An estimated DSGE model for the Netherlands with unemployment

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1 Introduction

DSGE models are used at many international institutions, especially at central banks (see, for example, Smets et al. (2010), Adfison et al. (2013), Andrés et al. (2010) and Ratto et al. (2009)). Central banks are mainly interested in inflation, economic growth and the transmission mechanisms of monetary policy and much of the development of DSGE models has followed these priorities (see Christiano et al. (2005)). They are less common in policy institutes such as the CPB, which are also interested in forecasting unemployment and in analysing the effects of various policy choices including fiscal policy.

In principal, though, DSGE models should be an attractive option for institutes such as the CPB, as they are less likely to suffer from the Lucas (1976) critique. In contrast to traditional macroeconometric models, one of the key features of DSGE models is that they are specified with respect to parameters describing the optimisation problems of economic agents, such as households’ rates of time preference or the disutility of labour, rather than with respect to parameters describing the outcomes of agents’ optimisation problems. The advantage of specifying models in terms of their ‘deep’ parameters is that these parameters should be robust to policy changes. This paper investigates whether DSGE models are indeed as attractive as suggested by developing a DSGE model for the Dutch economy including unemployment and tailored to analyse the effects of changes to fiscal policy.

To that end, we take the small open economy DSGE of Lafourcade and De Wind (2012), which was developed for De Nederlandsche Bank (DNB), and add a description of unemployment and credit-constrained households, a feature widely used in the literature to better model the effects of fiscal policy. For modelling unemployment we follow Galí (2011) and Gáf (2011) and the importance of credit constraints for aggregate consumption is discussed in Galí et al. (2007) and Coenen and Straub (2005). Finally, we adjust the tax structure to more accurately reflect the tax system in the Netherlands.

In total we estimate five models for the Dutch economy. The first model is a reproduction of the original non-stationary model of Lafourcade and De Wind (2012). The final model is a stationary model with unemployment and credit-constrained households. We also estimate three intermediate models to gauge the effect of each new aspect of the model. The first intermediate model is the original non-stationary model estimated with an updated data set over a more recent sample period. The second intermediate model is a stationary version of this model. The last intermediate model is also stationary and includes unemployment, but no credit-constrained households.

Although we would be able to analyse fiscal policy changes and the consequences for unemployment for various scenarios with either of our final two models, the five models display considerable differences in both parameter estimates and responses to shocks. We interpret this
as evidence that it is difficult to accurately estimate the deep parameters in models that are misspecified, something that all models are to some degree. Since we are unlikely to have accurately estimated the deep parameters, these models are also going to be susceptible to the Lucas critique. This negates one of the most attractive features of DSGE models.

This paper is structured as follows: Section 2 briefly describes the model of Lafourcade and De Wind (2012) whilst section 3 describes the changes we have made. Section 4 describes the data and the Bayesian estimation of the model parameters. We also demonstrate a method for judging the model fit in section 4.5. We then discuss the impulse response functions in section 4.6. In section 4.7 we use the models to decompose the historical time series: an additional tool made possible by the DSGE modelling approach, before ending with some concluding remarks in section 5. The appendices contain detailed derivations of the linearised model equations and diagnostics for the estimates.
2 The model

The small, open-economy DSGE of Lafourcade and De Wind (2012) contains a number of maximising agents. This section briefly introduces each of the agents. For a more detailed exposition, see Lafourcade and De Wind (2012). In essence, the model is an open economy extension of a core New Keynesian DSGE model with a large number of rigidities added to improve the ability of the model to fit the data. As such, the key mechanisms in the model are similar to much simpler versions of New Keynesian DSGE models. The key mechanisms in a closed-economy DSGE can be described by three equations (see, for example, Gertler et al. (1999) or Galí (2010)): a New Keynesian Phillips curve relating inflation to the output gap and expected inflation; a forward looking IS curve relating the output gap positively to the future output gap and negatively to real interest rates; and a monetary policy rule that satisfies the Taylor principle (ie. the central bank raises rates more than one-for-one with inflation, see Davig and Leeper (2007)). The key mechanisms behind the New Keynesian Phillips curve is that forward looking firms raise prices when they expect future prices to be high or if they are experiencing rising costs. The output gap is a measure of marginal costs - when output is below potential firms do not experience significant cost pressures. The forward looking IS curve follows from the forward looking behaviour of households. Households choose their current and future consumption levels in relation to the real interest - the higher the real interest rate the more willing households are to postpone consumption, which suppresses current demand. For more detailed models a similar mechanism works for investment. Finally, the monetary policy rule closes the model and ensures agent’s expectations are bounded by ruling out explosive paths for future inflation and output. As described at the start of this section, the model of Lafourcade and De Wind (2012) is a substantially expanded DSGE model. The following sections will introduce the agents of the model in more detail.

2.1 Final goods producers

Final goods producers maximise profits from taking the differentiated intermediate goods and bundling them into a homogenous final, domestically produced good using a constant elasticity of substitution (CES) technology. The final, domestically produced good is subsequently turned into consumption or investment goods by the consumption and investment good bundlers, which are described below. The profits of final goods producers is paid to the households.

2.2 Intermediate goods producers

Monopolistically competitive intermediate good producers maximise profits by hiring labour from labour unions and capital services from households to produce a differentiated product.
using a Cobb-Douglas production function. The production function is subject to both permanent and temporary technology shocks. Since the intermediate goods producers are monopolistically competitive they have the power to set the price of their products. Their price setting is subject to a Calvo-style (see Calvo (1983)) nominal rigidity. The profits of intermediate goods producers are paid to the households.

2.3 Households

A continuum of identical households maximise their expected lifetime utility by choosing consumption, hours worked, bonds, investment, capital and capital utilisation rates. Utility is subject to habit formation in consumption and is logarithmic in habit adjusted consumption whilst labour produces disutility. Habit formation has been found to be necessary to explain the smoothness of consumption across the business cycle (see Fuhrer (2000), for example). Households supply homogenous labour to the labour unions and are paid a wage by the unions. They consume a consumption bundle that contains all of the domestic and foreign types of consumption good - the proportions of which are decided by the consumption good bundlers. Households can smooth consumption across time by investing in government bonds or by investing in capital goods. Household demand for these assets depends on the exogenous interest rate set by the foreign central bank and the risk premium charged on bonds. Households rent out the capital services produced by the capital stock to the intermediate good producing firms and decide how hard the capital stock is worked by setting the utilisation rate. Working the capital stock harder entails higher costs for the household.

2.4 Labour unions

Labour unions are not unions in the normal sense of the word. They are a device for modelling monopolistic competition and wage bargaining in the labour market. Labour unions maximise profits by buying the labour input from households, before differentiating the labour types and selling the labour on to firms (via labour packers). Because labour unions are monopolistically competitive the differentiated labour bundles allows them to set the wages they charge firms. Those wages are subject to a Calvo-style nominal rigidity. Since wages are set monopolistically they are set at a level above households’ marginal rate of substitution. At the wage rate that firms have to pay, households would be willing to supply more labour than firms demand. It is this feature of wage setting that allows us to model unemployment. Since firms can only get labour through the labour unions, households are unable to increase employment by offering to work for lower wages. Profits of the labour unions are paid to households.
2.5 Government

The government levies distorting taxes on consumption, labour income and capital income as well as a lump-sum tax, which is used to target a debt-to-GDP ratio in the long-run. Government consumption is not productive and follows an autoregressive process. The small, open economy is in a currency union with the foreign country - monetary policy is set purely in the foreign country.

2.6 Foreign block

The foreign economy is exogenous to the domestic economy and is described by a simple three-equation DSGE model comprising an IS curve, a Phillips curve and a monetary policy rule similar to the one referred to at the start of this section. The foreign block describes an output gap, the inflation rate and the nominal interest rate in the foreign economy. The domestic economy is assumed to be in a currency union with the foreign country.

2.7 Consumption and investment bundlers

Consumption bundlers are profit maximising firms that bundle the domestically produced consumption good with the foreign produced consumption good using a CES technology subject to home bias. Since consumption bundlers are competitive they make zero profits. Investment bundlers are analogous to consumption bundlers.
3 Changes to Lafourcade and De Wind (2012)

In this section we describe in detail the changes we have made to the model of Lafourcade and De Wind (2012) and the new features we have added. In short, we add credit-constrained households to the model and utilise the difference between the marginal rate of substitution (what households are willing to work for) and the wage set by labour unions to produce a measure of unemployment following Galí (2011) and Galí (2011).

3.1 Households

There are two types of households in the model: optimising and credit-constrained. The latter household type can neither borrow nor lend which means they consume all of their income each period. In what follows, credit-constrained households are denoted by superscript $C$ and optimising households by superscript $O$. The share of credit-constrained households is $\lambda_C$. Both household types aim to maximise the expected lifetime utility subject to a sequence of budget constraints.

3.1.1 Credit-constrained households

Credit-constrained households gain utility from consuming the aggregate consumption good but lose utility from working. The utility they gain from consumption is subject to habit formation (see, for example, Fuhrer (2000)), which is governed by the parameter $\lambda$. They supply a continuum of labour types indexed by $l$ to labour unions and receive wages in return. The labour unions set a wage above the marginal rate of substitution, which means that households supply however much labour is demanded by the unions. Credit-constrained households pay taxes and, since they cannot lend or borrow, they consume their entire income each period. Household $q$ faces the following problem

$$\max_{C^C_t(q),L^C_t(q)} E \sum_{s=0}^{\infty} \beta^s \left( d_{t+s} \ln \left( C^C_{t+s}(q) - \lambda C^C_{t+s-1}(q) \right) - \phi_{t+s} \psi 1 + \eta \right)$$

subject to

$$\left( 1 + \tau_c \right) p^C_{t+s} C^C_{t+s}(q) = \left( 1 - \tau_w \right) \int_0^1 w(l) L^C_{t+s}(q,l) dl + T_{t+s} \gamma^d_{t+s},$$

where $\beta$ is a subjective discount factor, $d_t$ is a stationary preference shock, $C^C_t(q)$ is the consumption of household $q$ in period $t$, $\phi_t$ is a non-stationary shock to the disutility of labour used by Lafourcade and De Wind (2012) to model the upward trend in labour supply in the Netherlands between 1984 and 2007, $\psi$ and $\eta$ parameterise the disutility of labour and $L^C_t(q)$ is the total labour supplied to firms by household $q$ in period $t$. In the budget constraint, $\tau_c$ is a
consumption tax, \( p_t^c \) is the real price of the consumption good relative to the price of the domestic production good, \( \tau_w \) is a labour income tax, \( w_l(l) \) is the wage of labour type \( l \) and \( L_t^c (q, l) \) is household \( q \)'s supply of labour type \( l \). Finally, \( T_t y^d \) is a lump-sum transfer to or from the government.

Since the wages that are set by the labour unions are higher than the marginal rate of substitution, the household is willing to supply labour perfectly elastically at the market wage rate, \( w_l \). In other words, the quantity of labour supplied is not a choice variable of the household since they would always be willing to supply more than is demanded. Therefore, the right hand side of the credit-constrained household’s budget constraint is exogenous to the household. It therefore also follows that the credit-constrained household’s behaviour is entirely explained by the budget constraint, which when aggregated over all types of labour becomes

\[
(1 + \tau_c) p_t^c C_t^c (q) = (1 - \tau_w) w_t L_t^c (q) + T_t y^d. \tag{3.3}
\]

### 3.1.2 Optimising households

Optimising households maximise a comparable utility function to that for the credit-constrained households. Furthermore, they also supply a continuum of the same labour types as credit-constrained households for which they receive the same wage. In contrast to credit-constrained households, the optimising households also buy and sell bonds, \( B_t^o \), invest, \( I_t \), and also set the capital capacity utilisation rate, \( U_t^O \). Household \( j \) faces the following problem\(^1\)

\[
\max_{c_t^O (j), B_t^O (j), I_t, K_t^O (j), y, U_t^O (j), L_t^c (j)} \sum_{s=0}^{\infty} \beta^s \left( \int d_l \ln \left( C_{t+s}^O (j) - \lambda C_{t+s-1}^O (j) \right) - \phi_{t+s} \frac{\psi}{1 + \eta} L_{t+s}^O (j)^{1+\eta} \right) \tag{3.4}
\]

subject to the following budget constraint

\[
(1 + \tau_c) p_{t+s}^c C_{t+s}^O (j) + p_{t+s}^I I_{t+s} (j) + B_{t+s}^O (j) = (1 - \tau_w) \int_0^1 w_{t+s} (l) L_{t+s}^c (j, l) dl + \left( r_{t+s}^k U_{t+s}^O (j) - p_{t+s}^I (r_k + \Phi (U_{t+s}^O (j))) \right) K_{t+s-1}^O (j) + \frac{K_{t+s-1}^O H_{t+s-1} - \tau_e B_{t+s-1}^P O (j) + T_{t+s} y_{t+s}^d + D_{t+s}^P O + D_{t+s}^m O + D_{t+s}^E O}{\pi_{t+s}}
\]

and the law of motion for capital

\[
K_{t+s}^O (j) = (1 - \delta) K_{t+s-1}^O (j) + \mu_{t+s} \left( 1 - S \left( \frac{p_{t+s}^O (j)}{p_{t+s-1}^O (j)} \right) \right) I_{t+s}^O (j). \tag{3.6}
\]

\(^1\) We also replaced the capital income tax in Lafourcade and De Wind (2012) with a wealth tax, since that is the system in the Netherlands.
The optimising households solve this constrained optimisation problem in the usual way. The Lagrange multiplier on the optimising household budget constraint is denoted by $\Xi^O_t$ and the Lagrange multiplier on the law of motion for capital is give by $\Xi^k_t$. Once again, the union sets a wage higher than the marginal rate of substitution so the optimising household is also willing to supply labour perfectly elastically at the market wage rate and labour employed is decided by the demand for labour. The remaining first order conditions for the optimising households’ problem are given by

\[
(\partial C^O_t (j)) : \frac{d_t}{C^O_t (j) - \lambda C^O_{t+1} (j)} - \beta \lambda E_t \left( \frac{d_{t+1}}{C^O_{t+1} (j) - \lambda C^O_{t} (j)} \right) = \Xi^O_t (j) \rho^c_t (1 + \tau_c) \tag{3.7}
\]

\[
(\partial B^O_t (j)) : 1 = \beta E_t \left( \frac{\Xi^O_{t+1} (j)}{\Xi^O_t (j)} \frac{R^c_t H^c_t - \tau_c}{\pi_{t+1}} \right) \tag{3.8}
\]

\[
(\partial I^O_t (j)) : \frac{p^j_t}{Q^O_t (j)} = 1 - S \left( \frac{I^O_t (j)}{P^O_{t+1} (j)} \right) - S' \left( \frac{I^O_t (j)}{P^O_{t+1} (j)} \right) \frac{I^O_t (j)}{P^O_{t+1} (j)} + \beta E_t \left( \frac{\Xi^O_{t+1} (j) Q^O_{t+1} \mu_{t+1}}{\Xi^O_t (j) Q^O_t} \mu_t \right) \left( \frac{I^O_t (j)}{P^O_t (j)} \right) \tag{3.9}
\]

\[
(\partial K^O_t (j)) : Q^O_t (j) = \beta E_t \left[ \frac{\Xi^O_{t+1} (j)}{\Xi^O_t (j)} \left( Q^O_{t+1} (j) (1 - \delta) + r^k_{t+1} U^O_{t+1} (j) - p^j_{t+1} (\tau_k + \Phi (U^O_{t+1} (j))) \right) \right] \tag{3.10}
\]

\[
(\partial U^O_t (j)) : r^k_t = p^j_t \Phi' (U^O_t (j)), \tag{3.11}
\]

where

\[
Q_t (j) = \frac{\Xi^k_t (j)}{\Xi^O_t (j)} \tag{3.12}
\]

is Tobin’s Q. The first order condition for consumption relates the marginal utility of consumption today to the expected marginal utility of consumption tomorrow. How they relate to each depends on the after tax price of consumption goods tomorrow and, via the shadow price of wealth, the real interest rate. If the household is to forgo consumption today, the decision is based on how much consumption they can enjoy tomorrow after receiving interest on the amount they saved today. The first order condition for bonds relates the after tax return on holding bonds with, via the shadow price of wealth, the household’s expected marginal utility of consumption tomorrow. This risk premium, $H^c_t$, in the equation is used to ensure a stable path for the economy. The first order conditions for investment and capital relate the marginal benefit of investing in the capital stock to the marginal cost of undertaking the investment, taking into account the investment adjustment costs. Finally, the first order condition for the capital utilisation rate sets the marginal cost of working the available capital harder equal to the rental rate of capital.
3.2 Wage bargaining

Traditionally, DSGE models don’t model unemployment explicitly: the labour market is modeled with employment and wages. However, as proposed by Galí (2011) and Galí (2011) the Erceg et al. (2000) approach to modelling the wage bargaining process used in most DSGE models can be reinterpreted to give an intuitive measure of unemployment. This section will first explain the standard approach to wage bargaining in a setting with two household types before the following section will explain how the measure of unemployment is derived. As reported by Galí et al. (2004) wage setting with credit-constrained households presents some problems when model outcomes are compared to likely real world experience. Specifically, if credit-constrained households set their marginal rate of substitution equal to the wage rate then it is not possible for their labour supply, consumption and real wages to covary positively. To overcome this we employ the commonly used Erceg et al. (2000) labour market model where unions bargain for wages on behalf of households. Since the wage set by the union is a mark-up over the marginal rate of substitution, households are willing to supply labour perfectly elastically at the going wage rate. Applying the Erceg et al. (2000) labour market model with two household types requires some assumptions about how the union weighs the preferences of each type of household, which, for our model, is detailed below.

3.2.1 Erceg, Henderson and Levin wage bargaining

Two household types

With two types of household it is no longer clear what the best choice for the objective function for the labour unions is. For simplicity and following some of the literature we assume that wage bargaining maximises a weighted average of the two types of household’s utility. The problem for the union representing labour type $l$ is

$$\max_{\omega (l)} \sum_{s=0}^{\infty} \xi^s \beta^s \left[ \lambda^C C^C (q, L^C (q, l)) + (1 - \lambda^C) U^O (C^O (j, L^O (j, l)) \right]$$

subject to the demand for labour for type $l$ workers

$$L_{t+s} (l) = \left( \frac{\omega (l)}{w_t} X_{t+s}^w \right)^{-\gamma_{t+s}} L_{t+s},$$

(3.14)

---

2 The specification of the labour market in the Lafourcade and De Wind (2012) and the standard Erceg et al. setup used here are equivalent up to a point. Without taxes, the same wage is paid by firms, $w_t$, and the same quantity of labour is supplied, $L_t$. However, in Lafourcade and De Wind (2012) the labour income tax is levied on the wage income the worker takes home - specifically on $w^tl_t$. Because the remaining labour income is paid to households as the profits of the labour unions it is not taxed.
and the budget constraints of the two household types

\[(1 + \tau_r) p_{t+1}^C C_{t+1}^C (j) + p_{t+1}^l L_{t+1}^C (j) + B_{t+1}^O (j) = (1 - \tau_w) \int_0^1 \frac{\omega_k (l)}{w_t} X_w^w w_{t+1} L_{t+1}^O (j, l) \, dl\]

\[+ \left[ t_{t+1}^\varepsilon u_{t+1}^O (j) - p_{t+1}^l (\pi_k + \Phi (U_{t+1}^O (j))) \right] k_{t+1}^O (j) + R_{t+1}^\varepsilon H_{t+1}^O \left( \frac{\pi_k}{\pi_t} \right) B_{t+1}^P O (j) + T_{t+1}^\varepsilon L_{t+1}^C (j) \]

and

\[(1 + \tau_r) p_{t+1}^C C_{t+1}^C (q) + p_{t+1}^l L_{t+1}^C (q) + B_{t+1}^O (q) = (1 - \tau_w) \int_0^1 \frac{\omega_k (l)}{w_t} X_w^w w_{t+1} L_{t+1}^O (q, l) \, dl + T_{t+1}^\varepsilon y_{t+1}^D,\]

where \(X_w^w\) is a compound indexation term:

\[X_w^w = \frac{1}{\pi_t} \prod_{l=0}^{\infty} X_{l+1}^w.\]

and

\[x_q^w = \left( \frac{\pi_t - \pi_{t-1}}{\pi_t} \right) \frac{\pi_t \gamma_t}{\pi_{t-1}} x_q^{l-1}.\]

Note that \(\lambda_r^C\) is a weight based on the proportions of each type of household, but is not equal to \(\lambda_C^C\). Choosing a slightly different weight in the union optimisation problem\(^3\) allows us to easily derive the analytic steady state we will need for estimation. The first order condition is given by

\[E_t \sum_{s=0}^\infty \xi^s \beta^s \left( \lambda_C^C \frac{\partial U^C}{\partial X_s} \left( C_{t+1}^C (q), L_{t+1}^C (q, l) \right) + (1 - \lambda_C^C) \frac{\partial U^O}{\partial \theta_k (l)} \left( C_{t+1}^O (j), L_{t+1}^O (j, l) \right) \right) = 0.\]

(3.19)

This can be simplified to

\[\sum_{s=0}^\infty (\xi \beta)^s E_t L_{t+1} (l) \frac{\partial U}{\partial C} \left( \frac{1 - \tau_w}{1 + \tau_r} \frac{\omega_k (l)}{w_t} X_w^w W_{t+1}^C + \frac{\xi_{t+1}}{1 - \xi_{t+1}} MRS_{t+1}^* \right) = 0\]

(3.20)

where

\[\frac{\partial U}{\partial C} = \left( \lambda_C^C \frac{\partial U^C}{\partial C_{t+1}^C (q)} + (1 - \lambda_C^C) \frac{\partial U^O}{\partial C_{t+1}^O (j)} \right) \]

(3.21)

\(^3\) We define the weighted average marginal utility used by the labour union for setting wages to be the harmonic mean of the two households marginal utilities instead of the arithmetic mean. Specifically, we set \(\lambda_r^C\) such that

\[\Xi^r = (\lambda_r^C \Xi^C + (1 - \lambda_r^C) \Xi^O) = \frac{1}{\frac{1}{\Xi^C} + \frac{1}{\Xi^O}}.

12
and
\[ MRS_{t+s}^* = \left[ \lambda_C^* \frac{\partial U_C(C_{t+s}^0(q), L_{t+s}^0(l), p_{t+s}(l), q)}{\partial C_{t+s}(q)} + (1 - \lambda_C^*) \frac{\partial U_O(C_{t+s}^0(j), L_{t+s}^0(j), p_{t+s}(j), l)}{\partial L_{t+s}(l)} \right] \frac{\lambda^* C}{p_{t+s}} \]

(3.22)
can be thought of as a weighted average of the two households’ marginal rate of substitution between consumption and leisure. At this stage we define a new variable, let’s call it \( \omega_{t+s} \) such that
\[ MRS_{t+s}^* = \left[ \lambda_C^* \frac{\partial U_C(C_{t+s}^0(q), L_{t+s}^0(l), p_{t+s}(l), q)}{\partial C_{t+s}(q)} + (1 - \lambda_C^*) \frac{\partial U_O(C_{t+s}^0(j), L_{t+s}^0(j), p_{t+s}(j), l)}{\partial L_{t+s}(l)} \right] \frac{(1 - \tau_w)}{(1 + \tau_c)} p_{t+s} C_{t+s}^0 + \omega_{t+s}. \]

(3.23)

Finally, for simplification, let us define
\[ \Xi_{t+s}^* = \left[ \lambda_C^* \Xi_{t+s} + (1 - \lambda_C^*) \Xi_{t+s}^O \right], \]

(3.24)

then the condition for the optimal wage becomes
\[ E_t \sum_{s=0}^{\infty} \beta^s L_{t+s}^d(l) \frac{\Xi_{t+s}^*}{\Xi_t} \left[ \frac{\alpha_t(l)}{\omega_t} X_{t+s}^w \omega_{t+s} + \frac{\omega_{t+s}}{1 - \epsilon_{t+s}} \omega_{t+s} \right] = 0. \]

(3.25)

This is the same condition as in the Lafourcade and De Wind (2012), except we have new definitions of \( \omega_{t+s} \) and \( \Xi_{t+s}^* \). Intuitively this optimal wage setting equation says that over the expected duration of this wage, the wage set today \( \omega_t(l) \) should, on average, be a constant markup, \( \frac{\epsilon_{t+s}}{1 - \epsilon_{t+s}} \), over the marginal rate of substitution of households after taking account of taxes and expected changes in the price of the consumption good, \( \omega^h_{t+s} \), also taking into account the indexation process detailed above.

### 3.3 Unemployment

Following the Galí approach to unemployment, we define the level of unemployment as the difference between the number of hours that workers would be willing to work at the prevailing wage and actual employment. Let \( L_d^d \) be desired employment at the going wage rate, then unemployment, \( U_t \) is
\[ U_t = L_d^d - L_t, \]

(3.26)

where
\[ L_d^d = \lambda_C L_d^{d.C} + (1 - \lambda_C) L_d^{d.O}. \]

(3.27)

So how much labour do households want to supply? Setting the marginal disutility from working against the benefit from the wages received for each household type gives an expression for the desired labour supply of each household type:
\[ \phi_t \psi_t L_d^{d.O} = (1 - \tau_w) \Xi_t^O \omega_t \]

(3.28)
and
\[ \varphi_t L_t^\alpha \gamma_t = (1 - \tau_t) \Xi_t^\gamma w_t. \] (3.29)

These expressions are enough for us to calculate the measure of unemployment.

### 3.3.1 The rest of the model

In the model of Lafourcade and De Wind (2012) there is only one type of household. The relevant discount factor for the firms is then given by the household Lagrange multiplier. For our model the relevant discount factor is given by the optimising household Lagrange multiplier, since the optimising households own the firms. We also note that the reported estimation results in Lafourcade and De Wind (2012) are based on a specification of the Phillips curve with a minor error. Our estimates are based on a corrected specification of the Phillips curve.\(^4\)

### 3.4 The linearised model equations

All in all, the following additional equations are needed for the log-linearised model:

\[ \beta_t^c + \hat{C}_t^C = \frac{1}{(1 - \tau_t) w_t L_t + T_t y_t^d} \left[ (1 - \tau_t) w_t L_t (\hat{w}_t + \hat{L}_t) + T_t y_t^d \left( \hat{y}_t + y_t^d \right) \right]. \] (3.32)

\[ \hat{C}_t = \frac{\lambda_t C_t^C}{\lambda_t C_t^C + (1 - \lambda_t) C_t^O} \hat{C}_t^C + \frac{(1 - \lambda_t) C_t^O}{\lambda_t C_t^C + (1 - \lambda_t) C_t^O} \hat{C}_t^O \] (3.33)

\[ \hat{x}_t^C + \hat{p}_t = \frac{\gamma_t}{\gamma_t - \beta} \hat{x}_t^C + \frac{\beta \lambda}{\gamma_t - \beta} E_t \hat{C}_t^{C,1} \] (3.34)

\[ d_t - \hat{x}_t^C = \frac{\gamma_t}{\gamma_t - \lambda} \hat{x}_t^C + \frac{\lambda}{\gamma_t - \lambda} \left( \hat{C}_t^{C,1} - \hat{x}_t^{C,1} \right) \] (3.35)

\[ \hat{Z}_t^C = \lambda_t \hat{x}_t^C + (1 - \lambda_t) \hat{Z}_t^O \] (3.36)

and for the steady state:

\[ C_t^C = \frac{(1 - \tau_t) w_t L_t + T_t y_t^d}{(1 + \tau_t) \beta_t} \] (3.37)

---

\(^4\) The linearised version of the Phillips Curve is

\[ E_t (\sigma_t - u_t \sigma_t - 1 - (\sigma_t + 1 - u_t \sigma_t)) \hat{w}_t = \frac{(1 - \xi_t)(1 - \xi_t \beta_t)}{\xi_t} (\hat{w}_t - \hat{w}_t + \hat{\xi}_t), \] (3.30)

where \( \sigma_t = \hat{h} + \hat{w}_t - \hat{h}_t + \hat{\xi}_t \) and \( \hat{\xi}_t = \frac{1}{1 - \xi_t} \xi_t \). The reported estimation results in Lafourcade and De Wind (2012) are based on the following specification

\[ E_t (\sigma_t - u_t \sigma_t - 1 - (\sigma_t + 1 - u_t \sigma_t)) \hat{w}_t = \frac{(1 - \xi_t)(1 - \xi_t \beta_t)}{\xi_t} \left( \frac{\hat{w}_t - \hat{w}_t + \hat{\xi}_t}{1 + \eta_t} \right), \] (3.31)

where \( \sigma_t = \sigma_t - \hat{\xi}_t \).
\[ C_s^O = \frac{C_s - \lambda_C C_s^C}{1 - \lambda_C}. \]  

(3.38)

### 3.4.1 Additional parameters

The only additional parameter is \( \lambda_C \), the share of credit constrained households.
4 Bayesian estimation

In this chapter we discuss the estimation of our DSGE model. Estimation allows the researcher to obtain posterior distributions for the deep parameters given the information available in the data series used in the estimation. The deep parameters determine the properties and behaviour of the model for both forecasting and policy analysis. To better understand the parameter estimates we see, we estimate five model specifications, starting with a replication of the original results in Lafourcade and De Wind (2012) and ending with the model described in the previous chapter. The five model specifications we estimate are:

1. Model 1 is a replication of the original results of the non-stationary model in Lafourcade and De Wind (2012) based on their original data set for the sample period 1984Q1 to 2007Q4, only with two minor corrections to the model equations: a correction of the New Keynesian Phillips curve and a change from a capital income tax to a wealth tax.\(^1\)
2. Model 2 is the same as model 1, except that we re-estimate model 1 using a new dataset covering the period from 1988Q1 to 2014Q1.
3. Model 3 is a stationary version of model 2 estimated on HP-filtered data for the period 1988Q1 to 2014Q1.
4. Model 4 adds Galí unemployment to model 3 and uses a time-series of unemployment to estimate the model.
5. Model 5 adds credit-constrained households to model 4. Model 5 is the final version of the model derived in the previous chapter.

We begin our discussion of the estimation of the five models by describing the data we use in the next section. Thereafter we discuss the calibration of those parameters that we were unable to estimate, and then the priors on the remaining parameters for which we were able to obtain posteriors. We then review the estimation results: the posteriors, the impulse response functions and the shock decompositions. We end with some concluding remarks. The appendix reports on the estimation diagnostics.

4.1 Data

The data series we used to estimate the model parameters for the model versions 1-5 are shown in table 4.1. The number of series that can be used to estimate each model is limited by the number of stochastic processes in each model, hence the stationary models use fewer time series. The series used in the estimation of model 1 and 2 are in log levels. Most trending variables are

---

\(^1\) We also used slightly different priors, which enabled us to better check the estimated posteriors for any unwanted properties. This had no significant effect of the estimates themselves because the priors remained diffuse relative to the information in the likelihood.
on a per capita basis. These variables are GDP, consumption, exports, imports, wages, working hours and foreign GDP.

We estimate models 3, 4 and 5 using the stationary HP filter cycle of the logarithm of all data series, including those series which in theory are not subject to a trend. The one exception is the unemployment rate. We do not filter this series since we have a ready-made measure of the trend: the CPB’s definition of the NAIRU. We subtract that from the international definition of the Dutch unemployment rate.\(^2\) By making the model stationary we have removed the 3 non-stationary shocks, which means we cannot use as many time series for estimating models 3, 4 and 5 as we do for models 1 and 2. Models 4 and 5 use the unemployment rate instead of the wage rate as the labour market variable.\(^3\) Figures 4.1 through 4.4 show the HP cycles in black. The original levels used to obtain the stationary cycles are shown in red. We used this level data for the estimation of model 2. The sample period for this data runs from the first quarter of 1988 until the first quarter of 2014.

The level series in the figures shown in blue depict the data Lafourcade and De Wind (2012) used in their article. The sample period for this data covers the period of the first quarter of 1984 until the last quarter of 2007. We used this data to estimate model 1.

Note that the unemployment rate is not used in the estimation of either non-stationary version of the model. For this reason the graph in figure 4.2 for the unemployment rate shows only a green line. Also note that the series ending in the first quarter of 2014 for the wage in figure 4.3, foreign GDP and the foreign GDP deflator shown in figure 4.4 are shorter series. These series begin in the first quarter of 1995. This poses no problem for the estimation. The calculation of the likelihood in Dynare is based on the Kalman filter which can handle missing observations.\(^4\)

\(^2\) We then also slightly adjust the mean of the series to ensure that the sample mean is zero.

\(^3\) In a similar way to the binding constraint that one can only use as many variables as stochastic processes for the model as a whole, it is often advisable that within each ‘block’ of the model there are no more observed series than stochastic processes. Otherwise, the stochastic processes in other parts of the model will have to do the work of fitting, for example, some of the labour market developments if both wage rates and unemployment are observed but there is just one stochastic process in the labour market block.

\(^4\) Included in Dynare’s estimation output are graphs of the “fitted” posterior means of the observable series. If we have an idea about the plausibility of these “fitted” values covering the period when the data is missing, then this can help us to gauge the model fit.
Figure 4.1: The Data used in the model estimation

- Consumption
- GDP
- Foreign GDP
- Annual Wage rate

Figure 4.2: The Data used in the model estimation

- Exports
- Imports
- FTE's
- Unemployment rate
Figure 4.3: The Data used in the model estimation

- Government to GDP ratio
- Investment to Consumption ratio
- Interest rates
- Annual Wage to C + I ratio

Figure 4.4: The Data used in the model estimation

- GDP deflator
- Foreign GDP price level
- Relative Investment price level
- Export price level
Table 4.1: Data

<table>
<thead>
<tr>
<th>Observed data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 &amp; 2</td>
</tr>
<tr>
<td>Dutch real GDP per capita</td>
<td>✓</td>
</tr>
<tr>
<td>Dutch real consumption per capita</td>
<td>✓</td>
</tr>
<tr>
<td>Dutch investment/consumption</td>
<td>✓</td>
</tr>
<tr>
<td>Dutch CPI</td>
<td>✓</td>
</tr>
<tr>
<td>Dutch relative investment price</td>
<td>✓</td>
</tr>
<tr>
<td>Dutch Government expenditure/GDP</td>
<td>✓</td>
</tr>
<tr>
<td>Dutch interest rate</td>
<td>✓</td>
</tr>
<tr>
<td>Dutch real exports per capita</td>
<td>✓</td>
</tr>
<tr>
<td>Dutch export price index</td>
<td>✓</td>
</tr>
<tr>
<td>Dutch real imports per capita</td>
<td>✓</td>
</tr>
<tr>
<td>Dutch real wage</td>
<td>✓</td>
</tr>
<tr>
<td>Dutch unemployment rate</td>
<td>✓</td>
</tr>
<tr>
<td>Dutch total working hours per capita</td>
<td>✓</td>
</tr>
<tr>
<td>Dutch wage to consumption + investment ratio</td>
<td>✓</td>
</tr>
<tr>
<td>Foreign EU15 real GDP per capita</td>
<td>✓</td>
</tr>
<tr>
<td>Foreign EU15 GDP deflator</td>
<td>✓</td>
</tr>
</tbody>
</table>

4.2 Calibration

As is common in the DSGE literature, we begin the estimation process by first calibrating most of the steady states in the model based on the sample means of the available data. The foreign block is also fully calibrated. This is because the Netherlands is a small open country. As a result, we believe that the estimation of a model of the Dutch economy should not be allowed to effect the foreign block. Our calibration of the foreign block is based on the original calibration in model 1 used in Lafourcade and De Wind (2012). These calibrated values are shown in tables 4.2-4.4. Changing sample periods and using stationarised model versions also involved re-calibrating a number of steady state drift parameters, which are listed below in table 4.2.

In addition to the calibrated parameters governing the steady state of the model described...

---

5 Things such as steady state growth rates and the values of certain ratios of macro time variables are easily observed in our time-series. Moreover, these steady state parameters are typically determined by a single or a small number of parameters in the model. Calibrating these parameters reduces the dimension of the estimation problem making the estimation process for the remaining parameters considerably more efficient, without imposing any unrealistic restrictions on the model.
above, we also calibrate a number of other weakly identified parameters. In theory Bayesian estimation does not require that the remaining model parameters be identified in order to be able to obtain posteriors for the model parameters. This is because the prior will define the posterior in those cases in which a parameter is unidentified, or in other words in those cases in which the likelihood is uninformative about the parameter. In practice, using simulation techniques to obtain estimates of the model posteriors, as is done in Dynare, does require that the estimated set of parameters be well identified. To obtain a set of well identified parameters we determine which parameters are most weakly identified based on the output from the identification command in Dynare. This is an iterative procedure in which we calibrate the most weakly identified parameter remaining in the model and attempt to obtain a valid Hessian to use for the simulation algorithm in Dynare. If this fails, because the Hessian is still singular, then we re-run the identification command and again select the most weakly identified remaining parameter to calibrate. We repeat this process until we obtain a non-singular Hessian for use in Dynare. This procedure is explained in more detail in the document "Het Schatten van DSGE-modellen in Dynare".

4.3 Priors

The priors we used in the Bayesian estimation of the models 1-5 are listed in table 4.5. The priors are very similar to those used by Lafourcade and De Wind (2012) and we try to avoid imposing too much outside information on the estimates by using priors that are generally diffuse.

4.4 Estimation

The results of the model estimation consist of the posterior estimates for the parameters that we did not calibrate, the impulse response functions and the shock decompositions. We discuss each of these three aspects of the estimation in this section in turn.

---

6 This results from Dynare simulating parameter values from a multivariate random walk process with a covariance matrix defined by the inverse of the Hessian of the product of the likelihood and parameter priors. When the likelihood is uninformative for a particular parameter this tends to produce a singular or near singular Hessian and the simulation algorithm in Dynare breaks down because the algorithm no longer adequately covers the entire parameter space.

7 In some cases they were altered to make the plots of the estimated posteriors clearer, in the sense that a very diffuse prior and an informative likelihood leads to posteriors that look like spikes when plotted against a diffuse prior.
Table 4.2: Calibrated parameter values (part 1)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4 &amp; 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_L$</td>
<td>Per capita hours growth</td>
<td>0.0020</td>
<td>0.0015</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma_\Gamma$</td>
<td>Relative investment price growth</td>
<td>-0.0019</td>
<td>-0.0026</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma_\omega$</td>
<td>Real wage growth</td>
<td>0.0030</td>
<td>0.0028</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma_{f}$</td>
<td>Eurozone GDP growth</td>
<td>0.0042</td>
<td>0.0030</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma_y$</td>
<td>Domestic GDP growth</td>
<td>0.0055</td>
<td>0.0040</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma_m$</td>
<td>Import growth</td>
<td>0.0128</td>
<td>0.0113</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\pi^M$</td>
<td>SS import price inflation</td>
<td>0.0015</td>
<td>0.0018</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\pi^X$</td>
<td>SS export price inflation</td>
<td>0.0015</td>
<td>0.0020</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$I^<em>/Y^</em>$</td>
<td>SS investment-to-GDP ratio</td>
<td>0.214</td>
<td>0.197</td>
<td>0.197</td>
<td>0.197</td>
</tr>
<tr>
<td>$I^<em>/C^</em>$</td>
<td>SS investment-to-consumption ratio</td>
<td>0.431</td>
<td>0.421</td>
<td>0.421</td>
<td>0.421</td>
</tr>
<tr>
<td>$g^*$</td>
<td>SS government spending-to-GDP ratio</td>
<td>0.2355</td>
<td>0.240</td>
<td>0.240</td>
<td>0.240</td>
</tr>
<tr>
<td>$\epsilon^w$</td>
<td>Wage elasticity of labour supply</td>
<td>21</td>
<td>21</td>
<td>21</td>
<td>*</td>
</tr>
<tr>
<td>$u^*$</td>
<td>SS unemployment rate</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.048</td>
</tr>
</tbody>
</table>

* $\epsilon^w$ is calibrated to achieve the desired value of $u^*$.

4.4.1 Posteriors

Tables 4.6 to 4.7 contain the posterior means and 90% highest probability density (HPD) intervals for the five model specifications listed above. Table 4.8 shows the implied standard deviations of the AR disturbance processes in the model based on the posterior means and HPD intervals for the AR parameters and standard deviations of the AR errors.\(^8\) In all tables there is considerable variation in the parameter estimates across specifications, which is not what one would expect if these model parameters were truly identified deep parameters. However, from knowledge of the observed time series it is possible to explain the changes observed since the variation typically follows an intuitive pattern given the specification changes. In what follows we describe the changes observed as we move from one model specification to the next. We will refer to a significant change as being strongly significant when the 90% HPD intervals do not overlap and weakly significant when the posterior mean for one specification does not lie within the 90% HPD interval for the other.

\(^8\) Technically, therefore, the values in table 4.8 will only approximate the posterior means and 90% HPD intervals.
Table 4.3: Calibrated parameter values (part 2)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_*$</td>
<td>SS domestic debt-to-GDP ratio</td>
<td>0.60</td>
</tr>
<tr>
<td>$1 - \alpha$</td>
<td>SS labour share</td>
<td>0.52</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>Consumption tax</td>
<td>0.11</td>
</tr>
<tr>
<td>$\tau^w$</td>
<td>Labour income tax</td>
<td>0.34</td>
</tr>
<tr>
<td>$\tau^k$</td>
<td>Wealth tax*</td>
<td>0.006285</td>
</tr>
<tr>
<td>$e^T$</td>
<td>debt elasticity of fiscal transfers</td>
<td>5.e-4</td>
</tr>
<tr>
<td>$n^c$</td>
<td>Consumption home bias</td>
<td>0.3</td>
</tr>
<tr>
<td>$n^l$</td>
<td>Investment home bias</td>
<td>0.4</td>
</tr>
<tr>
<td>$n^x$</td>
<td>Share of domestic exports in foreign demand</td>
<td>0.04</td>
</tr>
<tr>
<td>$T_f$</td>
<td>Fiscal feedback</td>
<td>0.0005</td>
</tr>
<tr>
<td>$\pi_c$</td>
<td>Target inflation rate</td>
<td>0.0048</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.985</td>
</tr>
<tr>
<td>$R^*$</td>
<td>SS short-term nominal interest rate</td>
<td>0.008</td>
</tr>
<tr>
<td>$e^c$</td>
<td>Price elasticity of consumption</td>
<td>2</td>
</tr>
<tr>
<td>$e^i$</td>
<td>Price elasticity of investment</td>
<td>2</td>
</tr>
<tr>
<td>$e^p$</td>
<td>Price elasticity of output</td>
<td>11</td>
</tr>
<tr>
<td>$e^X$</td>
<td>Price elasticity of exports</td>
<td>11</td>
</tr>
<tr>
<td>$e^{c,m}$</td>
<td>Price elasticity of consumption imports</td>
<td>11</td>
</tr>
<tr>
<td>$e^{i,m}$</td>
<td>Price elasticity of investment imports</td>
<td>11</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Labour supply coefficient</td>
<td>7.5</td>
</tr>
<tr>
<td>$\iota_E$</td>
<td>Employment indexation</td>
<td>0</td>
</tr>
</tbody>
</table>

* This parameter was a capital income tax in the original version of the model, with $\tau^k = 0.0030$.

Model 1 to model 2: extending the sample period

In table 4.6, for two parameters, $\zeta_w$ (wage stickiness) and $\zeta_e$ (employment stickiness), out of fifteen the change in the posteriors is strongly significant when the sample period is changed. For another four, $\lambda$ (habit formation), $a_{b,b}$ (bond premium elasticity), $e_f$ (price elasticity of export demand) and $\zeta_p$ (price stickiness), the changes are more weakly significant. The later sample period includes the Great Recession, a period that was initially marked by unexpectedly resilient employment in the Netherlands.

Extending the sample period causes employment stickiness increases from 0.57 to 0.89 and wage stickiness falls from 0.94 to 0.83. Additionally, although the new estimate is just within the 90% HPD interval of the baseline model, the labour supply elasticity increases from 0.68 to
Figure 4.5: Wages levels and HP filter

![Wages levels and HP filter graph](image1)

Figure 4.6: FTE levels and HP filter

![FTE levels and HP filter graph](image2)
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_y$</td>
<td>IS curve for output gap</td>
<td>0.396</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>IS curve for real interest rate</td>
<td>-0.230</td>
</tr>
<tr>
<td>$\rho_y$</td>
<td>IS curve AR coefficient</td>
<td>0.534</td>
</tr>
<tr>
<td>$\alpha_\pi$</td>
<td>Phillips curve for inflation</td>
<td>0.736</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>Phillips curve for output gap</td>
<td>0.016</td>
</tr>
<tr>
<td>$\rho_\pi$</td>
<td>Phillips curve AR coefficient</td>
<td>-0.346</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>Monetary policy rule for interest rate</td>
<td>0.8</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Monetary policy rule for inflation</td>
<td>0.85</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Monetary policy rule output gap</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho_y^p$</td>
<td>Potential output AR coefficient</td>
<td>0.984</td>
</tr>
<tr>
<td>$\sigma_{\pi f}$</td>
<td>s.e. Phillips curve shock</td>
<td>0.0039</td>
</tr>
<tr>
<td>$\sigma_\pi$</td>
<td>s.e. shock to monetary policy rule</td>
<td>0.007</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>s.e. shock to trend EU output</td>
<td>0.0017</td>
</tr>
<tr>
<td>$\sigma_{y f p}$</td>
<td>s.e. shock to potential output</td>
<td>0.0035</td>
</tr>
</tbody>
</table>

1.12. These changes arise from model 1 not including the Great Recession and, as we can see from the HP filter cycle\(^9\) (in green) in figure 4.5, at the start of the crisis wages fell relative to trend, while in figure 4.6 we can see that employment initially kept increasing. Furthermore, in models 1 and 2, the labour supply elasticity governs the sensitivity of labour supply to changes in the real wage via the following log-linearised equation

$$\eta L_t = \hat{\xi}_t + \hat{\nu}_t.$$  \hspace{1cm} (4.1)

Therefore, as the labour supply elasticity, $\eta$, increases, labour supply becomes less sensitive to changes in the real wage. Given the size of the GDP fall in the Great Recession, the model parameterised as in model 1 would have predicted that firms would have reduced employment sharply. The change in wage stickiness has knock-on effects: the estimated price stickiness increases from 0.82 to 0.87. This is because inflation persistence in New Keynesian models depends on both wage stickiness, which generates inertia in marginal costs, and also on price stickiness directly. To match a similar inflation persistence with a lower wage stickiness, model 2 has to have higher price stickiness.

To understand the fall in the estimated bond premium elasticity, $\alpha_{b, h}$, from 1.58 in model 1 to 1.36 in model 2 can also be explained by looking at what happened in the Great Recession.

\(^9\) The right-hand vertical axis is for the HP filter cycle. The blue dashed line in the original DNB data in log levels, and the red dotted line is the more recent log level series until 2014.
Table 4.5: Prior distributions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Distribution</th>
<th>Prior</th>
</tr>
</thead>
<tbody>
<tr>
<td>λ</td>
<td>Habit formation</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>Φ</td>
<td>Utilisation cost</td>
<td>Gamma</td>
<td>25</td>
</tr>
<tr>
<td>η</td>
<td>Labour supply elasticity</td>
<td>Gamma</td>
<td>2.5</td>
</tr>
<tr>
<td>κ</td>
<td>Investment cost</td>
<td>Gamma</td>
<td>15</td>
</tr>
<tr>
<td>θ_{bf}</td>
<td>Bond premium feedback</td>
<td>Gamma</td>
<td>2</td>
</tr>
<tr>
<td>ε_{bf}</td>
<td>Price elasticity of export demand</td>
<td>Gamma</td>
<td>2</td>
</tr>
<tr>
<td>ζ_p</td>
<td>Price stickiness</td>
<td>Beta</td>
<td>0.6</td>
</tr>
<tr>
<td>ζ_w</td>
<td>Wage stickiness</td>
<td>Beta</td>
<td>0.6</td>
</tr>
<tr>
<td>ζ_e</td>
<td>Employment stickiness</td>
<td>Beta</td>
<td>0.6</td>
</tr>
<tr>
<td>ζ_s</td>
<td>Export price stickiness</td>
<td>Beta</td>
<td>0.6</td>
</tr>
<tr>
<td>ζ_{cp}</td>
<td>Consumption import price stickiness</td>
<td>Beta</td>
<td>0.6</td>
</tr>
<tr>
<td>τ_p</td>
<td>Price indexation</td>
<td>Beta</td>
<td>0</td>
</tr>
<tr>
<td>τ_w</td>
<td>Wage indexation</td>
<td>Beta</td>
<td>0</td>
</tr>
<tr>
<td>τ_s</td>
<td>Export price indexation</td>
<td>Beta</td>
<td>0</td>
</tr>
<tr>
<td>τ_{cp}</td>
<td>Consumption import price indexation</td>
<td>Beta</td>
<td>0</td>
</tr>
<tr>
<td>ρ_{gt}</td>
<td>AR(1) general technology</td>
<td>Beta</td>
<td>0</td>
</tr>
<tr>
<td>ρ_{gf}</td>
<td>AR(1) fiscal</td>
<td>Beta</td>
<td>0</td>
</tr>
<tr>
<td>ρ_{gb}</td>
<td>AR(1) investment-specific technology</td>
<td>Beta</td>
<td>0</td>
</tr>
<tr>
<td>ρ_{gb}</td>
<td>AR(1) bond premium</td>
<td>Beta</td>
<td>0</td>
</tr>
<tr>
<td>ρ_{gp}</td>
<td>AR(1) price mark-up</td>
<td>Beta</td>
<td>0</td>
</tr>
<tr>
<td>ρ_{gw}</td>
<td>AR(1) wage mark-up</td>
<td>Beta</td>
<td>0</td>
</tr>
<tr>
<td>ρ_{gp}</td>
<td>AR(1) export price mark-up</td>
<td>Beta</td>
<td>0</td>
</tr>
<tr>
<td>ρ_{gcp}</td>
<td>AR(1) consumption import price mark-up</td>
<td>Beta</td>
<td>0</td>
</tr>
<tr>
<td>σ_a</td>
<td>s.e. general technology</td>
<td>Gamma</td>
<td>0.015</td>
</tr>
<tr>
<td>σ_f</td>
<td>s.e. fiscal</td>
<td>Gamma</td>
<td>0.015</td>
</tr>
<tr>
<td>σ_b</td>
<td>s.e. investment-specific technology</td>
<td>Gamma</td>
<td>0.015</td>
</tr>
<tr>
<td>σ_b</td>
<td>s.e. bond premium</td>
<td>Gamma</td>
<td>0.015</td>
</tr>
<tr>
<td>σ_p</td>
<td>s.e. price mark-up</td>
<td>Gamma</td>
<td>0.015</td>
</tr>
<tr>
<td>σ_w</td>
<td>s.e. wage mark-up</td>
<td>Gamma</td>
<td>0.015</td>
</tr>
<tr>
<td>σ_e</td>
<td>s.e. export price mark-up</td>
<td>Gamma</td>
<td>0.015</td>
</tr>
<tr>
<td>σ_{cp}</td>
<td>s.e. consumption import price mark-up</td>
<td>Gamma</td>
<td>0.015</td>
</tr>
<tr>
<td>σ_{rf}</td>
<td>s.e. export demand</td>
<td>Gamma</td>
<td>0.015</td>
</tr>
<tr>
<td>σ_{gt}</td>
<td>s.e. trend general technology</td>
<td>Gamma</td>
<td>0.015</td>
</tr>
<tr>
<td>σ_p</td>
<td>s.e. trend labour supply</td>
<td>Gamma</td>
<td>0.015</td>
</tr>
<tr>
<td>σ_{gt}</td>
<td>s.e. trend investment-specific technology</td>
<td>Gamma</td>
<td>0.015</td>
</tr>
</tbody>
</table>

* Generalised beta distribution over the interval [0;0.95]  
** Generalised beta distribution over the interval [−1,1]  
*** Generalised beta distribution over the interval [−0.995,0.995]  

With the onset of the Great Recession expected inflation, real interest rates and consumption fell, which would affect the amount households would like to save and, hence, the current account surplus, which increased during the Great Recession. The link between these variables can be seen more clearly by looking at the equation for the risk premium on bonds, which is the only
place where the bond premium elasticity enters the log-linearised model. The bond risk premium is needed in a small open economy model with a common currency to close the model. Without this risk premium domestic households will want to build up infinite savings when their discount factor $\beta$ is lower than the interest rate on foreign bonds. If their discount factor is higher than the interest rate, then they will want to build up infinite debt. The log-linearised version of the bond risk premium equation is

$$\hat{H}_t = a_{h,b} \hat{b}_t + a_{h,e} \hat{e}_t^b.$$  (4.2)

The bond risk premium, $\hat{H}_t$, in turn only enters the model in equation (3.8). The log linearised version of (3.8) for the optimising consumer is given by

$$E_t \left( \hat{\Xi}_t - \hat{\Xi}_{t+1} \right) = \hat{R}_t - E_t \hat{\pi}_t + 1 + \hat{H}_t.$$  (4.3)

Since inflation, real interest rates and consumption are the elements in (4.3) that define $\hat{H}_t$, it follows that these falls in combination with the rising current account surplus can explain the change in the estimate of the bond premium elasticity.

In the case of the drop in the habit formation parameter $\lambda$ from 0.79 in model 1 to 0.74 in model 2, this is most likely the result in the drop in consumption following the Great Recession, which many feel is the result of the fall in housing prices. Since housing prices are not included in the model, the drop in consumption is therefore unexplained. This in turn requires a drop in habit formation to allow for the more volatile consumption.

The later sample period also increases the price elasticity of export demand from 0.40 to 0.51. This is because the price elasticity of exports, $\varepsilon^f$, controls how much exports, $X_t$, fall when their price, $p^x_t$, relative to the price in foreign markets, $p^f_t$, rises as in the following linearised equation:

$$\hat{X}_t = -\varepsilon^f \left( \hat{p}^x_t - \hat{p}^f_t \right) + \hat{\delta}_t^f + \hat{\varepsilon}_t^f.$$  (4.4)

As can be seen in figure 4.4 the export price level fell substantially more in the Great Recession than the foreign GDP deflator. Furthermore, given that exports experienced a sharper decline in the Great Recession than foreign GDP (see figure 4.2), the decline in exports is partly explained by making exports more sensitive to relative price differences.

**Shock processes**

As for the estimated shocks and their persistence, these are shown in tables 4.7 and ???. The Great Recession was clearly a large shock and this can be clearly seen when comparing the

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10 Model versions 1 and 2 have only optimising consumers. We therefore opted not to index the variables here with an “O”.
estimates of model 1 and model 2. Interestingly, model 2 accounts for a large portion of the Great Recession and subsequent period of low growth through temporary shocks. That can be seen from table 4.7 where none of the standard errors of the permanent shocks change significantly. In contrast, as shown in table 4.7, both the estimated standard error (from 0.02 to 0.05) and the persistence (from 0.01 to 0.53) of the temporary technology shock increase when the sample period is extended to include the Great Recession and its aftermath. The increase in the AR parameter for the fiscal policy shock is unsurprising given that fiscal policy is measured in the model as a proportion of GDP and that the fiscal policy response to the Great Recession was a large initial stimulus package followed by a persistent deficit.

Another shock process to change significantly is the wage mark-up process. The standard error of the wage mark-up increases from 0.004 to 0.012 whilst the AR parameter falls from 0.57 to 0.06. That a different shock process is needed to help explain the behaviour of wage and employment described above should not be surprising. Moreover, the AR process and the standard error of a process are typically only jointly well identified, since a larger standard error and smaller AR parameter will produce the same time series properties of the observed series.

For this reason we also show the implied standard deviations for the shock processes implied by the AR coefficients and standard deviations of the AR error term in table 4.8. The table shows that the standard deviation of the stationary technology shock nearly triples from 0.02 in model 1 to 0.06 in model 2. The approximate HPD’s do not overlap indicating that this change is strongly significant.

The implied standard deviations of the fiscal and wage mark-up shocks also increase from model 1 to model 2, although these changes are only significant in the weaker sense. The standard deviation of the shock to the bond risk premium also changes significantly in this weaker sense, but decreases from model 1 to model 2. Given the earlier discusses changes brought about by the Great Recession effecting (4.3) and (4.2), it is not surprising that this standard error changes. Whether the changes should imply and increase or a decrease is less clear, because all the factors change.

Model 2 to model 3: using filtered data and a stationary model

Stationarising the data by using the HP filter also causes a number of significant changes to the parameter estimates. The HP filter removes the lower frequency components of the data, which means the model no longer has to generate such persistence in response to shocks. As such, it is no surprise that most of the estimates of parameters governing real and nominal rigidities have fallen after stationarising the model. For instance, habit formation is typically used to make aggregate consumption more persistent and it falls from 0.74 to 0.60, with no overlap in the HPD intervals. Similarly, higher investment costs allow investment to deviate for longer from steady state. The decline in this parameter is also strongly significant. The same is the case for
price stickiness.

Three other parameters drop significantly in the weaker sense. These parameters are labour supply elasticity, which drops from 1.12 to 0.45, the price elasticity of export demand, which declines from 0.51 to 0.25, and price stickiness, which drop from 0.87 to 0.78.

The effects of using filtered data can also be seen in the estimated standard errors and the AR parameters. Unsurprisingly, many AR parameters fall indicating that the processes are less persistent. For example, the AR parameter for technology shocks has fallen from 0.53 to 0.15 so that the posterior mean falls outside the HPD interval from the other model. This is also the case for the AR parameter for the export price mark-up, which falls from 0.74 to 0.47. In the case of two AR parameters the fall is strongly significant: for the fiscal shocks from 0.92 to 0.77, and for export demand shocks from 0.96 to 0.72.

The HP filter removes a proportion of the volatility in the observed data series, so it is not surprising that the implied standard deviations of the shock processes have also mostly decreased. The standard deviations fiscal shocks, investment specific technology shocks, the bond premium shocks and export demand shocks all decline with no overlap in their HPD intervals. Only in the case of the consumption import price indexation shocks is there an increase in the standard deviation from 0.009 to 0.071 with no overlap in the HPD intervals. Technology’s standard deviation falls from 0.059 to 0.029. This is significant in the weaker sense. Furthermore, because model 3 no longer includes permanent shocks, the maximum number of time series used to estimate the model has decreased - real Dutch imports are no longer in the information set of the model, which no longer ties down the processes behind imports in model 3 to the same extent as in the non-stationary models.

Model 3 to model 4: fitting the unemployment time series

Estimating with unemployment data instead of wages also results in a number of large parameter changes. Neither labour supply nor the wage are used in the estimation of model 4. The change in posterior for four parameters is strongly significant: habit formation drops from 0.60 to an economically negligible 0.07, wage stickiness also falls considerably from 0.82 to 0.15, and the bond premium elasticity falls from 1.37 to 0.37. In the case of the fourth parameter, the labour supply elasticity, the posterior mean increases from 0.45 to 1.11. The unemployment series is very smooth which implies a smooth series for labour supply. From equation (4.1) we can see that a larger value for the labour supply elasticity, $\eta$, generates a less volatile labour supply series for a given wage volatility. This can also be seen in the impulse responses in the next section, where labour supply responses are smaller for models 4 and 5. The lower habit formation and wage stickiness also follow from this - smoother labour supply and employment implies less wage stickiness is needed to prevent the wage being too volatile and it also implies that household incomes are less volatile, so less habit formation is required to fit the smooth
consumption series. This also feeds through to the bond premium. Looking at equation (4.3) the left hand side has become smoother following the increased smoothness of the marginal utility of consumption, which implies that the free element on the right hand side must also become smoother. Hence the bond premium becomes much less sensitive to changes in the foreign asset position.

For the implied standard deviations of the shocks shown in table 4.8, two show changes that are strongly significant: the wage mark-up shock standard deviation increases strongly from 0.009 to 0.047, while the bond premium declines from 0.040 to 0.007. The fall in the volatility of the bond premium shock follows from the discussion above. The wage mark-up shock follows from the smoother implied labour supply series - if labour supply is smoother something else is needed to match the volatility of wages (the implied volatility of wages in this model is similar to the volatility of the actual wage data, as will be discussed in section 4.5, below). The change in three parameters is weakly significant: technology declines, investment specific technology declines and the price mark-up rises.

Model 4 to model 5: adding credit-constrained households

Ceteris paribus, adding credit-constrained households makes consumption follow current income more closely. However, since data on both consumption and income are observed for all of the previous models, the contemporaneous correlation between consumption and income is already fully explained by the previous models. Hence, adding credit-constrained households shifts at least some of the burden of explanation from the rest of the model and has consequences for a number of parameters. Adding credit-constrained households more than doubles the labour supply elasticity from 1.11 to 2.65. This follows from labour supply no longer having to respond so strongly to wage changes, which are often linked to changes in aggregate demand and income. Adding credit-constrained households also increases the bond premium elasticity from 0.37 to 0.82. This is because after adding credit-constrained households only half of the households in the domestic economy hold foreign bonds, so their reaction to the foreign asset position of the whole economy, which follows the current account, needs to be bigger to have the same effect at the economy level. There are also other changes with exports: the price elasticity of export demand increases considerably from 0.41 to 2.05, export price stickiness declines from 0.56 to 0.24, whilst both the standard deviation of export demand and the AR parameter also increase. Whilst equation (4.4) appears to pin down the price elasticity of demand for exports based on the observed series, this is not quite the case. The time series for both the foreign price level, $\hat{p}_f^t$, and foreign GDP, $\hat{y}_f^t$, only start in the first quarter of 1995. The model is, therefore, free to adjust these series at the start of the sample period. Furthermore, the home bias shifter, $\hat{\varepsilon}_n^f$, which the accumulation of foreign demand shocks, is negatively correlated with the relative price of exports. This allows the same series to be matched with both a more volatile response to
relative prices and a more volatile home bias shifter. The other significant change is price stickiness, which falls again from 0.69 to 0.41. This follows from the credit-constrained households not being price sensitive in their consumption decision - they consume their entire income. When product market conditions change, firms have less incentive to change their prices because half of the households will keep buying whatever the price. Therefore, less price stickiness is needed to match the smoothness in the inflation series.

Comments
Comparing the columns for models (1) and (5), none of the estimates of parameters governing the degree of real rigidities in model (5) bare any resemblance to those in model (1). For example, habit formation has fallen from 0.79 to 0.04 and investment costs have fallen from 15.2 to 5.3. Such lack of robustness calls into question the claim that these parameters are well identified deep parameters. On the other hand, some of the stickiness and indexation parameters show some consistency. Employment stickiness is estimated to be about 0.6 for four of the five model specifications as is consumption import price stickiness. Wage indexation, export price indexation and consumption import price indexation all have similar estimates in model (5) to model specification (1). This, however, still leaves price stickiness, wage stickiness, export price stickiness and price indexation being significantly different in specification (5) to specification (1).

4.5 Model fit
We evaluate the model fit of the various model versions in two ways. One is via the posterior odds ratios for the models, the other is by comparing model predictions of data series not used in the estimation with the actual data.

The posterior odds ratio is the standard Bayesian measure of model fit used to compare different models and is the ratio of two models posterior probabilities. This ratio is based on the marginal likelihood, $p(y|M)$, which is the density of the data conditional on the model used, but with the model parameters integrated out of the likelihood:

$$p(y|M) = \int p(y|\theta, M)p(\theta|M)d\theta.$$  (4.5)

The formula for the posterior odds ratio, $PO$, to compare $M_1$, with $M_2$, is then given by

$$PO = \frac{p(M_1|y)}{p(M_2|y)} = \frac{p(y|M_1)p(M_1)}{p(y|M_2)p(M_2)},$$  (4.6)

where $p(M_i)$ is the prior probability that model $i$ is the correct model. If both models are a priori deemed equally likely, then the posterior odds ratio simplifies to the ratio of the marginal
Table 4.6: Posterior estimates (part 1)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Values</th>
</tr>
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<tbody>
<tr>
<td>$\lambda$</td>
<td>Habit formation</td>
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<tr>
<td></td>
<td></td>
<td>[0.751, 0.820]</td>
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<tr>
<td>$\eta$</td>
<td>Labour supply elasticity</td>
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<td></td>
<td>[0.186, 1.144]</td>
</tr>
<tr>
<td>$\Phi'(1)$</td>
<td>Utilisation cost</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>[9.006, 53.157]</td>
</tr>
<tr>
<td>$\zeta^2 S'(z)'$</td>
<td>Investment cost</td>
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</tr>
<tr>
<td>$\alpha_{b,p}$</td>
<td>Bond premium elasticity</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>[1.379, 1.826]</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Price elasticity of export demand</td>
<td>0.398</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[0.337, 0.460]</td>
</tr>
</tbody>
</table>

Note: (1) indicates the non-stationary model of Lafourcade and De Wind (2012) with sample period from 1984Q1 to 2007Q4; (2) indicates the non-stationary model of Lafourcade and De Wind (2012) with sample period from 1988Q1 to 2014Q1; (3) indicates a stationary version of (2) estimated using HP-filtered data; (4) indicates (3) but estimated using unemployment data instead of wages; (5) indicates (4) but with the addition of credit-constrained households.

However, as equation (4.6) shows, a comparison of marginal likelihoods is only valid if the data, $y$, is the same for both models. That is only true for comparing models 4 and 5. The log marginal likelihood of model 4 is 3705 whilst for model 5 it is 3664. From this we can

\[ \text{log marginal likelihood of model 4} = 3705 \]
\[ \text{log marginal likelihood of model 5} = 3664 \]

The log marginal likelihoods of the other three models are: model 1, 4197; model 2, 4408; model 3, 3966. Whilst models 1, 2 and are estimated on different sample periods or data treatments (levels vs HP-filter) making interpretation difficulties.
conclude that model 4 performs better than model 5. At first sight this result is somewhat surprising, since model 5 is the same as model 4 but with the addition of an extra feature, which should make fitting the data easier. However, the extra parameter in model 5 is the share of credit-constrained households, which we calibrated at 50% (a number commonly used in the literature) rather than estimated. That model 5 has a smaller marginal likelihood suggests that the true share of credit-constrained households is less than 50%.

An alternative manner to obtain an idea of how well the model fits the data is by comparing the model predictions for data series not used in the estimation with the actual series. We do this here for model 4 and 5, using the wage and employment series. In figure 4.7 we show the comparison between the fitted wage series with the observed series for model 4 and 5. The original HP filter cycle series is shown in black. Note that this series is incomplete, beginning in 1995 instead of 1988. The fitted series estimated for the model is given by the blue dotted line. In general there does not seem to be much difference in the fit we obtain for the wage based on model 4 or 5. Both models seem to roughly match the amplitude of the observed series, and also for the most part match the peaks and troughs, for example the peaks around 1996 and 2000, and the trough around 2006. More recently the observed series has trended lower, whereas the fitted series have remained higher.

In figure 4.8 we can gauge the fit we obtain using the total hours worked series, $\hat{E}_t$ for models 4 and 5. Here too there does not seem to be much difference between the fit we obtain for models 4 and 5. The observed peaks in employment around 1990, 2000, and 2009 are also present in the fitted data. The troughs around 1995 and 2005 are also evident in the fitted series, although the fitted series does seem to lag behind the observed series. One difference between the fit we obtain from the two models is that model 5 is less volatile than the fit based on model 4. Both models predict that the level of employment should be lower than the current observed level.

### 4.6 Impulse response functions

The DSGE models 1 through 5 contain a large number of equations and it can therefore be difficult to tell the underlying economic story behind the model outcomes. Impulse response functions are useful for this. Here we discuss the impulse response functions we obtain for the of the relative marginal likelihoods difficult, the data sample differs between models 3, 4 and 5 only by the replacement of wage data in model 3 with unemployment data in models 4 and 5. Whilst not technically comparable, since the other 12 data series are identical across the three models, the lower marginal likelihoods for models 4 and 5 suggest that unemployment series is more difficult to fit than the wage series.

12 One of the advantages of the estimation procedure used to estimate these DSGE models is that we can obtain estimated values for the missing observations.
Figure 4.7: Fitted and actual wage, $\hat{w}_t$.

(a) Model 4

(b) Model 5

Figure 4.8: Fitted and actual employment, $\hat{E}_t$.

(a) Model 4

(b) Model 5
models for temporary shocks to technology, \( u^a_t \), to government spending, \( u^g_t \), and to the interest rate, \( \hat{\epsilon}_t \). We show the responses of output, consumption, wages, employment and labour supply to these three impulses. In addition we also show how these impulses affect the variables directly impacted by the shocks: technology, \( a_t \), government spending, \( \hat{g}_t \), and the interest rate, \( \hat{R}_t \). Each graph shows the impulse response function including confidence bands for all 5 models: model 1 in dark blue, model 2 in green, model 3 in red, model 4 in blue-green, and model 5 in purple.

**Responses to a temporary technology shock**

We show the impulse response functions in figure 4.9 for the temporary technology shock. After the minor alterations made to model 1, the impulse responses are no longer identical to those reported in Lafourcade and De Wind (2012), although the shapes remain the same. For comparison purposes the responses of model 1 will serve as the benchmark for the other models. The shock itself is very short-lived in model 1, returning to steady state almost immediately, which follows directly from the estimated AR coefficient of 0.013. The response of output follows a hump-shaped response and has the same magnitudes as in Lafourcade and De Wind (2012). The consumption response is also hump-shaped but more delayed than output. The magnitude of the consumption response is larger than reported in Lafourcade and De Wind (2012). The response of wages is also hump-shaped with the peak occurring shortly after the shock. The employment response is very similar to that reported in Lafourcade and De Wind (2012), although marginally more delayed. Labour supply falls immediately on impact before returning almost immediately to the steady state.

In contrast to model 1, the temporary technology shock in model 2 is more than twice as large and also much more persistent. This leads to much larger responses of output, consumption and wages. These responses are more than twice the magnitude of those for model 1. This is largely driven by the increased persistence of the technology shock: since the increased productivity lasts longer households have a significantly longer period of time during which they can benefit from the increased return on capital so they invest more. Since capital is long lasting this makes the responses more persistent. On top of that, households have a larger increase in lifetime wealth than in model 1 and, ceteris paribus, wish to increase leisure as a result, which can be seen by looking at the large labour supply response. To counteract this desire for increased leisure, firms have to increase wages more to keep their workforce and take advantage of the increased productivity.

The responses of model 3 are significantly smaller than model 2, but slightly larger than model 1. Once again, this stems from the size and persistence of the technology shock itself. By using HP-filtered data to estimate model 3 we removed a lot of the persistence in the data and both the estimated standard deviation and the autoregressive parameter are significantly lower than in model 2, although somewhat larger than model 1. Hence the responses of model 3 are
somewhat larger than model 1, but smaller than model 2. The responses of output and consumption in model 3 are marginally less persistent than model 1, which follows from the lower estimates of the rigidity parameters (such as lower habit formation or lower stickiness parameters) outweighing the marginally more persistent technology shock.

Model 4 is less precisely estimated than the other models, which can be seen by the wide confidence bands around the impulse response functions. The size of the technology shock is small, although very persistent. This persistence makes the responses more similar to model 2 than the other models. The wage and output responses are the most persistent, the lower estimated habit formation parameter allows consumption to fall back to the steady state relatively more quickly than output or wages.

Finally, model 5 has an almost negligible labour supply response driven by the higher estimate of the labour supply parameter. This means that employment also doesn’t respond much to the technology shock, which makes output respond instantly to the technology shock instead of displaying a hump-shaped response. The low estimate for habit formation also makes the consumption response return to steady state relatively quickly.

Figure 4.9: Responses to a temporary technology shock

**Responses to a temporary government spending shock**

We next show the impulse response functions in figure 4.10 for a temporary government spending shock. The government spending shock itself is of similar magnitude across all five
models although it dies out more slowly in models 1 and 2. That is not surprising given that the other three models are estimated on HP-filtered data and the estimates of the AR parameters for the fiscal process are higher in models 1 and 2. Models 1 and 2 generally produce similar responses to the fiscal shock with output increasing and consumption following a hump-shaped response. The main differences are the responses of employment and wages. In model 1, wages fall following a fiscal policy shock, whereas they increase in model 2. One of the mechanisms through which fiscal policy works in DSGE models is that forward looking households know that they will have to face increased future taxes to pay for the extra government spending following a shock. In order to pay for the increased future taxes the households supply more labour, as can be seen in the labour supply responses. The key driver of the differences between models 1 and 2 is that in model 1 more of the increased labour supply ends up being employed. In model 2 employment is more sticky and doesn’t increase as much as in model 1, which means the marginal product of labour rises more following the increase in demand in model 2 than in model 1.

With the smaller and less persistent shocks in models 3, 4 and 5 the magnitudes and the responses to the shocks are, typically, smaller in those models than in models 1 and 2. The response of model 3 is effectively a scaled down version of the response to model 2, except for the fact that the lower estimated employment stickiness in model 3 allows employment to rise more and wages to respond less than in model 2. Replacing the data on wages with data on unemployment results in the hump-shaped response of wages disappearing. Unemployment is also a smoother series than wages, which explains why the responses in models 4 and 5 are muted compared to the other three models. Furthermore, the high labour supply elasticity parameter estimated for model 5 means that labour supply doesn’t increase much following the government expenditure shock. This in turn results in the more muted response in the employment, output and consumption. Interestingly, although the addition of credit constrained household in model 5 makes the initial response of consumption larger than in model 4, this does not translate into a larger output effect. This is due to the smaller employment response in model 5.

Responses to a temporary interest rate shock

Finally in figure 4.11 we show the impulse response functions for a temporary interest rate shock. Note that the impulse responses for the interest rate are based on a fully calibrated equation and for this reason have no bands. Models 1, 2 and 3 show similar responses with hump-shaped responses of output, consumption, employment and labour supply. Once again, a large part of the differences between these responses can be explained by the variation in the estimated employment stickiness parameter. Model 2 has more sticky employment than model 1 so, despite the models having a similar labour supply response, the employment response is much more muted in model 2 than in model 1. This is mirrored in the response of wages with
model 1 displaying a wage increase following the shock whereas models 2 shows falling wages. Model 3 has slightly higher employment stickiness than model 1 and also a slightly smaller employment response. As described above, the increase in employment stickiness in model 2 is the result of the labour hoarding that took place during the Great Recession.

Models 4 and 5 show less similar responses. Both models 4 and 5 have much lower estimated habit formation in consumption as well as a much reduced wage stickiness parameter. This results in more volatile consumption and wage responses. In model 5, however, the labour supply and employment responses are negligible. This is related to the fact that the posterior mean for the labour supply elasticity in model 5 is more than double the mean in the other models. The smaller response in labour supply and employment in turn feeds through into smaller wage and output responses than in model 4.

We can see from the figures that there is considerable variation in the estimated impulse response functions across the five models. In particular the effects of the inclusion of the Great Recession in the sample period of model 2 seems to produces some significant changes. The HP-filter seems then able to partially remove from the effects of the Great Recession from model 3, as these responses tend to look more like those from model 1. The model estimation based on the unemployment series with Gali unemployment also seems to have a considerable impact on the estimated model behaviour. Finally we note that the inclusion of credit constrained consumers in model 5 only seems to only produce a slightly stronger positive response of
consumption to an impulse in government expenditure compared to the response from model 4. The response of output from model 5, however, to an impulse in government expenditure is weaker than that from model 4. The addition of credit constrained consumers to model 5 does not produce larger multipliers. The changes in the other estimated parameters of model 5 that are needed to fit the data outweigh the extra responsiveness of consumption.

### 4.7 Historical decompositions

Estimated DSGE models also allow the user to decompose the available time series into components due to each of the shocks. These historical decompositions allow the identification of which shock processes were driving economic developments in a given time period. A clear picture of which shocks have been playing an important role in recent times is useful for both forecasters and policy advice. However, the identification of the individual shocks depends on the estimated parameters of the DSGE model - if the DSGE parameters are unstable the historical decompositions may not be robust either. This section discusses the historical shock decompositions that we obtain from the model estimation. We show the decompositions for output, employment and consumption for models 2 and 5. Model 2 has permanent shocks, whereas model 5 does not. Comparing the decompositions from both models enables us to gauge how robust the decomposition of the shocks is when estimates are based on a
stationary version of the model using HP filtered data compared to decompositions based on non-stationary data. We can also see how significant the role of permanent shocks is in the movements of the original data series.

Model 2 has a total of 17 shocks, while model 5 has 13. Even in the case of model 5, which only has temporary shocks, the graphs of the shock decompositions are difficult to read and interpret. To simplify the figures and make them easier to interpret, we have grouped the shocks together into six categories. These are markup shocks, the fiscal shock, premium shocks, temporary technological shocks, foreign shocks and permanent shocks.

There are four shocks in the markup group: the price markup shock, the wage markup shock, the import price of consumption goods shock, and the export price markup shock. The fiscal shock consists of only the one shock for government expenditures shock. The premium shock is also just the one shock for the bond premium. There are 2 shocks in the group of technology shocks: the temporary technology shock and the investment price shock. The largest group of shocks is the foreign group which consists of five shocks: the world demand elasticity shock, the foreign price shock, the foreign output shock, the monetary policy shock and the domestic output potential shock. The last two are placed in the foreign group because the Netherlands is a small open economy and does not set its own interest rate. The domestic output potential is a function of the production frontier, which can reasonably be assumed to be determined by foreign technological developments. The permanent shocks in models 1 and 2 are the labour supply shock, the investment specific technology shock, the permanent technology shock and the world output shock.

The observed series is obtained by taking the initial value and adding the effect of the shocks as they occur. In the case of the stationary model, based on the HP filter cycle of the data, the initial condition is small and the HP cycle values of the observed series are near zero. This allows us to also include them in the graph of the decomposition. In contrast, we have not included the initial values or the data values themselves for the non-stationary model 2 decompositions, because the initial values and original level data are large and dominate the figures, obscuring the effects of the estimated shocks. We also opt to exclude the first 30 quarters of shock decomposition from the figures, because they are distorted by the initial conditions.

Figure 4.12 shows the shock decompositions for output based on model 2 and 5. The decomposition for consumption and employment in models 2 and 5 are shown in figures 4.13 and 4.14, respectively. In general the figures make clear that the HP filter removes a considerable fraction of the variation in the data. The spread in the decompositions for the stationary model 5 is less than half that of those for the levels model 2.

In general the decompositions based on model 5 are dominated by just three groups of shocks: the markups, the premium and temporary technology shocks. The fiscal and foreign shocks only play a small role in model 5. In contrast, while the markups and temporary
technology shocks still play a reasonably significant role in the decompositions for model 2, the permanent shocks, the fiscal shocks and the foreign shocks tend to dominate the figures. The role of the premium shock in the decompositions from model 2 is also considerably smaller. Furthermore, the relative contributions of the various shock groups to the evolution of the series often differ considerably between models 2 and 5. The sign of the net contribution is also often different between the two models. For example, model 2 tells us that temporary technology shocks were lowering GDP in model 2 but were having a positive effect on GDP in model 5.

However, if we look beyond the level of the decomposition and look at the changes, there are more common features between models 2 and 5. Consider the pattern in GDP in figure 4.12. Both figures show that there were mostly positive temporary technology shocks before about 2002, because the size of the effect of this shock is increasing in the figures. After 2002 until 2004 there were apparently predominately negative temporary technology shocks, which in turn became positive over the period 2004 until just before the Great Recession. With the onset of the Great Recession these shocks were evidently predominantly negative. Both figures also suggest that there was a cluster of negative shocks around 2013. The foreign shocks also follow a similar pattern in both figures, although the effect of the foreign shocks in model 5 is more muted.

The markup shocks also follow a fairly similar pattern. Particularly around the Great Recession we can see an initial group of positive shocks around 2009, which then in 2010 became negative. This pattern is perhaps the result of the initial relatively strong performance of the labour market, which eventually suffered a downturn. The same pattern can be seen in figure 4.14 for labour supply.

The effects of the fiscal shocks are small in model 5, but nonetheless their impact around the time of the Great Recession also seems consistent in the two models. Both decompositions show positive shocks in 2008 and 2009 turning negative around the time the Dutch government began imposing austerity measures in 2010. That the fiscal shocks in model 2 have remained positive in the last years of the sample period is a surprise. If we look at figure 4.3, however, we can see that the government expenditure to output ratio significantly increases at the end of the sample period. This must produce a sequence of positive shocks over this period which in turn will stimulate the labour supply and result in more output. This increase is mostly the result of the fall in output. This calls into question the assumed stationarity in model 2 of the government expenditure to GDP ratio over the sample period.

Being permanent, the effects of the group of permanent shocks continue to manifest themselves in the shock decompositions indefinitely. Their size only changes with the addition of a new shock. The changes in the pattern of the permanent shocks for model 2 also seem to

\[13\] Household want to work more in DSGE models when government expenditure increases, because they anticipate the need to pay higher taxes to finance this extra expenditure, and opt to work more to be able to pay these taxes.
Figure 4.12: Shock decomposition $\hat{y}_{t}$

(a) Non-stationary model 2

(b) Stationary model 5

Figure 4.13: Shock decomposition $\hat{C}_{t}$

(a) Non-stationary model 2

(b) Stationary model 5
follow the business cycle. In particular we can see that there were positive shocks leading up to the Great Recession, followed by several negative shocks at the beginning of the Great Recession in 2009. These outcomes, however, suggest that the size of these negative shocks was modest.

The build up of positive permanent shocks preceding the Great Recession is also visible in figures 4.13 and 4.14 of the decompositions from model 2 for consumption and labour supply. The shocks to labour supply, however, were apparently relatively modest. On the other hand, the negative shocks to labour supply which built up over the period of 2009 and 2010 are large, and their effects persist until the end of the sample period in 2014. The initial impact on consumption of the permanent shocks during the Great Recession is also negative, but in subsequent recent years there have apparently been a sequence of positive shocks which have almost reduced the net effect of the permanent shocks down to zero.

Comparing the shock decompositions in figure 4.14 for labour supply, we can see that the decomposition from model 5 is dominated by the markup shocks, while in the figure for model 2 the roles played by the permanent, fiscal, foreign and temporary technology shocks are equally large or larger than the markup shocks. The pattern of the foreign and fiscal shocks is similar to the one observed around the time of the Great Recession for output. This is true for model 2 and model 5, although the effects in model 5 are much smaller. A similar story can be told for the foreign and fiscal shocks in the decomposition of consumption: at the start of the crisis we can see evidence of negative foreign shocks and positive fiscal shocks, which then subsequently reverse sign around 2010. The decomposition for model 2 in fact indicates that foreign shocks are currently exerting a positive influence on consumption, presumably via the recent strong export demand.

In general the shock decompositions from models 2 and 5 differ in a number of ways. The HP filtering of the data used in model 5 results in smaller estimated shocks. The relative importance of the various shock groups is also different. The fiscal, foreign and temporary technology shocks tend to play a more significant role in shock decompositions from model 2. The permanent shock group from model 2 also seems to play a significant role in the decompositions. Finally, the sign of the net contribution of the shock groups is often different between the two models. These differences reduce the confidence with which one can use these shock decompositions for enlightening policy advice or forecasting, since the differences imply considerable uncertainty about what shocks were really impacting the economy. On the other hand, the changes in the shock decomposition do appear to be more robust.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_{a} )</td>
<td>AR(1) general technology</td>
<td>(0.013, 0.527, 0.154, 0.877)</td>
</tr>
<tr>
<td>( \rho_{g} )</td>
<td>AR(1) fiscal</td>
<td>(0.870, 0.919, 0.773, 0.715)</td>
</tr>
<tr>
<td>( \rho_{w} )</td>
<td>AR(1) wage mark-up</td>
<td>(-0.288, 0.314, 0.718, 0.715)</td>
</tr>
<tr>
<td>( \rho_{\mu} )</td>
<td>AR(1) investment-specific technology</td>
<td>(-0.395, -0.288, -0.205, -0.069)</td>
</tr>
<tr>
<td>( \rho_{p} )</td>
<td>AR(1) price mark-up</td>
<td>(0.057, 0.230, 0.310, 0.921)</td>
</tr>
<tr>
<td>( \rho_{b} )</td>
<td>AR(1) bond premium</td>
<td>(0.248, 0.017, -0.179, 0.446)</td>
</tr>
<tr>
<td>( \rho_{x} )</td>
<td>AR(1) export price mark-up</td>
<td>(0.815, 0.017, -0.353, -0.040)</td>
</tr>
<tr>
<td>( \rho_{f} )</td>
<td>AR(1) export demand</td>
<td>(0.952, 0.017, -0.069, 0.066)</td>
</tr>
<tr>
<td>( \rho_{c,m} )</td>
<td>AR(1) consumption import price mark-up</td>
<td>(0.903, 0.320, 0.153, 0.808)</td>
</tr>
</tbody>
</table>

| \( \sigma_{a} \) | s.e. general technology | (0.019, 0.050, 0.029, 0.007) |
| \( \sigma_{g} \) | s.e. fiscal | (0.012, 0.013, 0.011, 0.011) |
| \( \sigma_{w} \) | s.e. wage mark-up | (0.003, 0.004, 0.009, 0.017) |
| \( \sigma_{\mu} \) | s.e. investment-specific technology | (0.869, 0.021, 0.392, 0.280) |
| \( \sigma_{p} \) | s.e. price mark-up | (0.006, 0.005, 0.003, 0.003) |
| \( \sigma_{b} \) | s.e. bond premium | (0.200, 0.121, 0.039, 0.006) |
| \( \sigma_{x} \) | s.e. export price mark-up | (0.009, 0.008, 0.012, 0.013) |
| \( \sigma_{f} \) | s.e. export demand | (0.019, 0.017, 0.015, 0.015) |
| \( \sigma_{c,m} \) | s.e. consumption import price mark-up | (0.006, 0.009, 0.070, 0.076) |
| \( \rho_{\text{go}} \) | AR parameter | (0.278, 0.213) |
| \( \sigma_{t} \) | s.e. trend general technology | (0.013, 0.016, 0.013, 0.018) |
| \( \sigma_{y} \) | s.e. trend labour supply | (0.015, 0.018, 0.014, 0.022) |
| \( \sigma_{\text{r}} \) | s.e. trend investment-specific technology | (0.021, 0.024, 0.019, 0.017) |

Note: (1) indicates the non-stationary model of Lafourcade and De Wind (2012) with sample period from 1984Q1 to 2007Q4; (2) indicates the non-stationary model of Lafourcade and De Wind (2012) with sample period from 1988Q1 to 2014Q1; (3) indicates a stationary version of (2) estimated using HP-filtered data; (4) indicates (3) but estimated using unemployment data instead of wages; (5) indicates (4) but with the addition of credit-constrained households.
Table 4.8: Posterior estimates for shocks, approximate

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_a$</td>
<td>general technology</td>
<td>0.009</td>
<td>0.059</td>
<td>0.029</td>
<td>0.015</td>
<td>0.009</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>fiscal</td>
<td>0.024</td>
<td>0.033</td>
<td>0.017</td>
<td>0.017</td>
<td>0.016</td>
</tr>
<tr>
<td>$\sigma_w$</td>
<td>wage mark-up</td>
<td>0.005</td>
<td>0.012</td>
<td>0.009</td>
<td>0.047</td>
<td>0.069</td>
</tr>
<tr>
<td>$\sigma_\mu$</td>
<td>investment-specific technology</td>
<td>0.892</td>
<td>0.773</td>
<td>0.409</td>
<td>0.281</td>
<td>0.267</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td>price mark-up</td>
<td>0.006</td>
<td>0.005</td>
<td>0.003</td>
<td>0.008</td>
<td>0.013</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>bond premium</td>
<td>0.206</td>
<td>0.121</td>
<td>0.04</td>
<td>0.007</td>
<td>0.015</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>export price mark-up</td>
<td>0.016</td>
<td>0.012</td>
<td>0.014</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td>$\sigma_f$</td>
<td>export demand</td>
<td>0.062</td>
<td>0.064</td>
<td>0.022</td>
<td>0.022</td>
<td>0.05</td>
</tr>
<tr>
<td>$\sigma_c$, $\sigma_m$</td>
<td>consumption import price</td>
<td>0.014</td>
<td>0.009</td>
<td>0.071</td>
<td>0.076</td>
<td>0.068</td>
</tr>
</tbody>
</table>

Note: (1) indicates the non-stationary model of Lafourcade and De Wind (2012) with sample period from 1984Q1 to 2007Q4; (2) indicates the non-stationary model of Lafourcade and De Wind (2012) with sample period from 1988Q1 to 2014Q1; (3) indicates a stationary version of (2) estimated using HP-filtered data; (4) indicates (3) but estimated using unemployment data instead of wages; (5) indicates (4) but with the addition of credit-constrained households.

Figure 4.14: Shock decomposition $\hat{E}_t$

(a) Non-stationary model 2

(b) Stationary model 5
5 Concluding remarks

This paper summarises our attempts to develop a small open economy DSGE model for the Netherlands, that includes unemployment and is tailored to analyse the effects of fiscal policy. In principle, such a DSGE model should be very attractive for policy analysis since the DSGE modelling approach holds out the possibility of avoiding the Lucas critique. Our attempts took the form of estimating five variants of the model of Lafourcade and De Wind (2012), with different sample periods, different treatments of trending data and the addition of two desirable features for policy analysis to the model: an explicit measure of unemployment and credit-constrained households.

Interestingly, once estimated, the addition of credit-constrained households doesn’t increase the magnitude of the consumption response to government spending shocks. That, after all, is the main reason for adding credit constrained households. This finding suggests that the model without credit-constrained households was already doing a good job of capturing the response to fiscal shocks. That leaves the stationary version of the model including unemployment as potentially the most relevant version of the model for the CPB, because this model includes unemployment, is simpler than the final model with credit constrained consumers, and nonetheless manages to produce fairly similar behaviour. A potential second alternative to this model, is the original model in levels estimated with current data. The advantage of this model is that it includes permanent shocks, which can potentially be interesting for policy analysis. The disadvantage of this model, however, is that the cointegrating relationships it imposes may not be correct and the model does not model unemployment. In general, the models estimated on unemployment data showed markedly different labour market responses to shocks to those estimated using wage data. That is evidence that labour market specifications that describe wages well do not capture the process driving unemployment in the Netherlands.

Chief among the limitations of DSGE models is the size limitation imposed by any need to estimate the model. The size limitation reduces the level of detail in policy that a DSGE model can analyse. If, in addition, when smaller DSGE models are estimated they are not robust to some straightforward specification changes it is hard to argue that they are also not subject to the Lucas critique for important policy questions. That is exactly what we found when estimating our five model variants. Despite our ability to explain the differences between the estimates based on our knowledge of the time-series involved, this calls into question how deep these structural parameters really are. In fact, the close link between our knowledge of economic events and the changing parameter estimates suggest that the parameters are reduced-form proxies of some more complicated true model. Therefore, it is difficult to currently recommend a DSGE for a policy institute such as the CPB. Of course, since DSGE models are at the frontier of economic research, this conclusion may change with new and better approaches to modelling...
certain features of the macroeconomy within the DSGE framework.  

Our finding that the parameter estimates of our DSGE model change considerably across specifications is related to the literature on DSGE models, parameter instability and the Lucas critique. One strand of this literature relates to parameter instability to model misspecification. Both Cogley and Yagihashi (2010) and Chang et al. (2013) find that model misspecification leads to parameter instability. Another strand documents significant time variation in parameter estimates. For example, Inoue and Rossi (2008) find significant time variation in parameters widely distributed throughout the model. Hurtado (2014) finds that parameter instability affects a wide range of parameters including those that would be expected to be constant if they are well identified deep parameters. Our findings documented here are in line with this strand of literature.
6 Appendix: the linearised model equations

That means the additional equations needed for the log-linearised model are:

\[
\hat{p}_t + \hat{C}_t = \frac{1}{(1 - \tau_w) w_s L_s + T_s y_s^d} \left[ (1 - \tau_w) w_s L_s (\hat{w}_t + \hat{L}_s) + T_s y_s^d \left( \hat{T}_t + \hat{y}_s^d \right) \right]. \tag{6.1}
\]

\[
\hat{C}_t = \frac{\lambda C^C}{\lambda C^C + (1 - \lambda C) C^O} \hat{C}_t^C + \frac{(1 - \lambda C) C^O}{\lambda C^C + (1 - \lambda C) C^O} C^O_t \tag{6.2}
\]

\[
\hat{\Xi}_C^C + \hat{\xi}_C^C = \frac{zC}{zC - \beta \lambda} \hat{\xi}_C^C + \frac{\beta \lambda}{zC - \beta \lambda} E_{t+1} \hat{\xi}_C^C \tag{6.3}
\]

\[
\hat{d}_t - \hat{\xi}_C^C = \frac{zC}{zC - \lambda} \hat{\xi}_C^C + \frac{\lambda}{zC - \lambda} (\hat{C}_{t-1} - \hat{\xi}_C^C) \tag{6.4}
\]

\[
\hat{\Xi}_C^C = \lambda C \hat{x}_C^C + (1 - \lambda C) \hat{x}_C^C \tag{6.5}
\]

and for the steady state:

\[
C^C_s = \frac{(1 - \tau_w) w_s L_s + T_s y_s^d}{(1 + \tau_c) p_s^c}, \tag{6.6}
\]

\[
C^O_s = \frac{C_s - \lambda C^C_C}{1 - \lambda C}. \tag{6.7}
\]

6.1 Additional parameters

The only additional parameter is \(\lambda_C\), the share of credit constrained households.
Appendix: The steady state derivation

The elements of the steady state needed for running the Dynare code are:

\[ z_Y = z_L z_w \]  
(7.1)

\[ z_K = \frac{z_Y}{z^t} \]  
(7.2)

\[ z_C = z_Y \frac{I_s}{C_s} \]  
(7.3)

\[ z_{Pic} = \frac{z_Y}{z_C} \]  
(7.4)

\[ \delta = \frac{z_K}{\beta} - 1 + \tau_k - (zK - 1) \frac{\alpha}{\psi}. \]  
(7.5)

\[ mc_s = \frac{1}{\psi - 1} \]  
(7.6)

\[ r^*_k = \frac{zK}{\beta} - 1 + \delta + \tau_k \]  
(7.7)

\[ w^*_s = (1 - \alpha) mc \frac{1}{\psi} \left( \frac{r^*_k}{\alpha} \right)^{\frac{1}{1-\alpha}} \]  
(7.8)

\[ w^*_h = \frac{w^*_s}{\psi - 1} \]  
(7.9)

\[ F1 = \frac{z^h - 1 + \tau_c}{z^h - 1} \]  
(7.10)

\[ F2 = \frac{(w^* - 1)(1 - \tau_c)}{\psi} \]  
(7.11)

\[ L_s = \left( \frac{I_s/C_s (1 - \alpha)}{I_s/Y_s} F1 F2 \right)^{1/\psi} \]  
(7.12)

\[ Y_s = \frac{w^*_s L_s}{1 - \alpha} \]  
(7.13)

\[ K_s = \frac{z_K \alpha Y_s}{r^*_k} \]  
(7.14)

\[ I_s = I_s/Y_s Y_s \]  
(7.15)

\[ Id_s = n_i I_s \]  
(7.16)

\[ Im_s = \frac{I_s - Id_s}{pimp_s} \]  
(7.17)

\[ C_s = \frac{I_s}{I_s/C_s} \]  
(7.18)

\[ Cd_s = n_c C_s \]  
(7.19)
\[ C_{m*} = \frac{C_s - C_d s}{p c m p_s} \]  
(7.20)

\[ M_s = C_{m*} + I m_s \]  
(7.21)

\[ X_s = Y_s \left( 1 - g_s \right) - C_d s - I d_s \]  
(7.22)

\[ Y f_s = \frac{X_s}{n_s} \]  
(7.23)

\[ Lb_s = F1 \frac{C}{C_s} \]  
(7.24)

\[ \left( 1 - \frac{1}{\beta} \right) b_s = T_s + g_s - \tau_s \frac{C_s}{y_s^d} - \tau_w \frac{w_s L_s}{y_s^d} - \tau_\xi \frac{x_s}{\xi_s y_s} \]  
(7.25)

\[ C^C_s = \frac{(1 - \tau_w) w_s L_s + T_s y_s^d}{(1 + \tau_c) p_c^s} \]  
(7.26)

\[ C^O_s = \frac{C_s - \lambda C^C_s}{1 - \lambda C} \]  
(7.27)

\[ \Xi^C_s = \frac{F1}{C^C_s} \]  
(7.28)

\[ \Xi^O_s = \frac{F1}{C^O_s} \]  
(7.29)

\[ L^{d,C}_s = \left( \frac{(1 - \tau_w) \Xi^C_s w_s}{\psi} \right)^{\frac{1}{\eta}} \]  
(7.30)

\[ L^{d,O}_s = \left( \frac{(1 - \tau_w) \Xi^O_s w_s}{\psi} \right)^{\frac{1}{\eta}} \]  
(7.31)

\[ L^d_s = \lambda C L^{d,C}_s + (1 - \lambda C) L^{d,O}_s \]  
(7.32)

\[ u_s = 1 - \frac{L_s}{L^d_s} \]  
(7.33)
8 Appendix: Estimation diagnostics

Figure 8.1: Multivariate convergence criteria for Lafourcade and De Wind (2012), 1984Q1-2007Q4

Figure 8.2: Multivariate convergence criteria for Lafourcade and De Wind (2012), 1988Q1-2014Q1
Figure 8.3: Multivariate convergence criteria for stationary version of Lafourcade and De Wind (2012)

Figure 8.4: Multivariate convergence criteria for stationary model estimated with unemployment data
Figure 8.5: Multivariate convergence criteria for stationary model with credit-constrained households estimated with unemployment data
References


