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Auctioning Incentive Contracts: Application to Welfare-to-Work Programs[a]

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Abstract in English

This paper applies the theory of auctioning incentive contracts to welfare-to-work programs. In several countries, the government procures welfare-to-work projects to employment service providers. In doing so, the government trades off adverse selection (the winning provider is not the most efficient one) and moral hazard (the winning provider shirks in his effort to reintegrate unemployed people). We compare three simple auctions with the socially optimal mechanism and show that two of these auctions approximate the optimal mechanism if the number of providers is large. Using simulations, we observe that competition between three bidders is already sufficient for the outcome of these auctions to reach 95% of the optimal level of social welfare.

Keywords: Adverse selection; Auctions; Incentive contracts; Moral hazard; Welfare-to-work programs
JEL classification: D44; D82; J68

Abstract in Dutch

Dit Discussion Paper past de theorie van het veilen van prestatiecontracten toe op reïntegratieprogramma's. In verschillende landen kunnen reïntegratiebedrijven meedingen in publieke aanbestedingen van reïntegratietrajecten. De aanbestedende overheden moeten daarbij averechtse selectie (het winnende bedrijf is niet het meest geschikte) afwegen tegen moral hazard (het winnende bedrijf doet onvoldoende zijn best om werklozen aan het werk te helpen). We vergelijken drie eenvoudige veilingen met het sociaal optimale mechanisme en laten zien dat twee van deze veilingen het optimale mechanisme benaderen als het aantal biedende bedrijven groot wordt. Met behulp van simulaties observeren we dat voor deze veilingen concurrentie tussen drie bieders al voldoende is om 95% van het maximaal haalbare sociale welvaart te bereiken.

Steekwoorden: Averechtse selectie; Moral Hazard; Prestatiecontracten; Reïntegratietrajecten; Veilingen

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Summary

In several countries, including the Netherlands, governments use procurements for welfare-to-work programs. In these procurements, the government sells welfare-to- work projects to employment service providers. A welfare-to-work project typically consists of a number of unemployed people, and the winning provider is rewarded on the basis of the number of these people that find a job within a specified period of time.

In reaching their targets, governments may be confronted with two types of economic problems: adverse selection and moral hazard. Adverse selection occurs when the procurement does not select the 'best' employment provider, i.e., the provider that, relative to all other providers, is able to help the unemployed people back to work in the most cost efficient way. Moral hazard may occur as the winner of the procurement has no incentive to put much effort in the welfare-to-work project.

Most governments that procure welfare-to-work programs use a beauty contest. But, are beauty contests indeed optimal? In this paper, we study mechanisms that are simpler than beauty contests, auctions, and compare these with a socially optimal mechanism. Several economists claim that auctions perform better than beauty contests as they are more transparent, are less prone to favoritism, and give rise to less administrative burden for both the bidders and the procurer. In an auction, providers submit one-dimensional bids on a project, and the winner is chosen using a well-defined allocation rule.

We study three simple auctions. Two of these auction types are based on the beauty contests that we observe in practice. The first auction is the lowest- reward auction. In this auction, the provider that submits the lowest price per successful placement wins the project, and is rewarded according to its bid. Second, we focus on the highest-output auction. This auction rewards the project to the provider that promises the highest 'effort', i.e., the highest reduction in social benefits. The government pays the winner a reward for each unit of effort above its promise. The third auction type we consider is the constant-reward auction, which the OECD proposes as an alternative way to procure welfare-to-work projects. The government sells the project to the highest bidder. The winner is then paid a fixed reward for each unit of its effort.

We compare these auctions to the optimal mechanism, which has the following properties. First, the government selects the most efficient provider, provided that its efficiency level exceeds a threshold level. Second, the winning provider exerts effort that is below the full-information optimum. None of the three auctions turns out to be optimal.

However, when the number of bidding providers tends to infinity, or when the tax distortion approaches zero, the constant-rewards second-price auction and the highest-output auction approach the optimal outcome. Moreover, using simulations we observe that the outcomes of the constant-reward second-price auction and the highest effort auction rapidly approach the socially optimal mechanism. In the case of three competing providers, when the collection of $1 in taxes

costs $1, social welfare that is generated by these simple auctions is only 5% less than the highest possible level of social welfare.

In contrast, the lowest-reward auction performs poorly relative to the other two simple auctions. We find that this auction solves the adverse selection problem in the sense that the most efficient provider is selected. However, a strong moral hazard problem remains as the winning provider's equilibrium effort tends to zero when the number of bidders increases. This finding may place some doubt on the usefulness of having the price per successful placement as one of the dimensions in a beauty contest.

1 Introduction

In several countries, governments use procurements for welfare-to-work programs.[1] In these procurements, the government sells welfare-to-work projects to employment service providers. A welfare-to-work project typically consists of a number of unemployed people, and the winning provider is rewarded on the basis of the number of these people that find a job within a specified period of time. These procurements give flesh and blood to Demsetz' (1968) idea of competition 'for' the market.

What are the governments' targets in the procurements? The success of a procurement depends on (1) the number of people that find a job, (2) the costs incurred by the employment service provider, (3) the reduction in unemployment benefits, and (4) the payments made from the government to the employment service provider. The latter two are important as they imply that the government raises less distortionary taxes.[2]

In reaching these targets, governments may be confronted with two types of economic problems: adverse selection and moral hazard. Adverse selection occurs when the procurement does not select the 'best' employment provider, i.e., the provider that, relative to all other providers, is able to help the unemployed people back to work in the most cost efficient way. Moral hazard may occur as the winner of the procurement has no incentive to put much effort in the welfare-to-work project. An additional target may be a cheap procurement process.

Most governments that procure welfare-to-work programs use a beauty contest. Providers submit an offer that contains a bid on several pre-specified dimensions. In the Netherlands, some of these dimensions are well-defined (such as the price for a successful placement), others are rather vague (such as 'experience'). The government then signs a contract with the provider submitting the 'best' bid. This contract specifies how the government rewards the firm for its effort. Usually this is both input and output based: the government partly covers the cost of the provider, and in addition gives it is monetary award for each successful placement. However, are beauty contests indeed optimal? At least the administrative burden is usually high for these mechanisms: it is time consuming and costly for firms to write an offer and for the government to study and compare these offers.

In this paper, we study mechanisms that are simpler than beauty contests, auctions, and compare these with a socially optimal mechanism. Several economists claim that auctions perform better than beauty contests as they are more transparent, are less prone to favoritism, and give rise to less administrative burden for both the bidders and the procurer.[3] In an auction,

[1] See OECD (2001) and Productivity Commission (2002) for Australia, and Struyven and Steurs (2003) for The Netherlands. Zwinkels et al. (2004) provide a comparison of welfare-to-work procurements in Australia, Denmark, the Netherlands, Sweden, the UK, and the US.

[2] Ballard et al. (1985) estimate deadweight losses to lie between 17 and 56 cents for every extra \$1 raised in taxes.

[3] See, e.g., Binmore and Klemperer (2002).

providers submit one-dimensional bids on a project, and the winner is chosen using a well-defined allocation rule. In Section 2, we describe the setting in which we study several auction types. A fixed number of employment service providers submit bids on a project. The providers differ with respect to their efficiency level, which is private information to each provider, and about which its competitors and the government are incompletely informed.

In Section 3, we study three simple auctions. Two of these auction types are based on the beauty contests that we observe in practice. Usually, there are at least two objective dimensions in these beauty contests on which bidders submits bids: (1) the price per successful placement, and (2) the expected success rate, i.e., the fraction of people in the welfare-to-work project that finds a job. The first auction is the lowest-reward auction. In this auction, the provider that submits the lowest price per successful placement, wins the project. Imagine that the winner submitted a price equal to b. Then the government rewards the provider with b for every unit of effort it puts in the project, i.e., for each unit of savings in the unemployment benefits. We find that this auction solves the adverse selection problem in the sense that the most efficient provider is selected. However, a strong moral hazard problem remains as the winning provider's equilibrium effort tends to zero when the number of bidders increases. This finding may place some doubt on the usefulness of having the price per successful placement as one of the dimensions in the beauty contest.

Second, we focus on the highest-output auction. This auction rewards the project to the provider that promises the highest 'effort', i.e., the highest reduction in social benefits. The government pays the winner a reward $\tau\,(e-b)$, where τ is a constant, e the actual effort by the employment service provider, and b its promise in the auction. A negative reward is interpreted as a fine.

The third auction type we consider is the constant-reward auction, which OECD (2001) proposes as an alternative way to procure welfare-to-work projects. The government sells the project to the highest bidder, for instance in the second-price sealed-bid auction (the constant-reward second-price auction[4]).The winner is paid a fixed reward ρ for each unit of its effort. OECD (2001) argues that this auction is optimal, provided that the government awards the winner of this auction the marginal social value of each successful placement. However, the OECD's claim is based on McMillan (1992), who relies on the assumption of complete information regarding the efficiency of the provider, and who ignores the positive impact of a decrease in unemployment benefits on government finances.

In Section 4, we construct an optimal mechanism. This mechanism is an incentive compatible and individually rational direct revelation mechanism with the following properties. First, the government selects the most efficient provider, provided that its efficiency level exceeds a threshold level. The winning provider then exerts effort that is below the

[4] We will see later that the constant-reward *first-price* auction is strategically equivalent to the highest-output auction.

full-information optimum. The three auctions that we study in section 3 are not optimal. However, when the number of bidding providers tends to infinity, or when the tax distortion approaches zero, the constant-rewards second-price auction and the highest-output auction approach the optimal outcome.

Section 5 contains simulations on the three simple auctions, the socially optimal mechanism, and the socially optimal mechanism under complete information. We observe that the lowest-reward auction performs poorly relative to the two other simple auctions. Moreover, the outcomes of the constant-reward second-price auction and the highest effort auction rapidly approach the socially optimal mechanism. In the case of three competing providers, when the collection of $1 in taxes costs $1, social welfare that is generated by these simple auctions is only 5% less than the highest possible level of social welfare.

The following papers in the economic literature are related to ours. First of all, there is a substantial literature on the optimal design of auctions. Myerson (1981) and Riley and Samuelson (1981) show that the seller has an incentive to screen out the bidders with the lowest types, for instance by setting a reserve price below which he accepts no bids. For overviews of auction theory, see Klemperer (1999) and Krishna (2002). Secondly, the literature on incentive contracts is related. See McMillan (1992) and Laffont and Tirole (1993) for overviews.

McAfee and McMillan (1986, 1987) and Laffont and Tirole (1987, 1993) build a bridge between auction theory and incentive theory. Laffont and Tirole (1987, 1993) study a model in which the government auctions an indivisible project to one of several risk neutral firms. The government has to provide incentives for the selected firm to reduce the costs of the project. McAfee and McMillan (1986) study a similar setting, assuming risk averse bidders. The optimal contract in their model is usually an incentive contract, i.e., a contract that shares the risks among the government and the winning bidder. McAfee and McMillan (1987) is the most closely related to our paper. The most substantial difference between their paper and ours, is that McAfee and McMillan maximizes the principal's profit, whereas we maximize social welfare, taking into account the costs incurred by the winning agent. Qualitatively, the results of McAfee and McMillan (1987) and ours are the same: the optimal mechanism screens out all providers below a fixed threshold, and the winning provider's effort is lower than the full-information optimum.

2 The model

A risk neutral government wishes to procure a welfare-to-work project. We assume that n risk neutral employment service providers participate in the procurement. Each provider i, $i = 1, \ldots, n$, when winning the project, is able to exert effort e_i at the cost

$$C_i(e_i, \alpha_i) = \frac{1}{2}e_i^2 + e_i - \alpha_i e_i$$

where $\alpha_i \in [0,1]$ is provider i's efficiency level. The effort level e_i is observable, or the relevant output is the sum of e_i and a disturbance term with mean 0. The latter is irrelevant as by assumption, both the government and the providers are risk neutral. In the specific context of welfare-to-work programs, we interpret effort as the savings on social benefits when people in the project find a job. C_i is convex, which is a natural assumption in this context. In addition, we assume that the social welfare of each unit of savings on social benefits (apart from the impact on government finances) is equal to 1. The motivation for this assumption is that the marginal unemployed person is indifferent between working and not working, so that his additional utility for work equals 0. The society yields the output of his work, which is equal to the social benefits he received.[5]

The providers draw the α_i's independently from the same distribution with a cumulative distribution function F on the interval $[0,1]$ and a density function f. F is common knowledge. We assume that

$$\alpha_i - \frac{1 - F(\alpha_i)}{f(\alpha_i)}$$

is strictly increasing in α_i, which holds true for several standard distributions, including the uniform and the exponential distributions. Provider i has the utility function

$$U_i = t_i - C_i$$

where t_i is the monetary transfer that it receives from the government.

Let S denote the net social welfare of the project. We follow Laffont and Tirole (1987) in that the social cost of one unit of money is $1 + \lambda$, where $\lambda > 0$. Net social welfare is then given by

$$\begin{aligned} S &= (\sigma + \lambda)e_i - (1+\lambda)t_i + t_i - C_i(e_i, \alpha_i) \\ &= (\sigma + \lambda)e_i - \lambda U_i - (1+\lambda)\left[\frac{1}{2}e_i^2 + e_i - \alpha_i e_i\right] \end{aligned} \tag{2.1}$$

where i is the provider the government has selected for the welfare-to-work program. In other words, each unit of the provider's effort increases social welfare with $1 + \lambda$, while each unit of

[5] Some claim that the impact on total social welfare is higher than the net savings on social benefits. There may for instance arise positive externalities from an unemployed person finding a job, which are rooted in a decrease in crime and intergenerational welfare dependency (see OECD, 2001). However, there is little empirical evidence that these positive externalities are substantial.

its costs diminishes social welfare with one unit. An optimal mechanism maximizes S under the restriction that the providers play a Bayesian Nash equilibrium, and that it satisfies a participation constraint (each participating provider should at least receive zero expected utility).

The first-best optimum, i.e., the optimum under complete information, has the following properties. First of all, the government selects the most efficient provider, i.e., the provider with the highest type α_i, as this provider has the lowest C_i for a given effort level. Second, the government induces this provider to exert effort α_i. Finally, the government exactly covers the costs C_i. We will see that this first-best optimum cannot be reached in our setting with incomplete information: the government has to pay informational rents to the provider. In the optimal mechanism under incomplete information, the government 'pays' these informational rents by (1) only selecting the most efficient provider if its type exceeds a threshold level, (2) inducing effort level lower than α_i, and (3) covering more than the costs the provider actually incurs.

3 Simple mechanisms

We consider three simple mechanisms the government may use to allocate the welfare-to-work project to one of the employment service providers. These mechanisms are all two-stage games. In the first stage of each game, the government auctions the project. In the second stage, the winning provider chooses its effort level and the government rewards the provider depending on its effort choice.

3.1 The lowest-reward auction

The first mechanism is the lowest-reward auction. In this auction, the provider that submits the lowest reward, wins the project. Imagine that the winner submitted a reward level equal to b. Then the government rewards the provider with b for every unit of effort it puts in the project. The following proposition characterizes equilibrium bidding for this auction.[6]

Proposition 1. *Consider the following bidding function and effort level function.*

$$B(\alpha) = 1 - F(\alpha)^{-\frac{n-1}{2}} \int_0^\alpha x dF(x)^{\frac{n-1}{2}}, \text{ and}$$
$$L(b,\alpha) = b + \alpha - 1,$$

where b is the bid of the winner. B and L constitute a symmetric Bayesian Nash equilibrium of the lowest-reward auction. B is strictly decreasing in α and n.

From Proposition 1, we can draw the following conclusions. First of all, as B is strictly decreasing in α, the auction always rewards the project to the most efficient provider. In other words, this auction completely solves the adverse selection problem. Second, the provider chooses its effort at the level L at which the marginal benefits of effort (b) are equal to the marginal costs ($C_i'(L) = L + 1 - \alpha_i$). Third, both the expected effort level and social welfare converge to zero when the number of providers tends to infinity: more competition strengthens the moral hazard problem. This follows from the following trade-off. The more providers, the more efficient is the most efficient provider. However, the larger the number of providers in the auction, the more aggressively they have to bid. The winner, having submitted the lowest reward, then has little incentives to put much effort in the project as the marginal benefits from its effort are very low.

3.2 The highest-output auction

In order to avoid the moral hazard problem that is imminent in the lowest-reward auction, the government may pay the winning provider a constant reward τ for each unit of its effort. The

[6] All proofs are relegated to the appendix.

highest-output auction implements a mechanism with this property. This auction rewards the project to the provider that promises the highest output level. The government pays the winner the following amount of money, depending on its actual output level e and the output level b it promised in the auction:

$$t(e,b) = \tau(e-b)$$

where τ is a constant. We provide the equilibrium properties of this mechanism in the next proposition.

Proposition 2. *Let*

$$\underline{\alpha} = \max\{1-\tau, 0\},$$
$$k = \begin{cases} F(\underline{\alpha})^{n-1}\underline{\alpha} + \int_0^{\underline{\alpha}} x dF(x)^{n-1} & \text{if } \tau < 1 \\ 0 & \text{if } \tau \geq 1 \end{cases},$$
$$B(\alpha) = \begin{cases} \frac{\tau-1}{2\tau} + \frac{1}{2}\tau^{-1}F(\alpha)^{-n+1}\left[\int_0^{\alpha} x dF(x)^{n-1} - k\right] & \text{if } \alpha \geq \underline{\alpha} \\ 0 & \text{if } \alpha < \underline{\alpha} \end{cases}, \text{ and}$$
$$L(b,\alpha) = \begin{cases} \tau - 1 + \alpha & \text{if } \alpha \geq \underline{\alpha} \\ 0 & \text{if } \alpha < \underline{\alpha} \end{cases}$$

with B and L a bidding function and an effort function respectively. B and L constitute a symmetric Bayesian Nash equilibrium of the highest-output auction. B is strictly increasing in α for $\alpha \geq \underline{\alpha}$ and B is strictly increasing in n for $\tau \geq 1$.

The above proposition suggests that the highest-output auction is better than the lowest-reward auction for $\tau = 1$. Both auctions are efficient in the sense that the project is always rewarded to the provider with the highest efficiency parameter. However, the effort by the winning provider is higher in the highest-output auction. Moreover, as we will show later, if $\lambda = 0$ or if n tends to infinity, the highest-output auction is optimal for $\tau = 1$.

3.3 The constant-reward second-price auction

Alternatively, the government may use the constant-reward auction. OECD (2001) proposes this auction as an alternative to the beauty contest that are usually used in welfare-to-work programs. We focus on the constant-reward *second-price* auction. In this auction, the project is allocated in the second-price sealed-bid auction: the winner is the highest bidding provider, which has to pay the bid of the second highest bidder to the government. The winner is then rewarded ρ monetary units per unit of effort. Note that the constant reward *first-price* auctions is strategically equivalent to the highest-output auction. A bid b in this auction is equivalent to a bid b/ρ in the highest-output auction with $\tau = \rho$.

Proposition 3 provides the equilibrium properties of the constant-reward second-price auction.

Proposition 3. *Consider the following bidding function and effort level function.*

$$B(\alpha) = \begin{cases} \frac{1}{2}(\rho - 1 + \alpha)^2 & \text{if } \alpha \geq 1 - \rho \\ 0 & \text{if } \alpha < 1 - \rho \end{cases}, \text{ and}$$

$$L(b, \alpha) = \begin{cases} \rho - 1 + \alpha & \text{if } \alpha \geq 1 - \rho \\ 0 & \text{if } \alpha < 1 - \rho \end{cases},$$

B and L constitute a symmetric Bayesian Nash equilibrium of the constant-reward auction. B is constant in n and strictly increasing in α for all $\alpha \geq \rho - 1$.

For $\rho = 1$, the constant-reward second-price auction turns out to have the same properties as the highest-output auction with $\tau = 1$. Both auctions are equally efficient and share the same expected effort levels, expected payments, and expected social welfare. This result follows from a 'revenue equivalence theorem' which states that providers obtain the same expected payment from all mechanisms which allocate the project to the same provider and in which the winner provides the same effort level, provided that the utility of the lowest type equals zero. We prove this result in the Appendix in the proof of Proposition 4. Moreover, as we will show in the next section, when $\lambda = 0$ or n tends to infinity, the constant-reward second-price auction is optimal for $\rho = 1$.

4 The socially optimal mechanism

What is the socially optimal mechanism, i.e., the mechanism that maximizes (2.1)? According to Myerson (1981), we may, without loss of generality, restrict our attention to incentive compatible and individually rational direct revelation mechanisms. Let

$$\tilde{\alpha} = (\tilde{\alpha}_1, ..., \tilde{\alpha}_n)$$

be the vector of announcements by provider $1, ..., n$ respectively. We consider mechanisms $\mu = (x_i(\tilde{\alpha}), e_i(\tilde{\alpha}), t_i(\tilde{\alpha}))$ that induce a truth telling Bayesian Nash equilibrium, where, given the announcement $\tilde{\alpha}$, $x_i(\tilde{\alpha})$ is the probability that provider i wins the contract, and, given that provider i wins the contract, $e_i(\tilde{\alpha})$ is its effort and $t_i(\tilde{\alpha})$ is the monetary transfer it receives from the government.

Proposition 4. *The optimal mechanism $\mu^* = (x^*, e^*, t^*)$ has the following properties:*

$$x_i^*(\alpha) = \begin{cases} 1 & \text{if } \alpha_i > \max_{j \neq i} \alpha_j \text{ and } \alpha_i \geq \underline{\alpha} \\ 0 & \text{otherwise} \end{cases},$$

$$e_i^*(\alpha) = \alpha_i - \frac{\lambda}{1+\lambda} \frac{1 - F(\alpha_i)}{f(\alpha_i)}, \text{ and}$$

$$t_i^*(\alpha) = C_i(e_i^*(\alpha)) + \int_{\underline{\alpha}}^{\alpha_i} (e_i^*(y)) \frac{F(y)^n}{F(\alpha_i)^n} dy,$$

where $\underline{\alpha}$ is the unique solution to y in $y = \frac{\lambda}{1+\lambda} \frac{1-F(y)}{f(y)}$.

The optimal mechanism μ^* shows that the government optimally selects the most efficient provider, provided that its efficiency level exceeds $\underline{\alpha} > 0$. This provider exerts effort according to e_i^* and t_i^* determines the payments it receives from the government. Observe that the desired effort level $e_i^*(\alpha)$ and $\underline{\alpha}$ do not depend on the number of bidding providers.

Three types of inefficiency arise from this mechanism. First, since $e_i^*(\alpha) < \alpha_i$ for all $\alpha_i < 1$, the provider's effort is lower than in the full-information optimum. Second, the government will not contract with any provider whose efficiency level is below $\underline{\alpha}$, whereas in the full-information world, the government would contract with any provider. The latter is analogous to a reserve price in an optimal auction (see, e.g., Myerson, 1981). Third, as $t_i^*(\alpha) \geq C_i(e_i^*(\alpha))$, the government covers more than the costs that are actually born by the winning provider, which is inefficient as government finances are socially costly. These types of inefficiency give the government the opportunity to capture some of the informational rents that arise because of incomplete information.

Corollary 5. *Both the highest-output auction with $\tau = 1$ and the constant-reward second-price auction with $\rho = 1$ are optimal if (1) $\lambda = 0$ or (2) n tends to infinity.*

Recall that the constant-reward second-price auction with $\rho = 1$ and the highest-output auction with $\tau = 1$ always select the provider i with the highest efficiency level, and induce it to choose effort α_i. Corollary 5 follows immediate from the expressions for e_i^* and $\underline{\alpha}$. If $\lambda = 0$ then $e_i^*(\alpha) = \alpha_i$ and $\underline{\alpha} = 0$, so that both auctions are optimal. Moreover, as both do not depend on n, the only effect of increasing n is to change the distribution of the efficiency level of the selected provider. When n increases, the probability that the highest type exceeds $\underline{\alpha}$ equals one. In addition, $e_i^*(\alpha)$ tends to α_i for α_i approaching 1. As the highest type approaches 1 for n tending to infinity, the two auctions are optimal for large n. In the next section, we investigate in a simple setting with the uniform distribution how close both auctions come to the socially optimal mechanism.

5 Simulation

In this section, we simulate the outcomes of the model under the simplifying assumption that the providers draw their efficiency parameter from the uniform distribution on the interval $[0,1]$, i.e., $F = U[0,1]$. Let LRA denote the lowest-reward auction, CRA the constant-reward second-price auction with $\rho = 1$, and HOA the highest output auction with $\tau = 1$. Equilibrium bidding for the three mechanisms is respectively

$$\begin{aligned} B_{LRA}(\alpha) &= 1 - \frac{n-1}{n+1}\alpha, \\ B_{CRA}(\alpha) &= \frac{1}{2}\alpha^2, \text{ and} \\ B_{HOA}(\alpha) &= \frac{1}{2}\alpha^2 - \alpha\frac{n-1}{n}. \end{aligned}$$

We let OPT denote the optimal mechanism, which always selects the most efficient provider given that its efficiency parameter exceeds $\frac{\lambda}{1+2\lambda}$. The winning provider exerts the following effort level

$$e_i^*(\alpha) = \begin{cases} \frac{1+2\lambda}{1+\lambda}\alpha_i - \frac{\lambda}{1+\lambda} & \text{if } \alpha_i \geq \frac{\lambda}{1+2\lambda} \\ 0 & \text{if } \alpha_i < \frac{\lambda}{1+2\lambda} \end{cases}.$$

Table 5.1 contains expected effort and expected social welfare arising from these three simple mechanisms under $\lambda = 1$. Comparing the outcomes of these mechanisms with the socially optimal mechanism and the first-best mechanisms, we observe the following. First of all, CRA and HOA are equivalent in the sense that both induce the same effort from the winner of the auction and that the two mechanisms generate the same level of social welfare. Second, the outcomes of LRA deviate quite dramatically from the outcomes of the socially optimal mechanism: an increase in the number of providers may result in a decrease in the expected effort from the winning firm. The positive effect on effort arising from more bidders increasing the expected efficiency turns out to be diminished by the negative effect that more bidders decrease the per unit payment.

Third, the efforts under CRA and HOA are the same as in the social optimum. Fourth, the effort in the optimal mechanism is lower than the effort in a first-best world. The reason is that the government has to pay an informational rent, so that it induces less effort than in the complete information optimum. Fifth, the expected effort level and social welfare arising from both CRA and HOA converge to the social optimum. Sixth, the deviation from these two mechanisms from the social optimal is small even for a small number of bidders. Just a bit of competition (three bidders) is sufficient for CRA and HOA to perform well (reaching at least 95% of the maximum level of social welfare). And finally, the optimal mechanism converges to the first best. The intuition behind this observation is that for large n, the government can exploit competition between the providers, which reduces the level of informational rents it has to pay.

Table 5.1 Simulation results for $\lambda = 1$.

n	LRA	CRA	HOA	OPT	First-Best
	Expected effort				
1	0.500	0.500	0.500	0.333	0.500
2	0.444	0.667	0.667	0.519	0.667
3	0.375	0.750	0.750	0.630	0.750
4	0.320	0.800	0.800	0.701	0.800
5	0.278	0.833	0.833	0.750	0.833
6	0.245	0.857	0.857	0.786	0.857
7	0.219	0.875	0.875	0.813	0.875
8	0.198	0.889	0.889	0.833	0.889
9	0.180	0.900	0.900	0.850	0.900
	Expected social welfare				
1	0.167	0.167	0.167	0.222	0.333
2	0.333	0.333	0.333	0.370	0.500
3	0.375	0.450	0.450	0.474	0.600
4	0.373	0.533	0.533	0.550	0.667
5	0.357	0.595	0.595	0.607	0.714
6	0.337	0.643	0.643	0.652	0.750
7	0.316	0.681	0.681	0.687	0.778
8	0.296	0.711	0.711	0.717	0.800
9	0.278	0.736	0.736	0.741	0.818

For larger λ, the gap between CRA and HOA on one side, and the optimal mechanism on the other side becomes larger. This can be observed from table 5.2, which contains simulation results for $\lambda = 2$. Qualitatively, the observations in table 5.2 are the same as in table 5.1. However, note that social welfare is larger for $\lambda = 2$ than for $\lambda = 1$. This is straightforward as the higher λ, the more valuable the efforts by the employment service providers for society, as the impact on government finances is larger.

Table 5.2 Simulation results for $\lambda = 2$.

n	LRA	CRA	HOA	OPT	First-Best
	Expected effort				
1	0.500	0.500	0.500	0.300	0.500
2	0.444	0.667	0.667	0.480	0.667
3	0.375	0.750	0.750	0.594	0.750
4	0.320	0.800	0.800	0.670	0.800
5	0.278	0.833	0.833	0.723	0.833
6	0.245	0.857	0.857	0.762	0.857
7	0.219	0.875	0.875	0.792	0.875
8	0.198	0.889	0.889	0.815	0.889
9	0.180	0.900	0.900	0.833	0.900
	Expected social welfare				
1	0.167	0.167	0.167	0.300	0.500
2	0.444	0.417	0.417	0.510	0.750
3	0.525	0.600	0.600	0.662	0.900
4	0.533	0.733	0.733	0.777	1.000
5	0.516	0.833	0.833	0.865	1.071
6	0.490	0.911	0.911	0.934	1.125
7	0.462	0.972	0.972	0.991	1.167
8	0.435	1.022	1.022	1.037	1.200
9	0.409	1.064	1.064	1.076	1.227

6 Conclusion

In this paper, we have applied the theory of auctioning incentive contracts to welfare-to-work programs. In procurements for welfare-to-work projects to employment service providers, governments trade off adverse selection (the winning provider is not the most efficient one) and moral hazard (the winning provider shirks in its effort to reintegrate unemployed people). We have compared the optimal mechanism with three simple auctions (the lowest-reward auction, the highest-output auction, and the constant-reward second-price auction). We have shown that the latter two auctions approximate the socially optimal mechanism if the number of providers is large. In contrast to the optimal mechanism, the three auctions are 'simple' as they are not context dependent, i.e., 'the rules of the game' do not depend on λ, F and n. Using simulations, we have observed that competition between three bidders is already sufficient for the outcome of these auctions to reach 95% of the optimal level of social welfare. The lowest-reward auction, on the other hand, performs poorly. We show that if the number of providers is large, a 'race-to-the-bottom' will emerge, i.e., the winning bid converges to zero. As a consequence, the winner has little incentives to put much effort in the project as the marginal benefits from its effort are very low.

There are several interesting subjects for future research. First of all, as far as we know, auctions are rarely used in the practice of welfare-to-work programs. In some countries, beauty contests are used (e.g., in the Netherlands), and in others, the government awards contracts on the basis of the reputation the employment service providers gained in the past (e.g., Job Network in Australia). The question that arises is whether there are circumstances in which beauty contests or reputation mechanisms outperform auctions. Secondly, we have assumed that each unit of unemployment benefits saved increases social welfare with one unit plus the positive impact on government finances. In practice, social welfare may increase with more than one unit, for instance because positive externalities arise from people finding a job. What is the optimal mechanism in such a situation? How well do the constant-reward second-price auction and the highest-output auction perform? Finally, the effect of the winner's curse and risk aversion among the employment service providers may be interesting topics for further research.

Appendix A Proofs of propositions

A.1 Proof of Proposition 1

We construct the equilibrium using backward induction, first deriving the effort level by the winning firm, and then deriving the bids in the auction. The winner solves

$$\max_e be - \frac{1}{2}e^2 - e + \alpha e$$

where b is its bid in the auction and α its efficiency level. Straightforward calculations yield the equilibrium effort level

$$L(b, \alpha) = b + \alpha - 1.$$

To derive equilibrium bidding in the auction, we suppose that in equilibrium, all providers use the same bid function. By a standard argument, this bid function must be strictly increasing and continuous. Let $U(\alpha, \beta)$ be the utility for a provider with efficiency level α who behaves as if having signal β, whereas the other bidders play according to the equilibrium bid function. Then

$$U(\alpha, \beta) = F(\beta)^{n-1}\left[\frac{1}{2}(B(\beta) + \alpha - 1)^2\right]. \tag{A.1}$$

The first term in (A.1) refers to the probability that a provider announcing β wins the auction, and the second term refers to its expected profit when winning. A necessary equilibrium condition is that

$$\frac{\partial U(\alpha, \beta)}{\partial \beta} = 0$$

at $\beta = \alpha$, which results in the following differential equation:

$$\frac{dF(\alpha)^{\frac{n-1}{2}}[B(\alpha) - 1]}{d\alpha} + \alpha \frac{dF(\alpha)^{\frac{n-1}{2}}}{d\alpha} = 0.$$

The bidding function

$$B(\alpha) = 1 - F(\alpha)^{-\frac{n-1}{2}} \int_0^\alpha x dF(x)^{\frac{n-1}{2}}$$

is a solution. Let X_n be a stochastic variable with distribution function $F(\cdot)^{\frac{n-1}{2}}$. Note that B can be rewritten as

$$B(\alpha) = 1 - E(X_n | X_n \leq \alpha)$$

so that it is readily observed that B is strictly decreasing in α. Moreover, as X_{n+1} strictly first-order stochastically dominates X_n, B is strictly decreasing in n.

A.2 Proof of Proposition 2

We construct the equilibrium using backward induction, first deriving the effort level by the winning firm, and then deriving the bids in the auction. The winner solves

$$\max_e \tau e - \frac{1}{2}e^2 - e + \alpha e.$$

Straightforward calculations yield the equilibrium effort level

$$L(b, \alpha) = \max\{0, \tau + \alpha - 1\}.$$

To derive equilibrium bidding in the auction, we suppose that in equilibrium, all providers use the same bid function. By a standard argument, this function must be strictly increasing and continuous. Let $U(\alpha, \beta)$ be the utility for a provider with efficiency level α who behaves as if having efficiency level β, whereas the other bidders play according to the equilibrium bid function. Then

$$U(\alpha, \beta) = F(\beta)^{n-1}\left[\frac{1}{2}\max\{0, \tau + \alpha - 1\}^2 - \tau B(\beta)\right]. \tag{A.2}$$

The first term in (A.1) refers to the probability that a provider announcing β wins the auction, and the second term refers to its expected profit when winning. A necessary equilibrium condition is that

$$\frac{\partial U(\alpha, \beta)}{\partial \beta} = 0$$

at $\beta = \alpha$, which results in the following differential equation for $\alpha \geq \tau - 1$:

$$-\frac{dF(\alpha)^{n-1}\tau B(\alpha)}{d\alpha} + \frac{1}{2}(\tau + \alpha - 1)^2 \frac{dF(\alpha)^{n-1}}{d\alpha} = 0$$

with boundary condition

$$B(\underline{\alpha}) = 0.$$

where $\underline{\alpha} = \max\{1 - \tau, 0\}$.

The bidding function

$$B(\alpha) = \begin{cases} \frac{1}{2}\tau^{-1}(\tau + \alpha - 1)^2 - \tau^{-1}F(\alpha)^{-n+1}\left[\int_0^{\alpha}(\tau + x - 1)\,dF(x)^{n-1} - C\right] & \text{if } \alpha \geq \underline{\alpha} \\ 0 & \text{if } \alpha < \underline{\alpha} \end{cases}$$

is a solution, with

$$C = \begin{cases} \int_0^{\underline{\alpha}}(\tau + x - 1)F(x)^{n-1}dx & \text{if } \tau < 1 \\ 0 & \text{if } \tau \geq 1 \end{cases}.$$

Let Y_n be a stochastic variable with distribution function $F(\cdot)^{n-1}$. Note that B can be rewritten as

$$B(\alpha) = \frac{\tau - 1}{2\tau} + \frac{1}{2}\tau^{-1}E(Y_n|Y_n \leq \alpha) - \frac{1}{2}\tau^{-1}CF(\alpha)^{-n+1}$$

so that it is readily observed that B is strictly increasing in α for $\alpha \geq \underline{\alpha}$. Moreover, as Y_{n+1} strictly first-order stochastically dominates Y_n, B is strictly increasing in n if $\tau \geq 1$.

A.3 Proof of Proposition 3

We construct the equilibrium using backward induction, first deriving the effort level by the winning firm, and then deriving the bids in the auction. The winner solves

$$\max_e \rho e - \frac{1}{2}e^2 - e + \alpha e.$$

Straightforward calculations yield the equilibrium effort level

$$L(b,\alpha) = \max\{0, \rho + \alpha - 1\}.$$

In the auction, each provider has a dominant strategy, which is to submit a bid equal to its profits in the second stage, i.e.,

$$B(\alpha) = \frac{1}{2}\left(\max\{0, \rho + \alpha - 1\}\right)^2.$$

These dominant strategies constitute a Bayesian Nash equilibrium. It is readily observed that B is constant in n and strictly increasing in α for all $\alpha \geq \rho - 1$.

A.4 Proof of Proposition 4

The proof follows the same logic as Laffont and Tirole (1987) and McAfee and McMillan (1987). Without loss of generality, we may restrict our attention to direct revelation mechanisms in which each provider i announces an efficiency parameter $\tilde{\alpha}_i$. Let

$$\tilde{\alpha} = (\tilde{\alpha}_1, ..., \tilde{\alpha}_n).$$

We consider mechanisms $(x_i(.), e_i(.), t_i(.))$ that induce a truth telling Bayesian Nash equilibrium, where, given the announcement $\tilde{\alpha}$, $x_i(\tilde{\alpha})$ is the probability that provider i wins the contract, $e_i(\tilde{\alpha})$ is the effort exerted by provider i given that it wins the contract, and $t_i(\tilde{\alpha})$ is the monetary transfer to provider i if it wins the contract.

A.4.1 Providers' bidding behaviour

If all providers bid truthfully, provider i's interim utility (i.e., its expected utility given its efficiency parameter α_i) is equal to

$$U_i(\alpha_i) = E_{\alpha_{-i}}[t_i(\alpha) - x_i(\alpha)\{\varphi(e_i(\alpha)) - \alpha_i e_i(\alpha)\}] \tag{A.3}$$

where

$$\varphi(e) = \frac{1}{2}e^2 + e$$

Let $U_i(\alpha_i, \tilde{\alpha}_i)$ be provider i's utility when it has efficiency parameter α_i, it announces $\tilde{\alpha}_i$, and all other providers truthfully reveal their type. Then

$$\begin{aligned} U_i(\alpha_i, \tilde{\alpha}_i) = {} & E_{\alpha_{-i}}[t_i(\alpha_{-i}, \tilde{\alpha}_i) - x_i(\alpha_{-i}, \tilde{\alpha}_i)\varphi(e_i(\alpha_{-i}, \tilde{\alpha}_i))] \\ & + \alpha_i E_{\alpha_{-i}}[e_i(\alpha_{-i}, \tilde{\alpha}_i)x_i(\alpha_{-i}, \tilde{\alpha}_i)]. \end{aligned} \tag{A.4}$$

Incentive compatibility requires that

$$\frac{\partial U_i(\alpha_i,\tilde{\alpha}_i)}{\partial \tilde{\alpha}_i} = 0 \tag{A.5}$$

at $\tilde{\alpha}_i = \alpha_i$. From (A.3), (A.4), and (A.5) it immediately follows that

$$\begin{aligned}\frac{dU_i(\alpha_i)}{d\alpha_i} &= \left.\frac{\partial U_i(\alpha_i,\tilde{\alpha}_i)}{\partial \tilde{\alpha}_i}\right|_{\tilde{\alpha}_i=\alpha_i} + E_{\alpha_{-i}}[e_i(\alpha)x_i(\alpha)] \\ &= E_{\alpha_{-i}}[e_i(\alpha)x_i(\alpha)].\end{aligned}$$

The participation constraint then reduces to

$$U_i(0) \geq 0.$$

A.4.2 The government's problem

The government's problem is given by

$$\max_{(x_i(\cdot),e_i(\cdot),U_i(\cdot))} S = E_\alpha\left(\sum_i (1+\lambda)x_i(\alpha)e_i(\alpha) - \lambda U_i(\alpha_i) - (1+\lambda)x_i(\alpha)\left[\varphi(e_i(\alpha)) - \alpha_i e_i(\alpha)\right]\right)$$

$$\text{s.t.} \quad \begin{matrix} \dot{U}_i(\alpha_i) = E_{\alpha_{-i}}[e_i(\alpha)x_i(\alpha)] & \text{for all } \alpha_i,\, i \\ U_i(0) \geq 0 & \text{for all } i \end{matrix}.$$

According to a standard argument, $e_i(\alpha)$ only depends on α_i (see Laffont and Tirole (1987) and McAfee and McMillan (1987).). Let

$$X_i(a_i) \equiv E_{\alpha_{-i}}[x_i(\alpha)].$$

For given $X_i(a_i)$, the government's problem can be decomposed into the following n programs:

$$\max \int_0^1 \{(1+\lambda)X_i(a_i)\left[e_i(\alpha_i) - \varphi(e_i(\alpha_i)) + \alpha_i e_i(\alpha_i)\right] - \lambda U_i(\alpha_i)\} f(\alpha_i)\, d\alpha_i$$

$$\begin{aligned}\text{s.t. } & \dot{U}_i(\alpha_i) = e_i(\alpha_i)X_i(a_i), \\ & U_i(0) \geq 0.\end{aligned}$$

The Hamiltonian H_i of this program is given by

$$H_i(\alpha_i,e_i,U_i,\mu_i) = \{-\lambda U_i + (1+\lambda)X_i(a_i)\left[e_i - \varphi(e_i) + \alpha_i e_i\right]\} f(\alpha_i) + \mu_i e_i X_i(a_i).$$

Using the Pontryagin principle, we obtain

$$\begin{aligned}&\dot{\mu}_i(\alpha_i) = \lambda f(\alpha_i) \\ &\mu_i(\alpha_i) = -(1+\lambda)(1-\varphi'(e_i^*(\alpha)) + \alpha_i) f(\alpha_i) \\ &\mu_i(1) = 0\end{aligned}$$

Let $\underline{\alpha}$ is the unique solution to $y = \frac{\lambda}{1+\lambda}\frac{1-F(y)}{f(y)}$ w.r.t. y. Substituting $\varphi(e_i) = \frac{1}{2}e_i^2 + e_i$ together with some straightforward calculations yields

$$e_i^*(\alpha) = \begin{cases} \alpha_i - \frac{\lambda}{1+\lambda}\frac{1-F(\alpha_i)}{f(\alpha_i)} & \text{if } \alpha_i \geq \underline{\alpha} \\ 0 & \text{if } \alpha_i < \underline{\alpha} \end{cases}. \tag{A.6}$$

The government's problem is then reduced to

$$\max_{X_i(\cdot)} S = \sum_i \int_0^1 (1+\lambda) X_i(a_i) e_i^*(\alpha_i) \left(\alpha_i - \frac{1}{2} e_i^*(\alpha_i) \right) f(\alpha_i) d\alpha_i$$
$$- \sum_i \int_0^1 \lambda \int_0^{\alpha_i} e_i^*(y) X_i(y) dy\, f(\alpha_i) d\alpha_i$$

which is equivalent to

$$\max_{X_i(\cdot)} S = \sum_i \int_0^1 X_i(a_i) \left[\frac{1}{2} (1+\lambda) \left(e_i^*(\alpha_i) \right)^2 \right] dF(\alpha_i).$$

As $\frac{1}{2}(1+\lambda)(e_i^*(\alpha_i))^2$ is strictly increasing in a_i for $a_i \geq \underline{\alpha}$, the optimal mechanism $\mu^* = (x^*, e^*, t^*)$ has the following properties:

$$x_i^*(\alpha) = \begin{cases} 1 & \text{if } \alpha_i > \max_{j \neq i} \alpha_j \text{ and } \alpha_i \geq \underline{\alpha} \\ 0 & \text{otherwise} \end{cases},$$

$$e_i^*(\alpha) = \alpha_i - \frac{\lambda}{1+\lambda} \frac{1-F(\alpha_i)}{f(\alpha_i)}, \text{ and}$$

$$t_i^*(\alpha) = C_i(e_i^*(\alpha)) + \int_{\underline{\alpha}}^{\alpha_i} (e_i^*(y)) \frac{F(y)^n}{F(\alpha_i)^n} dy.$$

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