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Estimates of the Financial Cycle for Advanced Economies

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In this research we therefore propose a bi-variate state space model of credit and house prices that enables us to identify a single shared financial cycle. We obtain estimates of the financial cycle for a panel of 18 advanced economies.

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Abstract

Until recently macroeconomic theory provided at most a small role for the financial system to influence the real economy. This changed with the collapse of Lehman Brothers in 2008. Financial quantities such as credit and house prices are now believed to have real macroeconomic effects. In order to study these effects we need to quantify the influence of the financial system. The financial cycle, characterized by long period cyclical movements in financial variables, may provide such a measure. In this research we therefore propose a bi-variate state space model of credit and house prices that enables us to identify a single shared financial cycle. The financial cycle is modeled as an unobserved trigonometric cycle component with a long period. We identify one shared financial cycle by imposing rank reduction on the covariance matrix of the error vector driving the financial cycle component. This rank reduction can be justified based on a principal components argument. We obtain estimates of the financial cycle for a panel of 18 advanced economies.

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1 Introduction

Until recently macroeconomic theory provided at most a small role for the financial system to influence the real economy beyond the effects of the interest rate set by monetary authorities. However, since the collapse of Lehman Brothers in 2008 and the Euro crisis in 2010, it has become clear that the financial system has the power to greatly influence the real economy. The financial cycle might represent an important driver of the effect of the financial system on the rest of the economy. In this research we want to determine whether we can plausibly identify a single financial cycle for a panel of 18 advanced economies. In follow-up research we explore the possibility that these financial cycle estimates influence the fiscal multiplier Soederhuizen *et al.* (2019).

The principal stylized facts of the financial cycle is that it evolves in a similar fashion to the business cycle: it cycles between persistent periods of higher and lower activity, while over the long run the cycle has no net impact. Furthermore, the average period of the financial cycle is longer than that of the business cycle, typically lasting for 15 to 20 years, while the business cycle has a shorter average period of roughtly 7 to 10 year. We refer the reader to the recent work of Jordà *et al.* (2018), Borio (2014), Claessens *et al.* (2012) and Schüler *et al.* (2015) for details.

To estimate the financial cycle for each country we formulate a bivariate State Space Model, or SSM of credit and the housing price index. We based our estimates on a model of credit and housing prices because they are generally seen as the principal series behind the financial cycle, see for example de Winter *et al.* (2017) and Rünstler & Vlekke (2018). In our SSM we model the financial cycle as an unobserved trigonometric cycle component. This cycle component effectively represents a higher order autoregressive process that tends to exhibit persistent periods of down and upturns, but has no long-run impact on the level of a series. Although in the long run the average cycle length will be determined by an estimated parameter representing the cycle's period, the estimated down and upturns of the cycle are time-varying, being driven by the disturbance term of the cycle component. As a result the estimated cycle is determined by the data.

The use of an unobserved trigonometric cycle component in SSM's to capture cyclical dynamics is standard in the literature, see for example Harvey (1991) and Koopman *et al.* (1999). In fact, our SSM includes two unobserved trigonometric cycle components: one for the financial cycle and another for the business cycle. The inclusion of two cyclical components in the context of unobserved component time series models using Bayesian estimation techniques was earlier proposed in Harvey *et al.* (2007) to better model the business cycle. In this article the authors propose using two independently specified cycles with the same period, arguing that this allows the model to tend toward a band-pass filter as discussed in Baxter & King (1999). Our model differs from this research in that we specify two cyclical components each of which has its own period: one shorter period cycle to capture the business cycle and one longer-period cycle for the financial cycle.

We rely on rank reduction in our model to identify a single underlying financial cycle. In the model both credit and the housing price index have their own financial cycles. By imposing rank reduction on the covariance matrix of the stochastic error vector of the financial cycle component, we ensure that they share a single stochastic error process. This results asymptotically in the same financial cycle for both series. Our use of rank reduction to estimate a unique financial cycle for a country is as far as we know new to the literature.

We justify this rank reduction based on a principal components argument: the largest eigenvalue of the unrestricted covariance matrix of the disturbance vector driving the financial cycle components typically represent roughly 99% of the sum of the eigenvalues. This suggests that the covariance of rank one is sufficient to capture the most important aspects of both cycles. We note, however, that the rank reduction is not supported by a model test based on the Bayes factor.

In addition to the business and financial cycle, our model also includes unobserved components to capture time-varying seasonality, trends and growth rates. This results in slowly changing underlying trends and growth rates driving the development of the series. The estimated seasonal patterns and business cycles have no long-run impact on the level of the series, as is also the case for the estimated financial cycle. By explicitly modeling these underlying processes influencing the series, we can control for their effects when estimating the financial cycle. Note that this type of model is also referred to as an unobserved component time series model. We refer the reader to Harvey (1991) and Durbin & Koopman (2001) for further details on these types of models.

We perform our estimation using Bayesian methods based on Marco Chain Monte Carlo, or MCMC simulation. A Bayesian approach has the advantage that we can include prior information in our estimation to help identify the model. For example our priors assume that the financial cycle has a longer period than the business cycle, and that the underlying growth rate only gradually changes over time.

In the existing literature there are a number of articles in which the financial cycle is modeled as an unobserved trigonometric cycle component in a SSM. In Galati *et al.* (2016), Rünstler & Vlekke (2018) and WGEM (2018) the authors obtain financial cycle estimates for several financial series. In Koopman & Lucas (2005) and de Winter *et al.* (2017) the authors propose SSM's with both business and financial cycles modeled as unobserved trigonometric cycle components. These articles provide cycle estimates for various European countries. Of these articles only WGEM

(2018) makes use of Bayesian estimation methods. The other articles all employ maximum likelihood techniques for the estimation. Mostly importantly, however, none of the cited articles produce estimates of an unique financial cycle for each country.

Our model differs also in other ways from those referenced above. For one, the other SSM's are more restricted in the stochastic processes governing the trend and drift components. Secondly, we include seasonal components in our model, which allows us to base our estimates on seasonally unadjusted data. There have been a number of articles published in which the authors argue that estimates based on seasonally adjusted data are to be preferred. The problem with seasonally adjusted data is that it tends to introduce spurious cyclicality in the data, see for example Luginbuhl & Vos (2003), Harvey *et al.* (2007) and references therein.

An additional innovation involved in our estimation of the fincancial cycle is our mixed-frequency data set, which combines yearly data with more recent quarterly data. The annual data represents a fourth quarter measurement, while the first three quarters of the year are taken as missing. This results in a longer data set, which allows us to estimate over a sample period containing more completed cycles. Our model-based method facilitates the estimation with missing observation, because the estimation of SSM's with missing data is standard, see for example Koopman *et al.* (1999) for details.

Finally, we note that other researcher employ filter-based methods to estimate financial cycles. For example, Jordà *et al.* (2018) propose identifying financial cycles through the use of a bandpass filter using the same long-period annual data we use. Schüler *et al.* (2015) base their estimates of the financial cycle for European countries via a frequency domain based approach. Their data set begins in 1970. Rozite *et al.* (2016) propose a method of estimating a financial cycle for the US based on principal component analysis for data from 1973 to 2014. The Bank of International Settlements, or BIS publishes estimates of their financial cycle index based on Drehmann *et al.* (2012). These estimates involve the use of filtering as well as turning points.

We argue, however, that a model-based approach to the estimation of the financial cycle has a number of advantages. It allows us to simultaneously account for the effects of changing growth rates and seasonal patterns, and the business cycle. This model-based approach also allows us to easily include prior information about the unobserved components in the model and to produce model consistent forecasts both of the financial cycle as well as the other unobserved components and the observed series. These benefits are either lacking or difficult to realize with filter-based methods.

We begin by laying out our model in Section 2, after which we formulate the priors we use in Sections 3. This will be followed by Sections 4 and 5 in which we discuss the data and the estimation procedure. In Section 6 we present our results. We end the article in Section 7 with a discussion of our conclusions.

2 The SSM specification

We begin with the specification of the measurement equation. The measurement equation specifies how the unobserved components and measurement error combine to produce the measured data. Our measured data is the logarithm of the real level of the two financial series, which are assumed to follow a long run trend. This trend is in turn influenced by a growth rate that slowly varies over time. The business cycle and financial cycles cause longer frequency fluctuations around this slowly moving trend. Therefore when the financial cycle is larger than zero, financial market conditions are above their long-term trend. As a result the cycle components are assumed to produce no permanent changes to the level of the series, only temporary ones. Our model also includes seasonal factors to capture the seasonal pattern in the data.

For each country, therefore, we specify a measurement equation in which the observed data is denoted by y_{it} for $i = 1$ and 2 for the credit and house price index series, respectively, at period t. Each of the country's series is assumed to consist of a growing trend, μ_{it}, two stationary cyclical processes representing the business and financial cycles, a set of seasonal components and a measurement error, ε_{it}. The business cycle component is denote by ψ_{it}^{B}, and the financial cycle component by ψ_{it}^{F}. The seasonal components are denoted by $\gamma_{i,j,t}$. This results in the following measurement equation.

$$y_{it} = \mu_{it} + \psi_{it}^{F} + \psi_{it}^{B} + \sum_{j=1}^{[s/2]} \gamma_{i,j,t} + \varepsilon_{it}, \quad \vec{\varepsilon}_t \sim N\left(0, \Omega_{\varepsilon,t}\right) \tag{1}$$

Note that we adopt the notation $\vec{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t})$ here and throughout the paper. The measurement error covariance $\Omega_{\varepsilon,t}$ is assumed to be time-varying. This is because for most of the countries we analyze, the earlier parts of their sample periods suffers from a higher degree of variability due to the fact that the initial part of their sample periods consist of yearly data of lower quality, while the latter part of the sample periods for all countries consists of quarterly data. This leads us to specify the measurement covariance matrix as

$$\Omega_{\varepsilon,t} = \begin{bmatrix} \sigma_{\varepsilon,1} I\left(t \geq T_1^*\right) + \sigma_{h,1} I\left(t < T_1^*\right) & 0 \\ 0 & \sigma_{\varepsilon,2} I\left(t \geq T_2^*\right) + \sigma_{h,2} I\left(t < T_2^*\right) \end{bmatrix}. \tag{2}$$

We denote the indicator function here by $I(\cdot)$, therefore the date T_i^* is the date of the first quarter of the sample period with the lower variance $\sigma_{\varepsilon,i}$ for series i. In initial period is assumed to have the higher variance $\sigma_{h,i}$. This point is discussed below in more detail in Section 4.[1]

[1]An alternative formulation could involve allowing for this type of time-varying change in the covariance matrices of the other unobserved components in the model. Experimenting with a model version in which we impose the time-varying structure in (2) on the trend disturbance covariance instead of the measurement

Values for T_i^* are listed in Table B.2 in Appendix B.

The unobserved component μ_{it} in (1) represents a type of time-varying trend called a local linear trend:

$$\mu_{it} = \mu_{i,t-1} + \beta_{i,t-1} + \eta_{it}, \quad \vec{\eta}_t \sim N(0, \Omega_\eta), \quad \Omega_\eta = \begin{bmatrix} \sigma_{\eta_1} & 0 \\ 0 & \sigma_{\eta_2} \end{bmatrix} \tag{3}$$

Note that the covariance matrix Ω_η is restricted to be diagonal to achieve a more parsimonious model.[2] The β_{it} is an unobserved component that represents the time-varying growth rate of the trend. It evolves as a random walk:

$$\beta_{it} = \beta_{i,t-1} + \zeta_{it}, \quad \vec{\zeta}_t \sim N(0, \Omega_\zeta) \tag{4}$$

The two components of the trend μ_{it} and β_{it} together are responsible for the slowly changing, growing trend in the data. Together they make up what is known as a local linear trend component.

Both unobserved components in (1) ψ_{it}^F and ψ_{it}^B are cyclical components. In general a cyclical component ψ_{it}^C (where $C = F$ indicates a financial cycle, and $C = B$ a business cycle) evolves as follows.

$$\begin{pmatrix} \psi_{it}^C \\ \psi_{it}^{C*} \end{pmatrix} = \rho^C \begin{pmatrix} \cos\frac{2\pi}{\lambda^C} & \sin\frac{2\pi}{\lambda^C} \\ -\sin\frac{2\pi}{\lambda^C} & \cos\frac{2\pi}{\lambda^C} \end{pmatrix} \begin{pmatrix} \psi_{i,t-1}^C \\ \psi_{i,t-1}^{C*} \end{pmatrix} + \begin{pmatrix} \kappa_{it}^C \\ \kappa_{it}^{C*} \end{pmatrix} \tag{5}$$

Further we have that $\vec{\kappa}_t^C \sim N\left(0, \Omega_\kappa^C\right)$ and $\vec{\kappa}_t^{C*} \sim N\left(0, \Omega_\kappa^C\right)$. Note that the covariance matrices of both disturbance vectors $\vec{\kappa}_t^C$ and $\vec{\kappa}_t^{C*}$ are restricted to be equal. This restriction is standard, see Harvey (1991) for details. The cycle parameter ρ^C determines the persistence of the cycle ψ_{it}^C, and λ^C represents the period of the cycle.[3] We note that the unobserved component ψ_{it}^{C*} is only required for the construction of the cycle component ψ_{it}^C. The specification is stationary and ensures that when included in the measurement equation that the changes it induces in the data are temporary.

We are interested in the question of whether there is a single underlying financial cycle. In an attempt to answer this question, we take the approach of imposing a single underlying financial cycle in our model. We achieve this by restricting the rank of the covariance matrix of the financial cycle components Ω_κ^F to one instead of the full-rank value of two. In this manner both of the financial cycles for the series in the model are assumed to be driven by the same underlying stochastic process.

disturbance covariance makes no difference to the estimates we obtain for the rest of the model.

[2]The posteriors of Ω_η tend to be small, so this restriction is of little practical significance.

[3]The period of the cycle is given by $2\pi/\lambda^C$.

We also impose the restriction on both covariance matrices Ω_κ^B and Ω_κ^F to require that their implied correlation between credit and the housing price index is positive. In other words we assume that shocks to the financial cycle for credit and the housing price index produce movement in the same direction for both cycles. Economically this seems reasonable. In a financial boom, we would expect both credit and housing prices to increase. It seems reasonably to assume that this should also hold for the business cycle. We note that these restrictions seem to have little to no affect on our estimates.

An alternative approach to adding a trigonometric cycle component to a SSM is given in Koopman & Lucas (2005) and de Winter *et al.* (2017). We discuss this alternative further in Appendix A. In general, however, the central difference with our approach here is due to how the cycle components are formulated. In our model the measurement equation (1) includes cycle components that are specified with correlated disturbance terms. In the alternative model by comparison, there are two underlying cycle components which by construction are independent. It is also possible to formulate a single financial cycle in this alternative model. This would be based on the idea that the same underlying financial cycle affects both series in the model. This point is discussed in more detail in the appendix.

The unobserved seasonal components γ_{ijt} are also cyclical components with period $\lambda_j = \frac{2\pi j}{4}$ and are constructed together with γ_{ijt}^* components in the same manner as in (5). Note that $j = 1, \ldots, 2$ in the case of quarterly data, because the number of periods in a year is given by s, and $[s/2]$ represents the largest integer $\leq s/2$. Furthermore, for seasonal components it is standard to impose the restriction that the dampening coefficient $\rho_j = 1$. The seasonal component γ_{ijt} is then given by the following.

$$\begin{pmatrix} \gamma_{ijt} \\ \gamma_{ijt}^* \end{pmatrix} = \begin{bmatrix} \cos\lambda_j & \sin\lambda_j \\ -\sin\lambda_j & \cos\lambda_j \end{bmatrix} \begin{pmatrix} \gamma_{ij,t-1} \\ \gamma_{ij,t-1}^* \end{pmatrix} + \begin{pmatrix} \omega_{ijt} \\ \omega_{ijt}^* \end{pmatrix} \tag{6}$$

Note that $\vec{\omega}_{jt} \sim N(0, \Omega_\omega)$ and $\vec{\omega}_{jt}^* \sim N(0, \Omega_\omega)$, and that we use the standard restriction that the covariance matrices of $\vec{\omega}_{jt}$ and $\vec{\omega}_{jt}^*$ for $j = 1, \ldots, 2$ are diagonal and equal. The reader is referred to Harvey (1991) for further details.

To complete our model specification we must also specify priors for the initial values of the unobserved components. We therefore specify the priors we use in the following section before turning to the discussion of the estimation method.

3 Priors

The model we propose has a fair number of parameters, making the model quite flexible. There are therefore parameter regions that we would prefer to rule out. We achieve this using somewhat informative prior on some of the parameters. We also specify weakly informative priors to help achieve our business and financial cycle decompositions with cycle periods for the business cycle that are relatively short and for the financial cycle that are relatively long. For the other parameters we specify a low prior number of degrees of freedom and select the prior scaling factor centered around the main posterior density mass. In this way we specify fairly uninformative empirical Bayes priors. We discuss the various prior specifications we use for each unobserved component.

3.1 Cycles

Both cycle components require priors for the dampening coefficients ρ^C, the cycle periods λ^C, and the disturbance covariance matrices Ω_κ^C, for $C = B$ and F, see (5) above. Given that the dampening coefficients $\rho^C \in [0, 1)$, we specify a beta distribution for these priors. Note that apriori we want $\rho^C < 1$ to ensure that the cycle components are stationary and that the cycle disturbances have no permanent effects on the long run level of the series. The priors for the cycle periods $\lambda^C \in (4, \infty)$ for quarterly data, follow gamma distributions. The priors for the covariance matrices Ω_κ^C are inverse Wishart distributions.

The beta priors are parameterized as $Beta\left(\alpha_p^C, \beta_p^C\right)$, for $C = B$ and F.[4] For the business cycle component, ρ^B, we set $\alpha_p^B = 55.88$ and $\beta_p^B = 1.925$. This implies a prior mean of 0.967, with a standard deviation of 0.0234. This prior relatively diffuse and has little impact on the posteriors. The prior parameters for the financial cycle components' parameter ρ^F, are give by $\alpha_p^F = 321.3$ and $\beta_p^B = 4.617$. These parameters imply a posterior mean of 0.986 and standard deviation of 0.0065. Although this prior is more spread out than the posteriors, the posteriors tend to lie slightly above the prior. This prior is therefore somewhat informative in that it tends to pull the posterior away from the value of 1. Experimenting with differing prior parameters suggests that our results are not very sensitive to this prior.

The prior gamma distribution for the λ^C is denoted by $Gamma\left(a^C, b^C\right)$, for $C = B$ and F.[5] These priors are formulated using a Bayesian highest density region, or HDR. In the case of the business cycle, we make the prior assumption that the probability that the business cycle period is between five to ten years is 99%: $P\left(20 \text{ quarters} < \lambda^B \leq\ 40 \text{ quarters}\right) = 99\%$. This results in the prior parameter values of $a^B = 55.88$ and $b^B = 4.617$ for the gamma prior of λ^B. We formulate our prior for the financial cycle period λ^F in a similar fashion. Here we employ the 99% prior HDR of between 15 to 20 years: $P\left(60 \text{ quarters} < \lambda^F \leq\ 80 \text{ quarters}\right) = 99\%$. This implies the prior parameter values of $a^F = 321.3$ and $b^F = 1.925$ for the gamma prior of λ^F. Alternative priors based on the same HDR intervals, but with lower probabilities, such as 95%

[4]The density function of $Beta\left(\alpha_p, \beta_p\right)$ is then given by $f\left(x\right) = x^{\alpha_p - 1}\left(1 - x\right)^{\beta_p - 1}/B\left(\alpha_p, \beta_p\right)$.

[5]The density function of $Gamma\left(a, b\right)$ is then given by $f\left(x\right) = \frac{b^a}{\Gamma(a)}x^{a-1}\exp^{-b\,x}$.

or 90%, result in similar estimates. If, however, we increase these intervals to encompass longer periods, then this can alter our estimates. For example an HDR for λ^F based on the interval from 20 to 25 years tends to result in somewhat different financial cycle estimates. On the whole, however, we believe that our priors for the cycle periods represent the values most cited in the literature, see for example Drehmann *et al.* (2012) and Borio (2014). Although somewhat informative, these priors still allow the posteriors to be largely determined by the data.

We denote the prior inverse Wishart distribution for Ω_κ^C by $\mathcal{W}^{-1}\left(\nu^C, S^C\right)$, for $C = B$ and F.[6] The prior parameter ν^C represents the number of degrees of freedom. For both the business and financial cycle we set $\nu^B = \nu^F = 13$. Korea is an exception: in this case we use $\nu^F = 6$ to ensure that the prior is weak enough to allow the likelihood to dominate the prior in the posterior. We then select the positive (semi) definite matrix S to ensure that the mean of the posterior is unaffected by the prior. These values for S^C for $C = B$ and F are listed in Table B.1 in Appendix B.

To complete the prior specification for the cycle component we also need to specify priors for the initial values of the cycle components $\psi_{i,0}^C$ and $\psi_{i,0}^{C*}$ for $i = 1$ and 2 and $C = B$ and F. Provided that the dampeningen coefficients $\rho^B < 1$ and $\rho^F < 1$, which given our beta priors is the case, the cycle components' initial conditions are $\psi_{i,t}^C \sim N\left(0, \sigma_{\kappa_i^C} / \left(1 - \rho^{C^2}\right)\right)$ and $\psi_{i,t}^{C*} \sim N\left(0, \sigma_{\kappa_i^C} / \left(1 - \rho^{C^2}\right)\right)$, when we also specify that

$$\Omega_\kappa^C = \begin{pmatrix} \sigma_{\kappa_1}^C & \sigma_{\kappa_{12}}^C \\ \sigma_{\kappa_{12}}^C & \sigma_{\kappa_2}^C \end{pmatrix} \tag{7}$$

for $i = 1$ and 2 and $C = B$ and F.[7] We also make the standard assumption that the initial values of the cycles are uncorrelated.

3.2 Trend & Growth Rates

The two trend components $\mu_{i,t}$ in (3) and the two growth rates $\beta_{i,t}$ in (4) follow random walks. They are therefore non-stationary. As a result we assume diffuse priors for their initial values. We discuss the use of diffuse priors for the non-stationary elements of the state below in section 5.

[6] The density function of $\mathcal{W}^{-1}\left(\nu^C, S^C\right)$ is then given by

$$f(X) = \frac{|S|^{\nu/2}}{2^\nu \Gamma_2\left(\frac{\nu}{2}\right)} |X|^{-(\nu+3)/2} e^{-\frac{1}{2} tr\left(SX^{-1}\right)}$$

.

[7] Alternatively, Harvey & Streibel (1998) argue for an alternative specification where the prior variance of $\psi_{i,0}^C = \sigma_{\kappa,i}^C$, so that the variance of $\kappa_{i,t}^C = \sigma_{\kappa,i}^C\left(1 - \rho^{C\,2}\right)$. This allows the cycle component to remain stationary, although deterministic, as $\rho_i^C \to 1$.

The inverse Wishart prior degrees of freedom for the disturbance covariance matrices Ω_η for the trend component and Ω_ζ for the growth rate component are $\nu_\eta = 11$ and $\nu_\zeta = 83$, respectively. The values for the prior parameter matrices S_η and S_ζ are listed in Table B.2 of Appendix B.

In general 11 degrees of freedom for the inverse Wishart distribution produces a prior that is relatively uninformative. We select the values for S_η to ensure that the highest prior density region corresponds to that of the posterior.[8] In this way the priors for Ω_η are selected to have minimal impact on the form of the posteriors. This essentially an empirical Bayes approach.

Our prior specification for the Ω_ζ are more informative. We interpret the drift components $\beta_{i,t}$ as representing the underlying growth rates. As such we believe *apriori* that these rates will only change gradually over time. It is however common in SSM's of macroeconomic time series with a local linear trend, such as we have specified here, that the likelihood tends to favor larger values for the variance of the disturbance of the drift component. These larger values for the variance imply a relatively quickly changing growth rate. In the case of our model we believe that these changes ought to be captured by the cycles in the model. For this reason we specify the larger prior parameter value of $\nu_\zeta = 83$ for Ω_ζ of the growth rate component. This then represents a more informative prior. Compared with the information in the two sets of more than 200 observations of the sample periods, this number of degrees of freedom is still fairly modest. We specify diagonal elements of the prior parameter matrices S_ζ which correspond to modest changes over time in the growth rates $\beta_{i,t}$. The off-diagonal elements are assumed to be zero indicating a prior of no correlation between the growth rates of credit and the housing price index.

In those instances where the marginal posterior variance for ζ_{it} was lower than our initial prior specification would suggest[9], we lowered the corresponding value in S_ζ to match the posterior. In two instances, for the Dutch credit series and the Swedish housing price index, we adjusted the priors to correspond to larger values of these variances to accommodate for the more dramatic swings in these series during the Great Depression and Second World War.

3.3 Seasonal Components

The covariance matrices Ω_{ω_1} and Ω_{ω_2} in (6) are assumed to be diagonal. Therefore the prior parameter matrices S_{ω_1} and S_{ω_2} are as well. In all cases we set the number of degrees of freedom

[8] The off-diagonal elements of S_η are zero, because Ω_η is diagonal. These priors are therefore equivalent to inverse-gamma priors with the inverse gamma distribution parameters $\alpha_{\eta_i} = \nu_\eta/2$ and $\beta_{\eta_i} = s_{\eta_i}/2$.

[9] We initially specify a prior on Ω_ζ that implies an expected value of 0.08 for both σ_{ζ_1} and σ_{ζ_2}.

of these inverse Wishart priors to $\nu_{\omega_1} = \nu_{\omega_2} = 11$ and

$$S_{\omega_1} = S_{\omega_2} = \begin{bmatrix} 0.0002 & 0 \\ 0 & 0.0002 \end{bmatrix}. \tag{8}$$

Both the credit and housing price index series exhibit only a slight degree of seasonality. We specify diffuse priors on the initial values $\gamma_{i,j,0}$ and $\gamma^*_{i,j,0}$, because these components are non-stationary.

3.4 Measurement Error Covariance

To specify a prior on the covariance matrix $\Omega_{\varepsilon,t}$ of the measurement error as given in (2), we need to specify priors on Ω_η and Ω_h where

$$\Omega_\eta = \begin{bmatrix} \sigma_{\varepsilon_1} & 0 \\ 0 & \sigma_{\varepsilon_2} \end{bmatrix}, \quad \Omega_h = \begin{bmatrix} \sigma_{h_1} & 0 \\ 0 & \sigma_{h_2} \end{bmatrix}. \tag{9}$$

We use inverse Wishart priors: $P(\Omega_\eta) \sim \mathcal{W}^{-1}(\nu_\eta, S_\eta)$ and $P(\Omega_h) \sim \mathcal{W}^{-1}(\nu_h, S_h)$. We can define the matrices S_η and S_h as follows.

$$S_\eta = \begin{bmatrix} s_{\eta_1} & 0 \\ 0 & s_{\eta_2} \end{bmatrix} \quad S_h = \begin{bmatrix} s_{h_1} & 0 \\ 0 & s_{h_2} \end{bmatrix} \tag{10}$$

In Table B.3 of Appendix B we list the elements of the prior parameter matrices S_ε and S_h.[10] We also list in this table the dates T_i^* when our model transitions to the lower measurement error variance, see (2). We set the degrees of freedom $\nu_\varepsilon = \nu_h = 40$, with the exception of $\nu_{\eta_2} = 4000$ for Korea. The exceptional value for the Korean housing price index proved necessary to ensure the numerical stability of the Kalman filter in our MCMC estimation. The problem arrises due to the presence of missing values for the Korean housing price index in the beginning of the sample period. We discuss this point below in section 5 on the estimation of our model. First, however, we discuss the data.

4 The data

Our sample consists of credit data and housing price indices for 18 advanced economies: Australia, Belgium, Canada, Denmark, Finland, France, Germany, Ireland, Italy, Japan, the Nether-

[10]Given that the measurement errors between the two series are uncorrelated, these priors are equivalent to inverse-gamma priors on σ_{ε_i} and σ_{h_i} with the inverse-gamma distribution parameters $\alpha_{\varepsilon_i} = \nu_\varepsilon/2$, $\alpha_{h_i} = \nu_h/2$, $\beta_{\varepsilon_i} = s_{\varepsilon_i}/2$ and $\beta_{h_i} = s_{h_i}/2$, where $i = 1$ and 2.

lands, Norway, South Korea, Spain, Sweden, Switzerland, the United Kingdom, and the United States. To identify the main features of financial cycles we work with mixed yearly and quarterly data to include as many observations as possible and thereby obtain the maximum number of completed financial cycles in each country. In Table B.2 in Appendix B we list the starting date of the sample period for each country. All sample periods end in the fourth quarter of 2017.

The credit series for each country is for total credit to the private non-financial sector, measured as the stock of outstanding credit at the end of the quarter. This credit series and the housing price index are both published by the Bank of International Settlements, or BIS on a quarterly basis. For earlier values, when no quarterly values are available, we rely on the yearly credit data published in Jordà *et al.* (2017) and the yearly housing price indices published in Knoll *et al.* (2017). In this case the annual data represents a fourth quarter measurement, and the first three quarters of the year are missing. This requires us to estimate with missing quarterly observations. Our estimation method however is able to accommodate the missing values that the use of this yearly data necessarily entails.

Both the credit series and housing price indices are deflated using consumer price indices. To this end, we combine data from different sources on CPI measures. For all the countries we use monthly CPI from the OECD, and additionally, where prior data was not available, we use other sources. See Table C.1 in Appendix C for a full description of these sources and their starting dates.

Inspection of the data indicates that the earlier yearly data is more volatile. This motivated our decision to use the split measurement error variance in (2). We identify the transition dates T_i^* for $i = 1$ and 2 in (2) when the data transitions to a lower level of variability by determining at what point the data transition to more reliable sources from the documentation of the data series given in Jordà *et al.* (2017) for the credit data and in Knoll *et al.* (2017) for the housing price indices. These dates are listed in Table B.2 of Appendix B.

The BIS also produces a financial cycle index for each country in our panel, see Drehmann *et al.* (2012) for details.[11] The disadvantage of the BIS indices, however, is that they are only available starting in 1970. With the exception of Ireland, we are able to produce estimates that start earlier, typically before 1960. In fact in the case of Belgium, Canada, the Netherlands, Norway, Sweden, Switzerland, the UK and the US, our sample period starts in the early 1900's. As we show in Section 6, over the shorter period covered by the BIS financial index, our financial cycle estimates are substantially similar.

[11] The BIS provided us with their financial cycle estimates.

5 Estimation

Our data sets for each country consist of a combination of yearly and quarterly data. As a result, our estimation procedure must be able to accommodate missing observations in the first three quarters of each year in which we use annual data. We obtain our estimates of the financial cycles using Bayesian MCMC simulation methods. Fortunately the estimation of state space models with MCMC simulation methods in the presence of missing observations is possible and is now standard, see for example Koopman *et al.* (1999).[12] We wrote our own code to perform the MCMC estimation in the matrix programming language OX, see Doornik & Ooms (2007). MCMC simulation techniques are now standard, and we therefore do not discuss these sampling methods in detail. We refer the reader instead to any textbook on Bayesian statistics, such as Koop *et al.* (2007).

For most parameters it is possible to perform the simulation via the Gibbs sampler, or GS. The simulation of the cycle component dampening parameters ρ^B and ρ^F and period parameters λ^B and λ^F is not possible via the GS. In order to simulate these parameters we used the Metropolis-Hastings algorithm, or MH algorithm. The imposition of rank reduction on the covariance matrix Ω_κ^F also introduces an additional degree of complexity to the MCMC simulation. This involves both extra steps in the GS, as well as the use of the MH algorithm. We first briefly describe how the GS works with our model, and then discuss our implementation of the MH algorithm. We then describe how we tackle the problems introduced by the rank reduction in Ω_κ^F.

5.1 Gibbs Sampling

As is commonly done with SSM's, we augment the set of model parameters to simulate in the GS with the disturbance terms from our model. Given values for the model parameters, we can simulate the disturbances terms in our model using the disturbance smoother as implemented in SsfPack, see Koopman *et al.* (1999) for details. Once we have simulated the disturbance terms we then simulate new values of the covariance matrices of our model from their posterior distributions conditional on the drawn values of the disturbance terms. Given the assumed normality of the disturbance terms in the model and the conjugate inverse Wishart priors we specify on the covariance matrices of our model, the conditional posteriors from which we draw the new covariance values also follow an inverse Wishart distribution: $\mathcal{W}^{-1}(\nu, S)$. In this standard case, we have that the posterior degrees of freedom ν is given by the sum of the prior

[12]We have encountered stability issues with the Kalman filter and related algorithms in certain areas of the parameter space of our model, introduced by the presence of missing observation at the beginning of the sample period. However, in the relevant region of the parameter space for our estimation the Kalman filter-based algorithms remained well behaved.

degrees of freedom ν_p and the number of observations, T: $\nu = T + \nu_p$, and that the posterior parameter matrix S is equal to the sum of the prior matrix parameter S_p and the sum of outer product of the residual vector R: $S = S_p + R$.

In general the GS works by repeatedly circling through the two simulation blocks of drawing the disturbances and drawing the covariances. Asymptotically, by repeatedly re-simulating all the values, we obtain drawings from the unconditional joint posterior of the model parameters and disturbances.[13] This is however only true if we can also include a method to obtain updated drawings for ρ^B, ρ^F, λ^B and λ^F, as well as for the reduced rank covariance matrix Ω_κ^F. Drawing ρ^B, ρ^F, λ^B and λ^F is not feasible in the GS as we do not know any easily derived conditional posterior from which we could draw new values. Instead we use the MH algorithm.

5.2 Metropolis-Hastings Algorithm

We use the MH Algorithm when we are unable to draw new parameter values directly from the appropriate conditional posterior required by the GS. Instead we draw a new parameter value from a candidate distribution. We either accept this new draw, or reject it and keep the original value from the previous draw. The decision to reject or accept the candidate drawing is based on the value of δ_c:

$$\delta_c = \frac{P(\theta_n) L(Y|\theta_n, \theta_{-n}) f_c(\theta_{n-1}|\theta_n)}{P(\theta_{n-1}) L(Y|\theta_{n-1}, \theta_{-n}) f_c(\theta_n|\theta_{n-1})}. \tag{11}$$

When $\delta_c \geq 1$ we automatically accept the candidate value. When $\delta_c < 1$ we accept the candidate value with probability δ_c. Note that in (11) $P(\theta_n)$ represents the prior density of the parameter θ at the value given by the candidate drawing θ_n at step n of the MCMC algorthim. The value of the previous draw is denoted by θ_{n-1}. The value of the likelihood given the candidate parameter value θ_n and the other model parameters values in the MCMC algorithm θ_{-n} is denote by $L(Y|\theta_n, \theta_{-n})$. The density of the parameter value θ_n obtained from the candidate density function is then given by $f_c(\theta_n|\theta_{n-1})$. Note that the form of the candidate density can depend on the previously drawn parameter value θ_{n-1}. In our implementation this is the case.

For the cycle period parameters λ^B and λ^F we draw candidate values from the gamma distribution with an expected value equal to the previously drawn period value. Similarly for the dampening coefficients ρ^B and ρ^F we draw candidate values from the beta distribution also with an expected value equal to the previously drawn dampening coefficient value.[14] We obtain the required values of the likelihood from the diffuse Kalman Filter based on the prediction error decomposition of the likelihood. In our program we perform one Metropolis-Hastings rejection

[13]Via the disturbances we can also obtain drawings of the state vector: the trend, growth rate, cycles and seasonal components. The reader is referred to Koopman *et al.* (1999) for details.

[14]This leaves an additional distribution parameter to be fixed, both in the case of the gamma and of the beta candidate distributions. We tune this value to ensure a rejection rate of between 20% and 50%.

step for the four cycle parameters jointly.[15]

5.3 Sampling Ω_κ^F with Rank Reduction

In the presence of rank reduction, such as we impose on Ω_κ^F, drawing a new value for the covariance matrix is more complicated. Part of the covariance matrix can be simulated via the GS. The rest we draw using the MH algorithm. To see how we use the GS here, let us consider the general case of the covariance matrix Ω^C which has the reduced rank of $r < n$. We begin by first drawing a new value for Ω^C given the current simulated values of the associated disturbances $\vec{\kappa}_t^C$, $t = 1, \ldots, T$. Given the newly simulated value of Ω^C we then draw new values of the disturbances $\vec{\kappa}_t^C$, $t = 1, \ldots, T$ to complete the required GS steps.

We begin with the GS draw of Ω^C, and denote the conditional posterior of Ω^C in the GS by $\mathcal{W}^{-1}\left(\nu^C, S^C\right)$. Now consider the eigenvalue decomposition of the $n \times n$ parameter matrix, $S^C = E\Lambda E'$, where the matrix of eigenvectors E is given by $E = [\vec{e}_1, \ldots, \vec{e}_n]$ such that $E'E =$ the $n \times n$ identity matrix I_n, and Λ is a diagonal matrix with the eigenvalues λ_{Si}, $i = 1, \ldots, n$ along its diagonal. S^C has the reduced rank of $r < n$. If we order the eigenvalues from largest to smallest, then we have that $\lambda_{S,n-r+1} = \ldots \lambda_{Sn} = 0$. We can then denote the $n \times r$ matrix of r eigenvectors corresponding to the r non-zero eigenvalues as $E_r = [\vec{e}_1, \ldots, \vec{e}_r]$, and in the same manner the $r \times r$ diagonal matrix of non-zero eigenvalues as Λ_r. We can now re-write S^C as follows.

$$S^C = E_r \Lambda_r {E_r}' \tag{12}$$

To obtain a draw for the reduced rank covariance matrix Ω^C from the inverse Wishart distribution $\mathcal{W}^{-1}\left(\nu^C, S^C\right)$, we define the matrix Σ^C:

$$\Sigma^C = E_r \Lambda_r^{\frac{1}{2}}. \tag{13}$$

Then we draw the $r \times r$ full rank matrix $\hat{X}$ from the standard Wishart distribution: $X \sim \mathcal{W}\left(\nu^C, I_r\right)$ and obtain

$$\hat{Q} = \Sigma^C \hat{X} \Sigma^{C\prime}. \tag{14}$$

We now perform the eigenvalue decomposition of $\hat{Q}$, which is $n \times n$ and of rank r, so that $\hat{Q} = E_{Qr} \Lambda_{Qr} E'_{Qr}$ as in (12). The reduced rank drawing $\hat{\Omega}^C$ for the covariance Ω^C is then given by

$$\hat{\Omega}^C = E_{Qr} \Lambda_{Qr}^{-1} E'_{Qr}. \tag{15}$$

To complete the required steps of the GS for our model, we must now draw new values of for the disturbances $\vec{\kappa}_t^F$, $t = 1, \ldots, T$. However, this is also more complicated than for the other

[15]We repeat these joint MH drawings eight times in each cycle through the GS.

disturbances associated with the unrestricted covariances in the model. The reduced rank of Ω_κ^F causes statistical degeneracy in the joint distribution of the disturbances $\vec{\kappa}_t^F$, $t = 1, \ldots, T$. For this reason in our model of credit and the house prices where $n = 2$, we can only draw either κ_{1t}^F, $t = 1, \ldots, T$ or κ_{2t}^F, $t = 1, \ldots, T$ in the disturbance smoother, see Koopman *et al.* (1999) for a detailed discussion.

Once again we return to the more general case. To draw the $n \times 1$ disturbance vectors $\hat{\kappa}_t^C$, $t = 1, \ldots, T$ given the newly drawn covariance matrix $\hat{\Omega}^C$ with rank $r < n$, we assume that we have ordered the disturbance vectors $\vec{\kappa}_t^C$ and $\hat{\Omega}^C$ so that we have

$$\vec{\kappa}_t^C = \begin{pmatrix} \vec{\kappa}_{at}^C \\ \vec{\kappa}_{bt}^C \end{pmatrix}, \tag{16}$$

where $\vec{\kappa}_{at}^C$ represents the r elements of $\vec{\kappa}_t^C$ that we can simulate with the disturbance smoother, and $\vec{\kappa}_{bt}^C$ represents the $n - r$ remaining disturbances that we cannot obtain from the disturbance smoother due to the problem of statistic degeneracy caused by the rank reduction.[16] Similarly to (13), from the eigenvalue decomposition of $\hat{\Omega}^C$, where $\hat{\Omega}^C = \hat{E}_r \hat{\Lambda}_r \hat{E}_r'$, we then have that

$$\hat{\Sigma} = \hat{E}_r \hat{\Lambda}_r^{\frac{1}{2}}. \tag{17}$$

As a result, $\hat{\Omega}^C = \hat{\Sigma}\hat{\Sigma}'$. Therefore, with the unknown $r \times 1$ vector $\vec{\epsilon}_t \sim N(0, I_r)$, we have that the newly simulated values $\hat{\kappa}_{at}^C$ of the disturbances $\vec{\kappa}_{at}^C$, $t = 1, \ldots, T$ satisfy the following.

$$\hat{\kappa}_t^C = \begin{pmatrix} \hat{\kappa}_{at}^C \\ \hat{\kappa}_{bt}^C \end{pmatrix} = \hat{\Sigma}\hat{\epsilon}_t = \begin{bmatrix} \hat{\Sigma}_a \\ \hat{\Sigma}_b \end{bmatrix} \hat{\epsilon}_t, \tag{18}$$

where $\hat{\Sigma}_a$ is $r \times r$ and $\hat{\Sigma}_b$ is $(n - r) \times r$, both sub-matrices of $\hat{\Sigma}$, so that

$$\hat{\Omega}^C = \begin{bmatrix} \hat{\Sigma}_a \hat{\Sigma}_a' & \hat{\Sigma}_a \hat{\Sigma}_b' \\ \hat{\Sigma}_b \hat{\Sigma}_a' & \hat{\Sigma}_b \hat{\Sigma}_b' \end{bmatrix}. \tag{19}$$

Given the simulated values $\hat{\kappa}_{at}^C$ from the disturbance smoother, from (18) we have the following.

$$\hat{\epsilon}_t = \hat{\Sigma}_a^{-1} \hat{\kappa}_{at}^C, \tag{20}$$

Note that $\hat{\Sigma}_a^{-1}$ exists because the $r \times r$ sub-matrix $\hat{\Sigma}_a \hat{\Sigma}_a'$ from the top left corner of Ω^C in (19)

[16] The disturbance smoother in SsfPack requires the specification of the diagonal selection matrix Γ which is the same dimension as the state vector with either ones on the diagonal, or zeros for the corresponding stochastically degenerate elements of the state. Therefore, in our estimation procedure, Γ specifies the r elements of $\vec{\kappa}_{at}^C$, see Koopman *et al.* (1999) for details. We adjust the value of Γ so as to select the r series with the strongest cycle estimates.

has full rank by construction.[17] By combining the results from (18) and (20), we can see that we can recover $\hat{\kappa}_{bt}^{C}$ from the following.

$$\hat{\kappa}_{bt}^{C} = \hat{\Sigma}_{b}\hat{\Sigma}_{a}^{-1}\hat{\kappa}_{at}^{C} \tag{21}$$

We have now obtained the simulated disturbances $\hat{\kappa}_{t}^{C}$, $t = 1, \ldots, T$, which, together with the simulated covariance matrix $\hat{\Omega}^{C}$ completes the required steps of the GS. This leaves only the steps of the MH algorithm to ensure that Ω_{κ}^{F} is correctly simulated.

To see why we still require additional sampling, consider the rank reduction on Ω_{κ}^{F} where $r = 1$, In (14) the draw $\hat{X}$ is a scalar, whereas the complete draw $\hat{\Omega}_{\kappa}^{F}$ requires of two parameters: both variances, with the covariance being determined by the perfect correlation implied by the rank reduction. Clearly these GS steps only manage to simulate one of the two required parameters in Ω_{κ}^{F}. An additional set of steps using the MH algorithm is required to ensure that we fully sample a new value for Ω^{F}.

In the general case outlined above, the simulated value $\hat{X}$ in (14) is an $r \times r$ symmetric matrix, and therefore is implicitly only defined by $r\left(r+1\right)/2$ univariate elements. In general the $n \times n$ covariance matrix Ω_{κ}^{F} of rank $r < n$ is defined by

$$\frac{n\left(n+1\right) - \left(n-r\right)\left(n-r+1\right)}{2} > \frac{r\left(r+1\right)}{2} \tag{22}$$

univariate elements.

Similarly, if we examine (18), we can see that the disturbance smoother is only implicitly simulates the $r \times 1$ vector $\hat{\epsilon}_{t}$. Because $\hat{\kappa}_{at}^{C} = \hat{\Sigma}_{a\kappa}\hat{\epsilon}_{t}$, there is new information in the conditional posterior distribution of Ω_{κ}^{F} to define a new drawing of $\hat{\Sigma}_{a\kappa}$. We can also see, however, from (20) and (21), that the information in the $r \times 1$ drawing $\hat{\epsilon}_{t}$ is recycled to obtain the $(n-r)$ vector $\hat{\kappa}_{bt}$. There are therefore no new stochastic univariate elements used to construct the $(n-r) \times r$ matrix $\hat{\Sigma}_{b\kappa}$, which defines part of the conditional posterior of Ω_{κ}^{C} in the Gibbs sampling draw discussed above.

We have observed in practice that the term $\hat{\Sigma}_{b}\hat{\Sigma}_{a}^{-1}$ in (21) remains constant in our applications when $r = 1$. In general we denote this $(n-r) \times r$ matrix as B:

$$B = \hat{\Sigma}_{b}\hat{\Sigma}_{a}^{-1}. \tag{23}$$

We vectorize the elements of B and draw them as $\hat{B}_{n}$ from a multivariate normal candidate distribution, $N\left(\hat{B}_{n-1}, S_{B}\right)$, where S_{B} is a diagonal matrix of variances for the vectorized elements of B, and $\hat{B}_{n-1}$ is the previous draw of the elements of B. The variances in S_{B} must be set to

[17] This is due to the assumed ordering of the disturbance vector $\hat{\kappa}_{t}^{C}$ in (16).

be able to perform this application of the Metropolis-Hastings step.[18]

We note that to obtain a complete simulation of the financial cycle vector $\vec{\psi}_t^C$ for $t = 1, \ldots, T$ we require the simulated starting values $\hat{\psi}_0^C$, which we can straight-forwardly obtain from the simulation smoother. Draws for the other set of cycle disturbance vectors $\vec{\kappa}_t^{C*}$, as well as the cycle components $\vec{\psi}_t^{C*}$ for $t = 1, \ldots, T$ can be obtained in the same manner as outlined above. Once the MCMC algorithm has converged we continue to run the simulation steps to obtain a sample from the joint posterior distribution. We can then base our inference on this sample. Standard diagnostics can be used to check for the convergence of the MCMC algorithm.

Our results are based on a minimum of 100,000 replications for each country model, where we throw away the first 50,000 replications as burn-in to ensure that we only sample from the MCMC algorithm once convergence has been achieved. Convergence diagnostics indicate that our MCMC algorithm has converged, the details of which are available on request.

6 The results

For each country, plots of the estimated financial cycle, business cycle, trend and drift components are shown in Appendix D for both credit and the housing price index.[19] These plots show the posteriors over the full sample period for each country. In Figures 1-3 we display the estimated financial cycles based on the posterior mean of the financial cycle for the credit variable from 1950 together with the estimates produced by the BIS which start in 1970 (see Drehmann *et al.* (2012)).

We can gauge the plausibility of our financial cycle estimates based on known historical events such as the systemic banking crisis of the 1990's in Japan, the Swedish, Norwegian and Finish financial crises in the early 1990's, the US savings and loans crisis of 1986, and the Great Recession of 2007/2008, see Reinhart & Rogoff (2009) and Laeven & Valencia (2012) for further details on systemic banking crises. In Figures 1-3 we can see these events reflected by the drop in the respective cycle values during these crisis periods.

In Appendix D we can see the declines in the financial cycles due to the Great Depression as well as the recovery led by World War II and its aftermath in the plots for the eight countries with sample periods that begin before 1930: Belgium, Canada, The Netherlands, Norway, Sweden, Switzerland, The UK and the US. More recently, in the case of The Netherlands we can see the effects of the housing boom from 1976-78 and the crash that followed from 1979-1983.

We note that in Figures 1-3 we only display the financial cycle for credit, because we regard credit as the primary financial variable. Due to the rank reduction in Ω_κ^F, the estimated financial

[18]Through experimentation we tune these variances to produce a rejection rate of between 20% to 50% for the joint test of the elements of B.

[19]Plots of the estimated seasonal components are available on request. Both series exhibit only weak seasonality.

cycle based on the housing price index will be asymptotically identical to that of the financial cycle of credit. This is demonstrated in Table 1 where the first two columns list the correlation coefficients between the financial cycle medians we obtain for the credit and housing price index series. In the first column we see the correlation coefficient over the entire sample period, while the second column lists the correlations coefficient from 1980. Both on average, as well as for all individual countries, the correlation coefficients are higher in the second column and are typically nearly equal to 1. Only Finland (0.91), Italy (0.90), South Korea (0.80) and Norway (0.83) are under the average value of 0.96.

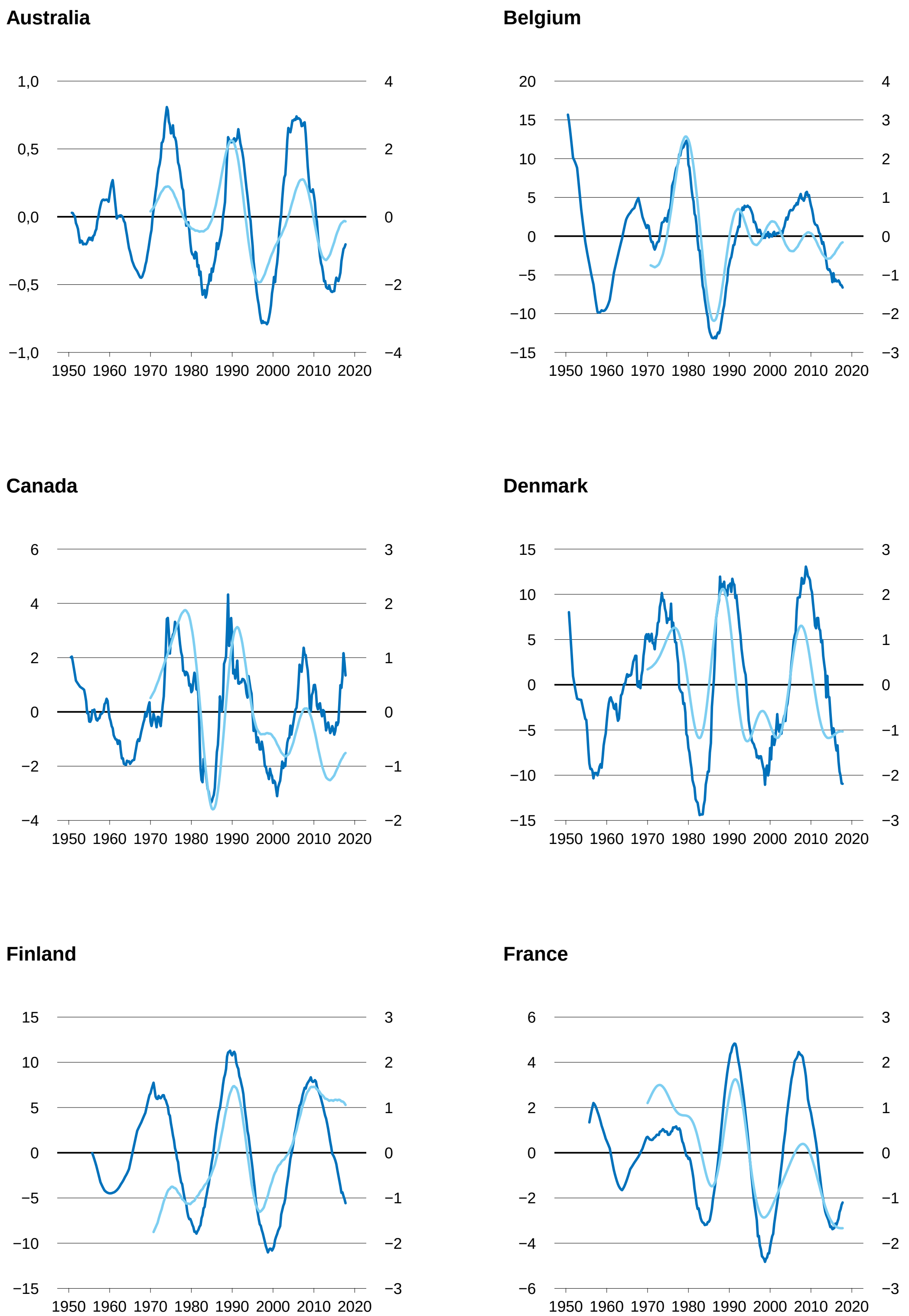
The estimated financial cycles based on the credit and housing price index differ initially due to the assumed independence between the initial values of these two cycles. It would be possible to use the financial cycle components' rank-reduced disturbance covariance matrix to impose the implied reduced-rank covariance on the initial value of these cycles in our estimation. In this manner the two financial cycle estimates would be identical with the exception of a scaling factor. We leave this, however, to future research.

In Table 1 we also list the correlation coefficients between the estimated financial cycles and that of the BIS. In the third column of the table we show the correlation coefficient with the estimated cycle based on the credit series, and in the forth column we show the coefficient based on the housing price index. The average correlation between the BIS cycle estimates and ours based on the credit series is 0.73, which indicates a substantial degree of agreement between the estimates.[20] Of the eighteen countries, only our estimates of the financial cycle for Germany and South Korea show a relatively weak correspondence with those of the BIS. The correlation between our financial cycle estimates and those of the BIS for the other countries is strong. Furthermore, for most countries both financial cycle estimates cross zero at approximately the same time.

Our estimates of a single financial cycle for each country we study rely on the rank reduction in the covariance matrix of the financial cycle components' disturbances. We test the validity of this rank restriction in two ways. First we calculate the log of the posterior data density both with and without the rank restriction. We use the same priors for the unrestricted model as we selected for the restricted model. This should tend to favor the restricted model, given that some of these priors are selected using empirical Bayesian priors. These values are listed in Table 2. We denote the rank of Ω_κ^F in the table by $r\left(\Omega_\kappa^F\right)$. The column denoted by BF, for Bayes Factor, shows the difference between the two log posterior data densities: the restricted value minus the unrestricted value. Positive values indicate support for the restricted model, while negative ones indicate support for the unrestricted model. We are clearly unable to justify

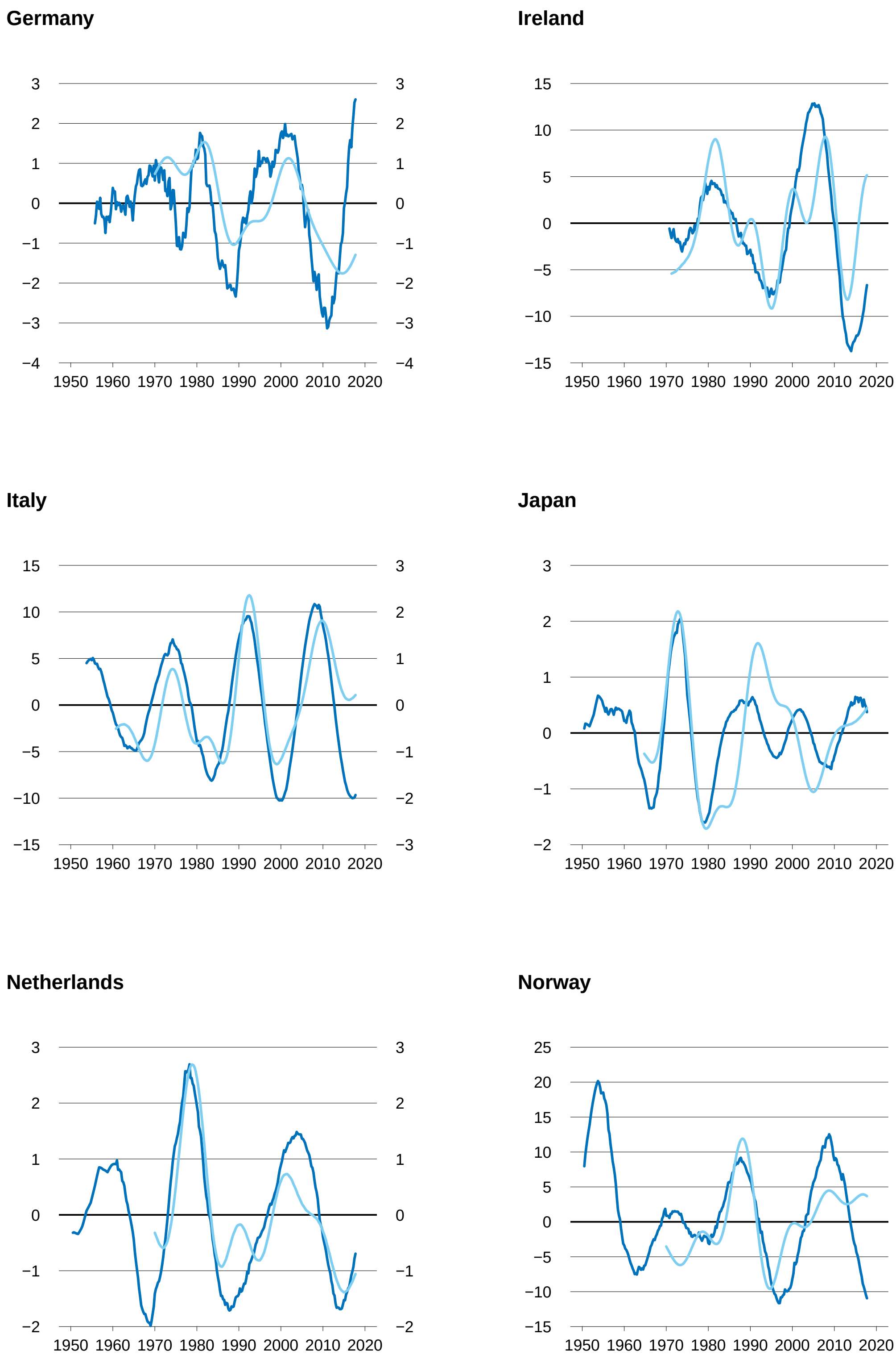
[20]In future work we intend to also compare our estimates with other estimates obtained in the literature such as in de Winter *et al.* (2017), Rünstler & Vlekke (2018) and WGEM (2018).

Figure 1: Financial cycle estimates for credit and from the BIS for advanced economies (i)



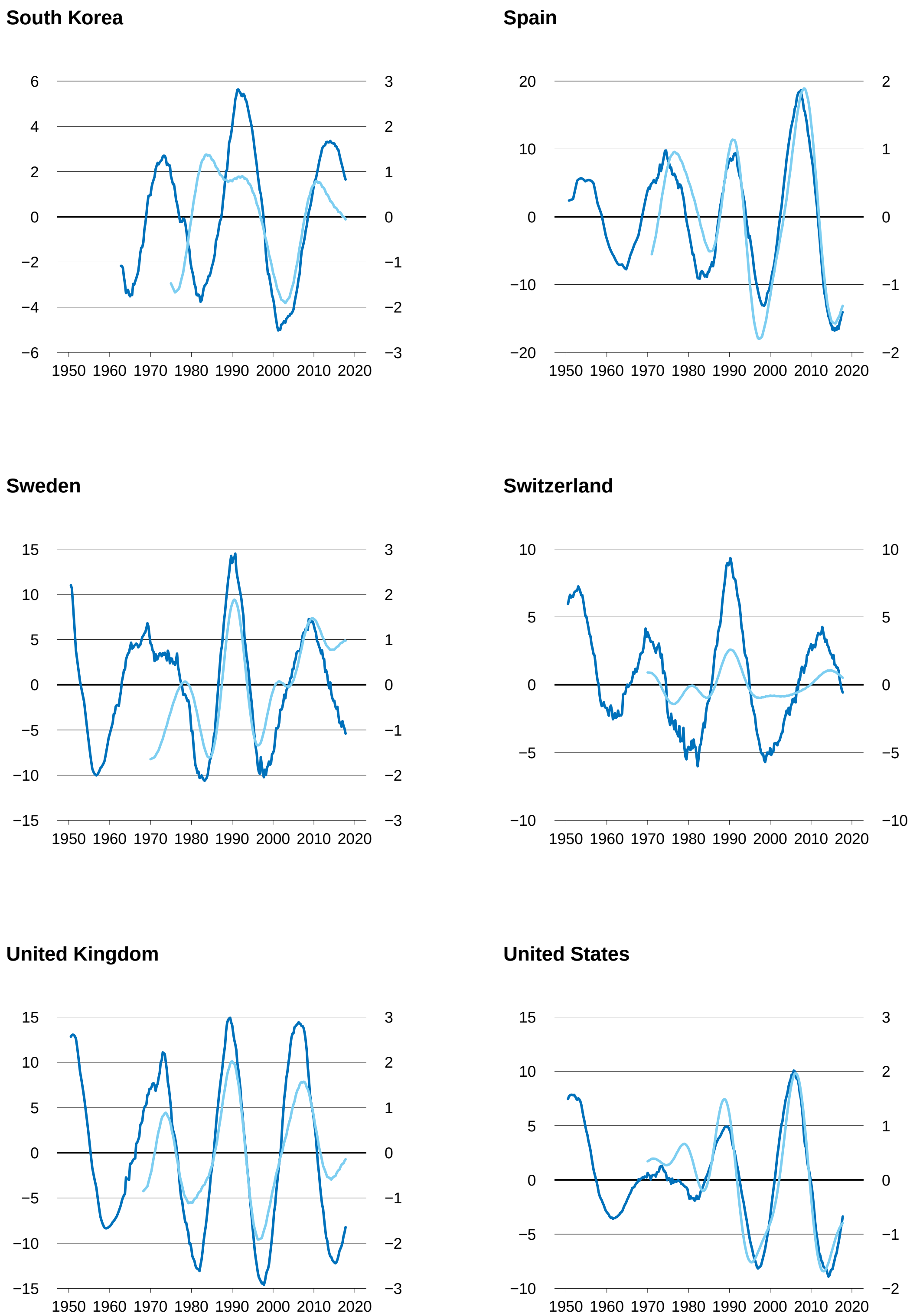
The right axis corresponds to our estimated financial cycle for credit (dark blue line) and the axis on the left corresponds to the estimated financial cycle of the BIS (light blue line).

Figure 2: Financial cycle estimates for credit and from the BIS for advanced economies (*ii*)



The right axis corresponds to our estimated financial cycle for credit (dark blue line) and the axis on the left corresponds to the estimated financial cycle of the BIS (light blue line).

Figure 3: Financial cycle estimates for credit and from the BIS for advanced economies (*iii*)



The right axis corresponds to our estimated financial cycle for credit (dark blue line) and the axis on the left corresponds to the estimated financial cycle of the BIS (light blue line).

Table 1: Correlation coefficients between financial cycle estimates

Between	Credit & House price		BIS &	
Country	all	≥ 1980	Credit	House price
Australia	0.94	0.99	0.81	0.82
Belgium	0.93	0.98	0.84	0.85
Canada	0.61	1.00	0.69	0.69
Denmark	0.93	1.00	0.74	0.72
Finland	0.85	0.91	0.57	0.52
France	0.80	0.97	0.71	0.61
Germany	0.83	0.99	0.52	0.48
Ireland	0.96	0.98	0.67	0.66
Italy	0.69	0.90	0.76	0.70
Japan	0.79	0.97	0.67	0.65
Netherlands	0.96	1.00	0.87	0.87
Norway	0.66	0.83	0.60	0.64
South Korea	0.62	0.80	0.51	0.66
Spain	0.95	0.99	0.89	0.89
Sweden	0.96	1.00	0.63	0.67
Switzerland	0.90	0.99	0.87	0.81
UK	0.75	0.99	0.88	0.88
US	0.84	0.96	0.93	0.88
Average	0.83	0.96	0.73	0.72

the rank reduction on Ω_κ^F based on this test.

In Table 2 however we also report on a second weaker test we employ based on the re-estimation of our model for each country with the unrestriction Ω_κ^F covariance with rank equal to two. For this test we take the value of the larger of the two eigenvalues of the unrestricted Ω_κ^F as a percentage of the eigenvalue sum. This value is shown under the last column of the table. These values are similar to those used in principal components analysis. The percentages shown indicate that nearly all of the variability in the estimated covariance matrices Ω_κ^F is due to the largest eigenvalue. Most values are above 99%, with the lowest value for Sweden still equal to the high value of 94.6%. These results suggest that the restricted model is picking up nearly all the variability in the data related to the financial cycle.

Although the estimation of the business cycle is not the primary purpose of this research, we nonetheless obtain estimates of cycle components with a business cycle periodicity. In Table 3 we list the correlation coefficients between the business cycle components we obtain for credit and the housing price index. These values are given in the first column. The second and third column list the correlation coefficients of a simple business cycle estimate we obtained using the HP filter of the GDP of each country with our credit and housing price index business cycles, respectively. Although the figures in the appendix of the estimated business cycles,

Table 2: Tests of Rank Reduction on Ω_κ^F

	log posterior data density			
Country	$r\left(\Omega_\kappa^F\right)=2$	$r\left(\Omega_\kappa^F\right)=1$	BF	$\frac{100\lambda_1}{\lambda_1+\lambda_2}$
Australia	-660	-696	-36	99.6
Belgium	-829	-850	-20	98.5
Canada	-843	-876	-33	99.9
Denmark	-643	-688	-45	99.9
Finland	-108	-141	-32	99.1
France	-285	-334	-49	99.9
Germany	-254	-285	-32	98.8
Ireland	-691	-718	-28	99.8
Italy	-690	-706	-16	98.8
Japan	-505	-584	-79	99.1
Netherlands	-1070	-1114	-44	99.9
Norway	-909	-933	-23	98.3
South Korea	-538	-557	-20	99.5
Spain	-641	-665	-23	98.4
Sweden	-893	-907	-14	94.6
Switzerland	-748	-775	-27	98.3
UK	-937	-960	-23	99.0
US	-623	-654	-31	99.4

The column header "BF" refers to the Bayes factor for the SSM with $r\left(\Omega_\kappa^F\right)=1$ vs. the SSM with $r\left(\Omega_\kappa^F\right)=2$.

which also show the HP filter business cycle estimated based on GDP, indicate a reasonable degree of co-movement, overall the degree of correlation is low. This suggests that the cyclical factors influencing credit and house prices at this frequency are not primarily determined by the business cycle in output.

The estimates of the trend and drift components, shown in Appendix D, are gradually changing and seem to reasonably capture the long-run trends shown in the data, which is also shown in the figures. The drift components, representing the underlying growth rates of the trends, also generally show the slowdown associated with the secular stagnation of the past decades.

Table 3: Correlation coefficients between business cycle estimates

Between	Credit & House price	HP filter GDP &	
Country		Credit	House price
Australia	0.22	0.06	0.38
Belgium	0.20	0.08	0.19
Canada	0.34	0.23	0.29
Denmark	0.78	0.50	0.55
Finland	0.13	0.10	0.53
France	-0.14	0.15	0.47
Germany	0.23	0.39	0.22
Ireland	0.38	0.44	0.55
Italy	0.12	0.14	0.07
Japan	0.19	0.33	0.46
Netherlands	-0.04	0.32	0.34
Norway	0.14	0.23	0.15
South Korea	0.36	0.08	0.42
Spain	-0.22	0.33	0.44
Sweden	0.24	0.04	0.31
Switzerland	0.29	0.47	0.36
UK	0.20	0.34	0.54
US	0.13	0.53	0.38
Average	0.20	0.26	0.37

The mean posterior values for the period of the financial cycles, λ^F of the 18 advanced economies in our sample were between 63 to 78 quarters, or roughly 16 to 20 years. Values for these posterior means are given in Table E.3 in Appendix E. For the business cycle we obtained posterior mean values for λ^B of between 6 to 10 years. These values are also listed in Table E.3. They are in close agreement with standard values for the business cycle in the literature. Appendix E also lists the posterior means and standard deviations for the other parameters in the models, see Tables E.1, E.2 and E.3.

7 Conclusion

We propose a bi-variate model based estimation of the financial cycle of 18 advanced economies. We model total credit to the private non-financial sector and the housing price index, the two series that are generally regarded as best reflecting the financial cycle. We employ a state space model based on unobserved components to capture the salient features in the data. In particular we specify the financial cycle as an unobserved trigonometric component, with rank reduction imposed on the covariance matrix of this cycle component's disturbance vector to help us to identify a single underlying financial cycle. This use of rank reduction to identify a country's financial cycle is new to the literature.

The rank reduction we impose on the covariance of the financial cycle components' disturbance terms can be justified in a manner that is similar to principal components analysis: the largest eigenvalue of the covariance is never less than 94% of the sum of both eigenvalues for all 18 economies in our sample. The reduction does not however seem to be supported by Bayesian model testing based on the Bayes Factor.

In future research we intend to base our estimates of the financial cycle on more series, such as output, industrial production, spreads and the price to earning ratio. We also will explore the use of the rank-reduced covariance of the financial cycle disturbances to obtain the covariance of the starting vector of the financial cycle. This should result in financial cycles for both the credit and house price series which are identical up to a scaling factor, whereas our estimation results here are only asymptotically identical.

The financial cycle estimates have periods lasting roughly 16 to 20 years. Financial events such as the Japanese banking crisis of 1991, the Scandinavian banking crises of the early 1990's, the US savings and loans crisis and the Great Recession are all reflected in the financial cycle estimates of the respective countries. Our estimates also largely agree with financial cycle estimates produced by the BIS. We conclude that our method succeeds in generating plausible estimates for a unique financial cycle for each country we analyze.

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A Alternative Specification of the Financial Cycle

An alternative specification for a model with a shared financial cycle can be formulated using the notation $\vec{y}_t = (y_1, y_2)$. First, however, note that using this notation, we can re-write (1) as follows.

$$\vec{y}_t = \vec{\mu}_t + \vec{\psi}_t^F + \vec{\psi}_t^B + \sum_{j=1}^{[s/2]} \vec{\gamma}_{j,t} + \vec{\varepsilon}_t \tag{A.1}$$

The alternative formulation of the second general state space model can then be expressed as follows.

$$\vec{y}_t = \vec{\mu}_t + A\,\vec{\psi}_t^F + B\,\vec{\psi}_t^B + \sum_{j=1}^{[s/2]} \vec{\gamma}_{j,t} + \vec{\varepsilon}_t, \tag{A.2}$$

where the matrices A and B are both lower triangular matrices with unity along the main diagonal. The matrix A then is a loading-matrix that determines how much each of the two cycles $\psi_{i,t}^F$ contributes to the data series $y_{i,t}$. The same is true for the loading-matrix B, which determines the weighted contribution of the two cycles $\psi_{i,t}^B$ to the data series $y_{i,t}$. In order to ensure that this model is identified, we must also restriction the covariance matrices Ω_κ to be a diagonal matrices. In other words, the underlying cycle components must be independent.

This second model specification is, with the exception of the alternate specification of the measurement equation shown in (A.2), based on the same equations for the unobserved components shown above in (3) - (5).[21] This model is similar to the models proposed in Koopman & Lucas (2005) and de Winter *et al.* (2017). The difference between our model and theirs is due to how the cycle components are formulated. In the first model the measurement equation (A.1) includes cycle components that are derived by correlated disturbance terms. This has the effect that the cyclical components can differ in their amplitude and phase even when the cycle component disturbances are perfectly correlated. The second model in (A.2) by comparison will be based on underlying cycle components which can only differ in their amplitude when each series selects the same underlying cycle component in the measurement equation (A.2).

We would like to identify a single financial cycle. There are two modeling options we can follow to achieve this. One option is based on (1) and requires the imposition of the restriction that the rank of the covariance matrix of the financial cycle component Ω_κ^f be reduced from two to one. In this manner both financial cycles for the series in the model are assumed to be derived from the same underlying stochastic process.

The second alternative is based on (A.2) with the restriction that the A matrix select the same underlying financial cycle for both series in the model. Although we have experimented with this second modeling technique to estimate a financial cycle, we found it to be too restrictive. Instead we base our estimates in this article on the model in which we impose rank reduction on Ω_κ^f. Our impression is that the extra flexibility of this method produces more plausible results, because the model still allows for differing phase shifts in the financial cycle of each series. On the other hand, the disadvantage of our approach is that it still requires us to chose which of the two estimated financial cycles represents the underlying financial cycle, because initially at least, they may not be the same.

[21] In our formulation of the measurement equation we have also included seasonal components in the model to accommodate seasonally unadjusted data.

We note, however, that these two modeling approaches can be made observationally equivalent. This can be achieved by specifying in our model a joint normal prior on the initial values of the financial cycle components with the prior covariance given by

$$\frac{\Omega_\kappa^F}{1-{\rho^F}^2}, \tag{A.3}$$

where Ω_κ^F is the reduced rank covariance matrix.[22]

B Prior Parameters

The inverse Wishart prior parameters S_κ^B and S_κ^F for the covariance matrices Ω_κ^B and Ω_κ^F, respectively, are listed in Table B.1. Note that $\nu^B = \nu^F = 13$ with the exception of Korea, where $\nu^F = 6$. The parameters $s_{\kappa 1}^B$ and $s_{\kappa 1}^F$ pertain to the credit series, $s_{\kappa 2}^B$ and $s_{\kappa 2}^F$ to the housing price index, and $s_{\kappa 12}^F$ is the scale factor for the covariance between the financial cycle disturbances for credit and the housing price index.[23] The comparable prior scale factor for the covariance of the business cycle disturbance is set to zero for all countries. We have therefore that

$$S_\kappa^B = \begin{bmatrix} s_{\kappa 1}^B & 0 \\ 0 & s_{\kappa 2}^B \end{bmatrix} \quad S_\kappa^F = \begin{bmatrix} s_{\kappa 1}^F & s_{\kappa 12}^F \\ s_{\kappa 12}^F & s_{\kappa 2}^F \end{bmatrix} \tag{B.1}$$

[22]In the case of the alternative formulation, the financial cycle components are independent, and therefore so are their initial values.

[23]The latter value is set to exactly ensure that the rank of $S^F = 1$.

Table B.1: Cycle Disturbance Covariance Priors

	S_κ^B		S_κ^F		
Country	$s_{\kappa_1}^B$	$s_{\kappa_2}^B$	$s_{\kappa_1}^F$	$s_{\kappa_2}^F$	$s_{\kappa_{12}}^F$
Australia	15.0	15.0	0.2	12.0	1.549
Belgium	50.0	15.0	6.0	6.0	6.000
Canada	25.0	1.0	1.0	60.0	7.746
Denmark	2.0	60.0	15.0	0.1	1.225
Finland	25.0	45.0	2.0	3.0	2.450
France	8.0	4.0	0.1	3.0	0.548
Germany	4.0	3.0	1.5	3.0	2.121
Ireland	120.0	40.0	2.0	30.0	7.746
Italy	25.0	20.0	2.0	2.0	2.000
Japan	10.0	14.0	0.2	2.0	0.632
Netherlands	20.0	20.0	0.2	6.0	1.095
Norway	20.0	60.0	8.0	4.0	5.657
South Korea	50.0	15.0	0.3	1.8	0.735
Spain	22.0	30.0	5.0	20.0	10.000
Sweden	20.0	50.0	12.0	7.5	9.487
Switzerland	12.0	22.0	1.5	2.5	1.937
UK	50.0	50.0	5.0	15.0	8.660
US	9.0	9.0	1.2	10.0	3.464

Table B.2: Trend & Drift disturbance Covariance Priors & Sample Starting Date

	S_η		S_ζ		begin
Country	s_{η_1}	s_{η_2}	s_{ζ_1}	s_{ζ_2}	sample period
Australia	0.300	0.300	0.512	0.512	1950 Q4
Belgium	0.400	0.300	0.512	0.512	1921 Q4
Canada	0.300	0.300	0.512	0.200	1914 Q4
Denmark	1.500	1.500	0.512	0.512	1950 Q4
Finland	0.600	0.300	0.008	3.2E-04	1955 Q4
France	0.600	0.300	0.512	0.512	1955 Q4
Germany	1.500	1.500	0.512	0.512	1955 Q4
Ireland	0.400	0.300	0.512	0.512	1970 Q4
Italy	0.600	0.300	0.392	0.512	1953 Q4
Japan	0.100	0.090	0.512	0.512	1949 Q4
Netherlands	0.100	0.100	0.800	0.512	1900 Q4
Norway	0.100	0.095	0.512	0.512	1925 Q4
South Korea	0.100	0.100	0.512	0.128	1962 Q4
Spain	0.500	0.200	0.512	0.512	1950 Q4
Sweden	0.180	0.150	0.512	0.800	1921 Q4
Switzerland	0.100	0.130	0.512	0.512	1921 Q4
UK	0.070	0.100	0.512	0.512	1921 Q4
US	0.016	0.060	0.512	0.200	1914 Q4

Table B.3: Measurement Error Covariance Priors

	S_ε		S_h			
Country	s_{ε_1}	s_{ε_2}	s_{h_1}	T^*_1	s_{h_2}	T^*_2
Australia	0.208	0.062	–	–	1.560	1970 Q4
Belgium	0.308	0.257	1.438	1960 Q4	6.085	1951 Q4
Canada	0.191	0.390	3.822	1954 Q4	7.800	1956 Q4
Denmark	0.117	0.273	1.950	1955 Q4	1.950	1972 Q4
Finland	0.047	0.047	7.800	1971 Q4	7.800	1970 Q4
France	0.020	0.020	9.750	1970 Q4	8.580	1970 Q4
Germany	0.078	0.039	–	–	11.700	1970 Q4
Ireland	0.780	0.780	–	–	–	–
Italy	0.390	0.390	–	–	–	–
Japan	0.047	0.009	9.750	1965 Q4	8.580	1955 Q4
Netherlands	0.234	0.273	9.750	1961 Q4	8.190	1970 Q4
Norway	0.156	0.273	9.750	1954 Q4	7.410	1970 Q4
South Korea	0.507	719.820*	–	–	–	–
Spain	0.078	7.800	1.950	1970 Q4	–	–
Sweden	0.090	0.156	10.335	1961 Q4	7.800	1970 Q4
Switzerland	0.057	0.101	3.510	1949 Q4	3.900	1970 Q4
UK	0.312	0.312	0.936	1963 Q4	0.936	1968 Q4
US	0.033	0.090	7.800	1952 Q4	2.340	1954 Q4

*The degrees of freedom $\nu_{\varepsilon_2} = 4000$ instead of 40. This larger value was required to ensure the numerical stability of the Kalman filter.

C Data series

Table C.1: CPI data for deflating

	Sources and starting dates			
Country	Start monthly OECD	Other source	Freqcuency	start
Australia	1950m7	Australian Bureau of Statistics	Quarterly	1948q3
Belgium	1955m1	STATBEL	Monthly	1920m1
Canada	1950m7	Statistics Canada	Yearly	1914
Denmark	1955m1	FRED St. Louis Fed	Yearly	1950
Finland	1955m1	-	-	-
France	1955m1	-	-	-
Germany	1955m1	Bundesbank	Yearly	1949
Ireland	1955m1	Central Statistics Office Ireland	Monthly	1922m1
Italy	1955m1	FRED St. Louis Fed	Yearly	1953
Japan	1955m1	Statistics of Japan (E-Stat)	Monthly	1946m8
Netherlands	1959m1	CBS - Statistics Netherlands	Yearly	1900
Norway	1955m1	Statistics Norway	Monthly	1925m1
South Korea	1951m8	-	-	-
Spain	1954m3	FRED St. Louis Fed	Yearly	1950
Sweden	1955m1	SCB, Statistics Sweden	Quarterly	1917q3
Switzerland	1955m1	Bundesambt fur statistik	Monthly	1921m1
United Kingdom	1955m1	FRED St. Louis Fed	Quarterly	1914q1
United States	1955m1	FRED St. Louis Fed	Monthly	1913m1

The series from the other sources are scaled to the monthly OECD series to have one full CPI series.

D Estimated Unobserved Components

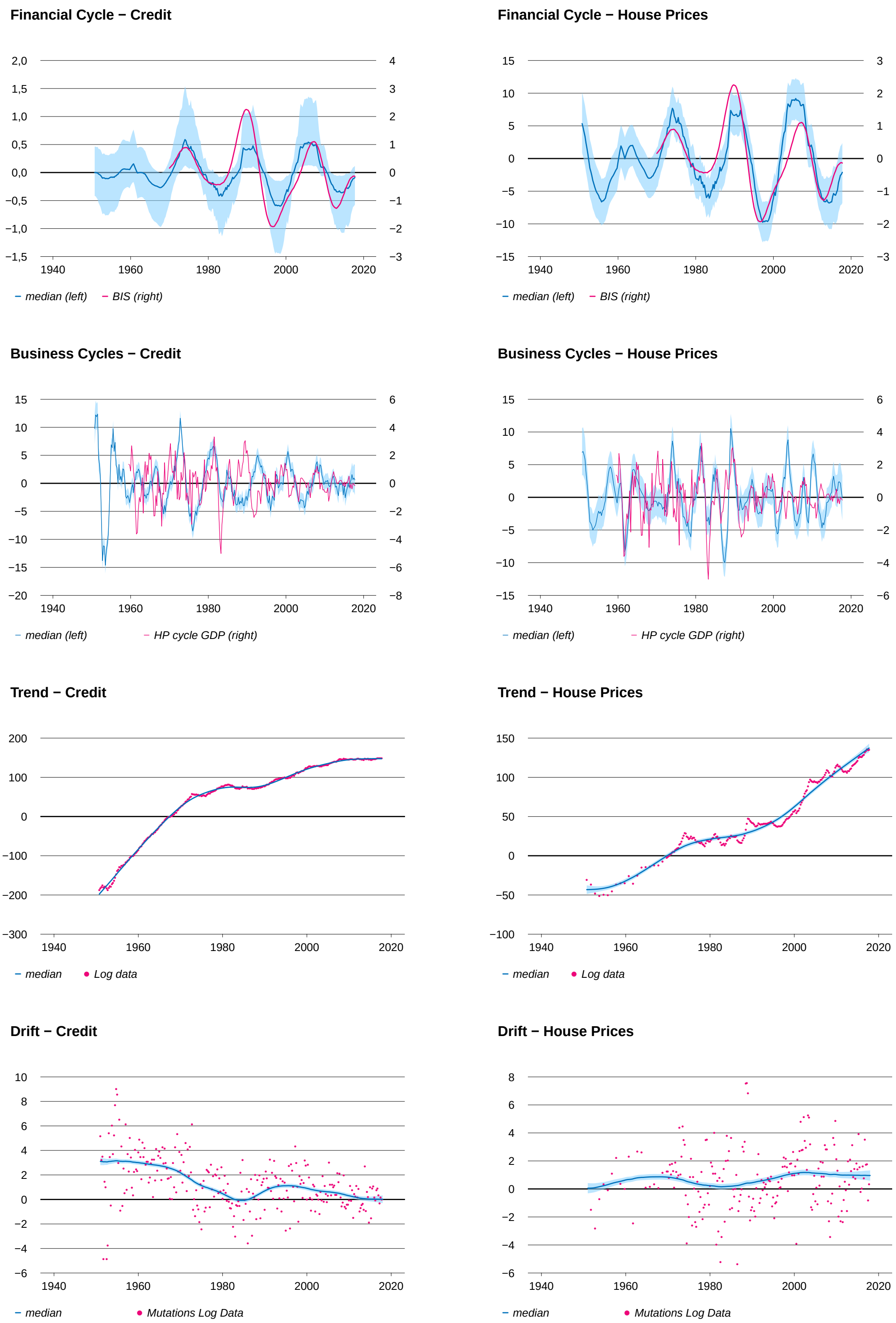
The Figures D.1 - D.18 in this appendix show plots of the medians[24] of the posterior distributions of the unobserved components in the model: the financial cycles, the business cycles, the trends and the drifts. The four plots on the left-hand side of each figure are the estimates of the unobserved components for credit. Those on the right-hand side are based on the housing price index. The plots also show the Bayesian credible interval of 68%, or in other words $\pm$ one standard deviation of the posteriors. For comparision, the plots of the financial cycle estimates also include the BIS estimates of the financial cycle. The same is true of the business cycle,

[24]Given the assumed normality of the disturbances, the median and mean of the posteriors will be equal.

where we also plot the HP-filter cycle of GDP for comparison. The plots of the estimated trends also show the logarithm of original data, while the drift, or growth rate plots show the first difference of the logarithm of the original data.

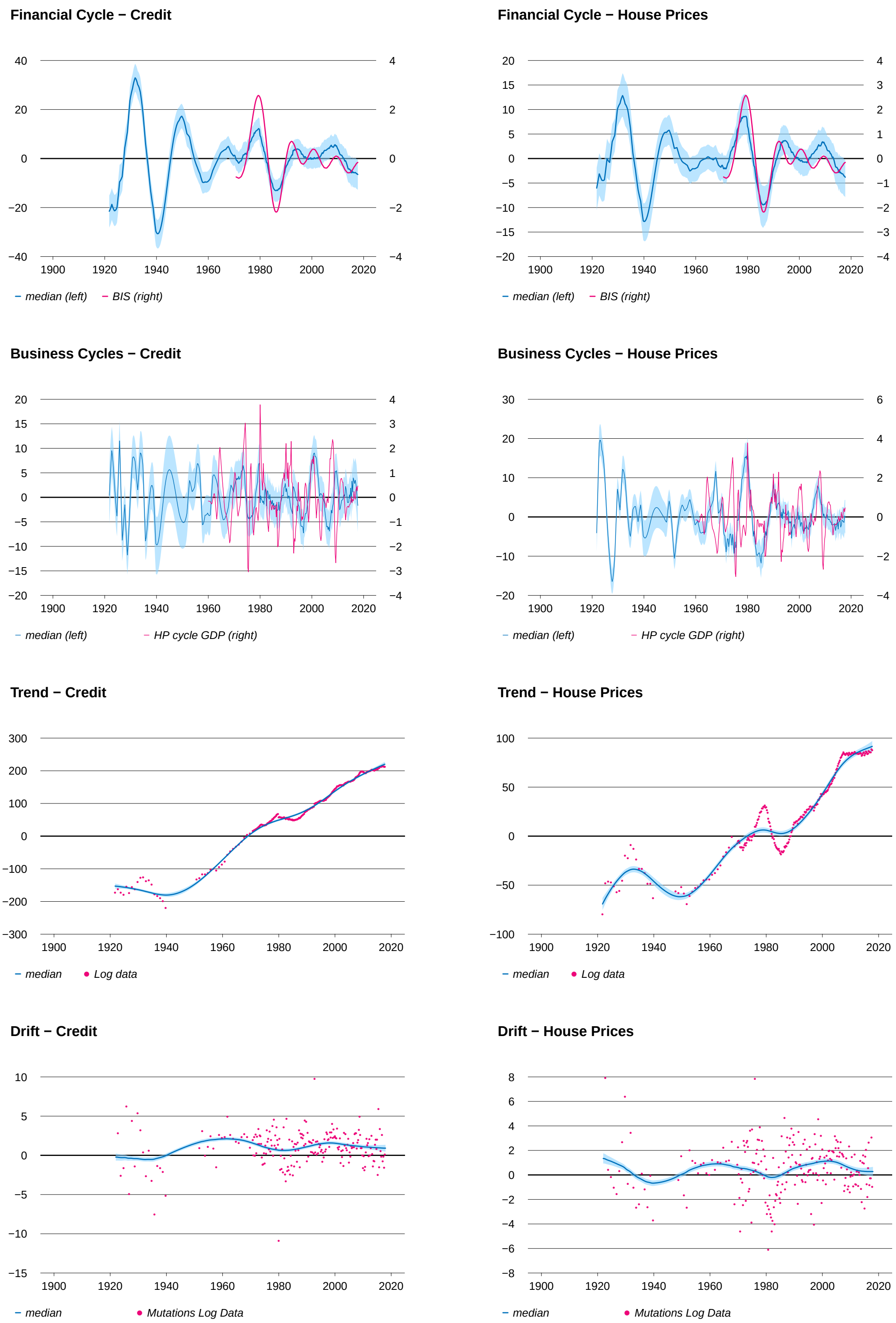
By comparing the financial cycle estimates in each figure based on credit with those based on the housing price index, we can see that in some cases one series clearly dominates in determining the cycle. See for example the case of The Netherlands in Figure D.11 where the credible interval on the financial cycle for credit is more diffuse, while the cycle based on the housing price index is nonetheless strongly defined. It is important to remember that the financial cycle estimates are jointly determined by the credit and the housing price series due to the rank reduction on the covariance matrix Ω_{κ}^{F}. Therefore these cycle estimates reflect the information in both series. These estimates suggest that the housing price index is the dominate factor in determining the financial cycle for The Netherlands.

Figure D.1: Underlying model estimates for Australia



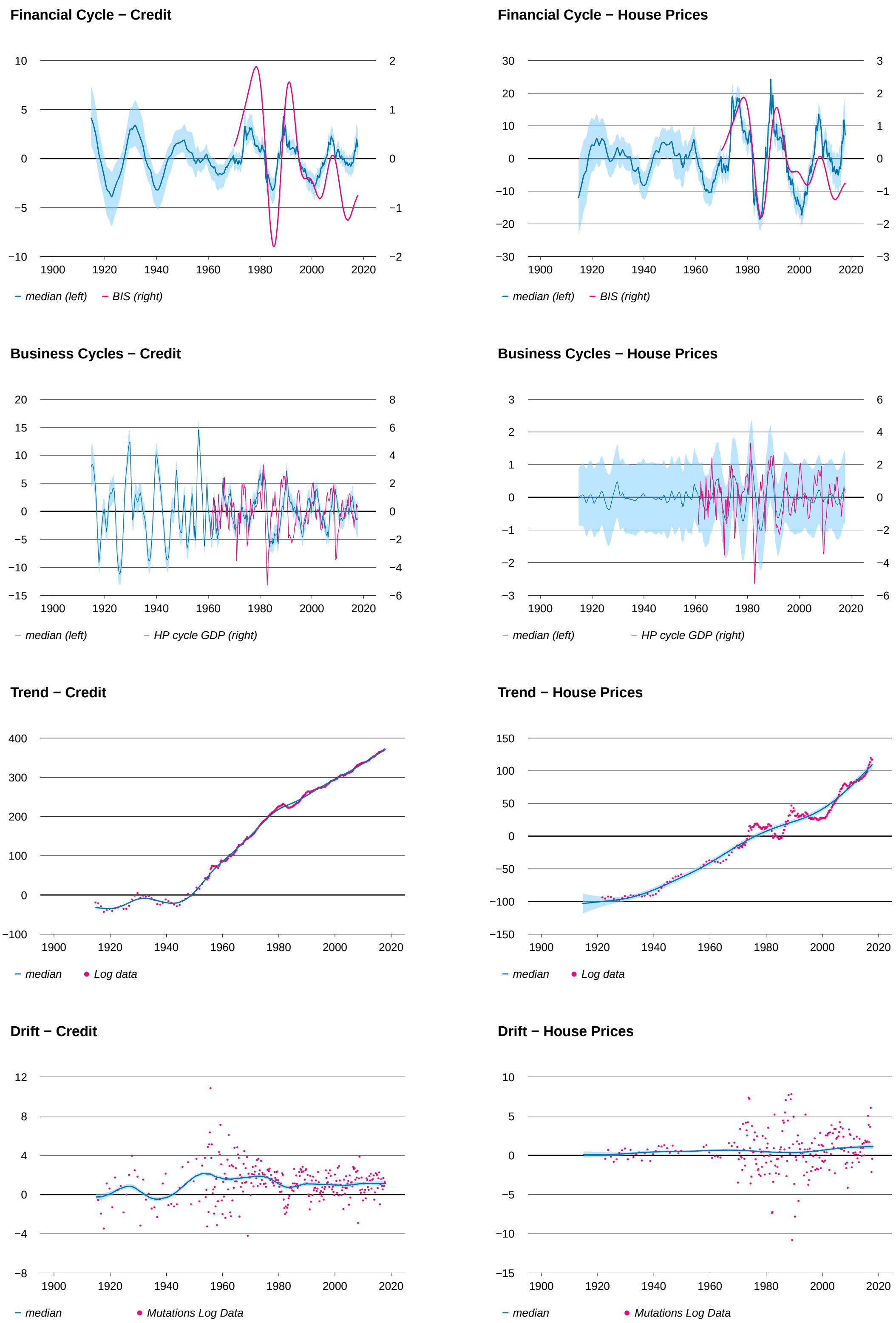
All confidence bands are one standard deviation.

Figure D.2: Underlying model estimates for Belgium



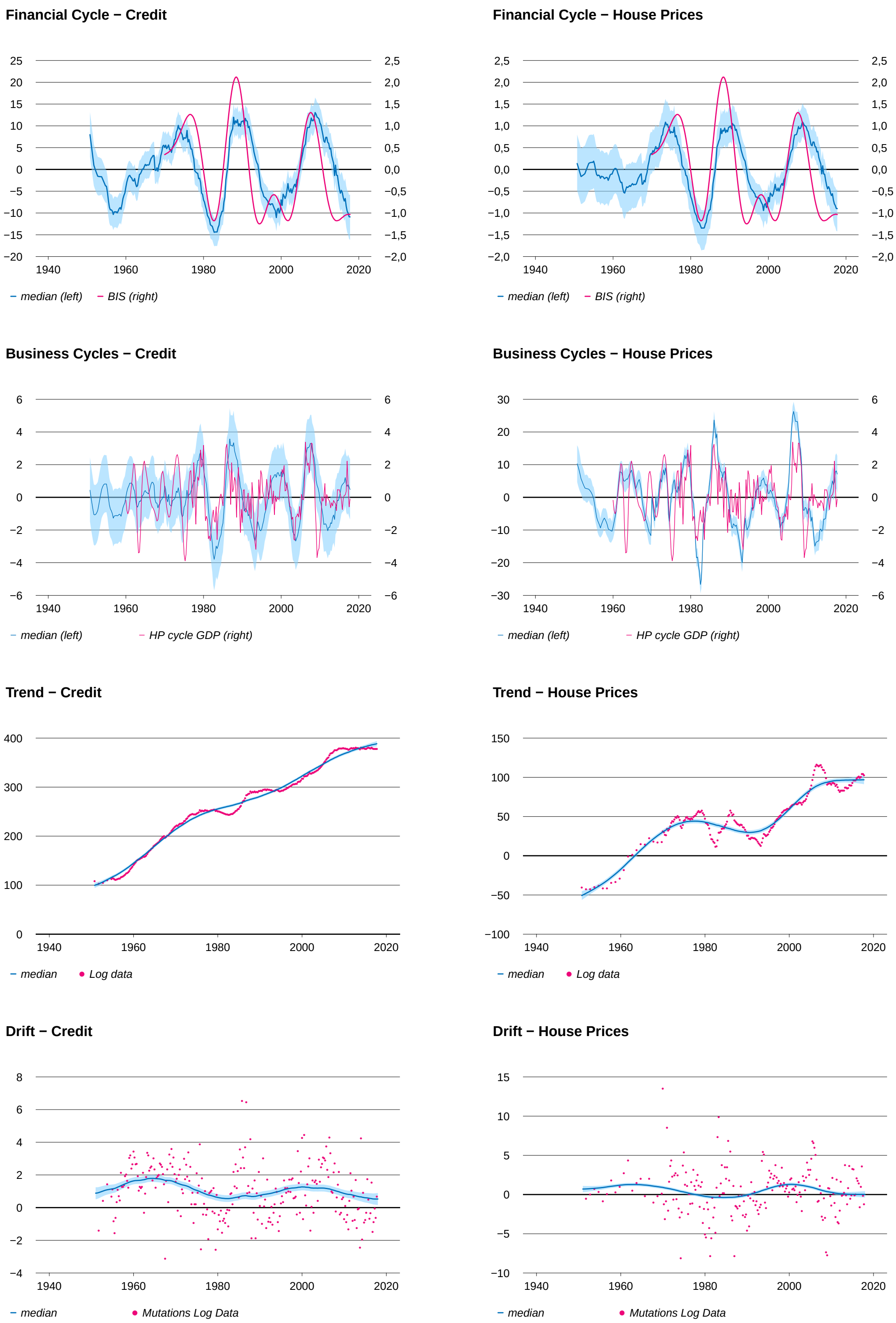
All confidence bands are one standard deviation.

Figure D.3: Underlying model estimates for Canada



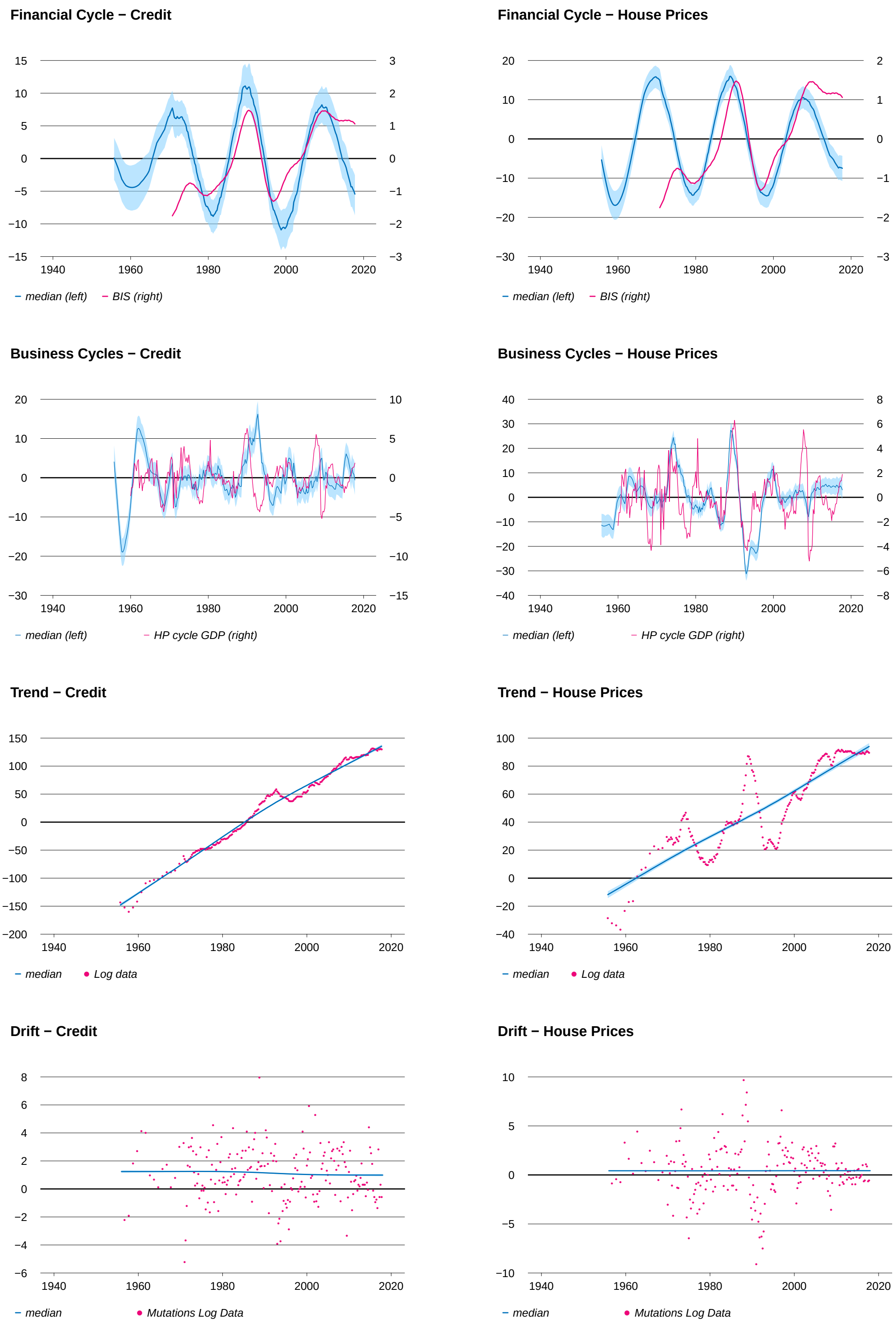
All confidence bands are one standard deviation.

Figure D.4: Underlying model estimates for Denmark



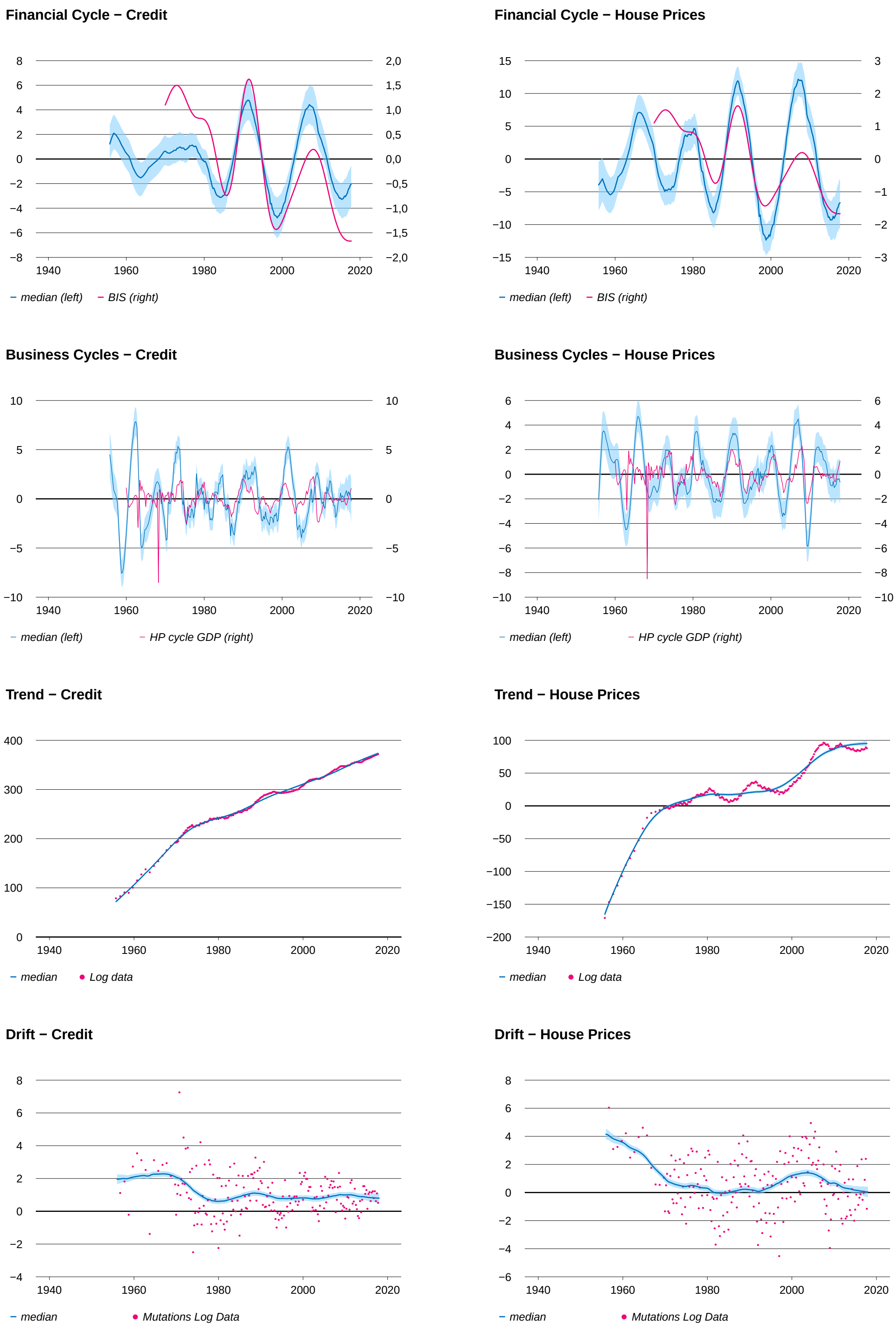
All confidence bands are one standard deviation.

Figure D.5: Underlying model estimates for Finland



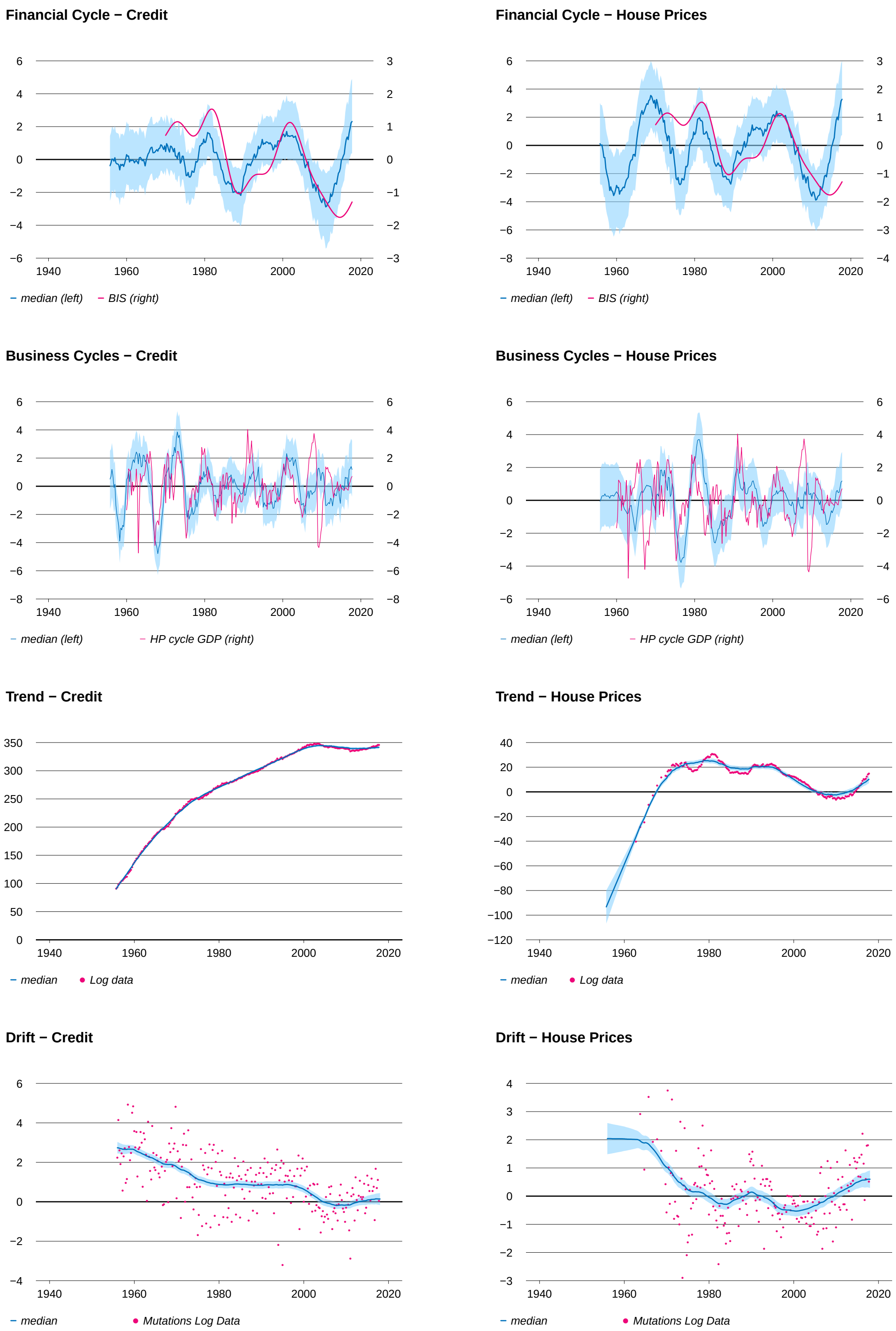
All confidence bands are one standard deviation.

Figure D.6: Underlying model estimates for France



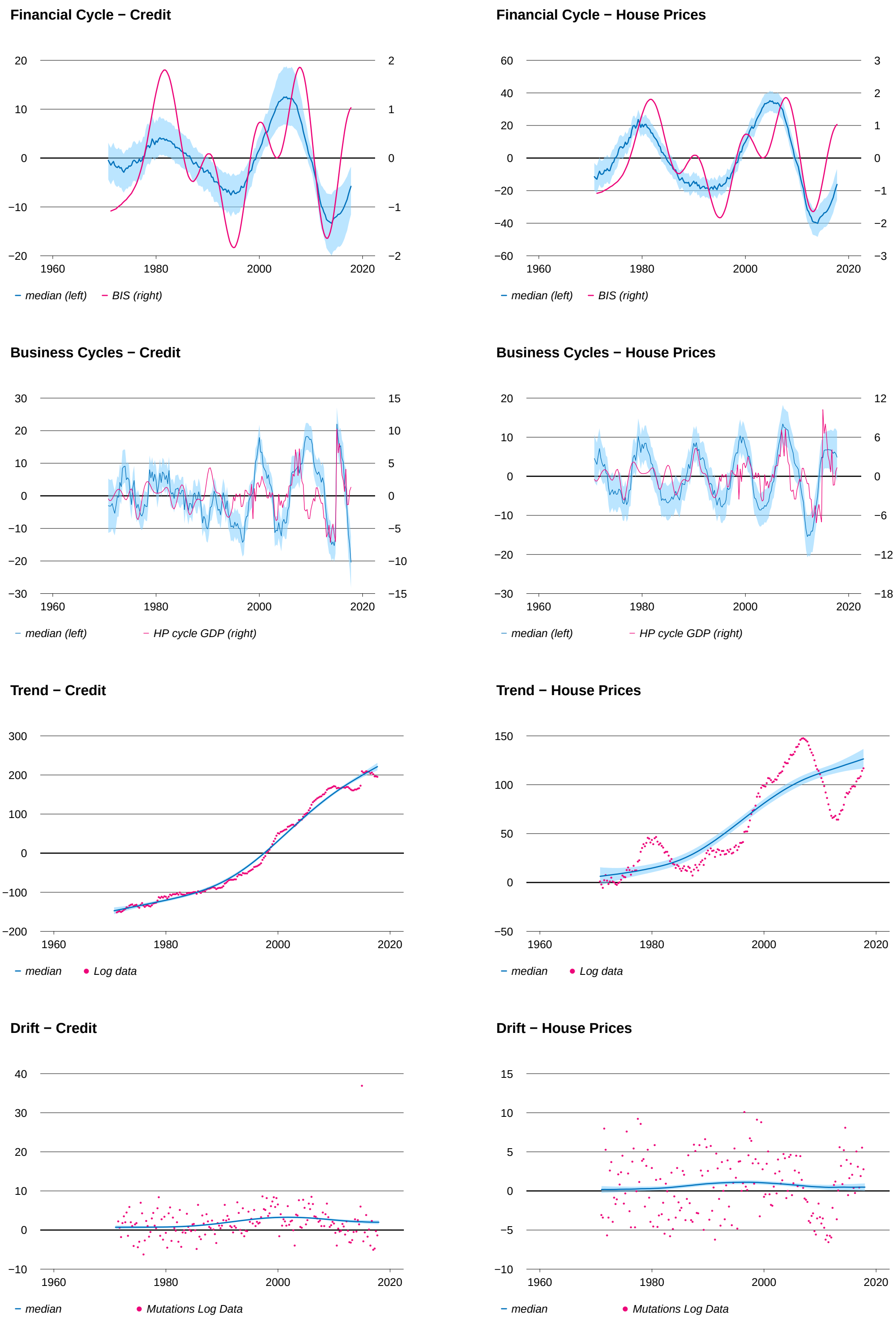
All confidence bands are one standard deviation.

Figure D.7: Underlying model estimates for Germany



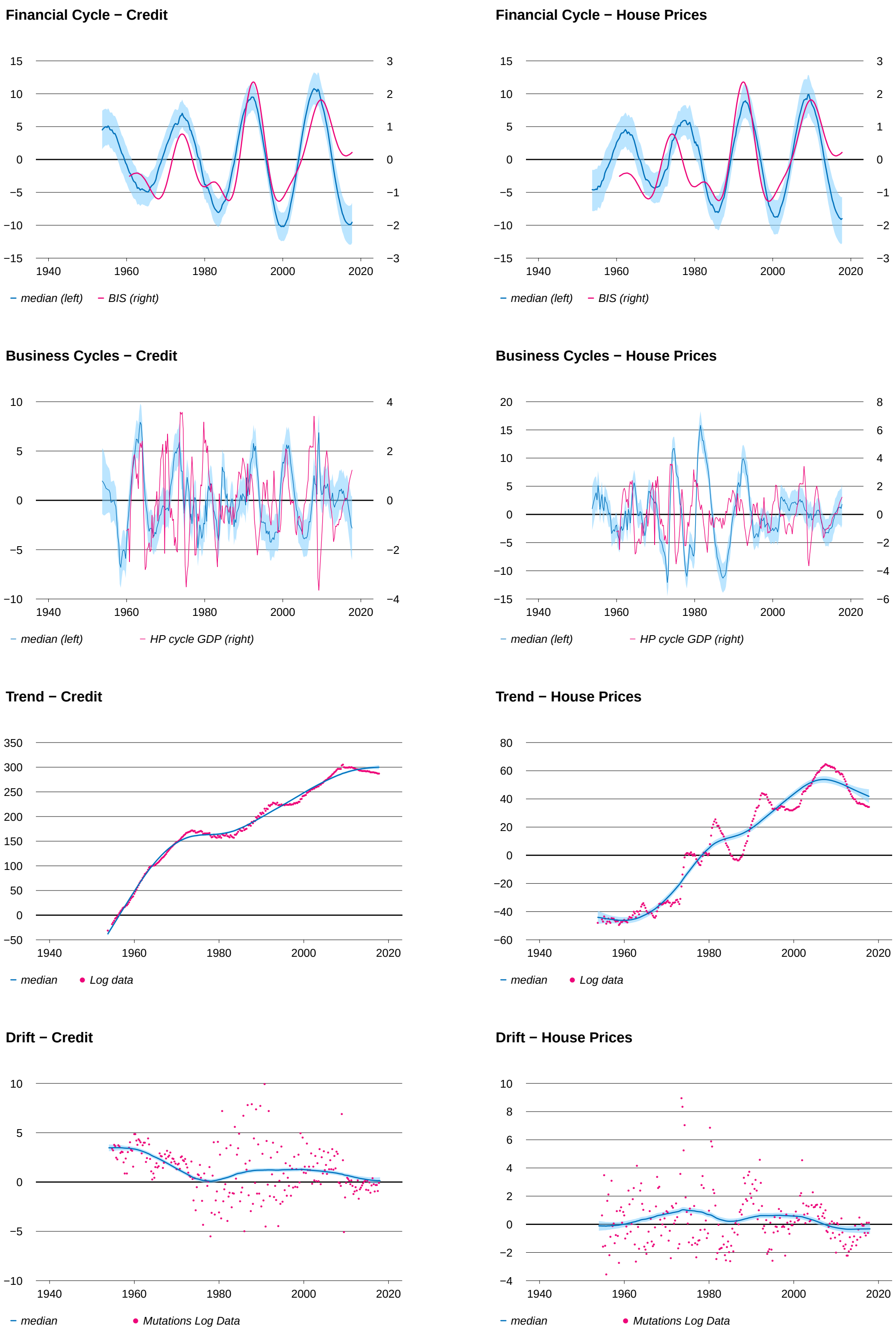
All confidence bands are one standard deviation.

Figure D.8: Underlying model estimates for Ireland



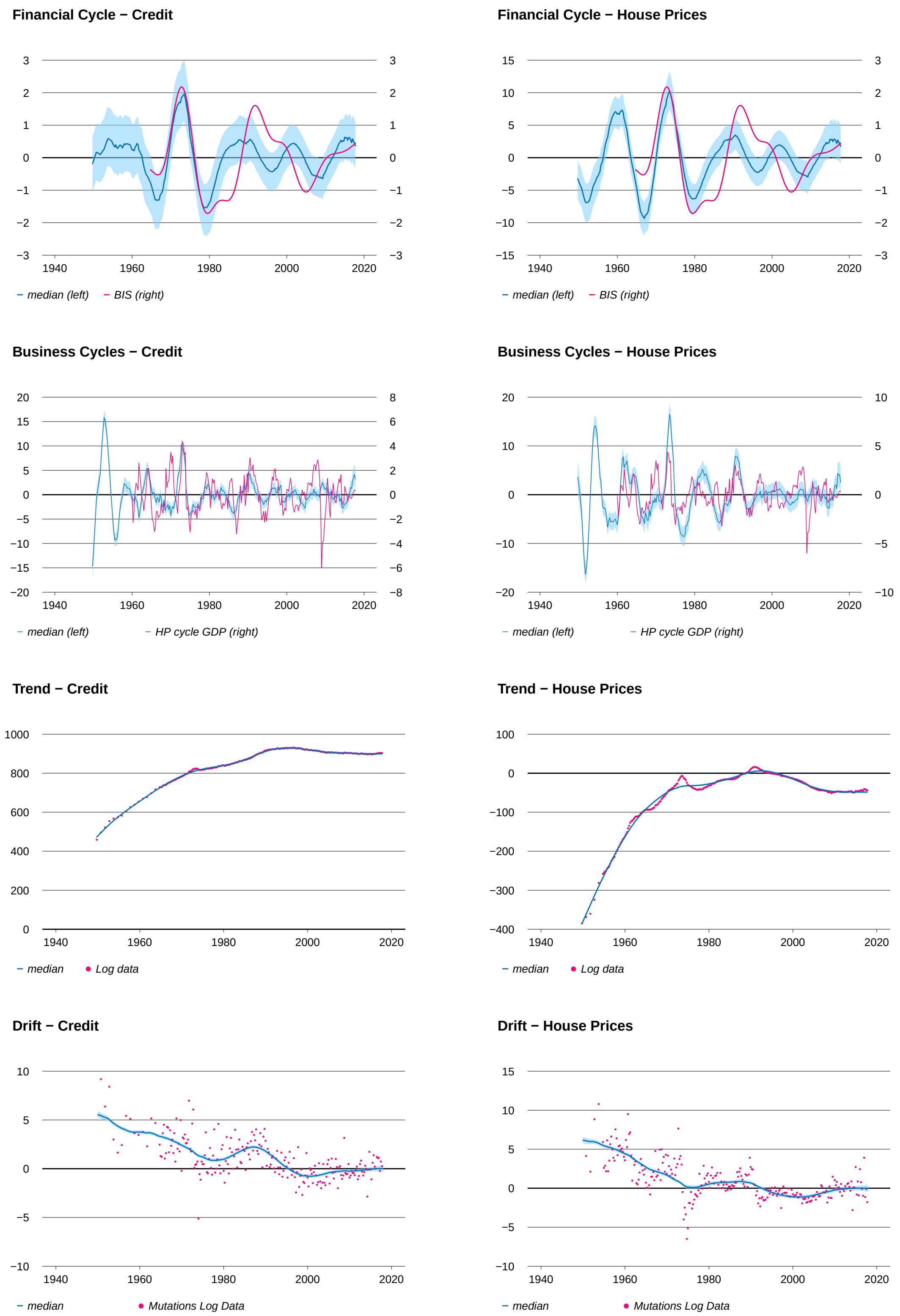
All confidence bands are one standard deviation.

Figure D.9: Underlying model estimates for Italy



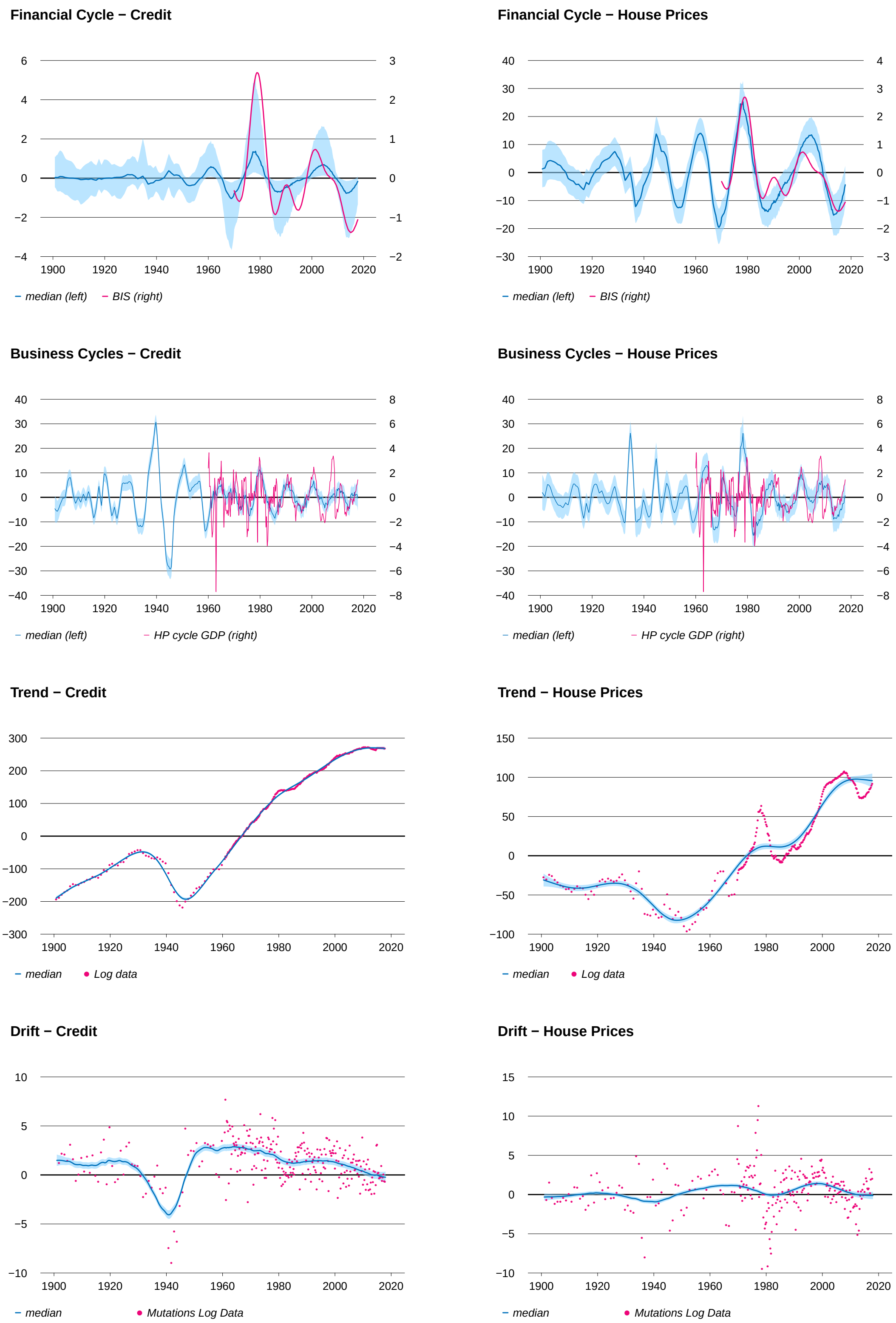
All confidence bands are one standard deviation.

Figure D.10: Underlying model estimates for Japan



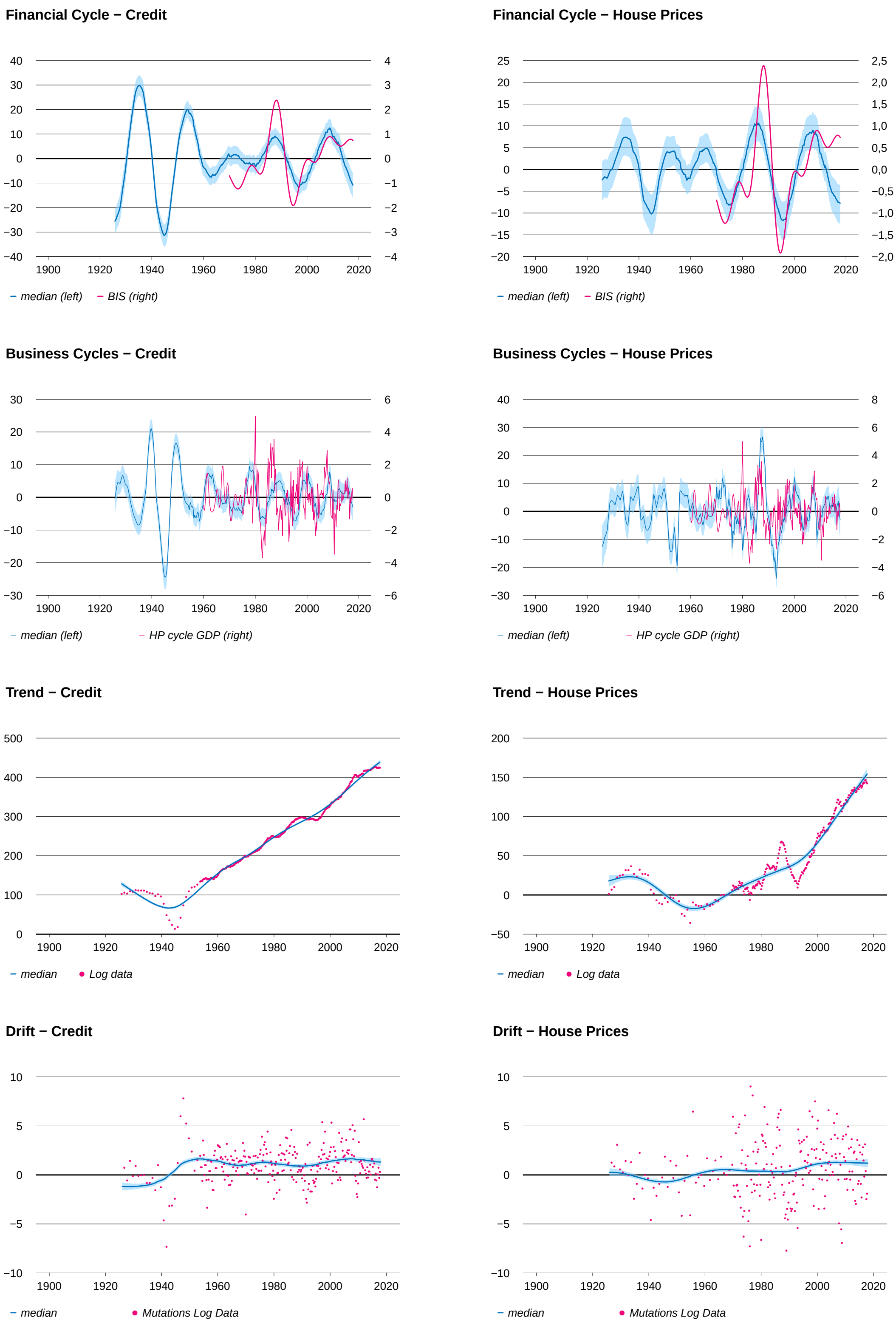
All confidence bands are one standard deviation.

Figure D.11: Underlying model estimates for the Netherlands



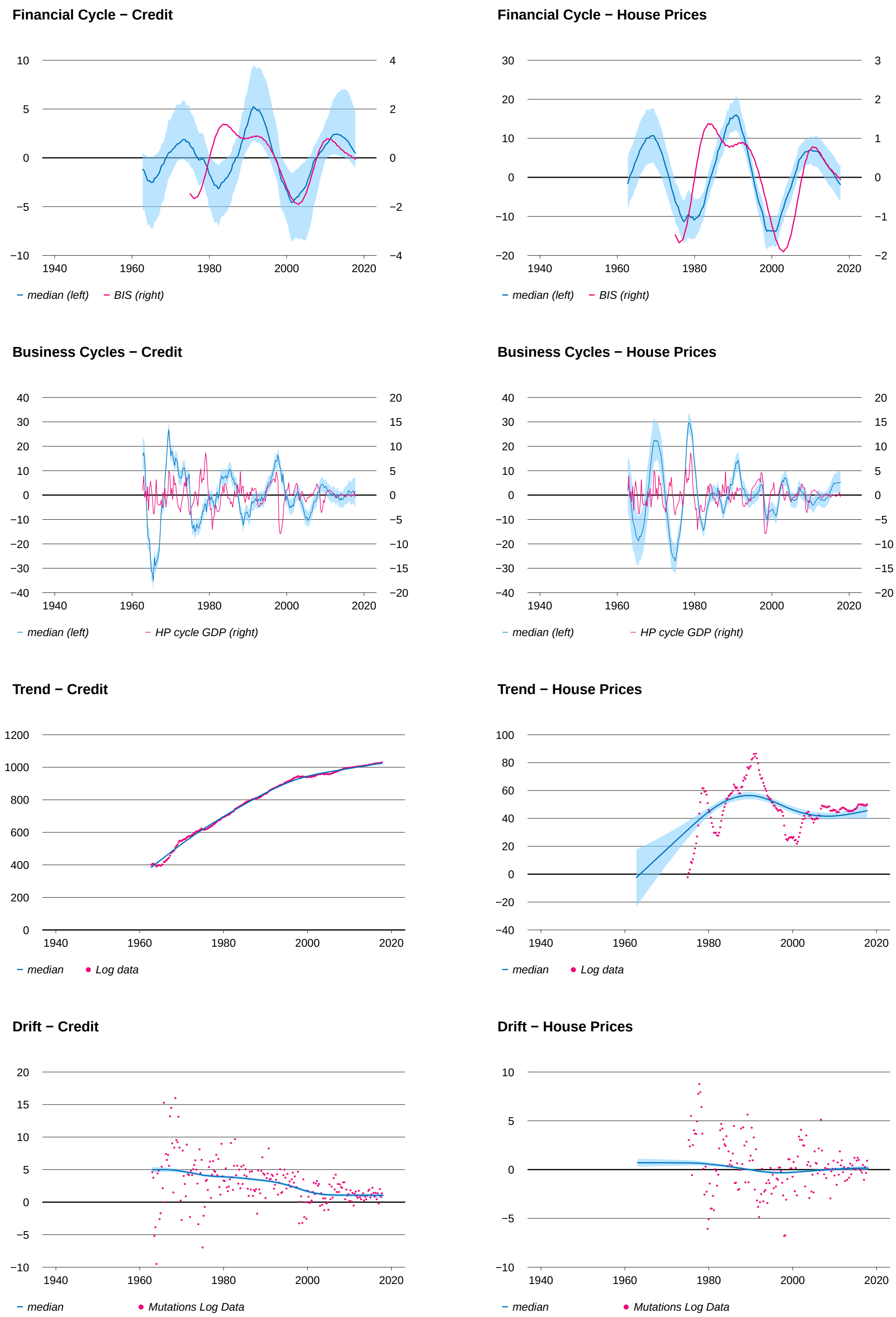
All confidence bands are one standard deviation.

Figure D.12: Underlying model estimates for Norway



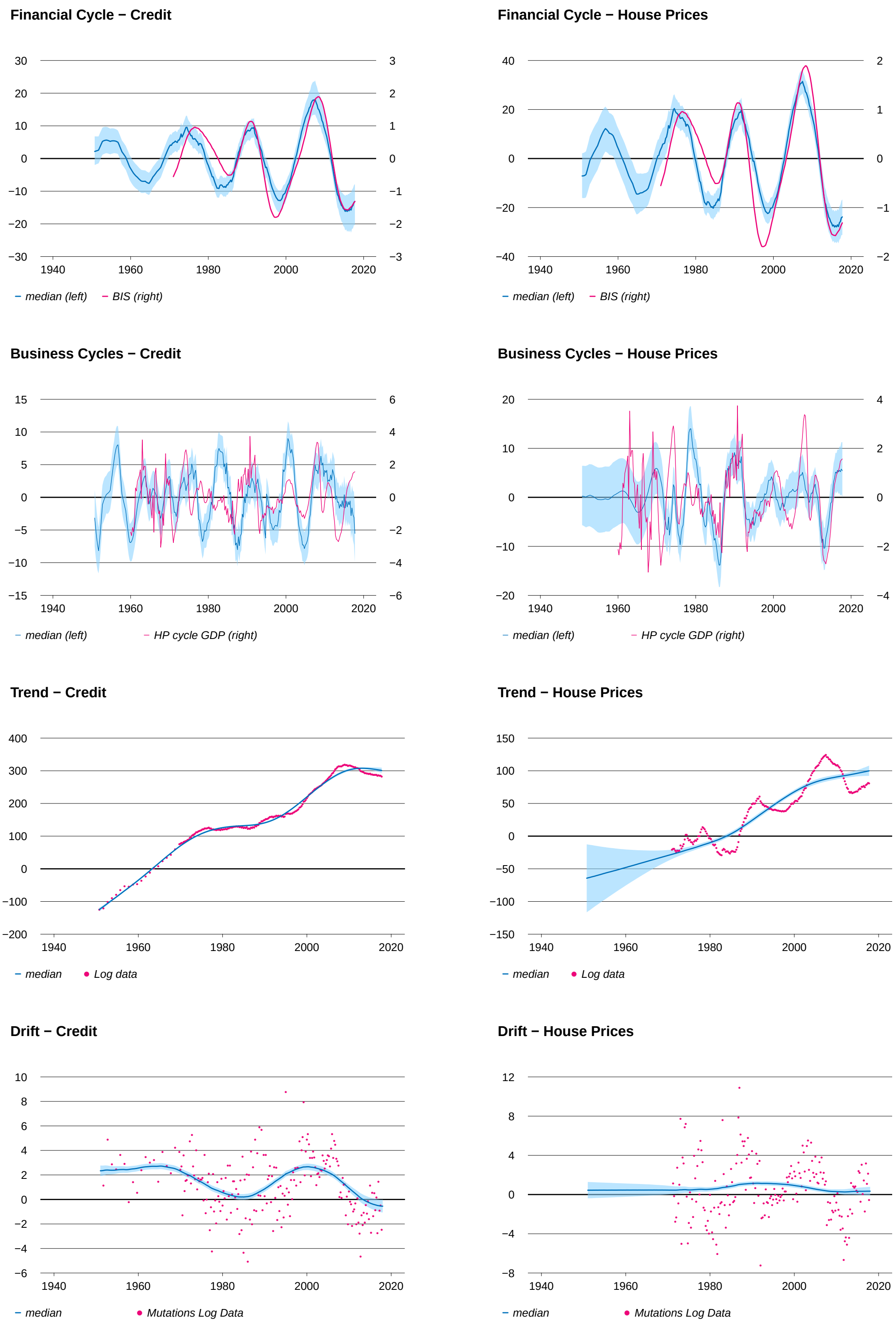
All confidence bands are one standard deviation.

Figure D.13: Underlying model estimates for South Korea



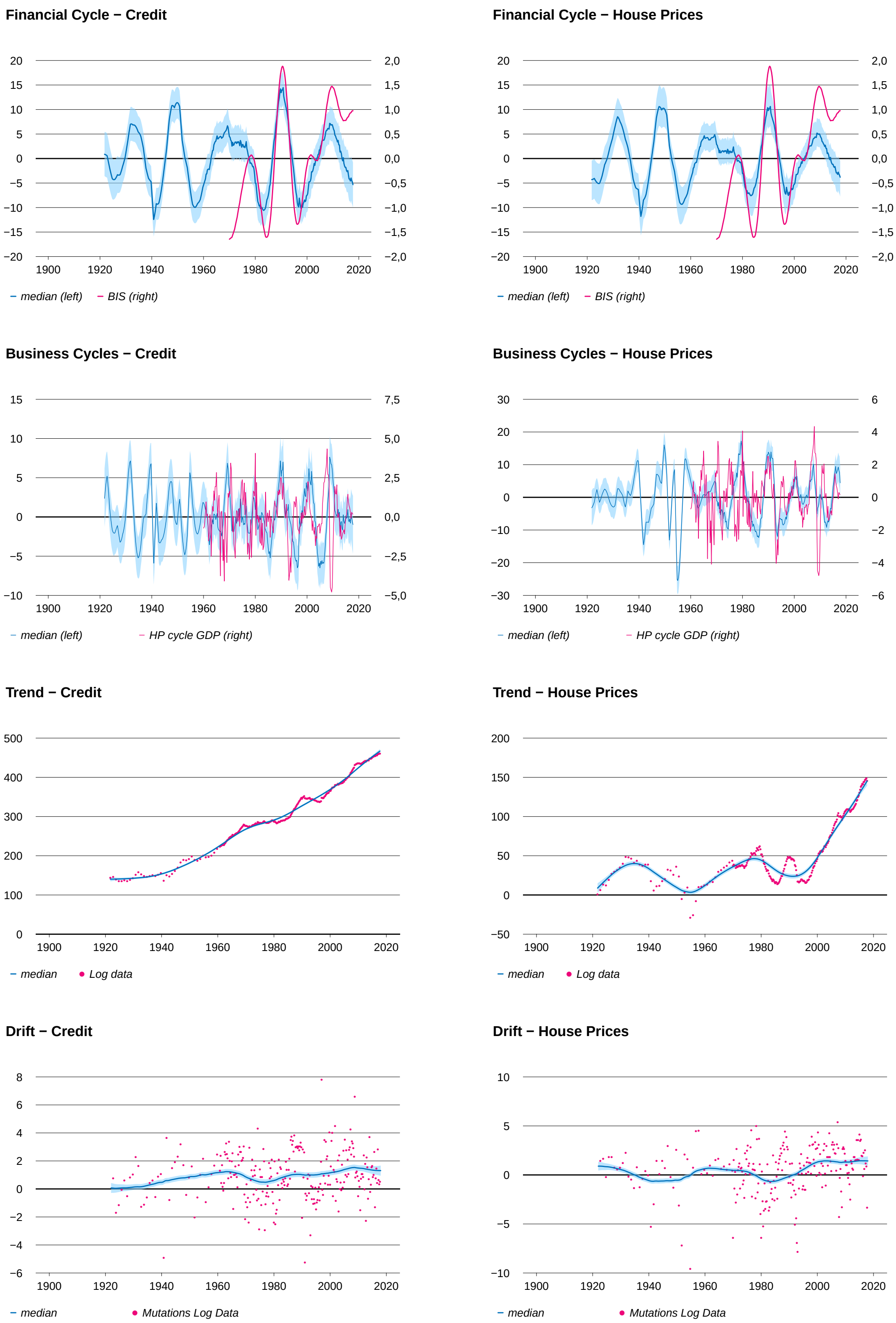
All confidence bands are one standard deviation.

Figure D.14: Underlying model estimates for Spain



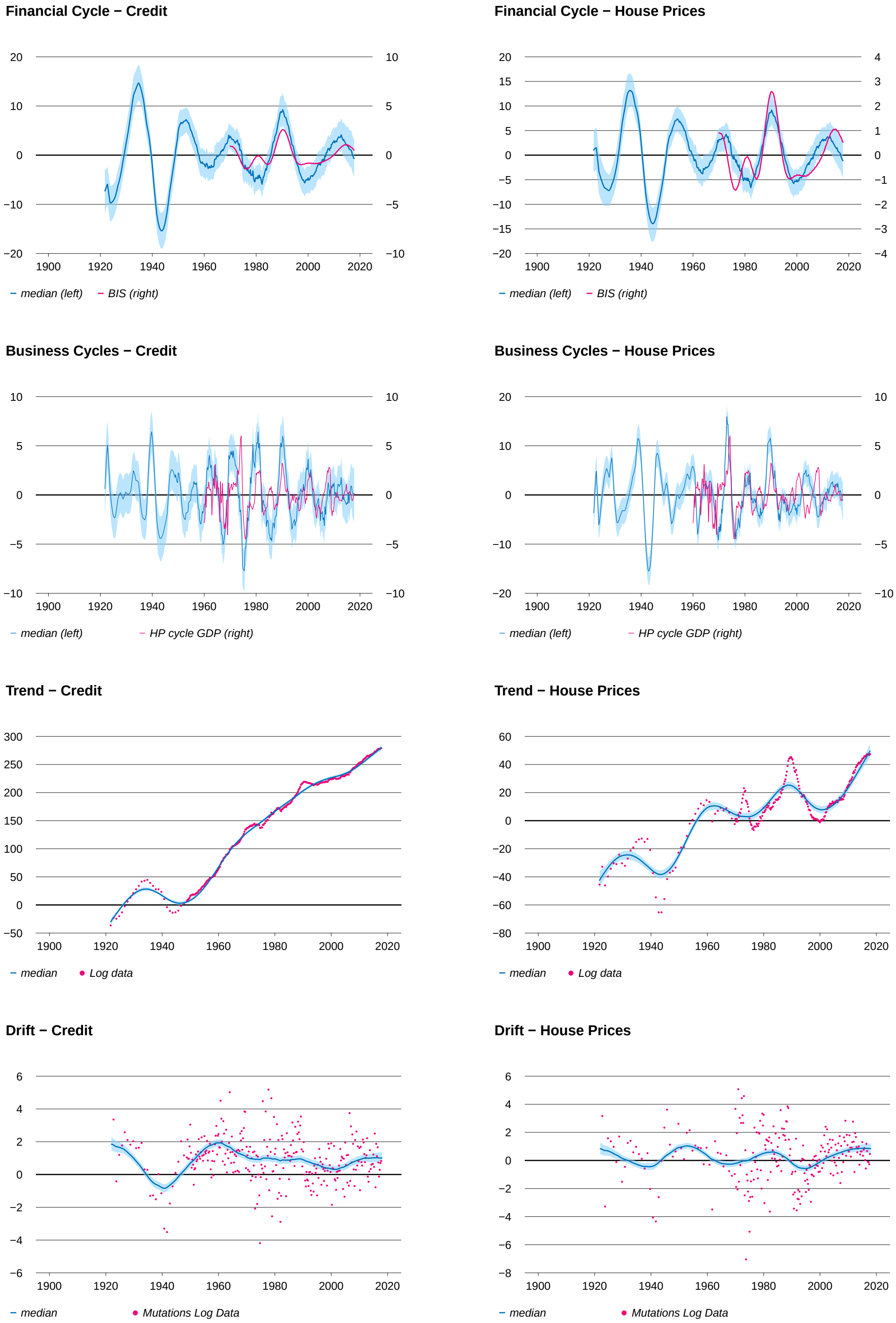
All confidence bands are one standard deviation.

Figure D.15: Underlying model estimates for Sweden



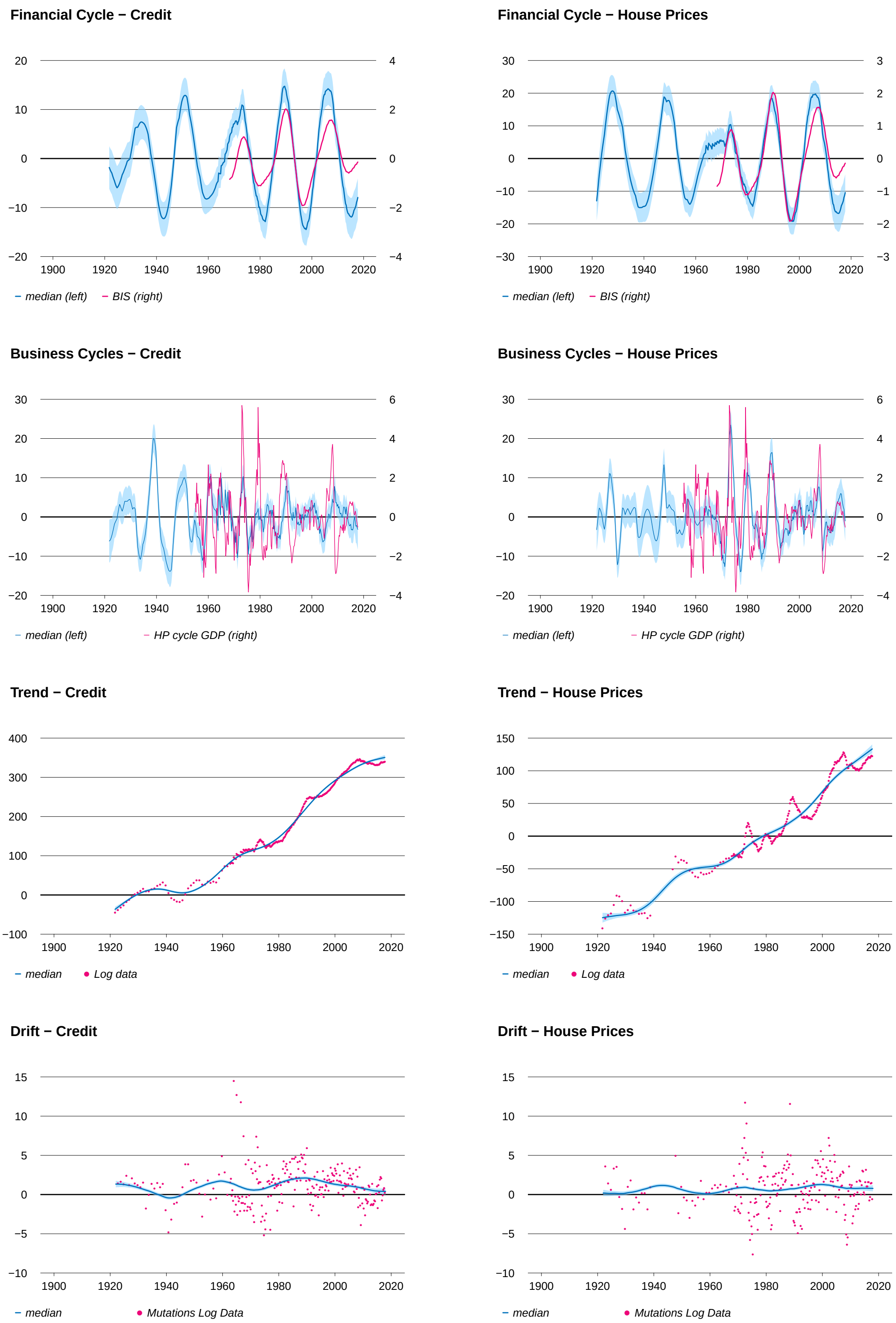
All confidence bands are one standard deviation.

Figure D.16: Underlying model estimates for Switzerland



All confidence bands are one standard deviation.

Figure D.17: Underlying model estimates for the United Kingdom



All confidence bands are one standard deviation.

Figure D.18: Underlying model estimates for the United States

All confidence bands are one standard deviation.

E Parameter posteriors

The posterior means and posterior standard deviations of the model parameters are shown below in Tables E.1, E.2 and E.3.

Table E.1: Parameter Posterior of Measurement and Trend Error Covariances

	Ω_ε				Ω_η	
Country	$\sigma_{\varepsilon 1}$	$\sigma_{\varepsilon 2}$	σ_{h1}	σ_{h2}	$\sigma_{\eta 1}$	$\sigma_{\eta 2}$
Australia	0.0056	0.0016	0.039	–	0.03	0.03
	0.0011	0.0003	0.006	–	0.01	0.01
Belgium	0.0085	0.0065	0.036	0.155	0.04	0.03
	0.0017	0.0010	0.006	0.025	0.01	0.01
Canada	0.0047	0.0105	0.096	0.194	0.03	0.03
	0.0007	0.0035	0.016	0.031	0.01	0.01
Denmark	0.0028	0.0069	0.050	0.049	0.15	0.15
	0.0004	0.0014	0.008	0.008	0.05	0.05
Finland	0.0012	0.0012	0.202	0.201	0.07	0.03
	0.0002	0.0002	0.033	0.033	0.02	0.01
France	0.0005	0.0005	0.269	0.227	0.06	0.03
	0.0001	0.0001	0.047	0.038	0.02	0.01
Germany	0.0020	0.0010	–	0.321	0.17	0.15
	0.0004	0.0002	–	0.055	0.06	0.04
Ireland	0.020	0.021	–	–	0.04	0.03
	0.004	0.004	–	–	0.01	0.01
Italy	0.009	0.009	–	–	0.06	0.03
	0.001	0.002	–	–	0.02	0.01
Japan	0.0012	0.0002	0.273	0.223	0.010	0.009
	0.0002	0.00004	0.048	0.037	0.003	0.003
Netherlands	0.006	0.007	0.255	0.204	0.010	0.010
	0.001	0.001	0.043	0.033	0.003	0.003
Norway	0.004	0.004	0.250	0.187	0.010	0.010
	0.001	0.001	0.041	0.030	0.003	0.003
South Korea	0.014	0.180	–	–	0.010	0.010
	0.004	0.004	–	–	0.003	0.003
Spain	0.0020	0.37	0.049	–	0.05	0.02
	0.0003	0.34	0.008	–	0.02	0.01
Sweden	0.003	0.004	0.271	0.199	0.018	0.015
	0.001	0.001	0.046	0.032	0.006	0.005
Switzerland	0.002	0.003	0.088	0.098	0.010	0.013
	0.000	0.001	0.014	0.016	0.003	0.004
UK	0.009	0.009	0.023	0.023	0.007	0.010
	0.003	0.002	0.004	0.004	0.002	0.003
US	0.0009	0.0023	0.252	0.060	0.002	0.006
	0.0002	0.0004	0.054	0.010	0.001	0.002

The table lists the posterior means and posterior standard deviations of the parameters. The first row for each country shows the mean, while the value directly under is the standard deviation.

Table E.2: Parameter Posteriors of Drift and Seasonal Error Covariances

	Ω_ζ			Ω_ω	
Country	$\sigma_{\zeta 1}$	$\sigma_{\zeta 2}$	$\sigma_{\zeta 12}$	$\sigma_{\omega 1}$	$\sigma_{\omega 2}$
Australia	0.0079	0.0066	0.0005	0.00003	0.00002
	0.0012	0.0010	0.0008	0.00001	0.00001
Belgium	0.0077	0.0078	0.0011	0.00002	0.00002
	0.0013	0.0013	0.0009	0.00001	0.00001
Canada	0.0087	0.0025	0.00003	0.00002	0.00002
	0.0015	0.0004	0.0006	0.00001	0.00001
Denmark	0.0068	0.0077	0.0009	0.00002	0.00002
	0.0011	0.0013	0.0008	0.00001	0.00001
Finland	0.0001	0.0000	0.0000	0.00002	0.00002
	0.00002	0.0000	0.0000	0.00001	0.00001
France	0.0070	0.0099	0.0003	0.00005	0.00002
	0.0011	0.0018	0.0010	0.00004	0.00001
Germany	0.0068	0.0075	0.0005	0.00003	0.00002
	0.0011	0.0012	0.0008	0.00002	0.00001
Ireland	0.0077	0.0065	0.0004	0.00002	0.00002
	0.0013	0.0011	0.0008	0.00001	0.00001
Italy	0.0072	0.0069	-0.0006	0.00454	0.00002
	0.0012	0.0011	0.0008	0.00135	0.00001
Japan	0.0117	0.0124	0.0048	0.00002	0.00002
	0.0019	0.0021	0.0016	0.00001	0.00001
Netherlands	0.0285	0.0082	0.0041	0.00002	0.00002
	0.0055	0.0015	0.0022	0.00001	0.00001
Norway	0.0078	0.0067	-0.0001	0.00002	0.00002
	0.0013	0.0011	0.0008	0.00001	0.00001
South Korea	0.0077	0.0018	0.0002	0.00002	0.00002
	0.0012	0.0003	0.0004	0.00001	0.00001
Spain	0.0108	0.0065	0.0002	0.00004	0.00002
	0.0020	0.0011	0.0011	0.00006	0.00001
Sweden	0.0064	0.0113	0.0001	0.00002	0.00002
	0.0010	0.0018	0.0010	0.00001	0.00001
Switzerland	0.0089	0.0082	0.0015	0.00003	0.00002
	0.0015	0.0015	0.0011	0.00001	0.00001
UK	0.0084	0.0069	-0.0011	0.00002	0.00002
	0.0014	0.0012	0.0009	0.00001	0.00001
US	0.0115	0.0025	-0.0004	0.00002	0.00002
	0.0023	0.0004	0.0007	0.00001	0.00001

The table lists the posterior means and posterior standard deviations of the parameters. The first row for each country shows the mean, while the value directly under is the standard deviation.

Table E.3: Parameter Posteriors of Cycle Components

	Ω_κ^F		Ω_κ^B						
Country	$\sigma_{\kappa 1}^F$	$\sigma_{\kappa 2}^F$	$\sigma_{\kappa 1}^B$	$\sigma_{\kappa 2}^B$	$\sigma_{\kappa 12}^B$	λ^F	λ^B	ρ^F	ρ^B
Australia	0.01	0.92	1.82	1.91	0.23	68.6	25.5	0.989	0.960
	0.02	0.30	0.17	0.33	0.16	3.7	1.8	0.004	0.010
Belgium	1.51	0.50	3.79	2.72	0.15	71.8	36.6	0.994	0.963
	0.66	0.23	0.63	0.41	0.14	2.7	3.2	0.002	0.010
Canada	0.17	4.96	2.19	0.09	0.07	72.5	30.0	0.988	0.964
	0.10	0.49	0.22	0.04	0.06	4.9	2.5	0.004	0.009
Denmark	1.14	0.01	0.24	5.85	0.48	71.1	37.8	0.991	0.973
	0.18	0.003	0.09	0.61	0.18	3.4	2.7	0.003	0.008
Finland	0.40	0.30	2.27	3.91	0.20	74.5	40.2	0.995	0.977
	0.30	0.15	0.37	0.46	0.16	2.2	2.4	0.002	0.006
France	0.03	0.30	0.71	0.28	0.04	64.7	28.4	0.994	0.974
	0.02	0.07	0.10	0.06	0.03	3.3	1.8	0.002	0.007
Germany	0.15	0.23	0.36	0.24	0.06	70.1	37.4	0.989	0.968
	0.10	0.07	0.09	0.06	0.05	4.5	3.8	0.004	0.011
Ireland	0.40	3.47	12.6	3.11	0.29	76.3	36.4	0.993	0.971
	0.33	0.90	1.39	0.82	0.27	3.5	2.8	0.003	0.009
Italy	0.17	0.25	1.21	1.64	0.08	69.2	34.9	0.995	0.970
	0.06	0.15	0.15	0.21	0.06	2.8	2.5	0.002	0.008
Japan	0.02	0.29	0.83	1.16	0.57	62.6	26.6	0.991	0.978
	0.01	0.13	0.09	0.16	0.09	4.5	1.3	0.003	0.005
Netherlands	0.06	1.48	2.47	3.80	0.57	72.7	41.3	0.993	0.979
	0.17	0.68	0.27	0.80	0.27	4.2	2.9	0.003	0.005
Norway	0.66	0.38	1.43	6.39	0.14	77.5	41.9	0.996	0.980
	0.19	0.23	0.21	0.65	0.12	2.2	2.1	0.001	0.005
South Korea	0.24	0.67	5.29	2.56	1.09	77.4	36.4	0.993	0.984
	0.42	0.43	0.65	0.54	0.52	4.4	2.7	0.003	0.005
Spain	0.65	1.68	1.87	2.65	0.15	69.0	36.2	0.993	0.971
	0.36	0.48	0.32	0.54	0.13	3.2	3.0	0.002	0.009
Sweden	0.93	0.61	1.55	4.09	0.18	69.1	35.0	0.991	0.963
	0.26	0.34	0.27	0.54	0.15	3.2	3.5	0.003	0.010
Switzerland	0.47	0.38	0.78	1.99	0.20	72.3	36.2	0.993	0.968
	0.21	0.19	0.17	0.26	0.13	3.8	2.3	0.002	0.007
UK	0.63	1.30	4.09	3.24	0.68	71.9	34.9	0.993	0.965
	0.37	0.36	0.51	0.49	0.38	2.3	2.8	0.002	0.008
US	0.13	0.67	0.65	0.72	0.05	77.7	34.7	0.996	0.986
	0.05	0.14	0.07	0.13	0.04	2.3	1.3	0.002	0.004

The table lists the posterior means and posterior standard deviations of the parameters. The first row for each country shows the mean, while the value directly under is the standard deviation.