var | | | | | | | | | | 0.69 |
| | | | | | | | | | | |

E Parameter posteriors

The posterior means and posterior standard deviations of the model parameters for the bivariate model are shown below in Tables E.1, E.2 and E.3. Details of the posteriors we obtain for the other model variants are available on request.

| $rank\left(\Omega_{\kappa}^{F}\right)$ | | | | ra | nk 2 | | | |
|--|---------------|--------|------|------|------|------|---------|------|
| | Variable | Credit | HP | IP | GDP | PE | Spreads | BIS |
| rank 1 | | | | | | | | |
| | Credit | 0.65 | 0.64 | 0.64 | 0.65 | 0.61 | 0.60 | 0.65 |
| | HP | 0.65 | 0.64 | 0.64 | 0.65 | 0.61 | 0.60 | 0.65 |
| | IP | 0.65 | 0.64 | 0.64 | 0.65 | 0.61 | 0.60 | 0.65 |
| | GDP | 0.65 | 0.64 | 0.64 | 0.65 | 0.61 | 0.60 | 0.65 |
| | PE | 0.61 | 0.61 | 0.61 | 0.61 | 0.58 | 0.57 | 0.61 |
| | Spreads | 0.61 | 0.61 | 0.61 | 0.61 | 0.58 | 0.57 | 0.61 |
| rank 2 | | | | | | | | |
| | Credit | | 0.65 | 0.65 | 0.65 | 0.62 | 0.60 | 0.65 |
| | HP | | | 0.64 | 0.65 | 0.61 | 0.60 | 0.65 |
| | IP | | | | 0.65 | 0.61 | 0.60 | 0.65 |
| | GDP | | | | | 0.62 | 0.60 | 0.65 |
| | \mathbf{PE} | | | | | | 0.58 | 0.62 |
| | Spreads | | | | | | | 0.60 |

Table D.3: Expected concordance US financial cycles, six-variable models with $rank\left(\Omega_{\kappa}^{F}\right) = 1$ and 6

Table D.4: Concordance Dutch financial cycles, 6 variable model with $rank\left(\Omega_{\kappa}^{F}\right) = 1$ and 2

| $rank\left(\Omega_{\kappa}^{F}\right)$ | | | | rank 2 | 2 | | | |
|--|----------|--------|------|---------|------|------|------|------|
| | Variable | Credit | HP | Spreads | IP | GDP | PE | BIS |
| rank 1 | | | | | | | | |
| | Credit | 97 | 0.97 | 0.04 | 0.73 | 0.88 | 0.05 | 0.93 |
| | HP | 97 | 0.97 | 0.04 | 0.73 | 0.88 | 0.05 | 0.93 |
| | Spreads | 3 | 0.03 | 0.97 | 0.27 | 0.13 | 0.95 | 0.07 |
| | IP | 97 | 0.97 | 0.05 | 0.76 | 0.90 | 0.07 | 0.92 |
| | GDP | 97 | 0.97 | 0.04 | 0.74 | 0.88 | 0.06 | 0.93 |
| | PE | 3 | 0.03 | 0.96 | 0.26 | 0.12 | 0.94 | 0.07 |
| rank 2 | | | | | | | | |
| | Credit | | 1.00 | 0.03 | 0.72 | 0.86 | 0.03 | 0.93 |
| | HP | | | 0.04 | 0.73 | 0.87 | 0.04 | 0.93 |
| | Spreads | | | | 0.30 | 0.15 | 0.95 | 0.08 |
| | IP | | | | | 0.86 | 0.31 | 0.73 |
| | GDP | | | | | | 0.17 | 0.88 |
| | PE | | | | | | | 0.07 |

| $rank\left(\Omega_{\kappa}^{F}\right)$ | | rank 2 | | | | | | |
|--|----------|--------|---------------------|---------|------|------|------|------|
| | Variable | Credit | HP | Spreads | IP | GDP | PE | BIS |
| rank 1 | | | | | | | | |
| | Credit | 0.66 | 0.66 | 0.63 | 0.66 | 0.68 | 0.63 | 0.69 |
| | HP | 0.66 | 0.66 | 0.63 | 0.66 | 0.68 | 0.63 | 0.69 |
| | Spreads | 0.62 | 0.62 | 0.60 | 0.63 | 0.64 | 0.60 | 0.65 |
| | IP | 0.66 | 0.66 | 0.63 | 0.67 | 0.68 | 0.63 | 0.70 |
| | GDP | 0.65 | 0.66 | 0.63 | 0.66 | 0.68 | 0.63 | 0.69 |
| | PE | 0.62 | 0.62 | 0.60 | 0.63 | 0.64 | 0.60 | 0.65 |
| | | | | | | | | |
| $\operatorname{rank} 2$ | | | | | | | | |
| | Credit | | 0.65 | 0.63 | 0.66 | 0.68 | 0.63 | 0.69 |
| | HP | | | 0.63 | 0.66 | 0.68 | 0.63 | 0.69 |
| | Spreads | | | | 0.64 | 0.65 | 0.61 | 0.66 |
| | IP | | | | | 0.69 | 0.64 | 0.70 |
| | GDP | | | | | | 0.65 | 0.72 |
| | PE | | | | | | | 0.66 |

Table D.5: Expected concordance Dutch financial cycles, six-variable models with $rank\left(\Omega_{\kappa}^{F}\right) = 1$ and 2

Table E.1: Parameter Posterior of Measurement and Trend Error Covariances

| | | Ω_{ϵ} | Ω_η | | | |
|-------------|--------------------------|--------------------------|---------------|---------------|-------------------|-------------------|
| Country | $\sigma_{\varepsilon 1}$ | $\sigma_{\varepsilon 2}$ | σ_{h1} | σ_{h2} | $\sigma_{\eta 1}$ | $\sigma_{\eta 2}$ |
| Netherlands | 0.006 | 0.007 | 0.255 | 0.204 | 0.010 | 0.010 |
| | 0.001 | 0.001 | 0.043 | 0.033 | 0.003 | 0.003 |
| US | 0.0009 | 0.0023 | 0.252 | 0.060 | 0.002 | 0.006 |
| | 0.0002 | 0.0004 | 0.054 | 0.010 | 0.001 | 0.002 |

The table lists the posterior means and posterior standard deviations of the parameters. The first row for each country shows the mean, while the value directly under is the standard deviation.

| | | Ω_{ζ} | Ω_{ω} | | | |
|-------------|--------------------|--------------------|---------------------|---------------------|---------------------|--|
| Country | $\sigma_{\zeta 1}$ | $\sigma_{\zeta 2}$ | $\sigma_{\zeta 12}$ | $\sigma_{\omega 1}$ | $\sigma_{\omega 2}$ | |
| Netherlands | 0.0285 | 0.0082 | 0.0041 | 0.00002 | 0.00002 | |
| | 0.0055 | 0.0015 | 0.0022 | 0.00001 | 0.00001 | |
| US | 0.0115 | 0.0025 | -0.0004 | 0.00002 | 0.00002 | |
| | 0.0023 | 0.0004 | 0.0007 | 0.00001 | 0.00001 | |

Table E.2: Parameter Posteriors of Drift and Seasonal Error Covariances

The table lists the posterior means and posterior standard deviations of the parameters. The first row for each country shows the mean, while the value directly under is the standard deviation.

| | Ω^F_{κ} | | Ω^B_{κ} | | | | | | |
|-------------|-----------------------|-----------------------|-----------------------|-----------------------|------------------------|-------------|-------------|---------|---------|
| Country | $\sigma^F_{\kappa 1}$ | $\sigma^F_{\kappa 2}$ | $\sigma^B_{\kappa 1}$ | $\sigma^B_{\kappa 2}$ | $\sigma^B_{\kappa 12}$ | λ^F | λ^B | $ ho^F$ | $ ho^B$ |
| Netherlands | 0.06 | 1.48 | 2.47 | 3.80 | 0.57 | 72.7 | 41.3 | 0.993 | 0.979 |
| | 0.17 | 0.68 | 0.27 | 0.80 | 0.27 | 4.2 | 2.9 | 0.003 | 0.005 |
| US | 0.13 | 0.67 | 0.65 | 0.72 | 0.05 | 77.7 | 34.7 | 0.996 | 0.986 |
| | 0.05 | 0.14 | 0.07 | 0.13 | 0.04 | 2.3 | 1.3 | 0.002 | 0.004 |

Table E.3: Parameter Posteriors of Cycle Components

The table lists the posterior means and posterior standard deviations of the parameters. The first row for each country shows the mean, while the value directly under is the standard deviation.