CPB Netherlands Bureau for Economic Policy Analysis

Energy prices, the distribution of income, and the effects of wage indexation

We construct a two-agent New Keynesian model with energy to analyze the effects of an energy price shock on real GDP, real GDI, and the distribution of income. A large increase in energy prices leads to a deterioration of the terms of trade, causing a fall in real GDI and a decrease in aggregate consumption.

Households with income from labor suffer a larger setback in than households that primarily receive capital income. Complete automatic wage indexation reduces consumption inequality, but has short and long run economic costs, as wage indexation reduces production, income, and aggregate consumption.

CPB - March 2023

Matthijs Katz

Energy prices, the distribution of income, and the effects of wage indexation

Matthijs Katz[*]

March 1, 2023

1 Introduction

The recent global increase in energy prices has lead to substantial increases in inflation and a large deterioration of the terms of trade for many countries; in particular those that are net importers of energy, such as the Netherlands. The Dutch economy seems to be weathering the storm reasonably from a macroeconomic perspective, although the growth rate of real gross domestic product (GDP) has slowed down. At the same time, energy price increases have increased the cost of production for firms, in particular energy-intensive industries. This reduction in macroeconomic purchasing power suggests that real gross domestic income (GDI) has fallen as a result of the deteriorating terms of trade. These developments have lead to a public debate about the distributional consequences of the energy price shock: which groups have been hit the hardest, and should the government redistribute income from one group to another? If firms are able to pass on these higher production costs to other firms and consumers, they should not necessarily have to face a decrease in profits. Firms that are able to both charge a markup over marginal costs and pass on higher production costs may potentially have seen their profits increase. Furthermore, when the cost of living increases as a result of higher prices and nominal wages are rigid, households should be confronted by a fall in real labor income. Stagnating nominal wages and potentially increasing profits may imply that groups more reliant on labor income have seen a larger drop in real income than those that rely primarily on capital income. Given stagnant nominal wage growth, this situation would open up the possibility for income redistribution through higher nominal wage rates by indexing wages to inflation. In turn, higher nominal wages could negatively impact future GDP growth by reducing expected future profits, which makes it less attractive for firms to invest.

We set up a macroeconomic model to rationalize the differential responses of real GDP and real GDI to an increase in energy prices, and analyze how an increase in energy prices might affect groups that rely on capital income differently than those that rely on labor income when nominal wages are rigid. Specifically, we examine the response of real GDP, real GDI, inflation, wages, real consumption, and how the relative difference between the consumption response of groups that rely on capital income vis-a-vis those that primarily receive labor income. Furthermore, we investigate the macroeconomic and distributional effects of automatic wage indexation, which is effectively a

[*]CPB Netherlands Bureau for Economic Policy Analysis. E-mail address: m.d.b.katz@cpb.nl.

form of redistribution between workers and firms. We do so in a two-agent New Keynesian (TANK) model of a small open economy that is a member of a currency union (Galí et al., 2007; Bilbiie, 2008). There are two kinds of households: firm owners, who receive income from holding firm equity and liquid assets, and workers, who receive labor income and cannot perfectly smooth out changes in consumption due to portfolio adjustment costs.[1] Labor is hired in a frictional labor market following Mortensen and Pissarides (1994) and nominal wages are highly rigid. Energy and imported goods are used by firms to produce domestic goods, and directly consumed in all final expenditure categories. We assume all energy is imported.[2] Both energy and imported goods can be re-exported to foreign countries. Firms in the domestic goods, exports, imported goods, and energy sectors operate under monopolistic competition, such that they make nonzero profits in equilibrium, and face nominal rigidities.[3] We calibrate the model to match the relevant import and energy shares in production and final expenditure categories and numerically solve the nonlinear model under perfect foresight to investigate the effects of an energy price shock.

Our two most important results are the following. First, we show that a large, temporary increase in energy prices drives a wedge between real GDP and GDI. Higher energy prices lead to a deterioration of the terms of trade, as import prices increase by more than export prices because more energy is imported than exported. Since other countries face the same increase in energy prices, domestic goods become relatively cheaper from the point of view of the rest of the world. This increases foreign demand for domestic goods, thereby increasing exports which leads to an increase in real GDP. At the same time, prices of all expenditure categories increase. As a result, real GDI decreases: the higher cost of both domestic goods and imports reduces the economy's effective purchasing power given the current consumption basket of domestic goods, imports, and energy. Nominal profits of firms increase due to price increases, while nominal wages are effectively rigid. In equilibrium, real profits fall, but real wages fall by more. As a result, aggregate consumption also falls. However, most of the burden of adjustment lies with workers: their consumption is reduced by more than firm owners' consumption. As a result, consumption inequality increases.

Second, we show that a redistributive scheme in the form of complete automatic wage indexation to consumption price changes is effective at changing the brunt of the burden of adjustment from workers to firm owners, but comes at macroeconomic costs in both the short run and the long run. Complete automatic wage indexation effectively creates a wage price spiral as defined by Blanchard (1986): while workers attempt to maintain their real wages, firms try to maintain their markup over marginal costs. These two factors drive price level dynamics.[4] As real wages are now effectively rigid, labor income falls by less, while capital income falls by more due to higher labor costs. This

1 Without portfolio adjustment costs, workers and firm owners would have the same level of consumption in every state of the world.

2 Although in reality the Netherlands imports more energy than it produces, a non-negligible part is produced domestically. We make this assumption for simplicity, as a significant part of the profits of domestic energy production accrue to the Dutch government. We leave this for future work.

3 In the numerical exercises, only prices in the domestic goods and imported goods sectors are sticky, to match that retail and wholesale energy and export prices have adjusted quickly after the increase in world energy prices.

4 Note that the model is stationary, such that nominal wages and prices eventually return to their steady state values.

results in a larger increase in prices, triggering expenditure switching from domestic to foreign goods, and a decrease in production and real GDP. Lower production and higher prices in turn reduce real GDI further relative to the simulation without wage indexation. Hence, there is a *trade-off* between *redistributing between households* and *speeding up the macroeconomic recovery* after a large exogenous price shock. This equity-efficiency trade-off also holds for incomplete automatic wage indexation, but is less severe. In Table 1 We summarize the responses of a number of key variables to an energy price shock without and with wage indexation. Furthermore, we find that this equity-efficiency trade-off also holds in the long run. The increase in energy price shocks leads to substantial welfare losses, measured as the discounted sum of utility of both workers and firm owners weighted by their population shares along the adjustment path back to the steady state. Automatic wage indexation further amplifies the welfare losses after an energy price shock, yet can have long lasting influences on consumption inequality. Hence, automatic wage indexation can help redistribute losses between groups.

TABLE 1 Qualitative response of key variables to a temporary energy price shock without and with complete automatic wage indexation. + indicates a positive effect, − indicates a negative effect, ++ (−−) indicates a larger positive (negative) effect, and 0 indicates no effect.

Variable	Response to shock	Response to shock with wage indexation
Real GDI	−	−−
Consumption inequality	+	−
Real GDP	+	−
Real wage	−	0
Aggregate consumption	−	−−
Worker consumption	−−	−
Firm owner consumption	−	−−
Consumer price inflation	+	++

Our analysis is related to the literature on the macroeconomic effects on terms of trade shocks (Schmitt-Grohé and Uribe, 2018), and the difference between the response of real GDP and real GDI after such a shock (Kohli, 2004; Reinsdorf, 2010; Kohli, 2022). We also build on the literature on TANK (Galí et al., 2007; Bilbiie, 2008) and tractable heterogeneous agent New Keynesian (THANK) (Bilbiie, 2020; Bilbiie et al., 2022) models. Pieroni (2022)'s analysis is related to mine: the paper finds that the economic burden of a reduction in energy supply is heaviest for low income households in a heterogeneous agents New Keynesian (HANK) model with an endogenous distribution of income and wealth. We find similar results, but instead focus on the distinction between households that rely on labor income and households that rely on capital income. The classic reference on wage price spirals is Blanchard (1986). Lorenzoni and Werning (2023) show that the standard New Keynesian model with price and wage rigidities already features a wage price spiral, as optimizing firms and workers try to outpace one another in setting nominal prices and wages.

From here on our analysis proceeds as follows. In Section 2 we present the model. We describe the calibration in Section 3. Section 4 contains the analysis of our numerical results. Finally, Section 5 concludes.

2 Model

The economy is populated by two kinds of households: workers and firm owners. Workers supply labor to the economy's firms, while firm owners own the economy's firms by holding firm equity shares. Hence, workers primarily receive labor income, while firm owners receive capital income.[5] The fiscal authority levies a lump sum tax, which it uses to finance its expenditure on consumption goods and benefits to unemployment workers. Firms produce domestic value-added using capital, labor and energy. Labor is hired in a frictional labor market, where firms post vacancies to attract workers and bargain with workers over wages (Mortensen and Pissarides, 1994). Domestic value-added is combined with a composite intermediate input to produce the domestic final good. Firms sell exports to other (unmodeled) countries. Private consumption, private investment, public consumption, intermediate goods, and exports are composite goods that consist of both domestic and imported final goods and energy. All consumed imports consist of an homogeneous foreign good. The price of this imported good is exogenous. We assume all energy is imported and that the price of energy is exogenous. Retailers of domestic goods, foreign goods, exports, and energy all operate under monopolistic competition, such that these firms make non-zero profits in equilibrium. The prices of domestic and foreign goods are rigid. The economy is part of a currency union, making monetary policy exogenous and the nominal exchange rate fixed.

Notation wise, all variables with upper-case letters are nominal, whereas lower case variables are either real quantities or relative prices. Foreign variables have a superscript asterisk, while domestic variables have none. In addition, goods that are produced domestically have subscript H, while goods that are imported have subscript F.

2.1 Worker household

Workers w make up a share ψ of the population, and receive income from supplying labor $W_t n_{w,t}$ to firms and repayment of a portfolio of risk free liquid assets $B_{w,t-1}$ bought in the previous period. This portfolio consists of both short term government bonds and an internationally traded asset.[6] A share $u_{w,t}$ of the worker household is unemployed and receive unemployment benefits B_u. We assume that there is perfect consumption insurance among all members of the worker household (Merz, 1995; Andolfatto, 1996). They spend their income on consumption goods $P_{c,t}c_{w,t}$, and purchases of the risk free asset $q_t^b B_{w,t}$ which comes at price q_t^b and promises to pay out one euro in the next period. Consumption expenditures are taxed at a constant rate T_c, such that total consumption expenditures are equal to $(1+T_c)\,P_{c,t}c_{w,t}$.[7] To capture limited asset market participation, purchasing additional

[5] In reality, a sizeable fraction of households are hybrid types: they receive both capital and labor income. Furthermore, Dutch household indirectly invest in firm equity through their pension fund. Fully capturing these features of the Dutch economy would likely require modeling the entire distribution of income and wealth. We leave this for future work.

[6] This can also be rationalized as households investing in risk free bank deposits when there are no financal frictions. The bank then uses these deposits to invest in government bonds and the internationally traded asset.

[7] We use this constant tax to target consumption as a share of GDP. Since the tax rate is constant it does not affect model dynamics.

bonds is subject to quadratic adjustment costs $f\left(B_{w,t}\right)$ (Cantore and Freund, 2021). They maximize their lifetime utility, which is increasing in consumption:

$$\max_{\{B_{w,t+s},c_{w,t+s}\}_{s=0}^{\infty}} \mathbb{E}_t\left\{\sum_{s=0}^{\infty}\beta^s\frac{c_{w,t+s}^{1-\sigma_w}-1}{1-\sigma_w}\right\},$$
$$\beta\in(0,1),\sigma_w\geq 0,$$

where $\mathbb{E}_t$ is an expectations operator conditional on the agent's information set, β is the worker's subjective discount factor, and σ_w is risk aversion. Workers do not receive disutility of supplying labor or searching for a job. The worker household's nominal budget constraint is given by:

$$\left(1+T_c\right)P_{c,t}c_{w,t}+q_t^bB_{w,t}+f\left(B_{w,t}\right)=W_tn_{w,t}+B_{w,t-1}+B_uu_{w,t}.$$

Workers optimal consumption and bond holdings. The first order conditions are standard and can be found in the Appendix.

2.2 Firm owner household

Firm owners d make up a share $1-\psi$ of the population, and receive income from repayment of their portfolio of risk free liquid assets $B_{d,t-1}$ bought in the previous period and from holding shares $\varsigma_{d,t-1}$ in the economy's production firms. Shares trade at price q_t^{ς} and are a claim on firm profits Π_t. They spend their income on consumption goods $\left(1+T_c\right)P_{c,t}c_{d,t}$, lump sum taxes $T_{d,t}$ and purchases of the risk free asset $q_t^bB_{d,t}$ which comes at price q_t^b and promises to pay out one euro in the next period. In addition, firm owners have to pay a constant tax T_r which ensures firm owners and workers have the same consumption in the deterministic steady state.[8] Firm owners maximize lifetime utility, which is increasing in consumption:

$$\max_{\left\{B_{d,t+s},c_{d,t+s},\varsigma_{d,t+s}\right\}_{s=0}^{\infty}} \mathbb{E}_t\left\{\sum_{s=0}^{\infty}\beta^s\frac{c_{d,t+s}^{1-\sigma_d}-1}{1-\sigma_d}\right\},$$
$$\beta\in(0,1),\sigma_d\geq 0.$$

The firm owner household's nominal budget constraint is given by:

$$\left(1+T_c\right)P_{c,t}c_{d,t}+T_{d,t}+T_r+q_t^bB_{d,t}+q_t^{\varsigma}\varsigma_{d,t}=\left(q_t^{\varsigma}+\Pi_t\right)\varsigma_{d,t-1}+B_{d,t-1}.$$

Firm owners choose optimal consumption, bond holdings, and firm shares. The first order conditions are standard and can be found in the Appendix.

8 We introduce this tax mostly to ease solving for the steady state. This is without much loss of generality given that the model is likely to be fairly linear, such that the model's initial conditions don't matter for how it responds to shocks.

2.3 Consumption allocation

Households consume a composite consumption good c_t that comes at price $P_{c,t}$. This consumption good consists of energy consumption $c_{e,t}$ and non-energy consumption $c_{x,t}$. Non-energy consumption is a composite good that consists of both domestically produced goods $c_{H,t}$ with price $P_{H,t}$ and imported goods $c_{F,t}$ with price $P_{F,t}$. The composite goods are constructed using the following CES production functions:

$$c_t \equiv \left[(1-\zeta_{c,e})^{\frac{1}{\eta_{c,e}}} c_{x,t}^{1-\frac{1}{\eta_{c,e}}} + \zeta_{c,e}^{\frac{1}{\eta_{c,e}}} c_{e,t}^{1-\frac{1}{\eta_{c,e}}} \right]^{\frac{\eta_{c,e}}{\eta_{c,e}-1}}, \tag{1}$$

$$c_{x,t} \equiv \left[(1-\zeta_{c})^{\frac{1}{\eta_{c}}} c_{H,t}^{1-\frac{1}{\eta_{c}}} + \zeta_{c}^{\frac{1}{\eta_{c}}} c_{F,t}^{1-\frac{1}{\eta_{c}}} \right]^{\frac{\eta_{c}}{\eta_{c}-1}}, \tag{2}$$

where $\zeta_{c,e} \in (0,1)$ is the share parameter of energy in total consumption, $\zeta_c \in (0,1)$ is the share parameter of the imported good in non-energy consumption, $\eta_{c,e} > 0$ is the elasticity of substitution between energy and non-energy consumption, and $\eta_c > 0$ is the elasticity of substitution between domestic and foreign goods in non-energy consumption. Total consumption expenditure is given by:

$$P_{c,t}c_t = P_{e,t}c_{e,t} + P_{x,c,t}c_{x,t},$$
$$P_{x,c,t}c_{x,t} = P_{F,t}c_{F,t} + P_{H,t}c_{H,t}.$$

The household minimizes total consumption expenditure subject to the CES production function. In equilibrium, this determines the allocation between energy and non-energy consumption $c_{e,t}$ and $c_{x,t}$, domestic and foreign goods $c_{H,t}$ and $c_{F,t}$, and the price of the composite goods $P_{c,t}$ and $P_{x,c,t}$.

2.4 Production and pricing

Production and pricing occur in the following manner. Monopolistically competitive retail goods importers purchase foreign goods from abroad. They differentiate these goods, determine the price on these goods and then sell them to import sellers. Foreign goods are used for final consumption and investment purchases, combined with domestic goods to produce intermediate goods, and combined with domestic goods to produce exports. Monopolistically competitive energy importers purchase energy from abroad and operate similarly to retail goods importers. Energy is used in all expenditure categories and to produce value added, intermediate goods and exports. We assume all energy is imported. Production firms use capital and labor to produce value-added, and subsequently sell their output to domestic goods producers. Labor is hired in a frictional labor market, as production firms need to post vacancies to attract workers and subsequently bargain over wages. Value-added is combined with intermediate goods to produce the domestic good, which is again sold to a monopolistically competitive firm that sets its price who sell the differentiated goods to final sellers. Domestic goods are used for final private consumption, investment, and government consumption purchases. Domestic goods are also combined with foreign goods and energy to produce intermediate goods, and combined with foreign goods and energy to produce exports. Finally, export producers purchase domestic goods, foreign goods and energy to construct exports. These are sold to a monopolistically competitive firm that determines the price of exports. Afterwards these differentiated exports are bundled and sold to foreign agents.

2.4.1 Investment decision

Firms use a composite investment good i_t with price $P_{i,t}$ to accumulate physical capital k_t, which is used in production in the next period. Just like the composite consumption good, the composite investment good is produced using energy $i_{e,t}$ and non-energy investment goods $i_{x,t}$, which in turn consists of domestic $i_{H,t}$ and foreign $i_{F,t}$ goods:

$$i_t \equiv \left[(1-\zeta_{i,e})^{\frac{1}{\eta_{i,e}}} i_{x,t}^{1-\frac{1}{\eta_{i,e}}} + \zeta_{i,e}^{\frac{1}{\eta_{i,e}}} i_{e,t}^{1-\frac{1}{\eta_{i,e}}} \right]^{\frac{\eta_{i,e}}{\eta_{i,e}-1}}, \tag{3}$$

$$i_{x,t} \equiv \left[(1-\zeta_i)^{\frac{1}{\eta_i}} i_{H,t}^{1-\frac{1}{\eta_i}} + \zeta_i^{\frac{1}{\eta_i}} i_{F,t}^{1-\frac{1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i-1}}, \tag{4}$$

where $\zeta_i \in (0,1)$ and $\zeta_{i,e} \in (0,1)$ are the shares of energy in total investment and imports in non-energy investment, and $\eta_i > 0$ and $\eta_{i,e} > 0$ are the relevant substitution elasticities. Investment expenditure is given by:

$$P_{i,t} i_t = P_{x,i,t} i_{x,t} + P_{e,t} i_{e,t},$$
$$P_{x,i,t} i_t = P_{H,t} i_{H,t} + P_{F,t} i_{F,t}.$$

The firm minimizes total investment expenditure subject to the relevant production functions. In equilibrium, this determines the allocation between domestic and foreign goods $i_{H,t}$ and $i_{F,t}$, the allocation between energy and non-energy goods $i_{e,t}$ and $i_{x,t}$, and the price of the relevant composite investment goods $P_{i,t}$ and $P_{x,i,t}$.

2.4.2 Value-added production

Perfectly competitive firms produce value-added v_t with price $P_{v,t}$ with a constant returns to scale CES production function using previously accumulated capital k_{t-1} and labor n_t:

$$v_t = z \left[(1-\alpha)^{\frac{1}{\sigma}} n_t^{1-\frac{1}{\sigma}} + \alpha^{\frac{1}{\sigma}} \left(\frac{k_{t-1}}{k_0} \right)^{1-\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}, \tag{5}$$

where α is the capital share in production, k_0 is a scaling parameter, σ is the elasticity of substitution between capital and labor, and z is a productivity scaling parameter. Labor is hired in a frictional market: value added producers post vacancies ν_t at cost $P_{v,t}\kappa$ to attract job seekers u_t^s. All unemployed workers look for jobs. Matches that are formed in period t start working in the same period. The number of matches $\mathcal{M}_t$ is defined by the following constant returns to scale matching function:

$$\mathcal{M}_t = \xi \left(u_t^s\right)^{\iota} \nu_t^{1-\iota}, \tag{6}$$

where ξ is a scaling parameter and ι is the elasticity of matches with respect to job seekers. The timing of labor market events is as follows. At the beginning of every period a share s of matches is terminated exogenously. Afterwards matches are formed and hiring takes place. The law of motion of employment n_t is given by:

$$n_t = (1-s)\, n_{t-1} + q_t \nu_t. \tag{7}$$

By normalizing the total population size to 1, the unemployment rate u_t is given by:

$$u_t = 1 - n_t,$$

while the amount of job seekers u_t^s is determined by agents who did not have a job in the previous period u_{t-1} or were separated from their match in the current period sn_{t-1}:

$$u_t^s = u_{t-1} + sn_t,$$

Firms accumulate physical capital by purchasing investment goods i_t. Firms select their capital stock after all hiring has taken place and that changing the level of investment relative to the previous period is subject to a quadratic adjustment cost. The law of motion of the capital stock is therefore given by:

$$k_t = (1-\delta)\, k_{t-1} + \left[1 - \frac{\kappa_i}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right] i_t, \tag{8}$$

where δ is the depreciation rate of physical capital and κ_i determines the strength of the quadratic adjustment costs. Labor is paid a nominal wage rate W_t. Firms maximize their value

$$V_t = \Pi_t^v + \mathbb{E}_t \left\{\beta \frac{\lambda_{d,t+1}}{\lambda_{d,t}} V_{t+1}\right\}, \tag{9}$$

with flow profits $\Pi_t^v = P_{v,t} v_t - W_t n_t - P_{i,t} i_t - P_{v,t} \kappa \nu_t$. Firms use the firm owner's stochastic discount factor $\beta \frac{\lambda_{d,t+1}}{\lambda_{d,t}}$ to discount future cash flows, as firm owners are the ultimate recipients of firm profits.. Firms maximize their value subject to the law of motion of capital and the law of motion of employment.

In a search and matching context, firms and workers have to bargain over wages. For simplicity, the nominal wage rate W_t is a linear function of the marginal product of labor and the worker's outside option (Jung and Kuester, 2011; Den Haan et al., 2021):

$$W_t = \gamma \left\{ \eta P_{v,t} \left[(1-\alpha) v_t / n_t \right]^{\frac{1}{\sigma}} + (1-\eta)(B_u + h) \right\} + (1-\gamma) W. \tag{10}$$

with bargaining weights η and $1-\eta$, where W is the nominal wage rate in the deterministic steady state, and h captures non-monetary aspects of the worker's outside option. This weight can be thought of as the worker's bargaining power.[9] We introduce additional wage rigidities by imposing that today's wage rate only adjusts by a factor γ, while it depends on the steady state wage rate with a factor $1-\gamma$.

2.4.3 Production of intermediate goods

Final goods are produced using a composite intermediate good m_t with price $P_{m,t}$. The composite intermediate good is produced using energy $m_{e,t}$ and non-energy intermediate goods $m_{x,t}$, which is produced using domestic $m_{H,t}$ and foreign $m_{F,t}$ goods:

$$m_t \equiv \left[(1-\zeta_{d,e})^{\frac{1}{\eta_{d,e}}} m_{x,t}^{1-\frac{1}{\eta_{d,e}}} + \zeta_{d,e}^{\frac{1}{\eta_{d,e}}} m_{e,t}^{1-\frac{1}{\eta_{d,e}}} \right]^{\frac{\eta_{d,e}}{\eta_{d,e}-1}}, \tag{11}$$

$$m_{x,t} \equiv \left[(1-\zeta_m)^{\frac{1}{\eta_m}} m_{H,t}^{1-\frac{1}{\eta_m}} + \zeta_m^{\frac{1}{\eta_m}} m_{F,t}^{1-\frac{1}{\eta_m}} \right]^{\frac{\eta_m}{\eta_m-1}}, \tag{12}$$

where the parameters have their usual meanings. Total expenditure on intermediate goods is given by:

$$P_{m,t} m_t = P_{x,m,t} m_{x,t} + P_{e,t} m_{e,t},$$
$$P_{x,m,t} m_{x,t} = P_{H,t} m_{H,t} + P_{F,t} m_{F,t}.$$

2.4.4 Production of domestic goods

The domestic good $P_{H,t}^f y_{H,t}$ is produced using domestic value-added $P_{v,t} v_t$ and an intermediate good m_t with price $P_{m,t}$ using a CES production function:

$$y_{H,t} \equiv \left[(1-\zeta_v)^{\frac{1}{\eta_v}} m_t^{1-\frac{1}{\eta_v}} + \zeta_v^{\frac{1}{\eta_v}} v_t^{1-\frac{1}{\eta_v}} \right]^{\frac{\eta_v}{\eta_v-1}}, \tag{13}$$

9 Nash bargaining is commonly used to determine wages in the search and matching literature. However, with heterogeneous agents Nash bargaining becomes less tractable because future utility flows matter and these are discounted using different stochastic discount factors. Said scheme can also be micro-founded as the outcome of an alternating offers game, see Hall and Milgrom (2008).

where the parameters have their usual meanings. Total expenditure on these two goods is given by:

$$P_{H,t}^{f} y_{H,t} = P_{v,t} v_t + P_{m,t} m_t.$$

The domestic good is sold to monopolistically competitive retailers, who differentiate these goods and set the price on these goods before the domestic good is sold to households, value added producing firms, the government, and intermediate goods producers.

2.4.5 Exporters

Suppose the economy supplies foreign countries with exports $\varkappa_t$ which come at price $P_{\varkappa,t}^{f}$, which are constructed using domestic goods $\varkappa_{H,t}$, foreign goods $\varkappa_{F,t}$, and energy $\varkappa_{e,t}$ with the following CES production functions:

$$\varkappa_t \equiv \left[(1-\zeta_{\varkappa,e})^{\frac{1}{\eta_{\varkappa,e}}} \varkappa_{x,t}^{1-\frac{1}{\eta_{\varkappa,e}}} + \zeta_{\varkappa,e}^{\frac{1}{\eta_{\varkappa,e}}} \varkappa_{e,t}^{1-\frac{1}{\eta_{\varkappa,e}}} \right]^{\frac{\eta_{\varkappa,e}}{\eta_{\varkappa,e}-1}}, \tag{14}$$

$$\varkappa_{x,t} \equiv \left[(1-\zeta_x)^{\frac{1}{\eta_x}} \varkappa_{H,t}^{1-\frac{1}{\eta_x}} + \zeta_x^{\frac{1}{\eta_x}} \varkappa_{F,t}^{1-\frac{1}{\eta_x}} \right]^{\frac{\eta_x}{\eta_x-1}}, \tag{15}$$

where the parameters have their usual meanings. Total expenditure on exports is given by:

$$P_{\varkappa,t}^{f} \varkappa_t = P_{x,\varkappa,t} \varkappa_{x,t} + P_{e,t} \varkappa_{e,t},$$
$$P_{x,\varkappa,t} \varkappa_{x,t} = P_{H,t} \varkappa_{H,t} + P_{F,t} \varkappa_{F,t}.$$

The export good is sold to monopolistically competitive retailers, who differentiate these goods and set their price before these goods are exported.

2.4.6 Nominal rigidities

We introduce monopolistic competition and nominal rigidities in the energy, imported goods, domestic goods, and exports sectors. For the sake of brevity, we only describe the optimization problem in the domestic goods sector. Consider a continuum of perfectly competitive final goods producers of measure one in the domestic goods sector. These final goods producers combine retail goods $y_{j,H,t}$, which they buy at price $P_{j,H,t}$ into a final domestic good $y_{H,t}$ which they sell for a nominal price $P_{H,t}$ according to the following production technology:

$$y_{H,t} = \left[\int_0^1 (y_{j,H,t})^{\frac{\epsilon_H - 1}{\epsilon_H}} dj \right]^{\frac{\epsilon_H}{\epsilon_H - 1}},$$

where ϵ_H is the elasticity of substitution between different kinds of retail goods. The demand function for retail goods from final goods producers is:

$$y_{j,H,t} = \left(\frac{P_{j,H,t}}{P_{H,t}} \right)^{-\epsilon_H} y_{H,t}. \tag{16}$$

Domestic retail goods producers j of measure one purchase homogenous output from perfectly competitive domestic goods producers at price $P^f_{H,t}$, which they subsequently convert into retail goods $y_{j,H,t}$. These are sold again to final goods producers for a price $P_{j,H,t}$. Retail goods firms in each sector operate in a monopolistically competitive environment, because they all produce a slightly different retail good and final goods producers cannot perfectly substitute between all different kinds of retail goods. Because retail goods firms operate in a monopolistically competitive environment they can charge a markup. This earns them nominal profits $\Pi^H_{j,t} = \left(P_{j,H,t} - P^f_{H,t} \right) y_{j,H,t}$. Firms face quadratic price adjustment costs a la Rotemberg (1982), such that prices adjust sluggishly.[10] A retail goods producer's optimization problem is therefore given by:

$$\max_{P_{j,H,t}} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{d,t+s}}{\lambda_{d,t}} \left[P_{j,H,t+s} y_{j,H,t+s} - P^f_{H,t+s} y_{j,H,t+s} - \frac{\kappa_H}{2} \left(\frac{P_{j,H,t+s}}{P_{j,H,t+s-1}} - 1 \right)^2 P_{H,t+s} y_{H,t+s} \right] \right\},$$

subject to the demand function for retail goods from final goods producers.

2.5 Fiscal authority

The fiscal authority issues bonds $B_{g,t}$ at price q^b_t that promise to pay out one euro in the next period. In addition, the fiscal authority levels (nominal) lump sum taxes T_t, a constant tax on firm owners T_r, and a constant distortionary consumption tax T_c. It consumes a composite good g_t with price $P_{g,t}$ and provides unemployment benefits B_u to unemployed workers $u_{w,t}$. Total government consumption is exogenous. The government budget constraint is therefore given by:

$$T_t + q^b_t B_{g,t} + T_r + T_c P_{c,t} c_t = P_{g,t} g_t + B_u u_{w,t} + B_{g,t-1}. \tag{17}$$

Total tax revenues T_t is the population weighted sum of taxes paid by firm owners $T_{d,t}$. The fiscal authority adjusts these taxes to ensure intertemporal solvency:

$$T_{d,t} - T_d = \kappa_T \left(B_{g,t-1} - B_g \right). \tag{18}$$

where T_d and B_g are steady state values.

[10] Following Eggertsson and Singh (2019), these adjustment costs are thought of as reputational costs instead of physical resource costs, such that they do not impact firms' final profits. With physical price adjustment costs, an increase in energy prices leads to a counterfactual increase in output. In that case, production needs to increase to satisfy the increased demand for goods necessary to facilitate the adjustment of prices.

Public consumption consists of $g_{H,t}$ domestic goods, $g_{F,t}$ foreign goods, and $g_{e,t}$ energy and is constructed using the following production technologies:

$$g_t \equiv \left[(1-\zeta_{g,e})^{\frac{1}{\eta_{g,e}}} g_{x,t}^{1-\frac{1}{\eta_{g,e}}} + \zeta_{g,e}^{\frac{1}{\eta_{g,e}}} g_{e,t}^{1-\frac{1}{\eta_{g,e}}} \right]^{\frac{\eta_{g,e}}{\eta_{g,e}-1}}, \tag{19}$$

$$g_{x,t} \equiv \left[(1-\zeta_g)^{\frac{1}{\eta_g}} g_{H,t}^{1-\frac{1}{\eta_g}} + \zeta_g^{\frac{1}{\eta_g}} g_{F,t}^{1-\frac{1}{\eta_g}} \right]^{\frac{\eta_g}{\eta_g-1}}, \tag{20}$$

where the parameters have their usual meanings. Total expenditure on public consumption is given by:

$$P_{g,t}g_t = P_{x,g,t}g_{x,t} + P_{e,t}g_{e,t},$$
$$P_{x,g,t}g_t = P_{H,t}g_{H,t} + P_{F,t}g_{F,t}.$$

2.6 Market clearing

Total demand for domestic goods, energy, and imported goods therefore given by:

$$P_{H,t}y_{H,t} = P_{H,t}c_{H,t} + P_{H,t}i_{H,t} + P_{H,t}g_{H,t} + P_{H,t}\varkappa_{H,t} + P_{H,t}m_{H,t},$$
$$P_{e,t}e_t = P_{e,t}c_{e,t} + P_{e,t}i_{e,t} + P_{e,t}g_{e,t} + P_{e,t}\varkappa_{e,t} + P_{e,t}m_{e,t} + P_{e,t}v_{e,t},$$
$$P_{F,t}^f y_{F,t} = P_{F,t}^f c_{F,t} + P_{F,t}^f i_{F,t} + P_{F,t}^f g_{F,t} + P_{F,t}^f x_{F,t} + P_{F,t}^f m_{F,t}.$$

Total imports consists of spending on private consumption goods, investment goods, public consumption goods, intermediate goods and energy. Let μ_t be the economy's total imports and let $P_{\mu,t}$ be the relevant price index for imports. Total imports are then given by:

$$P_{\mu,t}\mu_t = P_{F,t}^f y_{F,t} + P_{e,t}e_t,$$

where $P_{\mu,t}$ is a Laspeyres index for the price of imports from period $t-1$ to period t:

$$\frac{P_{\mu,t}}{P_{\mu,t-1}} = \frac{P_{F,t}^f y_{F,t-1} + P_{e,t}^f e_{t-1}}{P_{F,t-1}^f y_{F,t-1} + P_{e,t-1}^f e_{t-1}}.$$

Next, let $y_{D,t}$ be gross domestic final expenditures:

$$P_{D,t}y_{D,t} = P_{c,t}c_t + P_{i,t}i_t + P_{g,t}g_t,$$

where $P_{D,t}$ is a Laspeyres index for the price of domestic final expenditures from period $t-1$ to period t:

$$\frac{P_{D,t}}{P_{D,t-1}} = \frac{P_{c,t}c_{t-1} + P_{i,t}i_{t-1} + P_{g,t}g_{t-1}}{P_{c,t-1}c_{t-1} + P_{i,t-1}i_{t-1} + P_{g,t-1}g_{t-1}}.$$

The net foreign asset position evolves according to:

$$\left(q_t^b B_t^* - B_{t-1}^*\right) = P_{\varkappa,t}\varkappa_t - P_{\mu,t}\mu_t. \tag{21}$$

The inverse of the bond price $1/q_t^b$ corresponds to the risk free international interest rate. This variable is exogenously given. Finally, the terms of trade $\mathcal{T}_t$ is given by:

$$\mathcal{T}_t = \frac{P_{\varkappa,t}}{P_{\mu,t}}. \tag{22}$$

Foreign demand for the economy's exports is determined by the following demand function:

$$\varkappa_t = \left[\varkappa\left(\frac{P_{\varkappa,t}}{P_{\mu,t}^*}\right)^{-\eta_\varkappa^*}\right]^{\chi} \varkappa_{t-1}^{1-\chi}, \tag{23}$$

where $\eta_\varkappa^*$ is the price elasticity of foreign export demand and χ introduces inertia in the demand for exports. Other economies consume a similar import bundle as the domestic country with price $P_{\mu,t}^*$, such that a lower price of exports relative to imports increases foreign demand for exports. The relevant import price for other countries is given by:

$$\frac{P_{\mu,t}^*}{P_{\mu,t-1}^*} = \frac{P_{F,t}^f y^* + P_{e,t}^f e^*}{P_{F,t-1}^f y^* + P_{e,t-1}^f e^*},$$

where y^*, e^* are parameters. As is standard for small open economies, the prices of energy imports $P_{e,t}$ and non-energy imports $P_{F,t}$ are exogenous and do not respond to changes in domestic demand. The world interest rate responds to changes in the net foreign asset position relative to the steady state to guarantee stationarity of the net foreign asset position Schmitt-Grohé and Uribe (2003):

$$q_t^b = q^b + \kappa_q \left[\exp\left(B_t^* - B^*\right) - 1\right]. \tag{24}$$

2.7 Deflating macroeconomic income

How much an economy produces is measured by GDP, while its income, and thus how much it can consume, is measured by GDI. In nominal terms, GDI and GDP are identical. However, *real* GDI and GDP are typically not identical in open economies, as international trade allows an economy to consume a different set of goods and services than it produces. As a result, real GDP understates changes in real domestic income after changes in the terms of trade (Kohli, 2004). Since we want to differentiate between aggregate production and macroeconomic income, it is necessary to construct a measure of GDI in the model.

Essentially, by making a distinction between real GDP and real GDI, we have to take a stance on the appropriate price indices to deflate GDP and GDI. Typically, statistical agencies have some discretion in deciding what price index to use when calculating GDI.[11] We follow Reinsdorf (2010) and Kohli (2022) in using the price index of gross domestic final expenditures as the appropriate price index to deflate GDI. The intuition is that the relevant price index to deflate additional income that domestic agents gain by trading with other countries, is the price index for domestic absorption. Essentially, to obtain real income, one needs to deflate nominal income by the price index that that nominal income is used to purchase. In our model, the price index of domestic absorption is the price of gross domestic final expenditures $P_{D,t}$. The Paasche index for GDI, using the price index of total expenditures as a deflator, is then given by:

$$\text{gdi}_{P,t} = \frac{P_{D,t}y_{D,t} + P_{\varkappa,t}\varkappa_t - P_{\mu,t}\mu_t}{\left(P_{D,t-1}y_{D,t-1} + P_{\varkappa,t-1}\varkappa_{t-1} - P_{\mu,t-1}\mu_{t-1}\right)\mathcal{P}_{D,t}^{\text{Laspeyres}}}. \tag{25}$$

The Paasche index for GDP is given by:

$$\text{gdp}_{P,t} = \frac{P_{D,t}y_{D,t} + P_{\varkappa,t}\varkappa_t - P_{\mu,t}\mu_t}{P_{D,t-1}y_{D,t-1}\mathcal{P}_{D,t}^{\text{Laspeyres}} + P_{\varkappa,t-1}\varkappa_{t-1}\mathcal{P}_{\varkappa,t}^{\text{Laspeyres}} - P_{\mu,t-1}\mu_{t-1}\mathcal{P}_{\mu,t}^{\text{Laspeyres}}}, \tag{26}$$

where $\mathcal{P}_{z,t}^{\text{Laspeyres}}$ is the change in the Laspeyres price index of expenditure category $z \in D, \mu, \varkappa$ from period $t-1$ to period t. Additional derivations are provided in the Appendix.

3 Calibration

We calibrate the model to broadly match the Dutch economy, paying particular attention to the relevant shares of imports and energy, both in final expenditure categories and in their use as intermediate goods. One period in the model corresponds to a quarter. We set the subjective discount factor of both workers and firm owners $\beta = 0.9975$ to target an annual real interest rate of 1%. We set risk aversion σ_i, where $i = w, d$, to 2, which is a standard value in the literature. We set $\psi = 0.8$, such that the population consists of 80% workers and 20% firm owners.[12] Worker households' cost of adjusting their portfolio is given by $f\left(B_{w,t}\right) = \frac{\kappa_w}{2}\left(B_{w,t} - B_w\right)^2$, such that $f'\left(B_{w,t}\right) = \kappa_w\left(B_{w,t} - B_w\right)$. We set $\kappa_w = 0.01$. In addition, workers have zero liquid assets in the steady state. We normalize total outstanding firm equity shares $\hat{\varsigma}$ to unity. The tax adjustment parameter κ_T is set to 0.025, such that lump sum taxes adjust slowly to changes in government debt.

We set $B_u = 0.64$ to match a replacement rate of approximately 65% and then adjust h such that the cost of posting a vacancy is approximately equal to half the quarterly wage. We set $\gamma = 0.01$ to ensure nominal wages are almost completely rigid. The steady state unemployment rate u is 4%, the quarterly separation rate s is 3.5%, and the elasticity of matches w.r.t. vacancies ι is 0.5. The constant in the matching function ξ is adjusted to hit a quarterly job filling rate q of 35%. We set

[11] A common practice is to deflate the trade balance by an import price index when calculating GDI. Both Reinsdorf (2010) and Kohli (2022) argue against this practice.

[12] Luginbuhl and Smid (2021) report that the financial wealth of 80% of the Dutch population is less than €50,000. The workers in the model are meant to capture this particular group.

$\eta = 0.5$ such that the Hosios condition is satisfied. The elasticity of substitution between capital and labor σ is set to 0.5, while the share parameter α is set to 0.25. The investment adjustment cost parameter κ_i is set to unity, while the depreciation rate of physical capital δ is set to its standard value of 0.025. We normalize the marginal product of labor $P_{v,t}\left[(1-\alpha)\, v_t/n_t\right]^{\frac{1}{\sigma}}$ to unity in the steady state by adjusting the productivity scaling parameter z.

The elasticity of substitution between energy and non-energy in all expenditure bundles equal is set to 0.1, while the elasticity of substitution between home and foreign goods to is set to 1.5. We use input-output tables from Statistics Netherlands and Eurostat's FIGARO tables to calculate the import and energy shares in 2021.[13] Specifically, we calculate the shares of energy and imports used as intermediate goods, as private and public consumption goods, as investment goods, and as exports. These targeted shares are summarized in Table 2. We set the inertia in export demand $\chi = 1 - 0.75$ following Gertler et al. (2007) and the price elasticity of export demand $\eta^*_{\varkappa} = 2$. The parameters of the composite world import price bundle y^*, e^* are equal to the steady state values of imported domestic goods y_F and energy e, such that foreign countries effectively consume a similar basket of traded goods.

Table 2 Targeted expenditure shares

Targeted share	Value	Definition
c_F/c_x	0.1275	Imports in non-energy consumption
c_e/c	0.033	Share of energy in consumption
i_F/i_x	0.2115	Imports in non-energy investment
i_e/i	0.01316	Share of energy in investment
g_F/g_x	0.0077	Imports in non-energy government spending
g_e/g	0.00017	Share of energy in government spending
m_F/m_x	0.309	Imports in non-energy intermediates
m_e/m	0.064	Share of energy in intermediates
v/y_H	0.4709	Share of value added in domestic final goods
$\varkappa_F/\varkappa_x$	0.368	Imports in non-energy exports
$\varkappa_e/\varkappa$	0.041	Share of energy in exports

Steady state government debt is approximately 60% of annual GDP following the Maastricht Treaty, government spending and investment are approximately 20% of GDP, and consumption is around 50% of GDP. Total lump sum taxes T, distortionary consumption taxes T_c, the capital scaling parameter in production k_0, and the volume of exported home goods $\varkappa_H$ are adjusted to hit these targets.

The elasticity of substitution of retail goods in each expenditure category is set to 11, implying a 10% markup over marginal costs. Given observed quick adjustment in energy and export prices in the data, we impose that prices in those sectors are perfectly flexible.[14] Price adjustment costs in the domestic and foreign goods sectors are set such that price adjust on average once every three

[13] In Statistics Netherlands sector terms, we define both the imports and output of sectors B06, C19, and D35 as energy.

[14] Our qualitative results do not depend on the assumption that prices are perfectly flexible in the energy and exports sectors.

quarters. This yields $\kappa_j = 56.8177$, where $j = H, F$. We impose a corrective tax that ensures that markups are zero in the steady state: as a result all prices can be normalized to unity in the steady state, which greatly simplifies solving for the steady state. These tax revenues are transferred lump sum back to firms, such that they still earn positive profits in the steady state. Note that this tax does not affect the model's dynamic behavior.

All exogenous variables are driven by AR(1) processes with autocorrelation parameter $\rho_z = 0.95$ and standard deviation σ_z, where $z = F, e, g$. The price of risk free bonds q_t^b adjusts slowly with changes in holdings of the internationally traded asset to ensure a stationary net foreign asset position (Schmitt-Grohé and Uribe, 2003) by setting $\kappa_q = 0.0001$. This does not influence the model's short term dynamics, but does ensure that the model has a unique steady state. Given that we are interested in analyzing the effects of a large increase in energy prices, we solve the fully non-linear deterministic version of the model without aggregate risk. That is, after the arrival of the shock agents know that no further shocks will arrive.

4 Numerical results

In this section we present a number of numerical results. First, we analyze the macroeconomic and distributional impact of a temporary increase in energy prices of 100%. Second, we show how full, automatic indexation of wages to consumer price inflation after the same increase in energy prices affects macroeconomic prices and quantities, and consumption inequality between workers and firm owners. Finally, we will investigate both the short and long run trade-offs between stabilizing consumption inequality after an energy price shock and improving aggregate economic outcomes. Since ultimately households gain utility from consumption, we use the gap between consumption of firm owners and workers as the relevant inequality measure.

4.1 Effects of a temporary increase in energy prices

In Figure 1, the economy is initially in the deterministic steady state and then experiences a temporary increase in the world energy price $P_{e,t}^f$ of 100%. As prices in the domestic energy sector are perfectly flexible, this increase in the world energy price immediately translates into an equivalent increase in retail energy prices $P_{e,t}$. Since energy retailers operate under monopolistic competition, they charge a markup over marginal costs, thereby leading to higher profits in the energy sector. The increase in energy prices leads to an immediate increase in both the domestic and foreign import price indices $P_{\mu,t}$ and $P_{\mu,t}^*$. Recall that energy is also re-exported. As a result, the price of exports $P_{\varkappa,t}$ increases, but by less than the price of imports since the weight of energy is smaller in the export bundle than in the import bundle. As the increase in exports prices is smaller than the increase in import prices, the terms of trade $\mathcal{T}_t$ deteriorates. This smaller increase in the price of exports relative to imports increases demand for exports $\varkappa_t$ from abroad: the rest of the world is hit by the same increase in energy prices as the domestic economy, such that the increase in export prices is smaller than the price increase in the foreign import bundle $P^*_{\mu,t}$.[15] As a result, the volume of exports increases, leading to higher exporter profits. Higher demand for exports also increases foreign demand for domestic goods $y_{H,t}$, which in turn increases the demand for labor n_t, such that unemployment u_t falls. Higher foreign demand and a subsequent expansion in production leads to an immediate increase in the Paasche index for real GDP $gdp_{P,t}$.

Across the board, the increase in energy prices feeds into other prices, as energy e_t is used both as an intermediate good $m_{e,t}$ by production firms and in all final expenditure categories. As a result, the prices of all the final expenditure categories increase. Given this increase in prices, the Paasche index for real GDI $gdi_{P,t}$ falls: the large terms of trade shock essentially reduces real macroeconomic income. The intuition is that given a current consumption pattern of domestic goods, imports, and energy, the broad increase in prices reduces the economy's aggregate purchasing power. Recall that nominal wages W_t are virtually completely rigid (the blue, solid line in the bottom left panel). As the consumption price index $P_{c,t}$ increases in conjunction with other prices, consumer price inflation $\pi_{c,t} := \frac{P_{c,t}}{P_{c,t-1}}$ increases. The relevant price index for both households is the price index for the final consumption good, as both workers and firm owners gain utility from consumption c_t. As such, $w_t := W_t/P_{c,t}$ is the relevant real wage for workers. As nominal wages are rigid and consumption prices increase, real wages fall by more than 4% on impact (the blue circles in the bottom left panel). Even though the shock increases employment, real labor income falls, thereby leading to a decrease in worker consumption $c_{w,t}$ (the blue, solid line in the bottom center panel). At the same time, nominal profits Π_t increase, thereby leading to higher nominal capital income for firm owners. However, *real* capital income falls, as the consumption price index increases by more than nominal profits. The reduction in real capital income causes firm owner consumption $c_{d,t}$ to fall, albeit by less than worker consumption (the blue dots in the bottom center panel). Defining $\Gamma_t := \frac{c_{d,t}}{c_{w,t}}$ as the gap between consumption of firm owners and workers, we find that consumption inequality increases. The reduction in consumption of both households leads to a fall in aggregate consumption c_t.

So far we have shown that the model generates a positive response to real GDP and a negative response to real GDI after a shock that leads to a deterioration of the terms of trade. Our results so far show that although in the aggregate the economy is worse off, workers take a bigger hit than firm owners, leading to a larger fall in worker consumption than in firm owner consumption. This opens up possibilities to redistribute from one group to another. One possible redistribution mechanism is to allow for automatic wage indexation: workers are compensated for the increase in consumption prices by increasing nominal wages. We will investigate the effects of complete automatic wage indexation to consumption prices, a rather extreme scenario, in the next section.

[15] We assume that the domestic economy has zero weight in the rest of the world's consumption bundle, which is in line with the small open economy assumption.

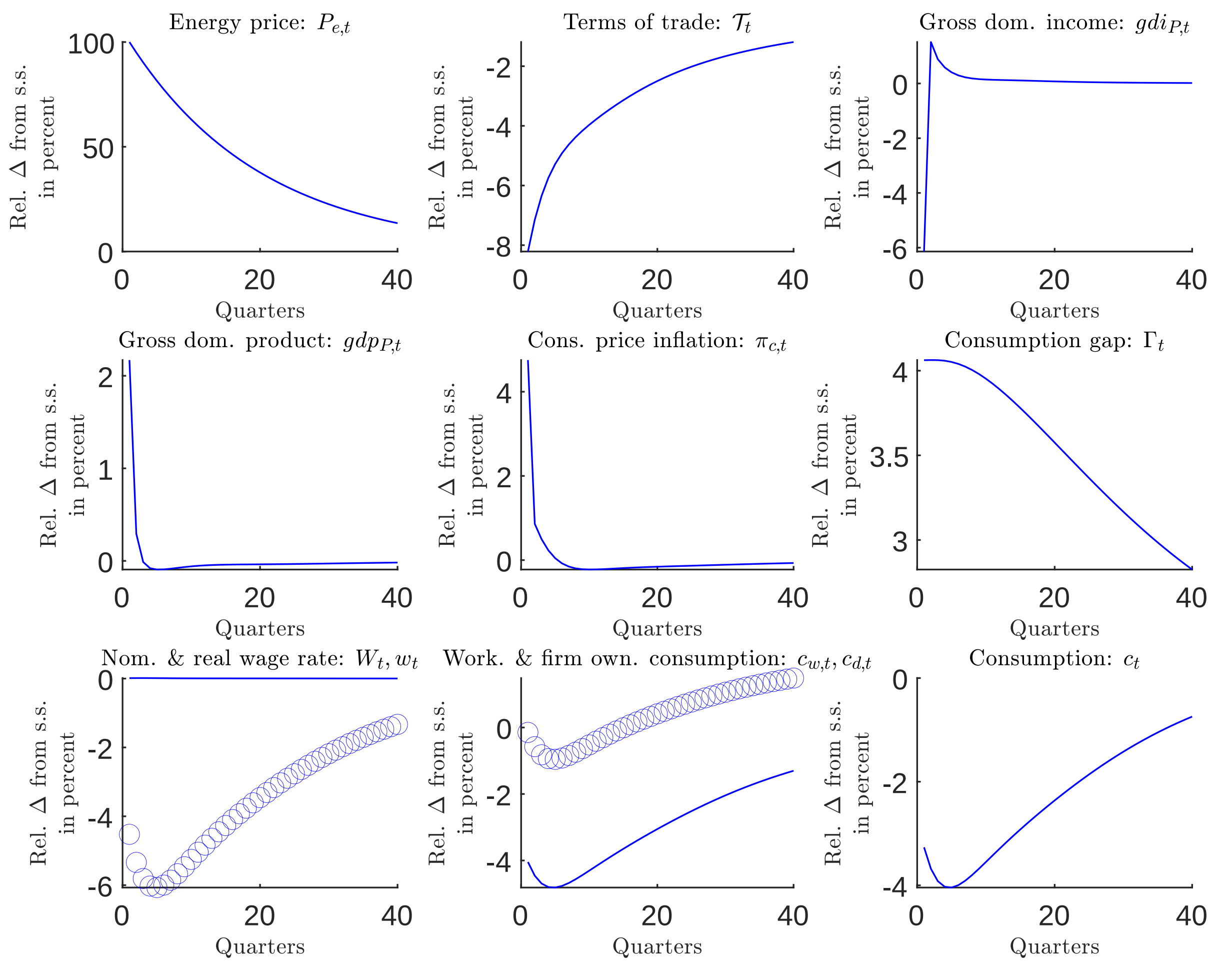

FIGURE 1. Impulse response functions after an increase in world energy prices of 100%. The dots correspond to the responses of the real wage rate $w_t := W_t/P_{c,t}$ and of firm owners' consumption $c_{d,t}$.

4.2 Automatic wage indexation after a large energy price shock

In this section we will analyze the economy's response to a large increase in energy prices that is accompanied by complete automatic wage indexation. To model automatic wage indexation, we allow nominal wages W_t to move with changes in the price index of consumption goods $P_{c,t}$ with a factor φ. As a result, the wage bargaining equation is augmented by the term $\varphi\left(P_{c,t} - P_c\right)$, where P_c is the steady state consumption price index. Hence, the bargained wage is now given by:

$$W_t = \gamma \left\{ \eta P_{v,t} \left[(1-\alpha) v_t / n_t \right]^{\frac{1}{\sigma}} + (1-\eta)(B_u + h) \right\} + (1-\gamma) W + \varphi \left(P_{c,t} - P_c \right).$$

In this section we set $\varphi = 1$, such that nominal wages move one-for-one with deviations of the consumption price index from its steady state level. Given that γ is small in the current calibration, full automatic wage indexation effectively renders the real wage rate constant. We compare a model simulation without (blue, solid) to a simulation with (red, dashed) automatic wage indexation in Figure 2. The core mechanisms at play in the simulation with automatic wage indexation remain the same as in the simulation without wage indexation. However, in contrast to the previous section, nominal wages now increase in tandem with the consumer price index. As a result, the real wage rate (red crosses in bottom left panel) are effectively constant. This increase in nominal wages increases labor costs of production firms, which translates into an increase in the price of domestic goods. The increase in domestic good prices caused by higher wages increases the price of exports, thereby leading to a smaller deterioration in the terms of trade relative to a simulation without automatic wage indexation and a smaller increase in foreign demand. The hike in prices leads to a further increase in wages, leading to yet another increase in prices. As a result, consumer price inflation increases by more than in the previous scenario.

Compared to a simulation without automatic wage indexation, real GDP now decreases instead of increases and real GDI falls by more. The reduction in real GDP and larger drop in real GDI are caused by a combination of higher prices and lower demand for exports relative to the previous simulation. In addition, higher labor costs reduce firms' expected future profits, making them reduce investment i_t and post less vacancies ν_t. As a result, unemployment increases over time. On the other hand, automatic wage indexation is effective at redistributing between workers and firm owners: worker consumption increases relative to the previous simulation (red, dashed line in bottom center panel), while the consumption of firm owners plummets (red crosses in bottom center panel). The relative increase in worker consumption is caused by a combination of (effectively) rigid real wages and slow adjustment of employment, as only a fraction s of workers are separated from their match every period. Since workers make up a larger share of the population, aggregate consumption increase relative to the simulation without automatic wage indexation. Furthermore, consumption inequality between firm owners and workers now falls.

Automatic wage indexation is effective at redistributing between workers and firm owners in the model simulations, at the cost of leading to a contraction in real GDP and a larger fall in real GDI. This raises the question whether we find this same trade-off between equity (defined as a

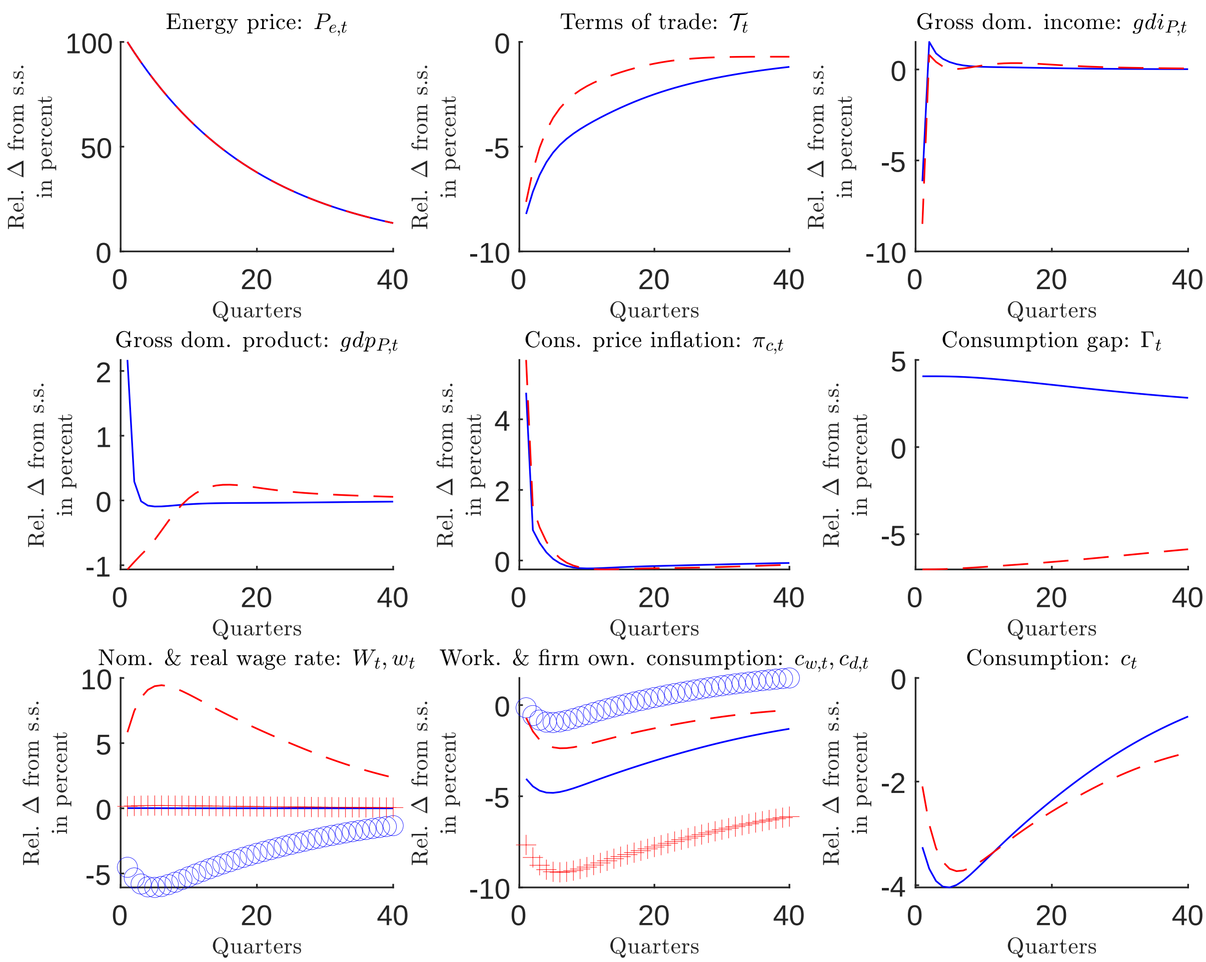

FIGURE 2. Impulse response functions after an increase in world energy prices of 100% without (blue, solid) and with (red, dashed) automatic wage indexation. The dots (crosses) correspond to the responses of the real wage rate $w_t := W_t/P_{c,t}$ and of firm owners' consumption $c_{d,t}$ without (with) automatic wage indexation.

consumption gap that favors workers) and efficiency for intermediate values of wage indexation. We construct a 101 point grid for φ that runs from 0 to 1 and calculate the model responses each time to a increase in energy prices of 100%. In Figure 3 we plot the change in consumption inequality relative to the deterministic steady in percent after one year, against the cumulative change in real GDP after one year for each value of φ on the grid. Each point on the graph corresponds to the response of real GDP and consumption inequality for a given φ. The top right point of the line corresponds to $\varphi = 0$, i.e. zero wage indexation, while the bottom left point corresponds to $\varphi = 1$, i.e. full wage indexation. Figure 3 shows that higher degrees of automatic wage indexation indeed reduce consumption inequality, at the cost of negative cumulative real GDP growth. As such, there is a trade-off between reducing consumption inequality between workers and firm owners on the one hand, and stimulating economic growth after a large terms of trade shock on the other hand.

Our results indicate that complete automatic wage indexation shifts the pain of adjustment after a large energy price shock from workers to firm owners, but stifles the subsequent macroeconomic recovery. This result holds for every non-zero value of wage indexation, albeit to a lesser extent. However, we have only looked at the short run trade-off in Figure 3. In the next section we will investigate whether wage indexation has similar effects in the long run.

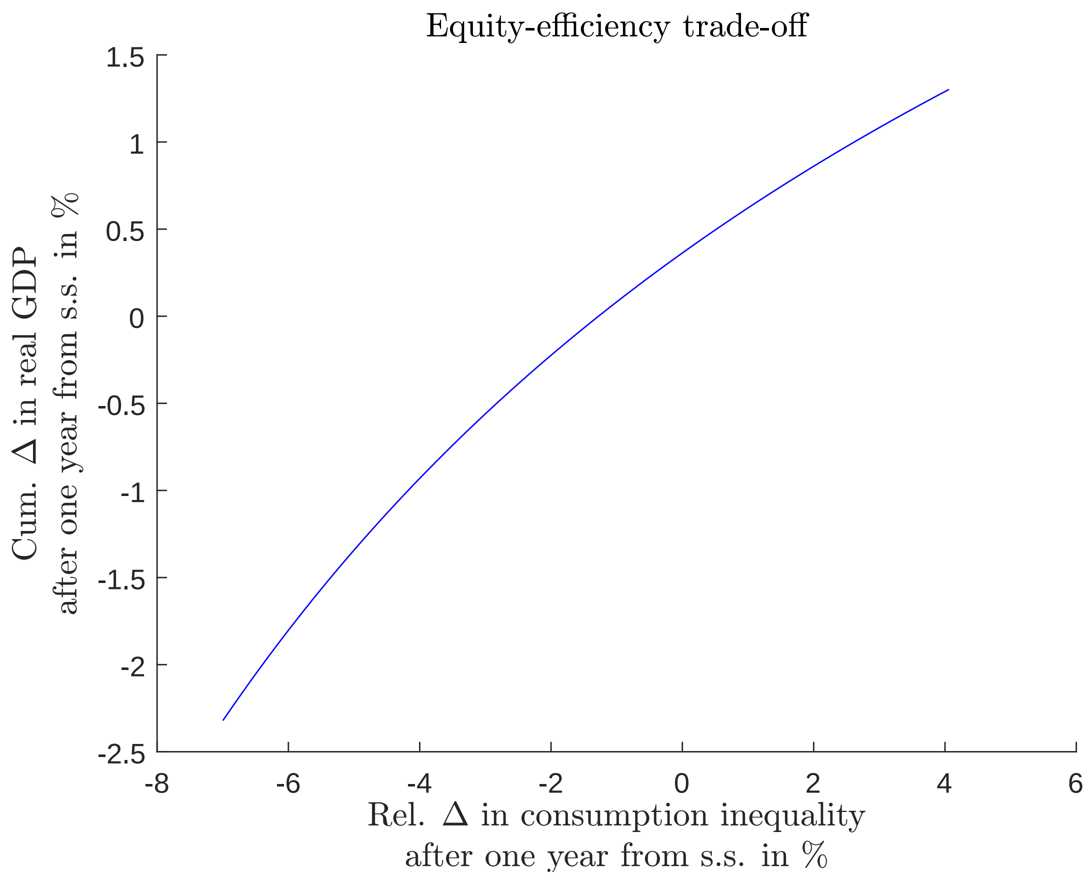

FIGURE 3. Response of consumption inequality and cumulative change in real GDP relative to the steady state for $\varphi = 0, 0.01, 0.02, ..., 1$ after one year.

4.3 Long run effects of (partial) automatic wage indexation

So far we have focused on the short run effects of energy price shocks and a wage indexation scheme that redistributes losses between workers and firm owners. In this section we will focus on the long run. To do so, we conduct a simple welfare analysis. Using the grid for the degree of wage indexation φ, we again run 101 model simulations where energy prices increase by 100%. Next, we calculate the sum of expected, discounted utility $\mathcal{U}_t$ over T periods for both worker and firm owner households, weighted by their respective population shares for each of these simulations:

$$\mathcal{U}_t = \psi \mathbb{E}_t \left\{ \sum_{s=0}^{T} \beta^s \frac{c_{w,t+s}^{1-\sigma_w} - 1}{1-\sigma_w} \right\} + (1-\psi)\, \mathbb{E}_t \left\{ \sum_{s=0}^{T} \beta^s \frac{c_{d,t+s}^{1-\sigma_d} - 1}{1-\sigma_d} \right\}.$$

We compare $\mathcal{U}_t$ with the same stream of discounted utility households would have gained if they would have stayed in the deterministic steady state for T periods. We do so for $T = 60$ (15 years) and $T = 200$ (50 years). We compare the deviation of $\mathcal{U}_t$ from its steady state value with the deviation in consumption inequality in period T relative to the steady state. As in Figure 3, the far right side of the curves corresponds to zero wage indexation, while the far left sides of the curves corresponds to full wage indexation.

In Figure 4 we plot the relative change in welfare compared to the change in consumption inequality after 15 years. First, notice that no matter the degree of wage indexation, the increase in energy prices and the resulting deterioration of the terms of trade leads to substantial aggregate utility losses. Next, consider the two extremes of no indexation and complete indexation. With zero indexation, consumption inequality is 3% higher than in the deterministic steady state, while the sum of discounted utility is approximately 5.4% lower. With complete wage indexation, consumption inequality is around 5% lower, while the sum of discounted utility is approximately 6.7% lower. For low values of wage indexation it is possible to both mitigate the increase in consumption inequality and slightly decrease the welfare losses that arise from the shock. However, the reduction in welfare losses for low values of φ compared to the simulation with zero indexation is small.

In Figure 5 we conduct the same analysis over a longer horizon by looking at the relative change in welfare and the change in consumption inequality after 50 years. First, notice that welfare losses from the shock have reduced compared to the shorter time horizon, as the economy has mostly recovered from the impact of the shock. However, the relative welfare losses of automatic wage indexation have increased relative to the shorter horizon. Recall from Figure 4 that welfare losses arising from complete wage indexation relative to no indexation was around 1.3 percentage points after 15 years. After 50 years, the additional welfare loss caused by complete wage indexation is around 3 percentage points. In this case, there is an almost linear relationship between higher degrees of wage indexation, additional welfare losses, and reduction in consumption inequality.

We can conclude that in the long run, the economy as a whole is worse off as a result of the shock, irrespective of the degree of wage indexation. This contrasts with the short run, as the increase in energy prices leads to short-lived gains in real GDP growth. Positive degrees of wage indexation are effective at redistributing losses arising from the shock, but make the economy as a whole worse off in the end.

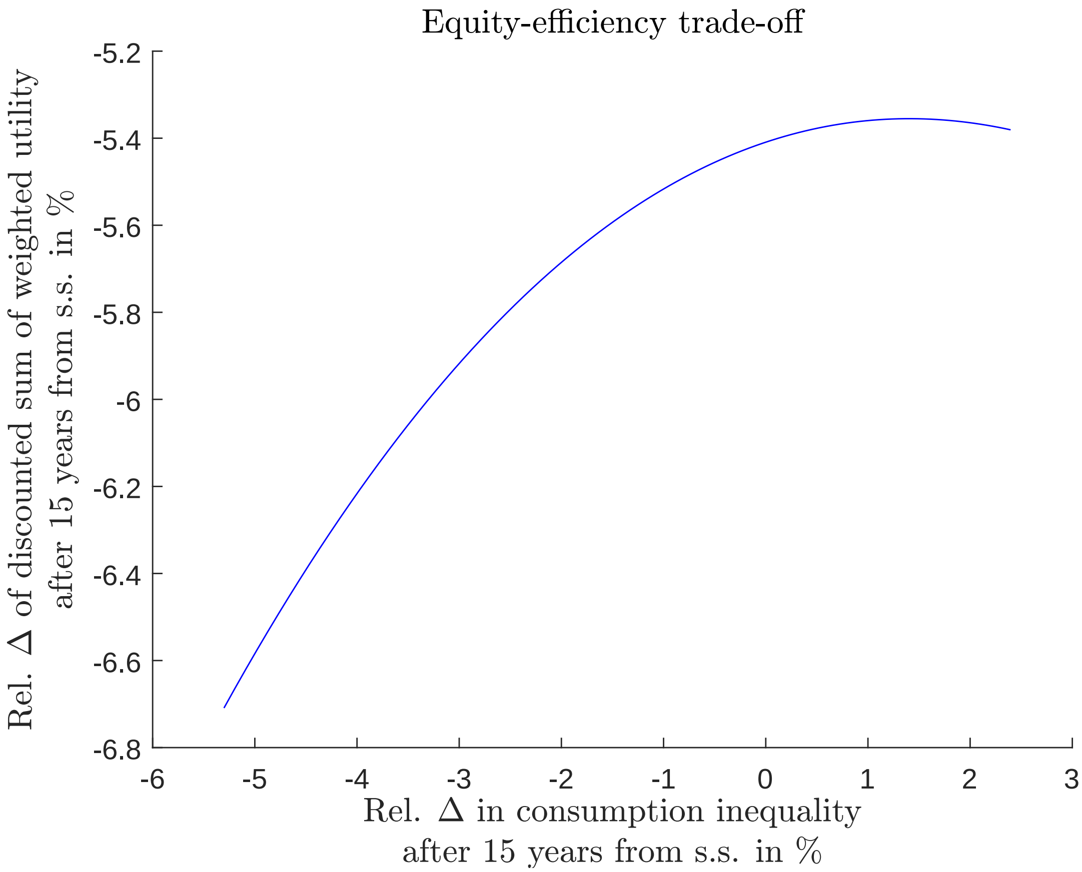

FIGURE 4. Response of consumption inequality and the relative change in the discounted sum of utility relative to the steady state for $\varphi = 0, 0.01, 0.02, ..., 1$ after 15 years.

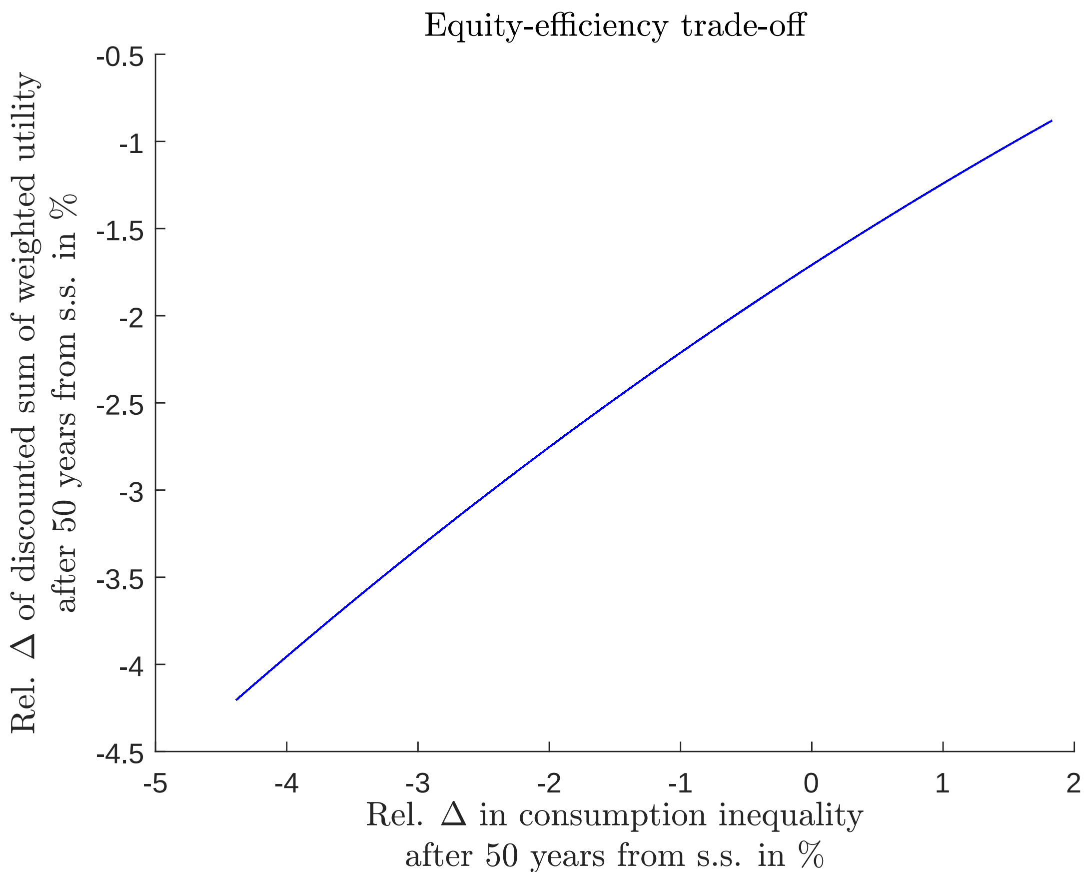

FIGURE 5. Response of consumption inequality and the relative change in the discounted sum of utility relative to the steady state for $\varphi = 0, 0.01, 0.02, ..., 1$ after 50 years.

5 Conclusion

We construct a two-agent New Keynesian (TANK) model of a small open economy in a currency union with energy consumption to analyze the effects of an energy price shock on real GDP, real GDI, and the distribution of income. We find that a large increase in energy prices leads to a deterioration of the terms of trade, which increases real GDP by stimulating exports. At the same time, the deterioration of the terms of trade leads to a fall in real GDI and a decrease in aggregate consumption. Rigid nominal wages and less rigid prices imply that households that primarily receive income from labor suffer a larger setback in real income and consumption than households that primarily receive capital income. As a result, the brunt of the downward adjustment of aggregate consumption is caused by lower real income of worker households. Complete automatic wage indexation is effective at reducing consumption inequality after such a shock, but comes at both short and long run economic costs by reducing production and income, increasing unemployment, and reducing real aggregate consumption.

The model features a binary distinction between households that primarily earn income from labor and households that receive capital income. In reality, many households receive income both from labor and from capital by indirectly holding firm shares, e.g. through an investment or pension fund. At the same time, foreign investors hold shares in Dutch firms, while Dutch investors also hold shares in foreign firms. Accurately capturing these features of the data would require a full blown HANK model and a more careful examination of cross-border firm equity holdings. Such an extension is beyond the scope of this work. Our focus has been to sketch the mechanisms at play: a rigorous quantitative analysis of the effects of an energy price shock would like require a full blown estimation exercise. However, we expect most of our qualitative conclusions to hold in that case. Finally, we have assumed that all energy is imported, while the Netherlands also produces energy. A substantial part of the profits of domestic energy production accrues to the Dutch government, thereby creating fiscal space to potentially compensate some households for the loss in real income. A HANK model that captures all the relevant details of the empirical distribution of income and wealth could be used to investigate the effects of highly targeted fiscal transfers relative to automatic wage indexation. Such an exercise is also beyond the scope of this work.

6 References

Andolfatto, D. (1996). Business cycles and labor-market search. *American Economic Review*, 86(1):112–132.

Bilbiie, F. O. (2008). Limited asset markets participation, monetary policy and (inverted) aggregate demand logic. *Journal of Economic Theory*, 140(1):162–196.

Bilbiie, F. O. (2020). The new keynesian cross. *Journal of Monetary Economics*, 114:90–108.

Bilbiie, F. O., Känzig, D. R., and Surico, P. (2022). Capital and income inequality: An aggregate-demand complementarity. *Journal of Monetary Economics*, 126:154–169.

Blanchard, O. J. (1986). The wage price spiral. *The Quarterly Journal of Economics*, 101(3):543–565.

Cantore, C. and Freund, L. B. (2021). Workers, capitalists, and the government: fiscal policy and income (re) distribution. *Journal of Monetary Economics*, 119:58–74.

Den Haan, W. J., Freund, L. B., and Rendahl, P. (2021). Volatile hiring: uncertainty in search and matching models. *Journal of Monetary Economics*, 123:1–18.

Eggertsson, G. B. and Singh, S. R. (2019). Log-linear approximation versus an exact solution at the zlb in the new keynesian model. *Journal of Economic Dynamics and Control*, 105:21–43.

Galí, J., López-Salido, J. D., and Vallés, J. (2007). Understanding the effects of government spending on consumption. *Journal of the European Economic Association*, 5(1):227–270.

Gertler, M., Gilchrist, S., and Natalucci, F. M. (2007). External constraints on monetary policy and the financial accelerator. *Journal of Money, Credit and Banking*, 39(2-3):295–330.

Hall, R. E. and Milgrom, P. R. (2008). The limited influence of unemployment on the wage bargain. *American Economic Review*, 98(4):1653–74.

Jung, P. and Kuester, K. (2011). The (un) importance of unemployment fluctuations for the welfare cost of business cycles. *Journal of Economic Dynamics and Control*, 35(10):1744–1768.

Kohli, U. (2004). Real gdp, real domestic income, and terms-of-trade changes. *Journal of International Economics*, 62(1):83–106.

Kohli, U. (2022). Trading gains: Terms-of-trade and real-exchange-rate effects. *Review of Income and Wealth*.

Lorenzoni, G. and Werning, I. (2023). Wage price spirals.

Luginbuhl, R. and Smid, B. (2021). De verscheidenheid van vermogens van nederlandse huishoudens: update. Technical report, CPB Achtergronddocument.

Merz, M. (1995). Search in the labor market and the real business cycle. *Journal of Monetary Economics*, 36(2):269–300.

Mortensen, D. T. and Pissarides, C. A. (1994). Job creation and job destruction in the theory of unemployment. *The Review of Economic Studies*, 61(3):397–415.

Pieroni, V. (2022). Energy shortages and aggregate demand: Output loss and unequal burden from hank. *European Economic Review*.

Reinsdorf, M. B. (2010). Terms of trade effects: Theory and measurement. *Review of Income and Wealth*, 56:S177–S205.

Rotemberg, J. J. (1982). Monopolistic price adjustment and aggregate output. *The Review of Economic Studies*, 49(4):517–531.

Schmitt-Grohé, S. and Uribe, M. (2003). Closing small open economy models. *Journal of International Economics*, 61(1):163–185.

Schmitt-Grohé, S. and Uribe, M. (2018). How important are terms-of-trade shocks? *International Economic Review*, 59(1):85–111.

A Mathematical derivations

A.1 Households

A.1.1 Workers

Workers receive income from supplying labor $W_t n_{w,t}$ to firms and repayment of a portfolio of risk free liquid assets $B_{w,t-1}$ bought in the previous period. This portfolio consists of both short term government bonds and an internationally traded asset. A share $u_{w,t}$ of the worker household is unemployed and receive unemployment benefits B_u. We assume that there is perfect consumption insurance among all members of the worker household (Merz, 1995; Andolfatto, 1996). They spend their income on consumption goods $(1+T_c) P_c c_{w,t}$ and purchases of the risk free asset $q_t^b B_{w,t}$ which comes at price q_t^b and promises to pay out one euro in the next period. To capture limited asset market participation, we assume that purchasing an additional bond is subject to quadratic adjustment costs $f(B_{w,t})$. They maximize their lifetime utility, which is increasing in consumption:

$$\max_{\{B_{w,t+s}, c_{w,t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \frac{c_{w,t+s}^{1-\sigma_w} - 1}{1-\sigma_w} \right\},$$
$$\beta \in (0,1), \sigma_w \geq 0,$$

where $\mathbb{E}_t$ is an expectations operator conditional on the agent's information set, β is the worker's subjective discount factor, and σ_w is risk aversion. Workers face no disutility of supplying labor or searching for a job. The worker household's nominal budget constraint is given by:

$$(1+T_c) P_{c,t} c_{w,t} + q_t^b B_{w,t} + f(B_{w,t}) = W_t n_{w,t} + B_{w,t-1} + B_u u_{w,t}.$$

The worker household chooses optimal consumption and bond holdings This is a standard optimization problem. The optimality conditions for the worker household are given by:

$$B_{w,t} : q_t^b + f'(B_{w,t}) = \mathbb{E}_t \left\{ \beta \frac{\lambda_{w,t+1}}{\lambda_{w,t}} \right\}, \tag{A.1}$$

$$c_{w,t} : (1+T_c) \lambda_{w,t} = u'(c_{w,t}) / P_{c,t}. \tag{A.2}$$

where superscript $'$ denotes a derivative, and $\lambda_{w,t}$ is the Lagrange multiplier on the worker's budget constraint.

A.1.2 Firm owners

firm owners d receive income from repayment of their portfolio of risk free liquid assets $B_{d,t-1}$ bought in the previous period and from holding shares $\varsigma_{d,t-1}$ in the economy's production firms. Shares trade at price q_t^{ς} and are a claim on firm profits Π_t. They spend their income on consumption goods $(1+T_c) P_c c_{d,t}$, lump sum taxes $T_{d,t}$ and purchases of the risk free asset $q_t^b B_{d,t}$ which comes at price q_t^b and promises to pay out one euro in the next period. In addition, firm owners have to pay a constant tax T_r which we use to ensure firm owners and workers have the same consumption in the deterministic steady state.[16] firm owners maximize lifetime utility, which is increasing in consumption:

$$\max_{\{B_{d,t+s},c_{d,t+s},\varsigma_{d,t+s}\}_{s=0}^{\infty}} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \frac{c_{d,t+s}^{1-\sigma_d}-1}{1-\sigma_d} \right\},$$
$$\beta \in (0,1), \sigma_d \geq 0.$$

The firm owner household's nominal budget constraint is given by:

$$(1+T_c) P_c c_{d,t} + T_{d,t} + T_r + q_t^b B_{d,t} + q_t^\varsigma \varsigma_{d,t} = (q_t^\varsigma + \Pi_t) \varsigma_{d,t-1} + B_{w,t-1}.$$

The firm owners household chooses optimal consumption, bond holdings, and firm shares. This is a standard optimization problem. The optimality conditions for firm owners are given by:

$$B_{d,t} : q_t^b = \mathbb{E}_t \left\{ \beta \frac{\lambda_{d,t+1}}{\lambda_{d,t}} \right\}, \tag{A.3}$$

$$c_{d,t} : (1+T_c) \lambda_{d,t} = u'(c_{d,t}) / P_{c,t}, \tag{A.4}$$

$$\varsigma_{d,t} : q_t^\varsigma = \mathbb{E}_t \left\{ \beta \frac{\lambda_{d,t+1}}{\lambda_{d,t}} (q_{t+1}^\varsigma + \Pi_{t+1}) \right\}, \tag{A.5}$$

where $\lambda_{d,t}$ is the Lagrange multiplier on the firm owners's budget constraint.

A.1.3 Consumption allocation

Households consume a composite consumption good c_t that comes at price $P_{c,t}$. This consumption good consists of energy consumption $c_{e,t}$ and non-energy consumption $c_{x,t}$. Non-energy consumption is a composite good that consists of both domestically produced goods $c_{H,t}$ with price $P_{H,t}$ and imported goods $c_{F,t}$ with price $P_{F,t}$. The composite goods are constructed using the following CES production functions:

$$c_t \equiv \left[(1-\zeta_{c,e})^{\frac{1}{\eta_{c,e}}} c_{x,t}^{1-\frac{1}{\eta_{c,e}}} + \zeta_{c,e}^{\frac{1}{\eta_{c,e}}} c_{e,t}^{1-\frac{1}{\eta_{c,e}}} \right]^{\frac{\eta_{c,e}}{\eta_{c,e}-1}}, \tag{A.6}$$

$$c_{x,t} \equiv \left[(1-\zeta_c)^{\frac{1}{\eta_c}} c_{H,t}^{1-\frac{1}{\eta_c}} + \zeta_c^{\frac{1}{\eta_c}} c_{F,t}^{1-\frac{1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c-1}}, \tag{A.7}$$

where $\zeta_{c,e} \in (0,1)$ is the share parameter of energy in total consumption, $\zeta_c \in (0,1)$ is the share parameter of the imported good in non-energy consumption, $\eta_{c,e} > 0$ is the elasticity of substitution

[16] We introduce this tax mostly to ease solving for the steady state. This is without much loss of generality given that the model is likely to be fairly linear, such that the model's initial conditions don't matter for how it responds to shocks.

between energy and non-energy consumption, and $\eta_c > 0$ is the elasticity of substitution between domestic and foreign goods in non-energy consumption. Total consumption expenditure is given by:

$$P_{c,t}c_t = P_{e,t}c_{e,t} + P_{x,c,t}c_{x,t},$$
$$P_{x,c,t}c_{x,t} = P_{F,t}c_{F,t} + P_{H,t}c_{H,t}.$$

The household minimizes total consumption expenditure subject to the CES production function. In equilibrium, this determines the allocation between energy and non-energy consumption $c_{e,t}$ and $c_{x,t}$, domestic and foreign goods $c_{H,t}$ and $c_{F,t}$, and the price of the composite goods $P_{c,t}$ and $P_{x,c,t}$. This is a relatively standard problem, and since every agent in the model faces a similar problem we will show how to derive the optimality conditions just once. Every one of these optimization problems can be solved using the method we will describe shortly.

The household's optimization problem is to minimize expenditures on domestic and foreign goods for a given level of non-energy consumption, and is given by:

$$\min_{\{c_{H,t},c_{F,t}\}} \quad P_{H,t}c_{H,t} + P_{F,t}c_{F,t}$$
$$\text{s.t.}$$
$$c_{x,t} \equiv \left[(1-\zeta_c)^{\frac{1}{\eta_c}} c_{H,t}^{1-\frac{1}{\eta_c}} + \zeta_c^{\frac{1}{\eta_c}} c_{F,t}^{1-\frac{1}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c-1}}.$$

The Lagrangian is then:

$$\mathcal{L} = P_{H,t}c_{H,t} + P_{F,t}c_{F,t} + \lambda_{c,t}\left(c_{x,t} - \left[(1-\zeta_c)^{\frac{1}{\eta_c}} c_{H,t}^{1-\frac{1}{\eta_c}} + \zeta_c^{\frac{1}{\eta_c}} c_{F,t}^{1-\frac{1}{\eta_c}}\right]^{\frac{\eta_c}{\eta_c-1}}\right),$$

where $\lambda_{c,t}$ is a Lagrange multiplier. The first order conditions for this problem are:

$$c_{H,t} : P_{H,t} = \lambda_{c,t}\left[(1-\zeta_c)^{\frac{1}{\eta_c}} c_{H,t}^{1-\frac{1}{\eta_c}} + \zeta_c^{\frac{1}{\eta_c}} c_{F,t}^{1-\frac{1}{\eta_c}}\right]^{\frac{1}{\eta_c-1}} (1-\zeta_c)^{\frac{1}{\eta_c}} c_{H,t}^{-\frac{1}{\eta_c}},$$
$$c_{F,t} : P_{F,t} = \lambda_{c,t}\left[(1-\zeta_c)^{\frac{1}{\eta_c}} c_{H,t}^{1-\frac{1}{\eta_c}} + \zeta_c^{\frac{1}{\eta_c}} c_{F,t}^{1-\frac{1}{\eta_c}}\right]^{\frac{1}{\eta_c-1}} \zeta_c^{\frac{1}{\eta_c}} c_{F,t}^{-\frac{1}{\eta_c}}.$$

Rewriting these first order conditions to get that:

$$P_{H,t} = \lambda_{c,t}c_{x,t}^{\frac{1}{\eta_c}}(1-\zeta_c)^{\frac{1}{\eta_c}} c_{H,t}^{-\frac{1}{\eta_c}} \Rightarrow c_{H,t} = (1-\zeta_c)\left(\frac{P_{H,t}}{\lambda_{c,t}}\right)^{-\eta_c} c_{x,t},$$
$$P_{F,t} = \lambda_{c,t}c_{x,t}^{\frac{1}{\eta_c}}\zeta_c^{\frac{1}{\eta_c}} c_{F,t}^{-\frac{1}{\eta_c}} \Rightarrow c_{F,t} = \zeta_c\left(\frac{P_{F,t}}{\lambda_{c,t}}\right)^{-\eta_c} c_{x,t}.$$

Dividing the first order condition for domestic goods by the first order condition for foreign goods yields the following optimality condition for the allocation between both kinds of goods:

$$\frac{c_{H,t}}{c_{F,t}} = \frac{1-\zeta_c}{\zeta_c} \left(\frac{P_{H,t}}{P_{F,t}} \right)^{-\eta_c} . \tag{A.8}$$

To find the corresponding index for the non-energy consumption price $P_{x,c,t}$, substitute the first order conditions for domestic and foreign goods back into the CES aggregator for total consumption:

$$c_{x,t} = \left[(1-\zeta_c) \left(\frac{P_{H,t}}{\lambda_{c,t}} \right)^{1-\eta_c} + \zeta_c \left(\frac{P_{F,t}}{\lambda_{c,t}} \right)^{1-\eta_c} \right]^{\frac{\eta_c}{\eta_c - 1}} c_{x,t}.$$

Then dividing by total consumption and solving for the Lagrange multiplier $\lambda_{c,t}$, we find the following solution for the price of non-energy consumption:

$$P_{x,c,t} = \left[(1-\zeta_c) P_{H,t}^{1-\eta_c} + \zeta_c P_{F,t}^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}} . \tag{A.9}$$

The optimal allocation between energy and non-energy consumption and the price of the consumption good can be found in a similar manner:

$$\frac{c_{x,t}}{c_{e,t}} = \frac{1-\zeta_{c,e}}{\zeta_{c,e}} \left(\frac{P_{x,c,t}}{P_{e,t}} \right)^{-\eta_{c,e}} , \tag{A.10}$$

$$P_{c,t} = \left[(1-\zeta_{c,e}) P_{x,t}^{1-\eta_{c,e}} + \zeta_{c,e} P_{e,t}^{1-\eta_{c,e}} \right]^{\frac{1}{1-\eta_{c,e}}} . \tag{A.11}$$

A.2 Production and pricing

Production and pricing occur in the following manner:

- Monopolistically competitive retail goods importers purchase foreign goods from abroad. They slightly differentiate these goods, determine the price on these goods and then sell them to import sellers. Foreign goods are used for final consumption and investment purchases, combined with domestic goods to produce intermediate goods, and combined with domestic goods to produce exports.

- Monopolistically competitive energy importers purchase energy from abroad. They slightly differentiate these goods, determine the price on these goods and then sell them to energy sellers. Energy is used in all expenditure categories and to produce value added, intermediate goods and exports.

- Production firms use capital and labor to produce value added excluding energy. Afterwards value added is sold to another firm that combine energy with value added. Labor is hired in a frictional labor market, as production firms need to post vacancies to attract workers and subsequently bargain over wages.

- Value added is combined with intermediate goods to produce the domestic good, which is again sold to a monopolistically competitive firm that sets its price who sell the differentiated goods to final sellers. Domestic goods are used for final consumption and investment purchases, combined with foreign goods to produce intermediate goods, and combined with foreign goods to produce exports.

- Export producers purchase domestic goods, foreign goods and energy to construct exports. These are sold to a monopolistically competitive firm that determines the price of exports. Afterwards these differentiated exports are bundled and sold to foreign agents.

A.2.1 Investment decision

Firms use a composite investment good i_t with price $P_{i,t}$ to accumulate physical capital k_t, which is used in production in the next period. Just like the composite consumption good, the composite investment good is produced using energy $i_{e,t}$ and non-energy investment goods $i_{x,t}$, which in turn consists of domestic $i_{H,t}$ and foreign $i_{F,t}$ goods:

$$i_t \equiv \left[(1-\zeta_{i,e})^{\frac{1}{\eta_{i,e}}} i_{x,t}^{1-\frac{1}{\eta_{i,e}}} + \zeta_{i,e}^{\frac{1}{\eta_{i,e}}} i_{e,t}^{1-\frac{1}{\eta_{i,e}}} \right]^{\frac{\eta_{i,e}}{\eta_{i,e}-1}}, \tag{A.12}$$

$$i_{x,t} \equiv \left[(1-\zeta_i)^{\frac{1}{\eta_i}} i_{H,t}^{1-\frac{1}{\eta_i}} + \zeta_i^{\frac{1}{\eta_i}} i_{F,t}^{1-\frac{1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i-1}}, \tag{A.13}$$

where $\zeta_i \in (0,1)$ and $\zeta_{i,e} \in (0,1)$ are the shares of energy in total investment and imports in non-energy investment, and $\eta_i > 0$ and $\eta_{i,e} > 0$ are the relevant substitution elasticities. Investment expenditure is given by:

$$P_{i,t} i_t = P_{x,i,t} i_{x,t} + P_{e,t} i_{e,t},$$
$$P_{x,i,t} i_t = P_{H,t} i_{H,t} + P_{F,t} i_{F,t}.$$

The firm minimizes total investment expenditure subject to the relevant production functions. In equilibrium, this determines the allocation between domestic and foreign goods $i_{H,t}$ and $i_{F,t}$, the allocation between energy and non-energy goods $i_{e,t}$ and $i_{x,t}$, and the price of the relevant composite investment goods $P_{i,t}$ and $P_{x,i,t}$. Going through the motions yields the following first order conditions:

$$\frac{i_{H,t}}{i_{F,t}} = \frac{1-\zeta_i}{\zeta_i}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta_i}, \tag{A.14}$$

$$P_{x,i,t} = \left[(1-\zeta_i)\, P_{H,t}^{1-\eta_i} + \zeta_i P_{F,t}^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}}, \tag{A.15}$$

$$\frac{i_{x,t}}{i_{e,t}} = \frac{1-\zeta_{i,e}}{\zeta_{i,e}}\left(\frac{P_{x,i,t}}{P_{e,t}}\right)^{-\eta_{i,e}}, \tag{A.16}$$

$$P_{i,t} = \left[(1-\zeta_{i,e})\, P_{x,i,t}^{1-\eta_{i,e}} + \zeta_{i,e} P_{e,t}^{1-\eta_{i,e}}\right]^{\frac{1}{1-\eta_{i,e}}}. \tag{A.17}$$

A.2.2 Value-added production (excluding energy)

Perfectly competitive firms produce value-added v_t with price $P_{v,t}$ with a constant returns to scale CES production function using previously accumulated capital k_{t-1} and labor n_t:

$$v_t = z\left[(1-\alpha)^{\frac{1}{\sigma}}\, n_t^{1-\frac{1}{\sigma}} + \alpha^{\frac{1}{\sigma}}\left(\frac{k_{t-1}}{k_0}\right)^{1-\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{A.18}$$

where α is the capital share in production, k_0 is a scaling parameter, σ is the elasticity of substitution between capital and labor, and z is a productivity scaling parameter.

Value added producers post vacancies ν_t at cost $P_{v,t}\kappa$ to attract unemployed workers u_t^s. We assume all unemployed workers look for jobs. Matches that are formed in period t start working in the same period. The number of matches $\mathcal{M}_t$ is defined by the following constant returns to scale matching function:

$$\mathcal{M}_t = \xi\,(u_t^s)^{\iota}\,\nu_t^{1-\iota}, \tag{A.19}$$

where ξ is a scaling parameter and ι is the elasticity of matches with respect to job seekers. Labor market tightness θ_t is then given by the vacancies to unemployment ratio $\theta_t = \nu_t/u_t^s$, the probability of finding a job is given by $f_t = \mathcal{M}_t/u_t^s$, and the probability of filling a vacancy is given by $q_t = \mathcal{M}_t/\nu_t$.

The timing of labor market events is as follows. At the beginning of every period a share s of matches is terminated exogenously. Afterwards matches are formed and hiring takes place. The law of motion of employment n_t is given by:

$$n_t = (1-s)\,n_{t-1} + q_t\nu_t. \tag{A.20}$$

By normalizing the total population size to 1, we find that the unemployment rate u_t is then given by:

$$u_t = 1 - n_t, \tag{A.21}$$

while the amount of job seekers u_t^s is determined by agents who did not have a job in the previous period u_{t-1} or were separated from their match in the current period sn_{t-1}:

$$u_t^s = u_{t-1} + sn_t, \tag{A.22}$$

Firms accumulate physical capital by purchasing investment goods i_t. Firms select their capital stock after all hiring has taken place and that changing the level of investment relative to the previous period is subject to a quadratic adjustment cost. The law of motion of the capital stock is therefore given by:

$$k_t = (1-\delta)\, k_{t-1} + \left[1 - \frac{\kappa_i}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right] i_t, \tag{A.23}$$

where δ is the depreciation rate of physical capital and κ_i determines the strength of the quadratic adjustment costs. Labor is paid a nominal wage rate W_t. Firms maximize their value

$$V_t = \Pi_t^v + \mathbb{E}_t \left\{ \beta \frac{\lambda_{d,t+1}}{\lambda_{d,t}} V_{t+1} \right\},$$

where they use the firm owner's stochastic discount factor $\beta \frac{\lambda_{d,t+1}}{\lambda_{d,t}}$ to discount future cash flows (because firm owners own firms through their holdings of firm shares). Flow profits of value added producers $\Pi_{v,t}$ are given by:

$$\begin{aligned}
\Pi_t^v &= P_{v,t} v_t - W_t n_t - P_{i,t} i_t - P_{v,t} \kappa \nu_t, \\
&= P_{v,t} \left[(1-\alpha)^{\frac{1}{\sigma}} n_t^{1-\frac{1}{\sigma}} + \alpha^{\frac{1}{\sigma}} \left(\frac{k_{t-1}}{k_0} \right)^{1-\frac{1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} - W_t n_t - P_{i,t} i_t - P_{v,t} \kappa \nu_t.
\end{aligned}$$

Firms maximize their value subject to the law of motion of capital and the law of motion of employment. We assume that there is an interior solution for vacancies ν_t. The firm's first order conditions are given by:

$$i_t : \frac{P_t^i}{q_t^k} = 1 - \frac{\kappa_i}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 - \kappa_i\left(\frac{i_t}{i_{t-1}} - 1\right)\frac{i_t}{i_{t-1}} + \mathbb{E}_t\left\{\beta\frac{\lambda_{d,t+1}}{\lambda_{d,t}}\frac{q_{t+1}^k}{q_t^k}\kappa_i\left(\frac{i_{t+1}}{i_t} - 1\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right\}, \tag{A.24}$$

$$k_t : q_t^k = \mathbb{E}_t\left\{\beta\frac{\lambda_{d,t+1}}{\lambda_{d,t}}\left[P_{v,t+1}\left[\alpha v_{x,t+1}k_0/k_t\right]^{\frac{1}{\sigma}} + (1-\delta)\, q_{t+1}^k\right]\right\}, \tag{A.25}$$

$$n_t : \lambda_{n,t} = P_{v,t}\left[(1-\alpha)\, v_t/n_t\right]^{\frac{1}{\sigma}} - W_t + (1-s)\,\mathbb{E}_t\left\{\beta\frac{\lambda_{d,t+1}}{\lambda_{d,t}}\lambda_{n,t+1}\right\}, \tag{A.26}$$

$$\nu_t : \lambda_{n,t} = P_{v,t}\frac{\kappa}{q_t}, \tag{A.27}$$

where q_t^k is the price of capital, and $\lambda_{n,t}$ is the Lagrange multiplier on the law of motion of employment. Combining the first order conditions for n_t and ν_t yields the following first order condition:

$$P_{v,t}\frac{\kappa}{q_{t+1}} = P_{v,t}\left[(1-\alpha)\, v_t/n_t\right]^{\frac{1}{\sigma}} - W_t + (1-s)\,\mathbb{E}_t\left\{\beta\frac{\lambda_{d,t+1}}{\lambda_{d,t}}P_{v,t+1}\frac{\kappa}{q_{t+1}}\right\},$$

which states that the marginal costs of posting a vacancy (corrected for the probablity of filling that vacancy q_t) must equal its marginal benefit, taking into account that today's match continues into the next period with probability $1-s$. For simplicity, we assume that the nominal wage rate W_t is a linear function of the marginal product of labor and the worker's outside option:

$$W_t = \gamma\left\{\eta P_{v,t}\left[(1-\alpha)\, v_t/n_t\right]^{\frac{1}{\sigma}} + (1-\eta)\,(B_u + h)\right\} + (1-\gamma)\, W.$$

with weights η and $1-\eta$, where W is the nominal wage rate in the deterministic steady state. This weight can be thought of as the worker's bargaining power.[17] We introduce the ad hoc parameter h that means to capture disutility from working. We introduce additional wage rigidities by imposing that today's wage rate only adjusts by a factor γ, while it depends on the steady state wage rate with a factor $1-\gamma$.

[17] Nash bargaining is commonly used to determine wages in the search and matching literature. However, with heterogeneous agents Nash bargaining becomes less tractable because future utility flows matter and these are discounted using different stochastic discount factors. This wage determination scheme is also used by Jung and Kuester (2011) and Den Haan et al. (2021). Said scheme can also be micro-founded as the outcome of an alternating offers game, see Hall and Milgrom (2008).

A.2.3 Production of intermediate goods

Final goods are produced using a composite intermediate good m_t with price $P_{m,t}$. This intermediate good is produced with energy $m_{e,t}$, domestic intermediate goods $m_{H,t}$ and foreign intermediate goods $m_{F,t}$ using the following production technologies:

$$m_t \equiv \left[(1-\zeta_{d,e})^{\frac{1}{\eta_{d,e}}} m_{x,t}^{1-\frac{1}{\eta_{d,e}}} + \zeta_{d,e}^{\frac{1}{\eta_{d,e}}} m_{e,t}^{1-\frac{1}{\eta_{d,e}}} \right]^{\frac{\eta_{d,e}}{\eta_{d,e}-1}}, \tag{A.28}$$

$$m_{x,t} \equiv \left[(1-\zeta_m)^{\frac{1}{\eta_m}} m_{H,t}^{1-\frac{1}{\eta_m}} + \zeta_m^{\frac{1}{\eta_m}} m_{F,t}^{1-\frac{1}{\eta_m}} \right]^{\frac{\eta_m}{\eta_m-1}}, \tag{A.29}$$

where the parameters have their usual meanings. Total expenditure on intermediate goods is given by:

$$P_{m,t} m_t = P_{x,m,t} m_{x,t} + P_{e,t} m_{e,t},$$
$$P_{m,x,t} m_{x,t} = P_{H,t} m_{H,t} + P_{F,t} m_{F,t}.$$

The first order conditions are familiar and are given by:

$$\frac{m_{H,t}}{m_{F,t}} = \frac{1-\zeta_m}{\zeta_m} \left(\frac{P_{H,t}}{P_{F,t}} \right)^{-\eta_m}, \tag{A.30}$$

$$P_{x,m,t} = \left[(1-\zeta_m) P_{H,t}^{1-\eta_m} + \zeta_m P_{F,t}^{1-\eta_m} \right]^{\frac{1}{1-\eta_m}}, \tag{A.31}$$

$$\frac{m_{x,t}}{m_{e,t}} = \frac{1-\zeta_{d,e}}{\zeta_{d,e}} \left(\frac{P_{x,m,t}}{P_{e,t}} \right)^{-\eta_{d,e}}, \tag{A.32}$$

$$P_{m,t} = \left[(1-\zeta_{d,e}) P_{x,m,t}^{1-\eta_{d,e}} + \zeta_{d,e} P_{e,t}^{1-\eta_{d,e}} \right]^{\frac{1}{1-\eta_{d,e}}}. \tag{A.33}$$

A.2.4 Production of domestic goods

The domestic good $P_{H,t}^f y_{H,t}$ is then produced using domestic value added $P_{v,t} v_t$ and an intermediate good m_t with price $P_{m,t}$ using a CES production function:

$$y_{H,t} \equiv \left[(1-\zeta_v)^{\frac{1}{\eta_v}} m_t^{1-\frac{1}{\eta_v}} + \zeta_v^{\frac{1}{\eta_v}} v_t^{1-\frac{1}{\eta_v}} \right]^{\frac{\eta_v}{\eta_v-1}}, \tag{A.34}$$

where the parameters have their usual meanings. Total expenditure on these two goods is given by:

$$P_{H,t}^f y_{H,t} = P_{v,t} v_t + P_{m,t} m_t.$$

The first order conditions are familiar and are given by:

$$\frac{m_t}{v_t} = \frac{1-\zeta_v}{\zeta_v}\left(\frac{P_{m,t}}{P_{v,t}}\right)^{-\eta_v}, \tag{A.35}$$

$$P^f_{H,t} = \left[(1-\zeta_v)\,P^{1-\eta_v}_{m,t} + \zeta_v P^{1-\eta_v}_{v,t}\right]^{\frac{1}{1-\eta_v}}. \tag{A.36}$$

The domestic good is sold to monopolistically competitive retailers, who differentiate these goods and set the price on these goods before the domestic good is sold to households, value added producing firms, the government, and intermediate goods producers.

A.2.5 Exporters

Suppose the economy supplies foreign countries with exports $\varkappa_t$ which come at price $P^f_{\varkappa,t}$, which are constructed using domestic goods $\varkappa_{H,t}$, foreign goods $\varkappa_{F,t}$, and energy $\varkappa_{e,t}$ with the following CES production functions:

$$\varkappa_t \equiv \left[(1-\zeta_{\varkappa,e})^{\frac{1}{\eta_{\varkappa,e}}}\,\varkappa_{x,t}^{1-\frac{1}{\eta_{\varkappa,e}}} + \zeta_{\varkappa,e}^{\frac{1}{\eta_{\varkappa,e}}}\,\varkappa_{e,t}^{1-\frac{1}{\eta_{\varkappa,e}}}\right]^{\frac{\eta_{\varkappa,e}}{\eta_{\varkappa,e}-1}}, \tag{A.37}$$

$$\varkappa_{x,t} \equiv \left[(1-\zeta_x)^{\frac{1}{\eta_x}}\,\varkappa_{H,t}^{1-\frac{1}{\eta_x}} + \zeta_x^{\frac{1}{\eta_x}}\,\varkappa_{F,t}^{1-\frac{1}{\eta_x}}\right]^{\frac{\eta_x}{\eta_x-1}}, \tag{A.38}$$

where the parameters have their usual meanings. Total expenditure on exports is given by:

$$P^f_{\varkappa,t}\varkappa_t = P_{x,\varkappa,t}\varkappa_{x,t} + P_{e,t}\varkappa_{e,t},$$
$$P_{x,\varkappa,t}\varkappa_{x,t} = P_{H,t}\varkappa_{H,t} + P_{F,t}\varkappa_{F,t}.$$

The first order conditions are familiar and are given by:

$$\frac{\varkappa_{H,t}}{\varkappa_{F,t}} = \frac{1-\zeta_\varkappa}{\zeta_\varkappa}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta_\varkappa}, \tag{A.39}$$

$$P_{x,\varkappa.t} = \left[(1-\zeta_x)\,P^{1-\eta_x}_{H,t} + \zeta_x P^{1-\eta_x}_{F,t}\right]^{\frac{1}{1-\eta_x}}, \tag{A.40}$$

$$\frac{\varkappa_{x,t}}{\varkappa_{e,t}} = \frac{1-\zeta_{\varkappa,e}}{\zeta_{\varkappa,e}}\left(\frac{P_{x,\varkappa,t}}{P_{e,t}}\right)^{-\eta_{\varkappa,e}}, \tag{A.41}$$

$$P^f_{\varkappa,t} = \left[(1-\zeta_{\varkappa,e})\,P^{1-\eta_{\varkappa,e}}_{x,\varkappa,t} + \zeta_{\varkappa,e} P^{1-\eta_{\varkappa,e}}_{e,t}\right]^{\frac{1}{1-\eta_{\varkappa,e}}}. \tag{A.42}$$

The export good is sold to monopolistically competitive retailers, who differentiate these goods and set their price before these goods are exported.

A.2.6 Nominal rigidities

We introduce nominal rigidities in the form of sticky prices in the energy, imported goods, domestic goods, and exports sectors. We will sketch how to solve the optimization problem in the energy sectors and will only state the resulting optimality conditions for the other sector.

Consider a continuum of perfectly competitive final goods producers of measure one in the energy sector. These final goods producers combine retail goods $e_{j,t}$, which they buy at price $P_{j,e,t}$ into a final energy good e_t which they sell for a nominal price $P_{e,t}$ according to the following production technology:

$$e_t = \left[\int_0^1 (e_{j,t})^{\frac{\epsilon_e - 1}{\epsilon_e}} \, dj \right]^{\frac{\epsilon_e}{\epsilon_e - 1}}, \tag{A.43}$$

where ϵ_e is the elasticity of substitution between different kinds of retail goods. Going through the motions yields the following demand function for retail goods from final goods producers:

$$e_{j,t} = \left(\frac{P_{j,e,t}}{P_{e,t}} \right)^{-\epsilon_e} e_t. \tag{A.44}$$

Energy retails goods producers j of measure one purchase homogenous output from perfectly competitive energy importers at price $P^f_{e,t}$, which they subsequently convert into retail goods $e_{j,t}$. Essentially, retailers (in all sector) purchase a homogenous good that they transform one to one into a retail good. These are sold again to final goods producers for a price $P_{j,e,t}$. Retail goods firms in each sector operate in a monopolistically competitive environment, because they all produce a slightly different retail good and final goods producers cannot perfectly substitute between all different kinds of retail goods. Because retail goods firms operate in a monopolistically competitive environment they can charge a markup. This earns them nominal profits $\Pi^e_{j,t} = \left(P_{j,e,t} - P^f_{e,t} \right) e_{j,t}$.

We introduce quadratic price adjustment costs a la Rotemberg (1982), such that prices adjust sluggishly. This cost is linear in aggregate energy output.[18] Following Eggertsson and Singh (2019), these adjustment costs should be thought of as reputational costs instead of physical resource costs, such that they do not impact firms' final profits. A retail goods producer's optimization problem is therefore given by:

$$\max_{P_{j,e,t}} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \beta^s \frac{\lambda_{d,t+s}}{\lambda_{d,t}} \left[P_{j,e,t+s} e_{j,t+s} - P^f_{e,t+s} e_{j,t+s} - \frac{\kappa_e}{2} \left(\frac{P_{j,e,t+s}}{P_{j,e,t+s-1}} - 1 \right)^2 P_{e,t+s} e_{t+s} \right] \right\},$$

subject to:

$$e_{j,t} = \left(\frac{P_{j,e,t}}{P_{e,t}} \right)^{-\epsilon_e} e_t.$$

[18] From the monetary policy literature, we know that the adjustment costs parameter κ_e can be parametrized such that the frequency of price adjustments can be matched.

Choosing the optimal $P_{j,e,t}$ and imposing symmetry such that all energy retailers choose the same price ($P_{j,e,t} = P_{e,t}\ \forall j$) results in the following New Keynesian Philips curve for energy prices:

$$(\epsilon_e - 1) - \epsilon_e \frac{P^f_{e,t}}{P_{e,t}} + \kappa_e \left(\frac{P_{e,t}}{P_{e,t-1}} - 1\right)\frac{P_{e,t}}{P_{e,t-1}} = \mathbb{E}_t \left\{\beta \frac{\lambda_{d,t+1}}{\lambda_{d,t}} \kappa_e \left(\frac{P_{e,t+1}}{P_{e,t}} - 1\right)\left(\frac{P_{e,t+1}}{P_{e,t}}\right)^2 \frac{e_{t+1}}{e_t}\right\}. \tag{A.45}$$

In the aggregate, profits for energy retailers are then given by:

$$\Pi^e_t = \left(P_{e,t} - P^f_{e,t}\right) e_t. \tag{A.46}$$

Going through the motions will result in the following New Keynesian Philips curves for domestic goods, imported goods, and exports:

$$(\epsilon_H - 1) - \epsilon_H \frac{P^f_{H,t}}{P_{H,t}} + \kappa_H \left(\frac{P_{H,t}}{P_{H,t-1}} - 1\right)\frac{P_{H,t}}{P_{H,t-1}} = \mathbb{E}_t \left\{\beta \frac{\lambda_{d,t+1}}{\lambda_{d,t}} \kappa_H \left(\frac{P_{H,t+1}}{P_{H,t}} - 1\right)\left(\frac{P_{H,t+1}}{P_{H,t}}\right)^2 \frac{y_{H,t+1}}{y_{H,t}}\right\}, \tag{A.47}$$

$$(\epsilon_F - 1) - \epsilon_F \frac{P^f_{F,t}}{P_{F,t}} + \kappa_F \left(\frac{P_{F,t}}{P_{F,t-1}} - 1\right)\frac{P_{F,t}}{P_{F,t-1}} = \mathbb{E}_t \left\{\beta \frac{\lambda_{d,t+1}}{\lambda_{d,t}} \kappa_F \left(\frac{P_{F,t+1}}{P_{F,t}} - 1\right)\left(\frac{P_{F,t+1}}{P_{F,t}}\right)^2 \frac{y_{F,t+1}}{y_{F,t}}\right\}, \tag{A.48}$$

$$(\epsilon_\varkappa - 1) - \epsilon_\varkappa \frac{P^f_{\varkappa,t}}{P_{\varkappa,t}} + \kappa_\varkappa \left(\frac{P_{\varkappa,t}}{P_{\varkappa,t-1}} - 1\right)\frac{P_{\varkappa,t}}{P_{\varkappa,t-1}} = \mathbb{E}_t \left\{\beta \frac{\lambda_{d,t+1}}{\lambda_{d,t}} \kappa_F \left(\frac{P_{\varkappa,t+1}}{P_{\varkappa,t}} - 1\right)\left(\frac{P_{\varkappa,t+1}}{P_{\varkappa,t}}\right)^2 \frac{\varkappa_{t+1}}{\varkappa_t}\right\}. \tag{A.49}$$

Profits for the corresponding retailers are given by:

$$\Pi^H_t = \left(P_{H,t} - P^f_{H,t}\right) y_{H,t}, \tag{A.50}$$

$$\Pi^F_t = \left(P_{F,t} - P^f_{F,t}\right) y_{F,t}, \tag{A.51}$$

$$\Pi^\varkappa_t = \left(P_{\varkappa,t} - P^f_{\varkappa,t}\right) \varkappa_t. \tag{A.52}$$

A.3 Fiscal authority

The fiscal authority issues bonds $B_{g,t}$ that come at price q^b_t that promise to pay out one euro in the next period. In addition, the fiscal authority levies lump sum taxes T_t, a constant distortionary consumption tax rate T_c, and a constant tax on firm owners T_r. It consumes a composite good g_t with price $P_{g,t}$ and provides unemployment benefits B_u to unemployed workers $u_{w,t}$. We assume total government consumption is exogenous. The government budget constraint is therefore given by:

$$T_t + q_t^b B_{g,t} + T_r + T_c P_{c,t} c_t = P_{g,t} g_t + B_u u_{w,t} + B_{g,t-1}. \tag{A.53}$$

Total tax revenues T_t is the population weighted sum of taxes paid by firm owners $T_{d,t}$. The fiscal authority adjusts this tax to ensure intertemporal solvency:

$$T_{d,t} - T_m = \kappa_T \left(B_{g,t-1} - B_g\right), \tag{A.54}$$

where T_m and B_g are steady state values.

Public consumption consists of $g_{H,t}$ domestic goods, $g_{F,t}$ foreign goods, and $g_{e,t}$ energy and is constructed using the following production technologies:

$$g_t \equiv \left[(1-\zeta_{g,e})^{\frac{1}{\eta_{g,e}}} g_{x,t}^{1-\frac{1}{\eta_{g,e}}} + \zeta_{g,e}^{\frac{1}{\eta_{g,e}}} g_{e,t}^{1-\frac{1}{\eta_{g,e}}}\right]^{\frac{\eta_{g,e}}{\eta_{g,e}-1}}, \tag{A.55}$$

$$g_{x,t} \equiv \left[(1-\zeta_g)^{\frac{1}{\eta_g}} g_{H,t}^{1-\frac{1}{\eta_g}} + \zeta_g^{\frac{1}{\eta_g}} g_{F,t}^{1-\frac{1}{\eta_g}}\right]^{\frac{\eta_g}{\eta_g-1}}, \tag{A.56}$$

where the parameters have their usual meanings. Total expenditure on public consumption is given by:

$$P_{g,t} g_t = P_{x,g,t} g_{x,t} + P_{e,t} g_{e,t},$$
$$P_{x,g,t} g_t = P_{H,t} g_{H,t} + P_{F,t} g_{F,t}.$$

The first order conditions are again familiar and are given by:

$$\frac{g_{H,t}}{g_{F,t}} = \frac{1-\zeta_g}{\zeta_g}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta_g}, \tag{A.57}$$

$$P_{x,g,t} = \left[(1-\zeta_g) P_{H,t}^{1-\eta_g} + \zeta_g P_{F,t}^{1-\eta_g}\right]^{\frac{1}{1-\eta_g}}, \tag{A.58}$$

$$\frac{g_{x,t}}{g_{e,t}} = \frac{1-\zeta_{g,e}}{\zeta_{g,e}}\left(\frac{P_{x,g,t}}{P_{e,t}}\right)^{-\eta_{g,e}}, \tag{A.59}$$

$$P_{g,t} = \left[(1-\zeta_{g,e}) P_{x,g,t}^{1-\eta_{g,e}} + \zeta_{g,e} P_{e,t}^{1-\eta_{g,e}}\right]^{\frac{1}{1-\eta_{g,e}}}. \tag{A.60}$$

A.4 Market clearing

In the aggregate, consumption, (un)employment, bond holdings and domestic equity holdings are given by:

$$
\begin{aligned}
B_t &= \psi B_{w,t} + (1-\psi)\, B_{d,t}, \\
c_t &= \psi c_{w,t} + (1-\psi)\, c_{d,t}, \\
n_t &= \psi n_{w,t}, \\
u_t &= \psi u_{w,t}, \\
\varsigma_t &= (1-\psi)\, \varsigma_{d,t}.
\end{aligned}
$$

Households' portfolios of liquid assets consist of government bonds $B_{g,t}$ and an internationally traded asset B_t^*:

$$B_t = B_{g,t} + B_t^*. \tag{A.61}$$

Note that letting households choose their holdings of both liquid assets would resulting in identical equilibrium conditions, as their expected returns would be equalized due to arbitrage. We normalize total outstanding firm equity to unity, such that total outstanding equity $\varsigma = 1\ \forall t$. Hence, total equity holdings are also going to be constant.

Foreign demand for the economy's exports is determined by the following demand function:

$$\varkappa_t = \left[\varkappa \left(\frac{P_{\varkappa,t}}{P_{\mu,t}^*}\right)^{-\eta_\varkappa^*}\right]^{\chi} \varkappa_{t-1}^{1-\chi}, \tag{A.62}$$

where $\eta_\varkappa^*$ is the price elasticity of foreign export demand and χ introduces inertia in the demand for exports. We assume that other economies consume a similar import bundle as the domestic country with price $P_{\mu,t}^*$, such that a lower price of exports relative to imports increases foreign demand for exports. The relevant import price for other countries is given by:

$$\frac{P_{\mu,t}^*}{P_{\mu,t-1}^*} = \frac{P_{F,t}^f y^* + P_{e,t} e^*}{P_{F,t-1}^f y^* + P_{e,t-1} e^*},$$

where y^*, e^* are parameters. Total profits are the sum of the profits of value added producers and retailers:

$$\Pi_t = \Pi_t^v + \Pi_t^e + \Pi_t^H + \Pi_t^F + \Pi_t^\varkappa. \tag{A.63}$$

In the aggregate, total after tax income of workers and firm owners is given by:

$$y_{w,t} = W_t n_{w,t} + B_{w,t-1} + B_u u_{w,t} - T_{w,t}, \tag{A.64}$$

$$y_{d,t} = \left(q_t^{\varsigma} + \Pi_t\right) \varsigma_{d,t-1} + B_{d,t-1} - T_r - T_{d,t}, \tag{A.65}$$

such that in the aggregate total household income is given by:

$$y_t = \psi y_{w,t} + (1-\psi)\, y_{d,t}. \tag{A.66}$$

Total demand for domestic goods is therefore given by:

$$P_{H,t} y_{H,t} = P_{H,t} c_{H,t} + P_{H,t} i_{H,t} + P_{H,t} g_{H,t} + P_{H,t} \varkappa_{H,t} + P_{H,t} m_{H,t}.$$

To streamline the accounting we assume all energy is imported. This is without much loss of generality, as in reality the price of energy is also determined in the world market.[19] Total demand for energy is given by:

$$P_{e,t} e_t = P_{e,t} c_{e,t} + P_{e,t} i_{e,t} + P_{e,t} g_{e,t} + P_{e,t} \varkappa_{e,t} + P_{e,t} m_{e,t} + P_{e,t} v_{e,t}.$$

Total imports consists of spending on private consumption goods, investment goods, public consumption goods, intermediate goods and energy. Let μ_t be the economy's total imports. Total imports come with price index $P_{\mu,t}$. Let us make a distinction between energy imports and non-energy imports, the latter of which are given by:

$$P^f_{F,t} y_{F,t} = P^f_{F,t} c_{F,t} + P^f_{F,t} i_{F,t} + P^f_{F,t} g_{F,t} + P^f_{F,t} x_{F,t} + P^f_{F,t} m_{F,t}.$$

Total imports are then given by:

$$P_{\mu,t} \mu_t = P^f_{F,t} y_{F,t} + P_{e,t} e_t,$$

where $P_{\mu,t}$ is a Paasche index for the price of imports from period $t-1$ to period t:

$$\frac{P_{\mu,t}}{P_{\mu,t-1}} = \frac{P^f_{F,t} y_{F,t} + P_{e,t} e_t}{P^f_{F,t-1} y_{F,t} + P_{e,t-1} e_t}. \tag{A.67}$$

Next, let $y_{D,t}$ be gross domestic final expenditures:

$$P_{D,t} y_{D,t} = P_{c,t} c_t + P_{i,t} i_t + P_{g,t} g_t, \tag{A.68}$$

[19] The price of electricity is linked to the price of natural gas, the price of which is not only determined in the Netherlands. A slightly weaker assumption is that the economy receives a constant endowment of energy every period, while any residual demand for energy is imported.

where $P_{D,t}$ is a Paasche index for the price of domestic final expenditures from period $t-1$ to period t:

$$\frac{P_{D,t}}{P_{D,t-1}} = \frac{P_{c,t}c_t + P_{i,t}i_t + P_{g,t}g_t}{P_{c,t-1}c_t + P_{i,t-1}i_t + P_{g,t-1}g_t}. \tag{A.69}$$

From here on out the exposition mostly follows Reinsdorf (2010). The Laspeyres volume index for GDP period $t-1$ to period t is then given by:

$$\text{gdp}_{L,t} = \frac{P_{D,t-1}y_{D,t} + P_{\varkappa,t-1}\varkappa_t - P_{\mu,t-1}\mu_t}{P_{D,t-1}y_{D,t-1} + P_{\varkappa,t-1}\varkappa_{t-1} - P_{\mu,t-1}\mu_{t-1}} \tag{A.70}$$

Let $\mathcal{P}_{z,t}^{\text{Paasche}}$, where $z \in D, \mu, \varkappa$, be the Paasche price index of expenditure category z from period $t-1$ to period t, such that:

$$\begin{aligned}
\mathcal{P}_{D,t}^{\text{Paasche}} &= \frac{P_{D,t}}{P_{D,t-1}}, \\
\mathcal{P}_{\varkappa,t}^{\text{Paasche}} &= \frac{P_{\varkappa,t}}{P_{\varkappa,t-1}}, \\
\mathcal{P}_{\mu,t}^{\text{Paasche}} &= \frac{P_{\mu,t}}{P_{\mu,t-1}}.
\end{aligned}$$

We can therefore rewrite the Laspeyres index for GDP as:

$$\text{gdp}_{L,t} = \frac{P_{D,t}y_{D,t}/\mathcal{P}_{D,t}^{\text{Paasche}} + P_{\varkappa,t}\varkappa_t/\mathcal{P}_{\varkappa,t}^{\text{Paasche}} - P_{\mu,t}\mu_t/\mathcal{P}_{\mu,t}^{\text{Paasche}}}{P_{D,t-1}y_{D,t-1} + P_{\varkappa,t-1}\varkappa_{t-1} - P_{\mu,t-1}\mu_{t-1}}. \tag{A.71}$$

Following the arguments in Reinsdorf (2010) that imports and exports should be deflated by a common deflator, the Laspeyres index for GDI, using the price index of total expenditures as a deflator, is given by:

$$\text{gdi}_{L,t} = \frac{P_{D,t}y_{D,t} + P_{\varkappa,t}\varkappa_t - P_{\mu,t}\mu_t}{\left(P_{D,t-1}y_{D,t-1} + P_{\varkappa,t-1}\varkappa_{t-1} - P_{\mu,t-1}\mu_{t-1}\right)\mathcal{P}_{D,t}^{\text{Paasche}}}. \tag{A.72}$$

The (deflated) Paasche index for GDP is given by:

$$\text{gdp}_{P,t} = \frac{P_{D,t}y_{D,t} + P_{\varkappa,t}\varkappa_t - P_{\mu,t}\mu_t}{P_{D,t-1}y_{D,t-1}\mathcal{P}_{D,t}^{\text{Laspeyres}} + P_{\varkappa,t-1}\varkappa_{t-1}\mathcal{P}_{\varkappa,t}^{\text{Laspeyres}} - P_{\mu,t-1}\mu_{t-1}\mathcal{P}_{\mu,t}^{\text{Laspeyres}}}. \tag{A.73}$$

Alternatively, the Laspeyres index for $P_{D,t}$ and $P_{\mu,t}$ are given by:

$$\begin{aligned}
\mathcal{P}_{D,t}^{\text{Laspeyres}} &= \frac{P_{D,t}}{P_{D,t-1}} = \frac{P_{c,t}c_{t-1} + P_{i,t}i_{t-1} + P_{g,t}g_{t-1}}{P_{c,t-1}c_{t-1} + P_{i,t-1}i_{t-1} + P_{g,t-1}g_{t-1}}, \\
\mathcal{P}_{\mu,t}^{\text{Laspeyres}} &= \frac{P_{\mu,t}}{P_{\mu,t-1}} = \frac{P_{F,t}^{f}y_{F,t-1} + P_{e,t}e_{t-1}}{P_{F,t-1}^{f}y_{F,t-1} + P_{e,t-1}e_{t-1}}.
\end{aligned}$$

We can use this to deflate the Paasche index for GDI:

$$\text{gdi}_{P,t} = \frac{P_{D,t} y_{D,t} + P_{\varkappa,t} \varkappa_t - P_{\mu,t} \mu_t}{\left(P_{D,t-1} y_{D,t-1} + P_{\varkappa,t-1} \varkappa_{t-1} - P_{\mu,t-1} \mu_{t-1}\right) \mathcal{P}_{D,t}^{\text{Laspeyres}}}. \tag{A.74}$$

The trading gains index for both cases is given by the ratio of GDP and GDI for both volume indices. In the case of the Paasche volume indices and Laspeyres price index, this yields the following expression:

$$\text{TGI}_{L,t} = 1 + s_{\varkappa,t} \left(\frac{\mathcal{P}_{\varkappa,t}^{\text{Laspeyres}}}{\mathcal{P}_{D,t}^{\text{Laspeyres}}} - 1 \right) - s_{\mu,t} \left(\frac{\mathcal{P}_{d,t}^{\text{Laspeyres}}}{\mathcal{P}_{D,t}^{\text{Laspeyres}}} - 1 \right),$$

where the Laspeyres price indices for exports and imports are identical to their Paasche indices, and $s_{\varkappa,t}$ and $s_{\mu,t}$ are respectively the export and import shares in GDP.

The net foreign asset position evolves according to:

$$\left(q_t^b B_t^* - B_{t-1}^* \right) = P_{\varkappa,t} \varkappa_t - P_{\mu,t} \mu_t.$$

The inverse of the bond price $1/q_t^b$ corresponds to the risk free international interest rate. We assume this variable is exogenously given. Finally, the terms of trade $\mathcal{T}_t$ is given by:

$$\mathcal{T}_t = \frac{P_{\varkappa,t}}{P_{\mu,t}}. \tag{A.75}$$

We assume that the prices of energy imports $P_{e,t}$ and non-energy imports $P_{F,t}$ are exogenous and do not respond to changes in domestic demand.

B Equilibrium conditions

Workers (4 equations):

$$q_t^b + f'\left(B_{w,t}\right) = \mathbb{E}_t \left\{ \beta \frac{\lambda_{w,t+1}}{\lambda_{w,t}} \right\}, \tag{B.1}$$

$$\left(1 + T_c\right) \lambda_{w,t} = u'\left(c_{w,t}\right) / P_{c,t}, \tag{B.2}$$

$$\left(1 + T_c\right) P_{c,t} c_{w,t} = W_t n_{w,t} + B_{w,t-1} + B_u u_{w,t} - q_t^b B_{w,t} - f\left(B_{w,t}\right), \tag{B.3}$$

$$y_{w,t} = W_t n_{w,t} + B_{w,t-1} + B_u u_{w,t}. \tag{B.4}$$

Firm owners (4 equations)[20]:

$$q_t^b = \mathbb{E}_t \left\{ \beta \frac{\lambda_{d,t+1}}{\lambda_{d,t}} \right\}, \tag{B.5}$$

$$(1 + T_c) \lambda_{d,t} = u' (c_{d,t}) / P_{c,t}, \tag{B.6}$$

$$q_t^\varsigma = \mathbb{E}_t \left\{ \beta \frac{\lambda_{d,t+1}}{\lambda_{d,t}} (q_{t+1}^\varsigma + \Pi_{t+1}) \right\}, \tag{B.7}$$

$$y_{d,t} = (q_t^\varsigma + \Pi_t) \varsigma_{d,t-1} + B_{d,t-1} - T_r - T_{d,t}. \tag{B.8}$$

Consumption demand (6 equations):

$$c_t \equiv \left[(1 - \zeta_{c,e})^{\frac{1}{\eta_{c,e}}} c_{x,t}^{1-\frac{1}{\eta_{c,e}}} + \zeta_{c,e}^{\frac{1}{\eta_{c,e}}} c_{e,t}^{1-\frac{1}{\eta_{c,e}}} \right]^{\frac{\eta_{c,e}}{\eta_{c,e}-1}}, \tag{B.9}$$

$$\frac{c_{x,t}}{c_{e,t}} = \frac{1 - \zeta_{c,e}}{\zeta_{c,e}} \left(\frac{P_{x,c,t}}{P_{e,t}} \right)^{-\eta_{c,e}}, \tag{B.10}$$

$$P_{c,t} = \left[(1 - \zeta_{c,e}) P_{x,t}^{1-\eta_{c,e}} + \zeta_{c,e} P_{e,t}^{1-\eta_{c,e}} \right]^{\frac{1}{1-\eta_{c,e}}}, \tag{B.11}$$

$$c_{x,t} \equiv \left[(1 - \zeta_c)^{\frac{1}{\eta_c}} c_{H,t}^{1-\frac{1}{\eta_c}} + \zeta_c^{\frac{1}{\eta_c}} c_{F,t}^{1-\frac{1}{\eta_c}} \right]^{\frac{\eta_c}{\eta_c-1}}, \tag{B.12}$$

$$\frac{c_{H,t}}{c_{F,t}} = \frac{1 - \zeta_c}{\zeta_c} \left(\frac{P_{H,t}}{P_{F,t}} \right)^{-\eta_c}, \tag{B.13}$$

$$P_{x,c,t} = \left[(1 - \zeta_c) P_{H,t}^{1-\eta_c} + \zeta_c P_{F,t}^{1-\eta_c} \right]^{\frac{1}{1-\eta_c}}. \tag{B.14}$$

Investment demand (6 equations):

$$i_t \equiv \left[(1 - \zeta_{i,e})^{\frac{1}{\eta_{i,e}}} i_{x,t}^{1-\frac{1}{\eta_{i,e}}} + \zeta_{i,e}^{\frac{1}{\eta_{i,e}}} i_{e,t}^{1-\frac{1}{\eta_{i,e}}} \right]^{\frac{\eta_{i,e}}{\eta_{i,e}-1}}, \tag{B.15}$$

$$\frac{i_{x,t}}{i_{e,t}} = \frac{1 - \zeta_{i,e}}{\zeta_{i,e}} \left(\frac{P_{x,i,t}}{P_{e,t}} \right)^{-\eta_{i,e}}, \tag{B.16}$$

$$P_{i,t} = \left[(1 - \zeta_{i,e}) P_{x,i,t}^{1-\eta_{i,e}} + \zeta_{i,e} P_{e,t}^{1-\eta_{i,e}} \right]^{\frac{1}{1-\eta_{i,e}}}, \tag{B.17}$$

$$i_{x,t} \equiv \left[(1 - \zeta_i)^{\frac{1}{\eta_i}} i_{H,t}^{1-\frac{1}{\eta_i}} + \zeta_i^{\frac{1}{\eta_i}} i_{F,t}^{1-\frac{1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i-1}}, \tag{B.18}$$

$$\frac{i_{H,t}}{i_{F,t}} = \frac{1 - \zeta_i}{\zeta_i} \left(\frac{P_{H,t}}{P_{F,t}} \right)^{-\eta_i}, \tag{B.19}$$

$$P_{x,i,t} = \left[(1 - \zeta_i) P_{H,t}^{1-\eta_i} + \zeta_i P_{F,t}^{1-\eta_i} \right]^{\frac{1}{1-\eta_i}}. \tag{B.20}$$

[20] Note that we can drop the firm owner's budget constraint because by Walras's law it is automatically satisfied.

Value added production (5 equations):

$$v_t = z\left[(1-\alpha)^{\frac{1}{\sigma}}\, n_t^{1-\frac{1}{\sigma}} + \alpha^{\frac{1}{\sigma}}\left(\frac{k_{t-1}}{k_0}\right)^{1-\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{B.21}$$

$$\frac{P_t^i}{q_t^k} = 1 - \frac{\kappa_i}{2}\left(\frac{i_t}{i_{t-1}}-1\right)^2 - \kappa_i\left(\frac{i_t}{i_{t-1}}-1\right)\frac{i_t}{i_{t-1}} + \mathbb{E}_t\left\{\beta\frac{\lambda_{d,t+1}}{\lambda_{d,t}}\frac{q_{t+1}^k}{q_t^k}\kappa_i\left(\frac{i_{t+1}}{i_t}-1\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right\}, \tag{B.22}$$

$$q_t^k = \mathbb{E}_t\left\{\beta\frac{\lambda_{d,t+1}}{\lambda_{d,t}}\left[P_{v,t+1}\left[\alpha v_{x,t+1}k_0/k_t\right]^{\frac{1}{\sigma}} + (1-\delta)\, q_{t+1}^k\right]\right\}, \tag{B.23}$$

$$P_{v,t}\frac{\kappa}{q_t} = P_{v,t}\left[(1-\alpha)\, v_t/n_t\right]^{\frac{1}{\sigma}} - W_t + (1-s)\,\mathbb{E}_t\left\{\beta\frac{\lambda_{d,t+1}}{\lambda_{d,t}}P_{v,t+1}\frac{\kappa}{q_{t+1}}\right\}, \tag{B.24}$$

$$k_t = (1-\delta)\, k_{t-1} + \left[1-\frac{\kappa_i}{2}\left(\frac{i_t}{i_{t-1}}-1\right)^2\right] i_t. \tag{B.25}$$

Wage determination (1 equation):

$$W_t = \gamma\left\{\eta P_{v,t}\left[(1-\alpha)\, v_{x,t}/n_t\right]^{\frac{1}{\sigma}} + (1-\eta)\,(B_u+h)\right\} + (1-\gamma)\, W. \tag{B.26}$$

Labor market flows (5 equations):

$$\mathcal{M}_t = \xi\left(u_t^s\right)^{\iota}\nu_t^{1-\iota}, \tag{B.27}$$

$$n_t = (1-s)\, n_{t-1} + q_t\nu_t, \tag{B.28}$$

$$q_t = \mathcal{M}_t/\nu_t, \tag{B.29}$$

$$u_t = 1 - n_t, \tag{B.30}$$

$$u_t^s = u_{t-1} + s n_t \tag{B.31}$$

Intermediate goods production (6 equations):

$$m_t \equiv \left[(1-\zeta_{d,e})^{\frac{1}{\eta_{d,e}}}\, m_{x,t}^{1-\frac{1}{\eta_{d,e}}} + \zeta_{d,e}^{\frac{1}{\eta_{d,e}}}\, m_{e,t}^{1-\frac{1}{\eta_{d,e}}}\right]^{\frac{\eta_{d,e}}{\eta_{d,e}-1}}, \tag{B.32}$$

$$\frac{m_{x,t}}{m_{e,t}} = \frac{1-\zeta_{d,e}}{\zeta_{d,e}}\left(\frac{P_{x,m,t}}{P_{e,t}}\right)^{-\eta_{d,e}}, \tag{B.33}$$

$$P_{m,t} = \left[(1-\zeta_{d,e})\, P_{x,m,t}^{1-\eta_{d,e}} + \zeta_{d,e}P_{e,t}^{1-\eta_{d,e}}\right]^{\frac{1}{1-\eta_{d,e}}}, \tag{B.34}$$

$$m_{x,t} \equiv \left[(1-\zeta_m)^{\frac{1}{\eta_m}}\, m_{H,t}^{1-\frac{1}{\eta_m}} + \zeta_m^{\frac{1}{\eta_m}}\, m_{F,t}^{1-\frac{1}{\eta_m}}\right]^{\frac{\eta_m}{\eta_m-1}}, \tag{B.35}$$

$$\frac{m_{H,t}}{m_{F,t}} = \frac{1-\zeta_m}{\zeta_m}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta_m}, \tag{B.36}$$

$$P_{x,m,t} = \left[(1-\zeta_m)\, P_{H,t}^{1-\eta_m} + \zeta_m P_{F,t}^{1-\eta_m}\right]^{\frac{1}{1-\eta_m}}. \tag{B.37}$$

Production of domestic goods (3 equations):

$$y_{H,t} \equiv \left[(1-\zeta_v)^{\frac{1}{\eta_v}} m_t^{1-\frac{1}{\eta_v}} + \zeta_v^{\frac{1}{\eta_v}} v_t^{1-\frac{1}{\eta_v}}\right]^{\frac{\eta_v}{\eta_v-1}}, \tag{B.38}$$

$$\frac{m_t}{v_t} = \frac{1-\zeta_v}{\zeta_v}\left(\frac{P_{m,t}}{P_{v,t}}\right)^{-\eta_v}, \tag{B.39}$$

$$P_{H,t}^f = \left[(1-\zeta_v) P_{m,t}^{1-\eta_v} + \zeta_v P_{v,t}^{1-\eta_v}\right]^{\frac{1}{1-\eta_v}}. \tag{B.40}$$

Export production (6 equations):

$$\varkappa_t \equiv \left[(1-\zeta_{\varkappa,e})^{\frac{1}{\eta_{\varkappa,e}}} \varkappa_{x,t}^{1-\frac{1}{\eta_{\varkappa,e}}} + \zeta_{\varkappa,e}^{\frac{1}{\eta_{\varkappa,e}}} \varkappa_{e,t}^{1-\frac{1}{\eta_{\varkappa,e}}}\right]^{\frac{\eta_{\varkappa,e}}{\eta_{\varkappa,e}-1}}, \tag{B.41}$$

$$\frac{\varkappa_{x,t}}{\varkappa_{e,t}} = \frac{1-\zeta_{\varkappa,e}}{\zeta_{\varkappa,e}}\left(\frac{P_{x,\varkappa,t}}{P_{e,t}}\right)^{-\eta_{\varkappa,e}}, \tag{B.42}$$

$$P_{\varkappa,t}^f = \left[(1-\zeta_{\varkappa,e}) P_{x,\varkappa,t}^{1-\eta_{\varkappa,e}} + \zeta_{\varkappa,e} P_{e,t}^{1-\eta_{\varkappa,e}}\right]^{\frac{1}{1-\eta_{\varkappa,e}}}, \tag{B.43}$$

$$\varkappa_{x,t} \equiv \left[(1-\zeta_x)^{\frac{1}{\eta_x}} \varkappa_{H,t}^{1-\frac{1}{\eta_x}} + \zeta_x^{\frac{1}{\eta_x}} \varkappa_{F,t}^{1-\frac{1}{\eta_x}}\right]^{\frac{\eta_x}{\eta_x-1}}, \tag{B.44}$$

$$\frac{\varkappa_{H,t}}{\varkappa_{F,t}} = \frac{1-\zeta_\varkappa}{\zeta_\varkappa}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta_\varkappa}, \tag{B.45}$$

$$P_{x,\varkappa.t} = \left[(1-\zeta_x) P_{H,t}^{1-\eta_x} + \zeta_x P_{F,t}^{1-\eta_x}\right]^{\frac{1}{1-\eta_x}}. \tag{B.46}$$

Nominal rigidities (4 equations):

$$(\epsilon_e - 1) - \epsilon_e \frac{P_{e,t}^f}{P_{e,t}} + \kappa_e \left(\frac{P_{e,t}}{P_{e,t-1}} - 1\right)\frac{P_{e,t}}{P_{e,t-1}} = \mathbb{E}_t\left\{\beta\frac{\lambda_{d,t+1}}{\lambda_{d,t}}\kappa_e\left(\frac{P_{e,t+1}}{P_{e,t}} - 1\right)\left(\frac{P_{e,t+1}}{P_{e,t}}\right)^2 \frac{e_{t+1}}{e_t}\right\}, \tag{B.47}$$

$$(\epsilon_H - 1) - \epsilon_H \frac{P_{H,t}^f}{P_{H,t}} + \kappa_H \left(\frac{P_{H,t}}{P_{H,t-1}} - 1\right)\frac{P_{H,t}}{P_{H,t-1}} = \mathbb{E}_t\left\{\beta\frac{\lambda_{d,t+1}}{\lambda_{d,t}}\kappa_H\left(\frac{P_{H,t+1}}{P_{H,t}} - 1\right)\left(\frac{P_{H,t+1}}{P_{H,t}}\right)^2 \frac{y_{H,t+1}}{y_{H,t}}\right\}, \tag{B.48}$$

$$(\epsilon_F - 1) - \epsilon_F \frac{P_{F,t}^f}{P_{F,t}} + \kappa_F \left(\frac{P_{F,t}}{P_{F,t-1}} - 1\right)\frac{P_{F,t}}{P_{F,t-1}} = \mathbb{E}_t\left\{\beta\frac{\lambda_{d,t+1}}{\lambda_{d,t}}\kappa_F\left(\frac{P_{F,t+1}}{P_{F,t}} - 1\right)\left(\frac{P_{F,t+1}}{P_{F,t}}\right)^2 \frac{y_{F,t+1}}{y_{F,t}}\right\}, \tag{B.49}$$

$$(\epsilon_\varkappa - 1) - \epsilon_\varkappa \frac{P_{\varkappa,t}^f}{P_{\varkappa,t}} + \kappa_\varkappa \left(\frac{P_{\varkappa,t}}{P_{\varkappa,t-1}} - 1\right)\frac{P_{\varkappa,t}}{P_{\varkappa,t-1}} = \mathbb{E}_t\left\{\beta\frac{\lambda_{d,t+1}}{\lambda_{d,t}}\kappa_F\left(\frac{P_{\varkappa,t+1}}{P_{\varkappa,t}} - 1\right)\left(\frac{P_{\varkappa,t+1}}{P_{\varkappa,t}}\right)^2 \frac{\varkappa_{t+1}}{\varkappa_t}\right\}. \tag{B.50}$$

Fiscal authority (8 equations):

$$T_t + q_t^b B_{g,t} + T_r + T_c P_{c,t} c_t = P_{g,t} g_t + B_u u_{w,t} + B_{g,t-1}, \tag{B.51}$$

$$T_{d,t} - T_d = \kappa_T \left(B_{g,t-1} - B_g\right), \tag{B.52}$$

$$g_t \equiv \left[\left(1-\zeta_{g,e}\right)^{\frac{1}{\eta_{g,e}}} g_{x,t}^{1-\frac{1}{\eta_{g,e}}} + \zeta_{g,e}^{\frac{1}{\eta_{g,e}}} g_{e,t}^{1-\frac{1}{\eta_{g,e}}} \right]^{\frac{\eta_{g,e}}{\eta_{g,e}-1}}, \tag{B.53}$$

$$\frac{g_{x,t}}{g_{e,t}} = \frac{1-\zeta_{g,e}}{\zeta_{g,e}} \left(\frac{P_{x,g,t}}{P_{e,t}}\right)^{-\eta_{g,e}}, \tag{B.54}$$

$$P_{g,t} = \left[\left(1-\zeta_{g,e}\right) P_{x,g,t}^{1-\eta_{g,e}} + \zeta_{g,e} P_{e,t}^{1-\eta_{g,e}} \right]^{\frac{1}{1-\eta_{g,e}}}, \tag{B.55}$$

$$g_{x,t} \equiv \left[\left(1-\zeta_g\right)^{\frac{1}{\eta_g}} g_{H,t}^{1-\frac{1}{\eta_g}} + \zeta_g^{\frac{1}{\eta_g}} g_{F,t}^{1-\frac{1}{\eta_g}} \right]^{\frac{\eta_g}{\eta_g-1}}, \tag{B.56}$$

$$\frac{g_{H,t}}{g_{F,t}} = \frac{1-\zeta_g}{\zeta_g} \left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta_g}, \tag{B.57}$$

$$P_{x,g,t} = \left[\left(1-\zeta_g\right) P_{H,t}^{1-\eta_g} + \zeta_g P_{F,t}^{1-\eta_g} \right]^{\frac{1}{1-\eta_g}}. \tag{B.58}$$

Aggregation (8 equations):

$$B_t = \psi B_{w,t} + (1-\psi) B_{d,t}, \tag{B.59}$$

$$c_t = \psi c_{w,t} + (1-\psi) c_{d,t}, \tag{B.60}$$

$$y_t = \psi y_{w,t} + (1-\psi) y_{d,t}, \tag{B.61}$$

$$n_t = \psi n_{w,t}, \tag{B.62}$$

$$u_t = \psi u_{w,t}, \tag{B.63}$$

$$T_t = (1-\psi) T_{d,t}, \tag{B.64}$$

$$\varsigma = (1-\psi) \varsigma_{d,t}, \tag{B.65}$$

$$B_t = B_{g,t} + B_t^*. \tag{B.66}$$

Aggregate profits (6 equations):

$$\Pi_t^v = P_{x,v,t} v_{x,t} - W_t n_t - P_{i,t} i_t - \kappa \nu_t, \tag{B.67}$$

$$\Pi_t^e = \left(P_{e,t} - P_{e,t}^f\right) e_t, \tag{B.68}$$

$$\Pi_t^H = \left(P_{H,t} - P_{H,t}^f\right) y_{H,t}, \tag{B.69}$$

$$\Pi_t^F = \left(P_{F,t} - P_{F,t}^f\right) y_{F,t}, \tag{B.70}$$

$$\Pi_t^{\varkappa} = \left(P_{\varkappa,t} - P_{\varkappa,t}^f\right) \varkappa_t, \tag{B.71}$$

$$\Pi_t = \Pi_t^v + \Pi_t^e + \Pi_t^H + \Pi_t^F + \Pi_t^{\varkappa}. \tag{B.72}$$

Market clearing (15 equations):

$$P_{H,t}y_{H,t} = P_{H,t}c_{H,t} + P_{H,t}i_{H,t} + P_{H,t}g_{H,t} + P_{H,t}\varkappa_{H,t} + P_{H,t}m_{H,t}, \tag{B.73}$$

$$P_{e,t}e_t = P_{e,t}c_{e,t} + P_{e,t}i_{e,t} + P_{e,t}g_{e,t} + P_{e,t}\varkappa_{e,t} + P_{e,t}m_{e,t} + P_{e,t}v_{e,t}, \tag{B.74}$$

$$P^f_{F,t}y_{F,t} = P^f_{F,t}c_{F,t} + P^f_{F,t}i_{F,t} + P^f_{F,t}g_{F,t} + P^f_{F,t}x_{F,t} + P^f_{F,t}m_{F,t}, \tag{B.75}$$

$$P_{\mu,t}\mu_t = P^f_{F,t}y_{F,t} + P^f_{e,t}e_t, \tag{B.76}$$

$$\frac{P_{\mu,t}}{P_{\mu,t-1}} = \frac{P^f_{F,t}y_{F,t-1} + P^f_{e,t}e_{t-1}}{P^f_{F,t-1}y_{F,t-1} + P^f_{e,t-1}e_{t-1}}, \tag{B.77}$$

$$P_{D,t}y_{D,t} = P_{c,t}c_t + P_{i,t}i_t + P_{g,t}g_t, \tag{B.78}$$

$$\frac{P_{D,t}}{P_{D,t-1}} = \frac{P_{c,t}c_{t-1} + P_{i,t}i_{t-1} + P_{g,t}g_{t-1}}{P_{c,t-1}c_{t-1} + P_{i,t-1}i_{t-1} + P_{g,t-1}g_{t-1}}, \tag{B.79}$$

$$\text{gdp}_{P,t} = \frac{P_{D,t}y_{D,t} + P_{\varkappa,t}\varkappa_t - P_{\mu,t}\mu_t}{P_{D,t-1}y_{D,t-1}\mathcal{P}^{\text{Laspeyres}}_{D,t} + P_{\varkappa,t-1}\varkappa_{t-1}\mathcal{P}^{\text{Laspeyres}}_{\varkappa,t} - P_{\mu,t-1}\mu_{t-1}\mathcal{P}^{\text{Laspeyres}}_{\mu,t}}, \tag{B.80}$$

$$\mathcal{P}^{\text{Laspeyres}}_{D,t} = \frac{P_{D,t}}{P_{D,t-1}} \tag{B.81}$$

$$\mathcal{P}^{\text{Laspeyres}}_{\mu,t} = \frac{P_{\mu,t}}{P_{\mu,t-1}}, \tag{B.82}$$

$$\mathcal{P}^{\text{Laspeyres}}_{\varkappa,t} = \frac{P_{\varkappa,t}}{P_{\varkappa,t-1}}, \tag{B.83}$$

$$\text{gdi}_{P,t} = \frac{P_{D,t}y_{D,t} + P_{\varkappa,t}\varkappa_t - P_{\mu,t}\mu_t}{\left(P_{D,t-1}y_{D,t-1} + P_{\varkappa,t-1}\varkappa_{t-1} - P_{\mu,t-1}\mu_{t-1}\right)\mathcal{P}^{\text{Laspeyres}}_{D,t}}, \tag{B.84}$$

$$\text{TGI}_{L,t} = 1 + s_{\varkappa,t}\left(\frac{\mathcal{P}^{\text{Laspeyres}}_{\varkappa,t}}{\mathcal{P}^{\text{Laspeyres}}_{D,t}} - 1\right) - s_{\mu,t}\left(\frac{\mathcal{P}^{\text{Laspeyres}}_{d,t}}{\mathcal{P}^{\text{Laspeyres}}_{D,t}} - 1\right), \tag{B.85}$$

$$s_{\varkappa,t} = P_{\varkappa,t}\varkappa_t/\text{gdp}_{P,t}, \tag{B.86}$$

$$s_{\mu,t} = P_{\mu,t}\mu_t/\text{gdp}_{P,t}. \tag{B.87}$$

Net foreign asset position (1 equation):

$$\left(q^b_t B^*_t - B^*_{t-1}\right) = P_{\varkappa,t}\varkappa_t - P_{\mu,t}\mu_t. \tag{B.88}$$

Foreign export demand (2 equations):

$$\varkappa_t = \left[\varkappa\left(\frac{P_{\varkappa,t}}{P^*_{\mu,t}}\right)^{-\eta^*_\varkappa}\right]^{\chi}\varkappa^{1-\chi}_{t-1}, \tag{B.89}$$

$$\frac{P^*_{\mu,t}}{P^*_{\mu,t-1}} = \frac{P^f_{F,t}y^* + P^f_{e,t}e^*}{P^f_{F,t-1}y^* + P^f_{e,t-1}e^*}. \tag{B.90}$$

World interest rate:

$$q_t^b = q^b + \kappa_q \left[\exp\left(B_t^* - B^*\right) - 1\right]. \tag{B.91}$$

Terms of trade (1 equation):

$$\mathcal{T}_t = \frac{P_{\varkappa,t}}{P_{\mu,t}}. \tag{B.92}$$

Exogenous variables (3 equations):

$$P_{F,t}^f, \tag{B.93}$$

$$P_{e,t}^f, \tag{B.94}$$

$$g_t. \tag{B.95}$$

Bringing the grand total to 95 equations. The model's recursive competitive equilibrium is then given by sequences of the following 95 variables:

- Prices (30): $\Big\{\lambda_{w,t+s}, W_{t+s}, \lambda_{d,t+s}, q_{t+s}^{\varsigma}, P_{c,t+s}, P_{x,c,t+s}, P_{H,t+s}, P_{i,t+s}, P_{x,i,t+s}, q_{t+s}, q_{t+s}^k, P_{v,t+s}, P_{m,t+s}, P_{x,m,t+s}, P_{H,t+s}^f, P_{x,\varkappa,t+s}, P_{\varkappa,t+s}^f, P_{\varkappa,t+s}, P_{e,t+s}, P_{F,t+s}, P_{g,t+s}, P_{x,g,t+s}, P_{\mu,t+s}, P_{\mu,t+s}^*, P_{D,t+s}, \mathcal{P}_{D,t+s}^{\text{Laspeyres}}, \mathcal{P}_{\mu,t+s}^{\text{Laspeyres}}, \mathcal{P}_{\varkappa,t+s}^{\text{Laspeyres}}, \mathcal{T}_{t+s}, q_{t+s}^b\Big\}_{s=0}^{\infty}$.
- Quantities (62): $\{c_{t+s}, c_{w,t+s}, c_{d,t+s}, n_{t+s}, n_{w,t+s}, u_{t+s}, u_{w,t+s}, u_{t+s}^s, B_{t+s}, B_{w,t+s}, B_{d,t+s}, T_{t+s}, T_{d,t+s}, \varsigma_{d,t+s}, \Pi_{t+s}, \Pi_{t+s}^v, \Pi_{t+s}^e, \Pi_{t+s}^H, \Pi_{t+s}^F, \Pi_{t+s}^{\varkappa}, c_{x,t+s}, c_{e,t+s}, c_{H,t+s}, c_{F,t+s}, i_{t+s}, i_{x,t+s}, i_{e,t+s}, i_{H,t+s}, i_{F,t+s}, k_{t+s}, \mathcal{M}_{t+s}, \nu_{t+s}, v_{t+s}, m_{t+s}, m_{x,t+s}, m_{e,t+s}, m_{H,t+s}, m_{F,t+s}, y_{H,t+s}, \varkappa_{x,t+s}, \varkappa_{e,t+s}, \varkappa_{H,t+s}, \varkappa_{F,t+s}, \varkappa_{t+s}, e_{t+s}, y_{F,t+s}, g_{x,t+s}, g_{e,t+s}, g_{H,t+s}, g_{F,t+s}, \mu_{t+s}, y_{D,t+s}, \text{gdp}_{P,t+s}, \text{gdi}_{P,t+s}, \text{TGI}_{L,t+s}, s_{\varkappa,t+s}, s_{\mu,t+s}, y_{w,t+s}, y_{d,t+s}, y_{t+s}, B_{g,t+s}, B_{t+s}^*\}_{s=0}^{\infty}$.
- Exogenous processes (3): $\Big\{P_{F,t+s}^f, P_{e,t+s}^f, g_{t+s}\Big\}_{s=0}^{\infty}$.