CPB Background document

‘Prudent debt level: a tentative calculation’

Accompanies the Dutch CPB Policy Brief 2013/05 ‘Naar een prudent niveau van de overheidsschuld’

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1 Summary[1]

What is a prudent public debt-to-GDP ratio? To answer this question, we provide a tentative calculation of the gross debt-to-GDP ratio for which the gains of holding a larger buffer to ward off negative shocks equal the cost of transitioning to a lower debt level. This buffer is held as a precaution as, above a certain ratio, the relationship between government debt and economic growth becomes increasingly negative. A smaller initial debt level, then, reduces the probability that debt becomes so large in the future that it adversely affects economic growth. Naturally, the gains of increasing the size of the buffer are lower when current debt is lower. These gains are then combined with the costs of debt reduction via fiscal consolidation to arrive at this prudent debt level.[2]

The focus in this calculation is on advanced economies. We thus rely on empirical research that assesses the effect of government debt on economic growth for advanced economies and we calculate the costs of debt reduction with Saffier, the structural model for the Dutch economy at CPB. We find the prudent debt level, being the level at which the gains equal the costs of reduction of the debt level, between 61% and 86% of GDP, with an average of 72%. The buffer is defined as the debt level above which negative effects materialize minus the prudent debt level and measures 20% of GDP. Historically, the probability that a negative effect materializes has been 18%.[3] Caution is advised in using these debt levels as anchors for policy, as the costs of deviating from these numbers are small in this range while the benefits may be significant.[4] Generally speaking, any debt level below this range seems prudent as well.

This CPB Background document is structured as follows: section 2 presents the precautionary savings motive for the government as an economic rationale for a buffer, in section 3 we perform some tentative calculations on the gains of having a larger buffer, section 4 presents the costs of debt reduction from the Saffier model, section 5 integrates both elements and calculates the prudent debt level and section 6 discusses these results and their limitations.

[1] This background document accompanies CPB Policy Brief 2013/05 *Naar een prudent niveau van de overheidsschuld* by Lukkezen and Suyker (2013).
[2] A more complicated question, that of the optimal debt level, is treated in Leith et al. (2011).
[3] Within our eight year horizon.
[4] See Delong (2013).

2 Precautionary savings motive

A government that responds to economic shocks by changing the debt level instead of increasing the tax rate, as advocated by Barro (1979), has a debt-to-GDP ratio that follows a random walk. If the government debt level has no impact on the economy, either directly or via the expectations it generates, our analysis would end here. Unfortunately this is not the case. Recently, several studies[5] have evaluated the effects of the gross debt level on GDP in advanced economies and found that above some high threshold level, debt has a negative impact on economic growth.[6] This could be because debt drives up interest rates or expected future taxes, which reduce incentives to work and invest.

As Dehejia and Rowe (1995) show, uncertainty and a convex loss function generate a precautionary savings motive for government. Such a motive provides an economic rationale for holding a buffer (being accumulated precautionary savings). The intuition behind this is as follows: if government debt has an increasingly negative effect on economic growth at higher levels (thus the loss in GDP is a strict convex function of the debt level), and the future is uncertain (meaning these high debt levels are not fully improbable), debt reduction brings gains as it lowers the probability of entering into a situation where these high debt levels reduce economic growth.

With the GDP losses being a function of the debt level and current and future generations being treated on an equal footing, we formalize this as:

(1) Losses(current debt level) = E[Losses(future debt level)].

Notice that by Jensen's inequality (see the text box *Jensen's inequality* for a graphical explanation) for a strictly convex loss function[7]:

(2) E[Losses(future debt level)] > Losses(E[future debt level]).

By combining Equation (1) and (2) and using the fact that the loss function is strictly convex and increasing in the debt level, we find that the expected future debt level should be smaller than the current debt level, or, in other words, the policy maker should plan to reduce debt from a precautionary motive alone.

Reducing the debt level however, also brings costs in the form of temporary output losses as the government cuts expenditure and raises taxes. Then, the debt-to-GDP ratio at which the policy maker is indifferent between remaining at the current debt level and reducing it further is considered the prudent debt level. It is given by:

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[5] See Lukkezen and Suyker (2013) or Table 8 in IMF (2013) for an overview. The studies we use for our analysis are listed in Table 1.

[6] Some of these studies also find a positive relationship between debt and growth at low debt levels. This effect is not relevant here, as countries can always increase gross debt and government assets.

[7] Note that the same holds for a concave function that describes gains. In section 3 we will have such a function.

(3) Losses(current debt level) - Losses(E[future debt level]) = Cost(current debt level - E[future debt level])

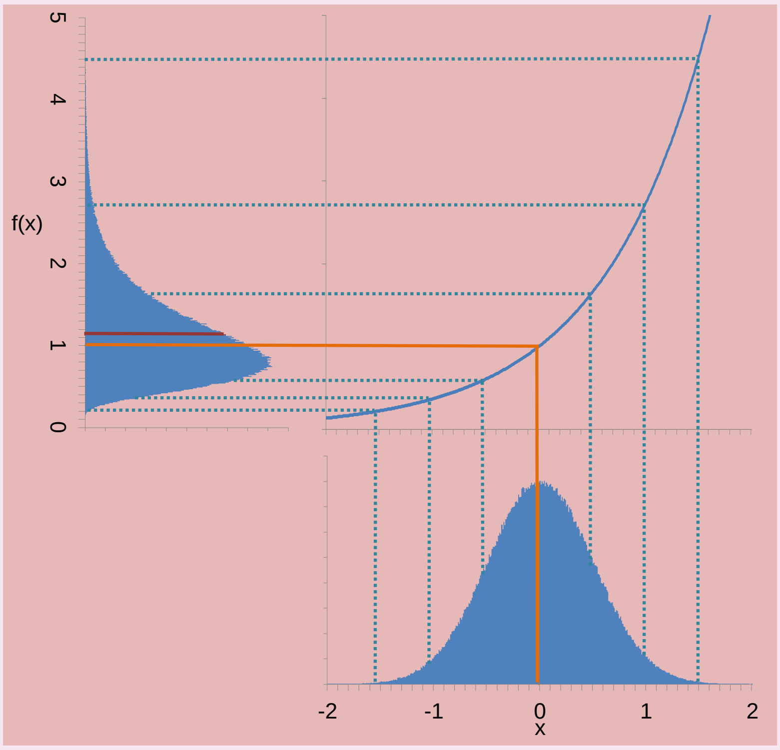
Jensen's inequality

Jensen's inequality was derived by the Danish mathematician Johan Jensen in 1906 and has implications for statistical physics, information theory and economic policy analysis. Section 2 in this background document shows such an application to economic policy analysis. The inequality states in its simplest form that the transformation by a convex function of a mean of a random variable is less than or equal to the mean of the transformation of that random variable. Or mathematically:

(1) $E[f(x)] \geq f(E[x])$ for f() a convex function.

The figure below illustrates this point. There a convex function, f(x) = exp(x), is shown in the top right. This convex function transforms a normally distributed variable x (mean 0, standard deviation 0.5) in the bottom into the distribution f(x) on the left. The mean of x is zero and is transformed into exp(0)=1. Thus the transformation of the mean is 1 and is shown by the orange line. The transformation of the distribution of x is denoted by the dotted blue lines. Clearly, the drawings of x below 0 get closer to each other after the transformation as they are mapped between 0 and 1. The drawings of x larger than 0 drift apart after the transformation as they are mapped between 1 and infinity. The transformed distribution is skewed. The mean of the transformation of x is denoted by the dark red line and equals 1.13.

Figure A graphic representation of Jensen's inequality



Both the convexity of f and the fact that the variable x has a distribution is relevant for the inequality to hold. If f were a straight line, the transformed distribution would be a normal distribution again. And if x were known with certainty, the inequality would become an equality.

For our purposes, this means that the precautionary motive is present if there is uncertainty and if the marginal costs associated with a rise in the debt level are an increasing function of the initial debt level. Both assumptions do not seem unreasonable.

3 Gains of debt reduction

To calculate the gains of reducing the debt level by 1% we subtract expected discounted future GDP at the current debt level minus 1% from that at the current debt level. Discounted future GDP is calculated as follows:

$$(4) \qquad Y_t = \sum_{s=t}^{\infty} GDP_s \left(\frac{1}{1+\mathrm{d}}\right)^{\mathrm{t}} = GDP_t \sum_{s=t}^{\infty}(1-c_s)\left(\frac{1+\mathrm{g}}{1+\mathrm{d}}\right)^{\mathrm{t}},$$

where d is the annual time discount rate of 3%, which is 1.5% higher than the annual real growth rate g of 1.5%[8] and c_s the reduction in GDP growth as a function of the debt-to-GDP ratio. We obtain c_s from the literature, specifically the four studies in Table1.

Table 1 Studies on the relationship between debt and economic growth

Author	Data	Specification	Effect
Baum et al. (2012)	Euro area, 1990-2010	Piecewise linear	>96%, -0.059*d
Cecchetti et al. (2011)	OECD, 1980-2010	Piecewise linear	>84%, -0.013*d
Checherita-Westphal and Rother (2012)	Euro area, 1970-2008	Quadratic	>98%, const.+0.120*d-0.00062*d^2
Égert (2012)	Reinhart en Rogoff (2010) dataset	Piecewise linear	>90%#, -0.019*d

The dependent variable is economic growth in percent, the debt-to-GDP ratio is the most important explanatory variable.
This threshold is imposed exogenously, in contrast to the other studies which estimate it endogenously.

To continue, we have to make some assumption on the variability of the debt level. For sake of simplicity, we assume the historic volatility of the public debt ratio for OECD countries is informative for current volatility. The time horizon at which we use this volatility is 8 years, which corresponds to the horizon we will use for debt reduction in calculating the costs later.[9]

We obtain the distribution ($\mathcal{F}$) of 8-year changes in public debt levels ($\Delta_8 d_t = d_t - d_{t-8}$) for OECD countries from the historic public debt database of the IMF[10]. Figure 1 presents the 2852 observations in a histogram. The average increase in the debt level over 8 years has been 2.1% and its standard deviation is 27.7%. Its skew is 0.30 and its kurtosis 9.5, which indicates fat tails, especially on the right. Table 2, which shows the debt increases of at certain percentiles of the distribution $\mathcal{F}$, confirms this.

[8] We follow the literature here: 1.5% and 3% are applied in sustainability analyses in ageing studies. See for a discussion for the Dutch economy Van Ewijk et al. (2006).
[9] We use an eight year horizon as it corresponds to two subsequent governments that serve their entire term. In the OECD only 10% of governments are able to stay in power longer since 1975, by data from the World Bank Political institutions database (Beck et al., 2001, update 2013).
[10] Contains long time series on the debt-to-GDP ratio, starting date however differs per country. For 16 out of 34 OECD-members there is data for 1890 or before. See Abbas et al. (2010) for a full description.

Figure 1 Debt level changes in 8 years for OECD countries

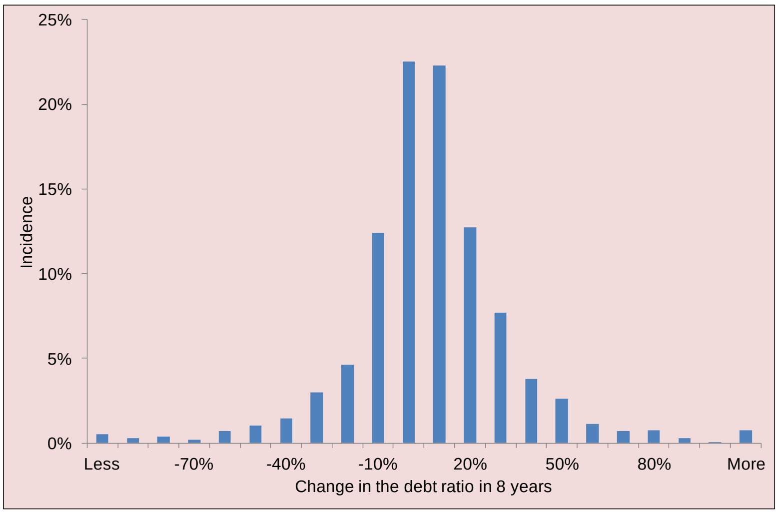


Table 2 Percentiles of the distribution of debt changes

Standard deviation	Percentile	Debt change ($\Delta_8 d_t$)
1.00	84%	22%
1.28	90%	30%
1.64	95%	44%
2.00	98%	66%
3.00	100%	169%

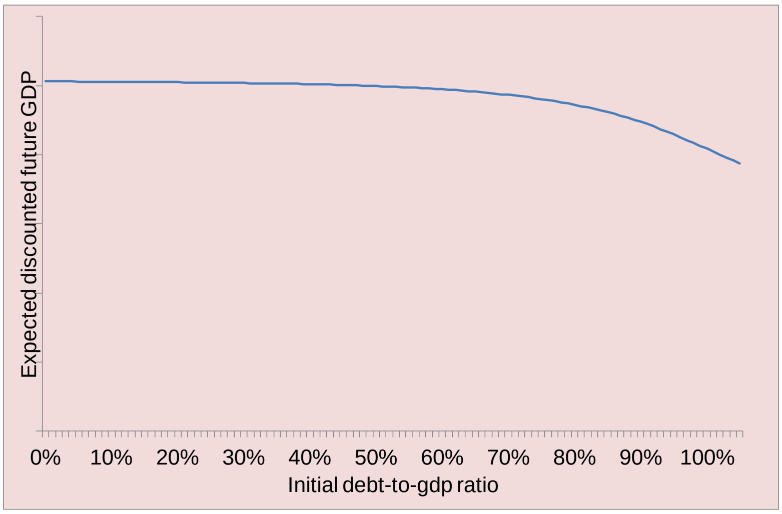
Then, for a given initial debt level we add these 2852 changes in the debt ratio one-by-one to the initial debt level and calculate the value of discounted future GDP using equation (4) and c_s from the last column of Table 1. The expected value of discounted future GDP in 8 years is then simply the average value of discounted future GDP using these changes:

$$(5) \qquad Y_t(d_t) = GDP_t \frac{1}{2852} \sum_{\Delta_8 d_t \in \mathcal{F}} \sum_{s=t}^{\infty} \left(1 + c_s(d_t + \Delta_8 d_t)\right) \left(\frac{1+g}{1+d}\right)^t.$$

As this gives the expected value of discounted future GDP 8 years from now (i.e. after the debt changes), we discount it to get the current value. For each of the specifications in Table 1, the expected value of discounted future GDP is a declining function of the initial debt level. Figure 2 shows the expected value of discounted future GDP by Equation (5) for the first specification in Table 1.[11]

[11] The other specifications generate qualitatively similar graphs. We omit them from Figure 2 to facilitate the presentation.

Figure 2 Expected discounted future GDP as a function of the initial debt-to-GDP ratio



Note: The vertical axes have a regular scale, only the relative differences are relevant, hence we omit the units.

4 Costs of debt reduction

We calculate the costs of debt reduction by a package of budget cuts and tax increases using Saffier, a structural model for the Dutch economy (Verbruggen et al., 2010). The package reduces the deficit by 1% of GDP ex ante and has a permanent character. Half of the ex ante deficit reduction is due to a reduction in government consumption, a quarter by an increase in VAT and a quarter by an increase in income and corporate taxes. Table 3 shows the impact of this package on GDP, CPI prices, unemployment and the debt level. As a robustness check we will use a deficit reduction purely financed by each of these measures individually, their effects on the economy are given in Tables 5.9 - 5.11 of Verbruggen et al. (2010).[12]

Table 3 Effects of a 1% of GDP ex ante austerity package

	1	2	3	4	8
GDP	-0.6%	-0.8%	-0.8%	-0.8%	-0.6%
Unemployment	0.2%	0.5%	0.5%	0.5%	0.2%
CPI	0.3%	0.2%	0.1%	0.0%	-0.2%
Debt level	-0.7%	-0.9%	-1.0%	-1.3%	-2.9%

Using Equation (4) and taking the effects a deficit adjustment temporary has on GDP into account we find that discounted future GDP is expected to reduce by 0.47%. After eight years this package reduces the debt level by 2.9% of GDP. Hence, the ex ante improvement of the budget balance due to the package, is partly offset by lower tax revenue and higher unemployment benefits as a result of the negative impact of the package on GDP. Per percentage point debt-decrease, this yields an expected reduction of discounted future GDP of 0.17%-points.

[12] The effects on the debt-to-GDP ratio after eight years are not shown there. They are -3.1%, -2.4% and -2.9% respectively.

5 Comparing costs and benefit

The debt level at which the policy maker is indifferent between debt reduction and taking no action depends on the size and shape of the non-linear effect of debt on GDP-growth as well as the debt level above which it materializes. At the indifference debt level, the benefits of debt reduction are equal to the costs. Table 4 lists these debt level for the four studies in Table 1 using the deficit reduction package of the previous section (in bold) and its three constituent measures (regular).

Table 4 Indifference debt levels

	2.5% Discount rate				3% Discount rate				3.5% Discount rate			
	pack.	cons.	VAT	IT/CIT	**pack.**	cons.	VAT	IT/CIT	**pack.**	cons.	VAT	IT/CIT
Baum	**61**	53	65	71	**66**	56	67	75	**69**	61	72	78
Cecc.	**65**	57	67	75	**72**	63	74	82	**76**	69	79	85
C&R	**76**	67	78	86	**82**	73	84	91	**86**	78	89	95
Égert	**66**	59	68	76	**72**	64	75	82	**77**	69	80	86

Note: units in % of GDP, pack. is the deficit reduction package, cons. is government consumption.

We find that that the indifference debt level for the package lies between 61 and 86% of GDP, with an average of 72%. For a higher discount rate, or lower steady state growth rate, the indifference debt level is higher, as the future is valued less, whereas for lower discount rates (or higher growth rates) the opposite applies. Naturally, if deficit reductions bring higher cost, which is the case for reductions via increases in the income tax, this range shifts upwards (71-95% of GDP), whereas if the costs are lower, this range shifts downward (53-78% of GDP). For an indifference debt level between 61 and 86% of GDP, the buffer, being defined as the debt level at which negative effects kick in minus the indifference debt level, is between 8 and 35% of GDP, with an average of 20% of GDP.

6 Discussion

Some caveats apply. The set-up of this cost-benefit analysis is a partial equilibrium approach compares the gains of having more precautionary savings with the costs of acquiring more savings. It does so irrespective of the current debt level and the economic circumstances. There is, for example, no feedback from high government debt onto the budget deficit or the initial volatility of the government debt level. Furthermore, once debt is high, it is assumed to remain at that level (the same goes for low debt). Also, we do not take cross-country heterogeneity into account in the response of growth to debt and the costs of debt reduction.

And lastly, the results of this analysis depend on the time frame under consideration. Here we compare the benefits of having a lower debt level in eight years with the costs of reducing the debt level in eight years. For longer time horizons, the costs of debt reduction reduce as there is more time to reduce the debt, whereas the gains increase as uncertainty becomes larger further in the future. Thus, the longer the horizon is, the lower the prudent debt level becomes. For shorter horizons the reverse applies.

Nevertheless, with these limitations, our tentative calculation shows that a prudent debt level is approximately 20% lower than the debt level at which debt is expected to have a negative impact on the economy. Historically, the probability that the debt level has increased by more than 20% within eight years has been 18% (from the distribution $\mathcal{F}$). Given that the 90% confidence interval of the debt level at which negative effects of debt on growth materialize is 80-100% (from Lukkezen and Suyker (2013)), any debt level up to 60-80% seems prudent. The 60% debt level from the Stability and Growth Pact lies within that range.

Caution is advised in using these prudent debt levels to anchor policy. Note for example that, by Table 1, an increase in debt of a 10% from such a prudent level reduces lifetime earnings by just a few percentage points. This is relatively small if the debt increase averts a financial crisis. Also, adverse economic circumstances may lead to temporarily higher debt levels, which can be prudent.[13]

On the other hand, saving now to finance ageing related expenditure later, might be intergenerationally fair and is a good reason for debt reduction below this range. Also, at a much lower debt level, political economy considerations could be strong arguments against an increase of the debt level. Generally thus, we consider debt levels below the 60-80% range prudent as well; prudent debt levels from these calculations should be seen as upper bounds under normal economic circumstances absent intergenerational fairness and political economy considerations.

[13] This can be understood by seeing that the cost of debt reduction increase in a recession. Then, the prudent debt level, that at which the gains of debt reduction equal the costs, rises as well.

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