Moving Behaviour in the Dutch Owner Occupied Housing Market

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Abstract

We develop a method to model moving behaviour in the Dutch owner occupied housing market and investigate to what extent the model can account for moving behaviour in the Dutch housing market. The demand for housing services of an agent is uncertain, and changing the actual consumption involves lump sum moving costs. The model is based on an impulse control of Brownian motion. From the database 'WoON2006' we obtain a cross-section of the actual consumption of housing services. The desired consumption of housing services is derived by applying the model in Donders et al. (2010) to income and family size data in WoON2006. We fitted the model to this cross-section in order to find the parameters of housing behaviour and the moving costs. The model is used to analyse the effects of reduction of the transfer tax that is currently paid when purchasing a new home. We find that the associated reduction in moving costs by one percent point increases the number of moves by about 3% to 10% and gives a welfare gain of around 0.1 billion euro.

Chapter 1

Introduction

In this study, we develop a model for moving behaviour in the Dutch owner occupied housing market. Moving behaviour is affected by moving costs which prevent households from adapting their actual consumption to the desired consumption of housing services. Previous studies consider a hazard rate describing the probability of moving as done in Van Ommeren and Van Leuvensteijn (2003). Romijn (2000) considers a dynamic model where the agent optimizes its consumption considering the moving costs. This model is solved using a control band policy as stated in Harrison et al. (1983). Romijn (2000) applied this model to the Dutch office space market and calibrated it using aggregate time series. The model was also used for the housing market, but the results were unsatisfactory and the value of the moving costs was of an unrealistic order. We will use the continuous time micro-model from Romijn (2000) but fit it this time to cross-sectional data. The distribution of the gap between the actual consumption of housing services is fitted to the data of the owner-occupied housing market in the Netherlands from WoON2006. After that, we can calculate the moving costs for home owners in the Netherlands. The government levies a transfer tax of 6% of the value of real estate. These costs are part of the moving costs and are known. Furthermore there are broker costs which leads us to expect the moving cost to be around 10% of the value of the new home. Above these cost, we can imagine search costs, the costs of moving the movable property of the household and loss of the social neighbourhood network. With our model, we aim to get an idea of these total moving costs. We will do policy experiments by removing the transfer tax from the moving costs and evaluate how this affects moving behaviour and how welfare will increase.

We study the moving behaviour of households. Households have a disposable income that they can spend on their house and other goods and services. Houses are highly heterogeneous. Some are small and others are large, some have a garden and others do not. Some are located in The Hague and others in Groningen. Some houses have a luxurious bathroom and others simple ones. To make houses comparable, we follow Donders et al. (2010) who assume that the quality of a house is fully reflected by its price. As a unit we use 'standard house'. Households can e.g. own a home with a quality of 2.3 standard houses. We use the average price of a house in 2005 in the Netherlands as the price and therefore quality of one standard house. Houses deliver services to the occupier, the household, and we call these housing services. One standard house supplies the occupier with one housing service per year. Donders et al. (2010) model the households demand for housing services to depend positively on their disposable income and the size and composition of the households. Details about this derivation are given in section 2.1.

The data for the empirical analysis of our model is derived from the database WoON2006. WoON2006 contains data about 60.000 households in the Netherlands. It is a cross section over these households in the year 2005. [1] Using the model of demand for housing services by Donders et al. (2010), the desried level of housing service consumption of households (h^*) can be calculated.

[1] The data is weighted such that is it representative for all households in the Netherlands. This means that every record of the data has a number that indicates how many household this record represents. The sum of this numbers is the total number of households in The Netherlands.

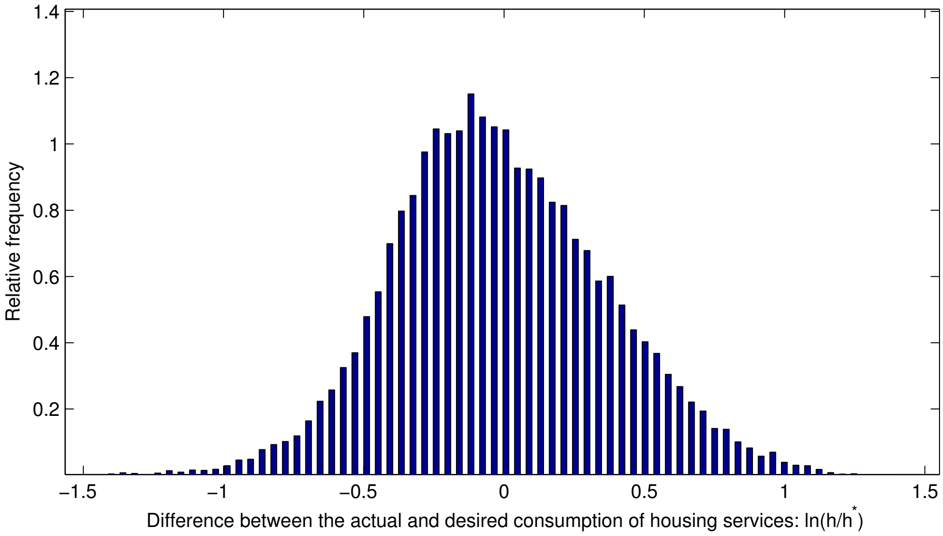

Figure 1.1: Weighted histogram of the observed gap between optimal (h^*) and actual consumption (h) of housing services gap as approximate percentage of the desired level ($\ln \frac{h}{h^*} = \ln \left(1 + \frac{h-h^*}{h^*}\right) \approx \frac{h-h^*}{h^*}$). It is scaled such that the surface sums to one.

This value is compared with the actual consumption of housing services (h). A histogram of the gap between optimal and actual consumption is given in figure 1.1 . The histogram shows that there can be considerable gaps between actual and desired housing services consumption. In this paper, we consider to what extent moving costs prevent people from moving to their desired house. This could explain the distribution of gaps that is apparent in the histogram.

It is easy to understand that the higher the moving costs, the longer a household will take to adapt its consumption. Therefore, we are interested in the moving frequency as well. From the data we can measure the time since a household's last move into its current home. This is shown in figure 1.2. We use this data and data from the CBS with the amount of transfer tax transaction to find the average time of residence in one house which is estimated at 17 years. Details about this derivation are given in section 3.1.

The gap between actual and desired housing services consumption entails a loss to the household due to a suboptimal consumption pattern. If moving costs are an explanation for the observed gap, reducing the moving cost will reduce the welfare loss by making it less costly for households to move. The present discussion in the housing market on the role of the transfer tax can be addressed by considering moving costs and its effect on moving behaviour. Van Ommeren and van Leuvensteijn (2003) estimated these effects. This paper provides further evidence based on an explicit behavioural model and micro data.

In chapter 2, we model the utility maximizing behaviour of a household as well as the decision to move. In chapter 3, we estimate the parameters by fitting the theoretical distribution to the observed distribution of actual and desired consumption. This allows us to evaluate the moving costs. In chapter 4, we turn to policy analysis and welfare effects. The results, and the extensions and limitations of the model, are discussed in chapter 5.

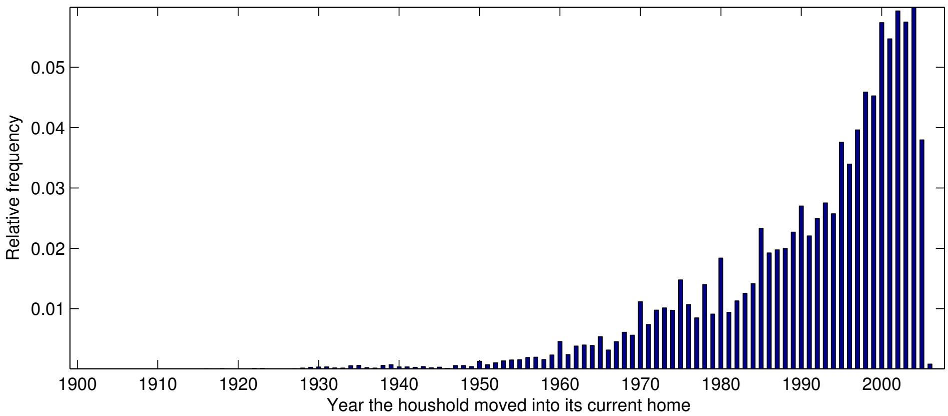

Figure 1.2: Year of move into current house. On the vertical axis the percentage of households for each year.

Chapter 2

The model

In this section, we first describe how a household decides how much of its income it will spend on housing, and how much on other goods in a frictionless world. Next, we introduce moving costs and we explain a dynamic micro model of moving behaviour for an individual household under uncertainty as developed in Romijn (2000). Then we derive an optimal control band policy to find the optimal decision rule for moving, which can be characterized as a stopping time[1]. After that, we aggregate the model and derive the cross-sectional distribution of the gap between desired and actual housing services consumption. Finally, we link the model of utility optimization with the dynamic moving behaviour model to calculate the welfare losses of moving costs.

2.1 Housing services demand

A household has a disposable income b that it spends on housing services h and other goods x. Housing services have price p_h and other goods have price p_x. Hence, the households budget constraint is given by:

$$b = p_h h + p_x x \tag{2.1}$$

To make a decision between the consumption of housing services or other goods and services, we use an utility function of the Cobb-Douglas type[2]. Here, we suppose that every households needs at least a minimal level of housing consumption $\bar{h}$ (one can't live in a box). The size of this minimum depends on the size of the households and is further specified in Donders et al. (2010). The household therefore decides about the supra minimal housing consumption. We assume a household will, given its budget constraint, maximize its utility given by:

$$\begin{aligned} U(h,c) &= \left(h-\bar{h}\right)^{\delta} x^{1-\delta} \\ \text{s.t. } b &= hp_h + p_x x \end{aligned} \tag{2.2}$$

where δ is the preference parameter for housing services. Utility is maximized for:

$$h^* = \bar{h} + \delta \frac{b - \bar{h}p_h}{p_h}. \tag{2.3}$$

2.1.1 Welfare loss

Because there are costs of adapting the consumption of housing services[3], not every household will consume exactly h^*. This suboptimal housing situation generates a loss of welfare. In this

[1] Stopping times are loosely speaking "rules" by which we interrupt the process without looking at the process after it is interrupted. (Gamarnik (2005))

[2] Utility is a measure of relative satisfaction. We use it to measure how satisfied a household is with a combination of goods. See for example Himmelweit et al. (2001) for a textbook introduction to utility and consumer choice.

[3] We assume that adjusting the consumption of other goods and services has no friction.

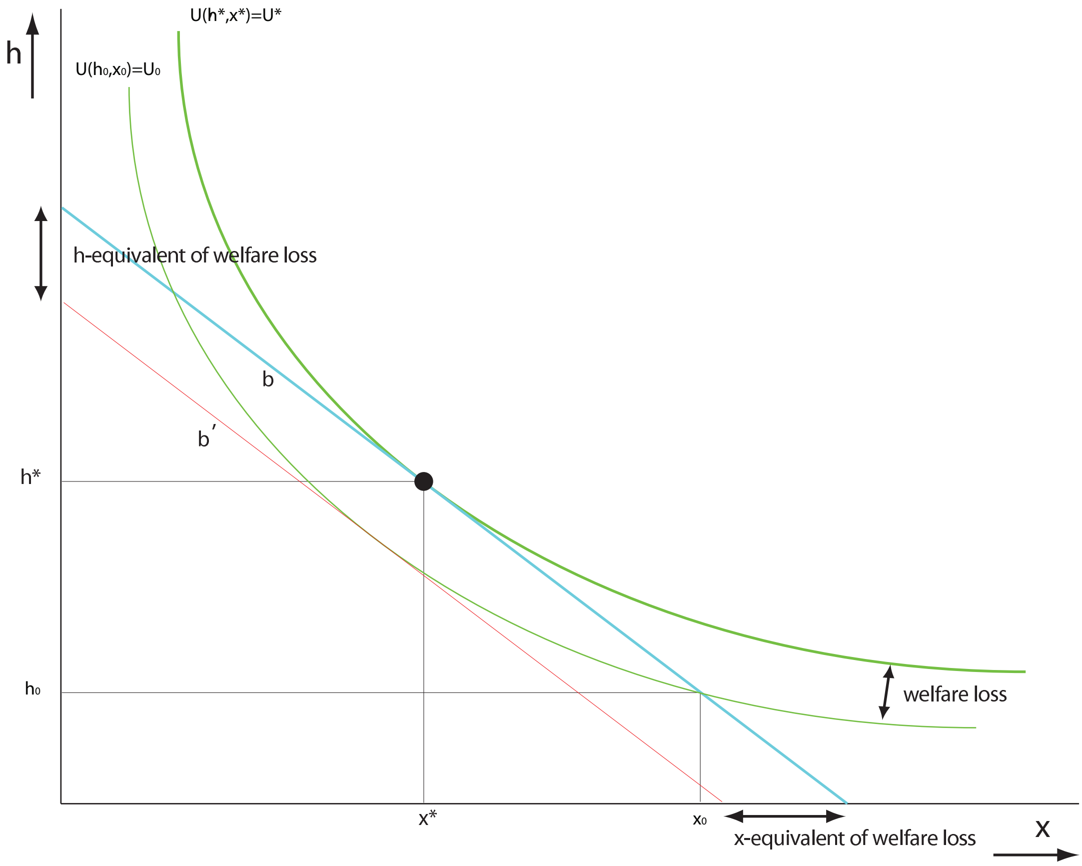

Figure 2.1: Method of calculating the loss of welfare

subsection, we will present a method to measure this loss of welfare. This is illustrated in figure 2.1. Assume that for a certain supra minimal income $\hat{b} = b - \bar{h}p_h$ and a utility function as given in equation 2.2, the utility maximizing consumption of housing services and other goods and services is given by h^* and x^*. Now, the actual consumption i.e. h_0, x_0 differs from the optimum. Restricted by its budget, the household has a lower utility. Its loss of utility is given by $U^* - U$ with $U^* = U^*(h^*, x^*)$ and $U_0 = U_0(h_0, x_0)$. If the household could choose its consumption of housing services freely he could reach utility U_0 with a lower budget $\hat{b}'$. The difference between $\hat{b}'$ and $\hat{b}$ is the income equivalent of the welfare loss. The income equivalent answers the question how much income the household is willing to maximally sacrifice in exchange for the opportunity to choose its consumption optimally. This is given by minimizing the budget for a given utility $(\underbrace{h - \bar{h}}_{\hat{h}})^{\delta} x^{1-\delta}$. Measured in this way, the welfare loss is given by the following relation:

$$
\begin{aligned}
b' - b &= p'_c \left(U_0 - U^*\right), \text{ where } p'_c = \left(\frac{p_h}{\delta}\right)^{\delta} \left(\frac{p_x}{1-\delta}\right)^{1-\delta}, \\
p'_c U &= \left(\frac{p_h(h - \bar{h})}{\delta}\right)^{\delta} \left(\frac{b - hp_h}{1-\delta}\right)^{1-\delta}.
\end{aligned}
$$

Details about the derivation can be found in appendix C. Thus we see that the income equivalent of the welfare loss corresponds to the loss of utility (as expected). The constant of proportionality is the aggregate consumption price p'_c which also equals the inverse of the marginal utility of income[4] i.e. how much extra utility does an extra euro of income buy.

2.2 Dynamic model of moving behaviour

In the previous section we saw that if some households do not adjust their housing services consumption to changing circumstances, the resulting sub-optimality of their consumption pattern results in a welfare loss to the household. These changing circumstances are the result of e.g. change of disposable income, change of the size of the household etc. We assume that a household has knowledge about his current circumstances and has an expectation, but *no* certainty, about its future circumstances. A household will thus want to minimize its welfare loss for example by moving to another house that better fits its circumstances. However, moving entails lump sum (or 'fixed') moving costs. Because of this, a household will not move every time when its circumstances change. To derive a decision rule for moving we use a model of moving behavior as proposed by Romijn (2000). We consider the desired consumption of housing services for a household in time. The evolution of desired consumption of housing services can be driven by many factors. Households can grow or shrink (e.g. the birth of a child or a divorce), the disposable income of the household can change, etcetera. For simplicity, this is modelled by a non-negative random process denoted with the non-negative random variable $Z^d(t), t \in [0, \infty)$. This random process is meant to depict the evolution of $h^*(t)$ as described in the previous paragraph. In this report we use the notation of h, h^* to describe and model static or cross sectional values. For the dynamic model, we use the notation Z, Z^d. The dynamics of the natural logarithm of $Z^d(t)$ is described with a standard $(-\mu, \sigma)$ Brownian motion, i.e.:

$$dz^d(t) = -\mu dt + \sigma dw(t), \ z^d(t) = \ln Z^d(t), \ t \geq 0. \tag{2.4}$$

which has solution:

$$\begin{aligned} z^d(t) &= z^d(0) - \mu t + \sigma w(t) &(2.5)\\ &\sim \mathcal{N}(z^d(0) - \mu t, \sigma^2 t) &(2.6)\end{aligned}$$

Note that $Z^d(t)$ is called a geometric Brownian motion. The birth and death of households is for reasons of convenience left out of the model. Households are supposed to live infinitely long. Or one could look at households as dynasties (with the assumptions that they don't split or die, however they can become infinitelly small). The natural logarithm of the actual consumption of housing services $Z(t)$ is denoted by $z(t) = \log Z(t)$. Because of moving costs, households may not always consume $Z^d(t)$ but $Z(t)$ instead. This entails a welfare loss which is modelled to be quadratic in deviation[5], i.e. $1/2 \left[z(t) - z^d(t)\right]^2$. This can be viewed as the result of a linear quadratic approximation of the welfare loss in section 2.1.1. We will elaborate on this relation in section 2.5. A household can adapt its actual consumption to its desired consumption by moving from one house to another. In our model, we consider this as a change in its consumption of housing services which can be chosen by the household. When the household moves it pays a fixed moving costs $\gamma > 0$. If the agent continuously adapts his actual consumption, he would pay infinitely high moving costs. Therefore, he moves only at certain moments in time, the so called stopping times T_n, $n \in \{0, 1, ...\}$, $0 = T_0 < T_1 < ... \to \infty$.

A change in consumption of housing services at a stopping time T_n is denoted by ξ_n. Since $T_0 = 0$, it must be possible that $\xi_0 = 0$. Now the agent has to find an optimal policy of stopping times and jumps. The cost function of moving is denoted by:

$$\phi(\xi) = \begin{cases} 0 & \text{for}\, \xi = 0 \\ \gamma & \text{for}\, \xi \neq 0 \end{cases} \tag{2.7}$$

[4]In economic literature, the marginal A of B means the partial derivative $\frac{\partial A}{\partial B}$ which is more common in engineering literature.

[5]In subsection 2.1.1 we derived a function for loss of utility for a household. Romijn (2000) used a quadratic cost function. One could see this as the Taylor Expansion of the Cobb-Douglas function. We'll stick to this quadratic approximation for a convenient solution. See subsection 2.5 for how we linked these two models.

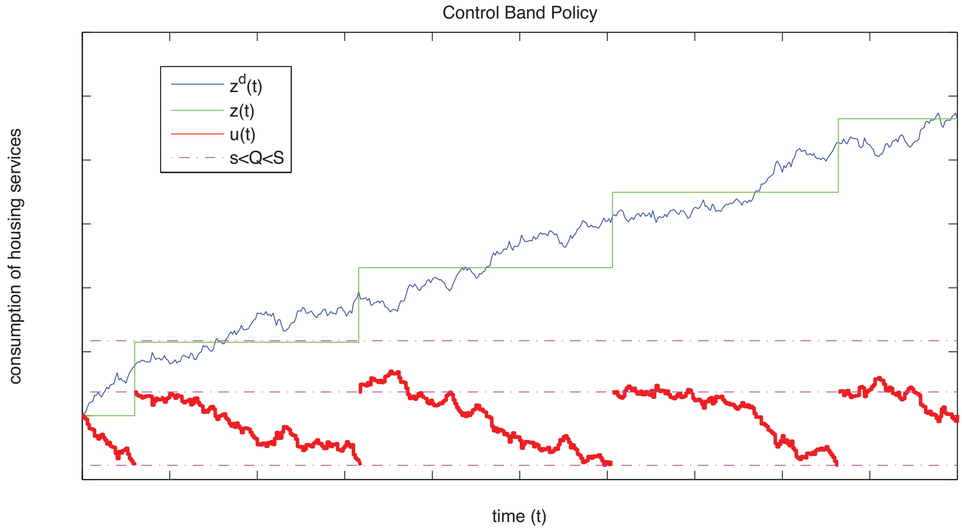

Figure 2.2: A control band policy for the optimal moving problem. Shown is the desired consumption $z^d(t)$ and the actual consumption $z(t)$. Now the process $u(t) = z(t) - z^d(t) = x(t) + y(t)$ is controlled by the control band characterised by two barriers s and S and an optimal consumption Q.

Next, define $N(t) = \max\{n \geq 0 : T_n \leq t\}$ and $y(t) = \xi_0 + ... + \xi_{N(t)}$ as the accumulated change in the consumption of housing services from time zero to time t. Furthermore, define $x(t) = -z^d(t) + z(0)$ a Brownian motion with drift μ, variance σ^2 and starting state $x(0) = x$, and $u(t) = z(t) - z^d(t)$ the gap between actual and desired consumption. The latter process follows $u(t) = x(t) + y(t)$. Then the jump is $\xi_n = u(T_n) - u(T_{n^-})$. If the household doesn't take any action, i.e. doesn't move, the expected value of loss of utility will be $1/2 x(t)^2$ and will grow to infinity. Hence the household will adapt its consumption by ξ_n and pay γ at certain stopping times. When we discount the costs by interest rate r, the expected total costs up to $t = \infty$ from starting value $x = x(0)$ is given by the cost function:

$$C(x) = \mathbb{E}\left\{\int_0^\infty 1/2 u(t)^2 e^{-rt} dt + \sum_{n=1}^{\infty} \phi(\xi_n) e^{-rT_n}\right\} \tag{2.8}$$

for a policy of stopping times and jumps $\{(T_n, \xi_n)\}$. This policy is the control method of the household. The household is assumed to behave rationally and therefore wants to choose its policy in such a way, that the cost function is minimized. In figure 2.2 we see an example of the desired consumption of a household and how he optimally adapts his actual consumption by a policy $\{(T_n, \xi_n)\}$ for a certain μ, σ, γ, r.

2.3 The optimal policy

We assume that a household chooses a policy of stopping times and jumps in consumption at these stopping times $\{(T_n, \xi_n)\}$ that is optimal if it minimizes the cost function over all feasible policies. The policy is characterized by three numbers: the optimal level Q, and the lower and upper barrier s and S and is therefore referred to as a 'control band policy'. When $u(T_{n^-})$ hits one of the barriers, the household will immediately change its consumption by paying moving costs γ and moving. For $n \geq 1$ the jump in u_t is given by:

$$\xi_n = \begin{cases} Q - s > 0 & \text{if } u(T_{n^-}) \to s \\ Q - S < 0 & \text{if } u(T_{n^-}) \to S \end{cases} \tag{2.9}$$

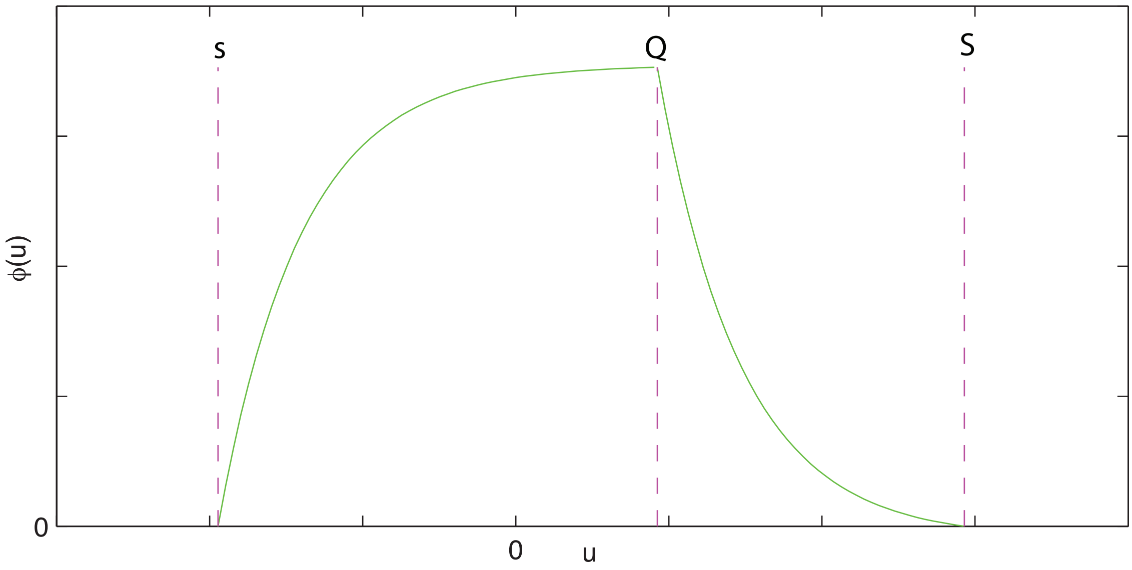

Figure 2.3: Theoretical distribution, with $\theta > 0$. The dashed vertical lines show the control band $s < Q < S$

and for time zero we have to define:

$$\xi_0 = \begin{cases} 0 & \text{if } s < x < S \\ Q - x & \text{otherwise} \end{cases} \tag{2.10}$$

The cost function is given by equation 2.8. In the appendix, subsection D, it is shown how the Bellman equation is used to derive a set of decision rules for moving, characterised by s, S, Q such that the cost function is minimized for a given drift μ, a volatility (uncertainty) σ, the lump sum moving costs γ, and a discount rate r.

2.4 Distribution

From the introduction we know the distribution of the gap between actual and desired consumption in 2005 as obtained from the database WoON2006. We will use this cross section to estimate the parameters of the moving behaviour model. Therefore, we need the theoretical density function of gap between actual and desired consumption, in order to compare it with the data. For the parameter estimation we are interested in the the probability density function of the gap. It is given by:

$$\phi(u) = \begin{cases} A_1 \left[e^{\theta u} - e^{\theta s} \right], & s < u < Q \\ A_2 \left[e^{\theta u} - e^{\theta S} \right], & Q < u < S \end{cases} \tag{2.11}$$

with the ratio between A_1 and A_2:

$$\frac{A_1}{A_2} = \frac{e^{\theta Q} - e^{\theta S}}{e^{\theta Q} - e^{\theta s}} = c < 0$$

and:

$$A_2 = -\frac{1}{c e^{\theta s}(Q - s) + e^{\theta S}(S - Q)}.$$

Details about the derivation of this distribution are given in appendix E. An example of the distribution is shown in figure 2.3 . This distribution is in line with our intuition. The Brownian motion between the barriers has a negative drift hence the large mass of the p.d.f to the left of Q. Furthermore, it is unlikely to have large probability density at s and S since $u(t)$ will jump to Q immediately and the values at the barrier indeed go to zero.

2.4.1 Derivation of the distribution with noise

Above, we assumed there is no noise in the determination of h^*. However, h^* is derived within a relatively simple model and it is likely that different individual circumstances imply that actual desired housing services consumption differs from our model's prediction. Reasons can be manifold and include heterogeneity in minimal housing service consumption $\bar{h}$, heterogeneity in housing preference parameter δ, or even a mismatch between observed income and permanent income as perceived by the household due to incidentally lower or higher earnings.

In addition, even if we could construct a completely accurate description of desired housing services consumption for everybody, people may differ in their propensity to move for a given gap between actual and desired housing services consumption.

Both measurement error in desired housing services consumption and heterogeneity in moving propensity imply that the gap between actual housing services consumption and estimated desired housing services consumption is not purely distributed as shown in figure 2.3, but also contains a noise component. Therefore, we add an independent random variable $\varepsilon \sim \mathcal{N}(\mu_\varepsilon, \sigma_\varepsilon)$ such that our measurement becomes $\ln \frac{h}{h^*} + \varepsilon$. The random variable $\varepsilon(t)$ is assumed to be Gaussian white noise. Gaussian white noise is normally distributed with probability density function:

$$f_\varepsilon(\varepsilon) = \frac{1}{\sqrt{2\pi\sigma_\varepsilon^2}} e^{-\frac{(\varepsilon-\mu_\varepsilon)^2}{2\sigma_\varepsilon^2}}.$$

We assume that we observe the sum of the *independent* processes $u(t)$ and the noise $\varepsilon(t)$. Given the distributions of those variables, we are looking for the distribution of the sum of this process i.e. the observed gap. To show this, consider the cumulative density of $u(t) + \varepsilon(t)$ at x:

$$\hat{u}(t) = u(t) + \varepsilon(t)$$

Then the density function $f_{\hat{u}}(\hat{u})$ is given by the convolution of the density functions $\phi(u)$ and $f_\varepsilon(\varepsilon)$:

$$\begin{aligned} \mathbb{P}(u + \varepsilon \leq x) &= \int_{-\infty}^{+\infty} \mathbb{P}(u \leq x - y) f_\varepsilon(y) dy \\ &= \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{x-y} \phi(z) dz \right] f_\varepsilon(y) dy \end{aligned}$$

The probability density function is obtained by differentiating the cumulative density function w.r.t. x:

$$\frac{\partial}{\partial(x-y)} \left[\int_{-\infty}^{+\infty} \left[\int_{-\infty}^{x-y} \phi(z) dz \right] f_\varepsilon(y) dy \right] = \int_{-\infty}^{+\infty} \phi(x-y) f_\varepsilon(y) dy \,\text{(convolution)}$$

The latter equation gives the probability density function of the gap with noise. Further details about the derivation are in appendix E.

2.5 Link between the moving model and the utility model

To link the dynamic moving model with Donders et al. (2010) we need to define the technical relationship. First of all, we consider U as a *flow* of utility. It gives the household U utility per year. We consider a desired consumption of housing services $Z^d(t)$. This is the result of the utility optimization at a certain time instance hence, $Z^d(t) = h^*$. In our model we considered the logarithm of the desired consumption $z^d(t) = \ln Z^d(t)$. The actual consumption is denoted by $Z(t)$ and the logarithm $z(t) = \ln Z(t)$. Now we perform a coordinate change $h = Z(t) = e^{z(t)}$ and substitute the consumption of other goods by the budget constraint $x = \frac{b - hp_h}{p_x}$, which is just equation 2.1 rewritten with x explicitly. Now we can write the loss of utility $L(z(t), z^d(t))$ for an imposed $z(t)$ as:

$$L(z, z^d) = U(z) - U(z^d) \tag{2.12}$$

And we take the Taylor expansion of $U(z)$ around $z^d(t)$:

$$L = \left[U(z^d) + \underbrace{U'(z^d)}_{=0}(z - z^d) + \frac{1}{2}U''(z^d)(z - z^d)^2 + \mathcal{O}^3 \right] - U(z^d) \tag{2.13}$$

$$= \frac{1}{2}U''(z^d)(z - z^d)^2 + \mathcal{O}^3 \tag{2.14}$$

In equation 2.8, we defined the losses of utility that a households suffers from the moving problem. This consist of the accumulated moving costs, and a quadratic approximation of the loss of utility. This loss of utility was given by $\frac{1}{2}u(t)^2$, with $u(t) = z(t) - z^d(t)$. When we omit all terms of order 3 and multiply it with a factor $U''(z^d)$ we arrive at equation 2.13.

From the subsection 2.1.1 we know that the income equivalent of a loss of utility is given by $p'_c(U - U^*) = p'_c L(z - z^d) = \frac{1}{2}p'_c U''(z^d)u(t)^2$. In the cost function equation 2.8 we can easily connect the models by setting:

$$\frac{1}{2}u(t)^2 = \frac{L(z - z^d)}{U''(z^d)}. \tag{2.15}$$

The problem we face now is that we have $\frac{1}{U''(z^d)}$ within the integral of the cost function, equation 2.8. Since the moving costs γ are in the same unit of measurement as the cost function and not in euros, as we would like it, it would be a lot more convenient if we could treat this factor as a constant and therefore place it outside the integral. Then we can just multiply γ with this factor to get the moving costs in euros, instead of 'utility'. The price of other goods and services p_x is unknown, but it is not relevant for our problem. To get the price of utility loss and the price of moving in euros, we have to multiply $U''(z^d)$ with the factor p'_c. In this multiplication, p_x will disappear. Most households have an h^* between 0.5 and 1.5. This results in a second derivative that has numbers between $p'_c \cdot U''(z^d = \ln 0.5) = -2.4 \cdot 10^4$ and $p'_c \cdot U''(z^d = \ln 1.5) = -2.7 \cdot 10^4$. For convenience we choose the number constant at $p'_c \cdot U''(z^d = \ln 1) = -2.1 \cdot 10^4$. Hence the value is evaluated for a household consuming one housing service per year, which is also the average. Since we can treat $U''(z^d)$ as a constant the formula for the price of moving costs becomes:

$$\gamma_{\text{euro}} = p'_c U''(z^d)\gamma. \tag{2.16}$$

Here we used the calibrated values from the table of stylized facts in appendix B. Now we have a full characterization of the welfare loss of moving costs in euros.

Chapter 3

Parameter Estimation

In this section, we aim to estimate the parameters of our model of moving behaviour for the Dutch owner occupied housing market. We have two sets of data, namely the gap between actual and desired consumption and the year the household moved into its current house. First we will estimate the parameters with the frequencies of gaps in the data using the theoretical probability density function of the control band policy. We will see that it is difficult to identify a set of parameters that is clearly optimal. The reason is that the model is too simple to adequately describe the data. To remedy this, we extend the model to allow noise in the estimation of optimal housing consumption. This results in a more realistic range of parameters, but not a clear optimum. We can calculate an estimation of the average moving time of a household. If we fix this moving time, we can find parameters to do policy analysis.

We have certain expectations from the outcomes of the parameters, based on literature such as Ekamper and Van Huis (2002) and common sense. The transfer tax is 6% so with search, brokerage and other moving costs, our intuition tells us to find moving costs between 6% and 20%. Furthermore, the data on the year that the household moved into its current home tell us that households on average move once in 17 years. Finally, we expect that the jumps in housing services consumption when people move are not bigger than factor two.

3.1 Residence time of households

From our data we can derive the time that people have lived in their current house. The histogram of these data is shown in figure 1.2. We would like to use these data learn something about the average time between moves. In model terms: the time elapsing between 'stopping times', i.e. the points in time that a household 'hits' one of the boundaries of the control band policy. One way to get an estimate of the expected time of occupation is using the number of moves in one year. Assuming that the probability that someone moves after one year is close to zero, we can count the number of people that moved the last year before the data set was constructed.

The data contain information about 3.6 million households. Furthermore, the data were based on questionnaires that were conducted during 2005 and the beginning of 2006. These two years are therefore not useful to make an estimate about the number of households moved in that year since many may have moved after the questionnaire. The number of households that last moved in 2002, 2003 or 2004 is about 210-220 thousand per year. For earlier years, the number of moves decreases, since households may have moved again after a move in those years. Therefore, we assume that every year, 220 thousand households move. According to this data, we assume that households, on average move every 17 year. This number is supported by CBS data with the amount of housing transactions. Ekamper and Van Huis (2002) state that people move every ten years, and households, that do not change in composition move every twenty years. However they consider rental and owner occupied data together hence it is impossible to find the the moving time of owner occupied households. However, we know that the moving time is between 10-20

#	Uncertainty σ	Moving costs γ	Moving costs (%) $\gamma_{\%}$	Expected moving time $\mathbb{E}T^*$	Residual norm or squared error
1	0.53	0.16	1.3%	2.6	0.3894
2	0.089	3.0	24%	89	0.3259

Table 3.1: Results of parameter estimation on the gap without noise.

from Ekamper and Van Huis (2002) hence the residence time calculated here is in line with our expectation. This will be used as a validation of the data.

3.2 Parameter Estimation using the distribution of the gap

The control band policy (s, S, Q) is determined by four parameters (μ, σ, γ, r). We have to find these four numbers such that the theoretical probability density of the gap gives the optimal fit with the observed distribution. Some parameters are considered to be known. We set the risk free interest rate at 2%[1] per annum. However, buying a house is not risk free, because the price of housing services is not certain in the future. Therefore, a risk premium of 3% is added which results in an interest rate of 5% per annum. Furthermore, the model in Donders et al. (2010) implies a steady state growth of the entire volume of housing services consumption of 0.65% per year. However, this 0.65% consists of an expected rise in housing services consumption of 0.33% per household and the remaining 0.32% is made up by the rise in the number of households. Since our model considers the individual expected growth, we will use a μ of -0.0033 which is the expected rise of housing services consumption for the individual households[2]. This leaves us with estimating a volatility of the desired consumption σ and the moving costs γ. We start with the parameter estimation by searching for the best fit with the theoretical p.d.f by means of Least Square Estimation (LSE) i.e. minimizing the squared distance between height of the histogram and the shape of the p.d.f.. The reason we use LSE instead of Maximum Likelihood estimation (MLE) is that the theoretical p.d.f is defined as zero outside the control band. Consequently, MLE will estimate the parameters such that s is equal to the minimum of the sample and S the maximum. For this reason, LSE will be used for this estimation. The fit of this distribution can be seen in figure 3.1. The figure shows that the theory appears to capture certain salient aspects of actual behaviour. As we shall see the estimates do not fit with our expectations. The reason for that can be that our model doesn't explain all possible underlying reasons for a gap. To account for other factors we will add a noise component as explained in section 2.4.1. We assume that this noise is normally distributed. We assume that the mean of this noise is $\mu_\varepsilon = 0$ but leave the variance σ_ε to be estimated. Here, we can use MLE to find our parameters.

3.2.1 Parameter estimation without noise

The p.d.f without noise has value zero outside the control band. We use least square curve fitting to find the σ, and γ. The curve of the p.d.f is fitted on a histogram of the data. As mentioned, the data are weighted in such a way that the data set of 60.000 households is representative for all households in The Netherlands. We divided the histogram in 100 bars. We minimize the squared distance between the height of the bars and the value of the distribution function. The curve fitting is done by Matlab. This problem is non-convex, hence we have to use a multistart algorithm to find the global minimum. We find two local minima, depending on our starting values. The results are given in table 3.1 and figure 3.1. To test the 'goodness of fit', we use the Kolmogorov-Smirnov

[1]This follows the advice of the "Werkgroep actualisatie discontovoet" and the "Commissie risicowaardering" from 2007. Their advice was a risk free interest rate of 2-3% per annum. To follow the capital markets, the first commission advised to use the lower side of the bandwidth.

[2]The WLO scenario Transatlantic Markets (TM) is the basis under the steady state growth of Donders et al. (2010). There is a growth of 0.3174% per year of the number of households. The real income of a household grows with 1.3249% per year. The real growth in price of housing services grows with 0.9953% per year. Together this

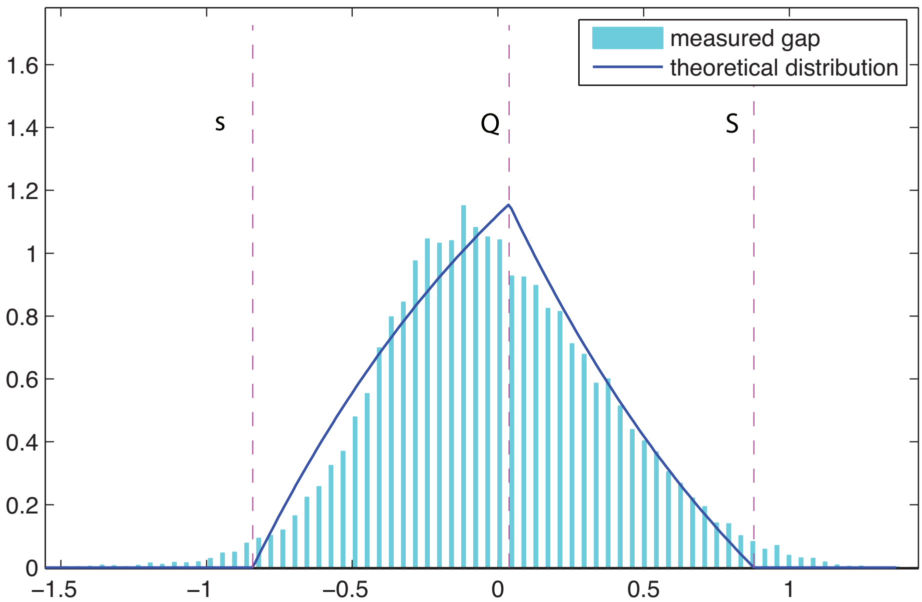

Figure 3.1: Least square curve fitting (#2 of table 3.1) of the distribution without noise. $\sigma = 0.09, \gamma = 3.0$

test. This test rejects the hypothesis that the data comes from this distribution at a level of 5%[3]. The results also do not correspond with our intuition since the moving costs γ of the first estimation is rather low. We would expect them to be in a $10\% - 20\%$ range. The estimates of the moving time T^* are respectively higher and lower than we expect. The second estimation has the lowest residual norm[4] but is very unrealistic. For this reason, we leave this p.d.f. and go on with parameter estimation with a p.d.f. with a model that includes noise.

3.2.2 Parameter estimation with noise

Since the distribution with noise allows for observations outside the control band, we can use maximum likelihood estimation. Also, we have an other degree of freedom in our MLE i.e. the variance of the noise. Estimation of the parameters is difficult because different starting values for the optimization procedure give different outcomes. These outcomes are all plausible with moving costs between 5% and 17%, and the moving times between 10 and 42 years as shown in table 3.2 and figure 3.2. However there is not a clear optimum. This list could have been made very long, because there are many local optima. This table shows some examples. The results are note satisfying for several reasons. First of all, there are too many optima that differ too much. The choice of the optimum can therefore not be made. Furthermore, there is no combination of values that gives a realistic set. A combination of moving time and moving costs that satisfies our expectations such as estimation #17 results in jumps to a three times bigger house when moving, which we think is not realistic.

gives the real growth in consumption of housing services per year of 0.326% per year.

[3]Because we have quite a large set of data, 'goodness of fit' test demand an almost perfect fit with the distribution. The p-value in this case was $p \to 0$.

[4]The residual norm is the value of the squared 2-norm of the residual at the set of parameters: $\sum [\text{'p.d.f. function'}([\sigma, \gamma], \text{UDATA}) - \text{FDATA}]^2$ where UDATA are the range of gaps and FDATA the associated frequencies from the data.

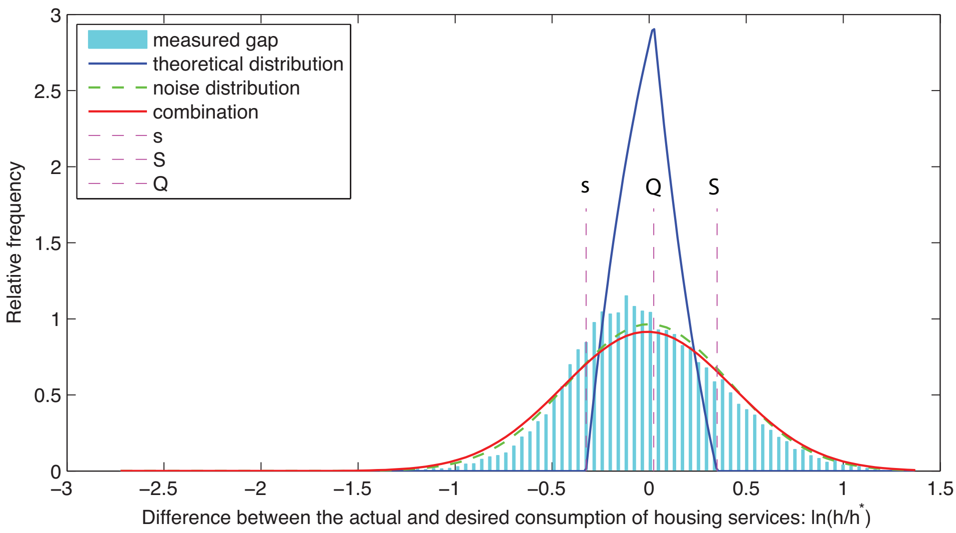

Figure 3.2: p.d.f.'s with parameter estimation of the gap (#1, highest likelihood) with noise

#	Uncertainty σ	Moving costs γ	Noise σ_ε	Moving costs (%)	Expected moving time $\mathbb{E}T^*$	Likelihood	Control band e^s	e^S	e^Q
1	0.050	0.30	0.41	2.3%	44 years	-16,144	0.7	1.4	1.0
2	0.064	0.48	0.40	3.8%	44 years	-16,204	0.7	1.6	1.0
3	0.044	0.69	0.39	5.4%	80 years	-16,260	0.7	1.6	1.0
4	0.13	0.35	0.36	2.7%	17 years	-16,297	0.6	1.7	1.0
5	0.096	0.32	0.35	2.4%	22 years	-16,311	0.6	1.5	1.0
6	0.14	0.33	0.39	2.6%	15 years	-16,540	0.6	1.7	1.0
7	0.16	0.36	0.37	2.8%	14 years	-16,415	0.6	1.8	1.0
8	0.058	1.48	0.37	12%	96 years	-16,443	0.6	1.8	1.0
9	0.061	1.45	0.36	11%	89 years	-16,475	0.6	1.8	1.0
10	0.16	0.36	0.38	2.8%	14 years	-16,706	0.6	1.8	1.0
11	0.094	1.04	0.33	8.2%	45 years	-16,709	0.5	1.9	1.0
12	0.15	0.64	0.39	5.0%	20 years	-17,014	0.5	2.0	1.0
13	0.19	0.95	0.33	7.5%	20 years	-17,373	0.4	2.3	1.0
14	0.09	2.03	0.26	16%	69 years	-17,746	0.5	2.2	1.0
15	0.18	1.05	0.42	8.2%	22 years	-18,181	0.4	2.3	1.0
16	0.20	1.67	0.34	13%	25 years	-18,676	0.4	2.7	1.0
17	0.28	1.7	0.36	13%	17 years	-20,312	0.3	3.2	1.0
...									

Table 3.2: Parameter estimations of the gap using the distribution with noise for some starting values. Also the size of the control band is given, where s and S are the lower and upper barrier respectively, and Q is the optimal level. The exponential is taken such that it gives the ratio $\frac{h}{h^*}$

	Moving costs γ	Moving costs (%)	Uncertainty σ	Noise σ_ε	Likelihood	Control band e^s	e^S	e^Q
1	0.38	3%	0.135	0.36	-16,492	0.6	1.8	1.0
2	0.76	6%	0.192	0.31	-17,524	0.4	2.3	1.0
3	1.27	10%	0.248	0.30	-19,023	0.4	2.8	1.0
4	1.90	15%	0.303	0.31	-20,820	0.3	3.5	1.0
5	2.53	20%	0.350	0.33	-22,895	0.2	4.3	1.0

Table 3.3: Results with fixed moving time T^* at 17 years for several given moving costs.

3.2.3 More restrictions

The problem is highly non convex hence the optimum has not been found. Therefore, we use another stylized fact, obtained from the data with the times that people moved last time. To do this we add the restriction that the expected moving time T^* is 17 years. It was not possible to add this in the optimization loop. Therefore we searched for amount of volatility in the desired consumption σ at several given moving costs. We chose for a range from 3% to 20%. This is a purely algebraic operation. Then we searched for an optimal fit with the data, with σ, γ restricted. The only free variable in this case is then the noise σ_ε. The likelihood of the estimations is given. As already noted in the previous paragraph, there is no combination of parameters that give realistic moving costs, moving times *and* moving jumps. The moving jumps are determined by the size of the control band which tends to be very wide at higher moving costs and moving time of 17 years. In each case that we try to obtain a plasuible set of moving time, moving jumps and moving costs, one of those three is not according to our expectations. In table 3.3 we observe unrealistic jumps from moving costs of 10% and higher. The model is apparently too simple to find a clearly optimal set of parameters that yield plausible results. However, the results in terms of moving costs, moving time and moving jump are within a reasonable range. Further refinements to the model need to be made to yield more plausible results, but this falls outside the scope of the paper. Chapter 5 discusses shortcomings and refinements to the model. To reflect the uncertainties surrounding the model we will not settle for a single estimate, but consider a range of estimates. This range is reported in table 3 with moving costs γ set at 3%, 6%, 10%, 15%, 20%.

Chapter 4

Policy Analysis

The goal of this chapter is to use the model to analyse the welfare effects of the transfer tax. In the previous sections we developed a dynamic model for how households choose their optimal time of moving given their moving costs. We will now use this model to estimate the welfare loss associated with moving costs. First we will illustrate the method and calculate the total welfare loss of moving costs in general. In the second section, we will calculate the welfare gain if we adjust the transfer tax.

4.1 Calculation of the welfare loss

In this section we calculate the welfare loss of the moving costs. The welfare loss is expressed in terms of the cost function, equation 2.8. When a household could continuously adapt its consumption without any costs i.e. the moving costs would be zero, then the gap between actual and optimal living would be zero and so the value of the cost function. The total welfare loss of the Netherlands is the sum of all the evaluations of costs functions for all households in the Netherlands. This means that welfare loss consists of utility loss because of suboptimal living, as well as utility loss of paying moving costs. Also the cost function gives the net present value of the costs till infinite. We are interested in the costs per year and therefore amortize[1] the cost function. We can evaluate the cost function, equation 2.8, for all households in WoON 2006 with parameters μ, σ, r, γ and as a starting point the gap in 2005. But then we attribute a cost to values of the gap which might be caused by other factors which we lumped together in the noise component. For example, in the data we observe gaps where the logarithm of the gap was smaller than -2.5, but with none of the estimated sets of parameters, we arrive at a control band that allows these values. We assume that this gap is at least partly the result of other factors than the moving costs, and therefore, it should not be included in evolution of the cost function. To avoid this, we will evaluate the amortized cost function $rC(x)$ over the p.d.f. of the control band and scale it such that it represents all the owner occupied households in the Netherlands. Summarized, we don't want to use the data to evaluate the welfare loss because could assign a welfare loss to gaps that are corrupted by noise. Therefore, we use the model to calculate the welfare loss. In the Netherlands there are 3,644,000 home owners in 2005. We have to solve the following integral and multiply it with the total number of households to get the total welfare loss in the Netherlands:

$$W_{loss} = p_c' \cdot U''(Z^d = 1) \cdot \int_{-\infty}^{+\infty} rC(x)\phi(x)dx. \tag{4.1}$$

This integral is integrated over all possible gaps x from $-\infty$ to $+\infty$ and $\phi(x)$ the p.d.f. of the gap *without* noise. We also looked at the possibility of an U'' which varies over the gap, but it

[1] If A is the annual payment and P the net present value with n periods with interest rate i, we can write if $n \to \infty$ $P = \frac{A}{(1+i)} + \frac{A}{(1+i)^2} + \frac{A}{(1+i)^3} ...$ which can be written as $P = \frac{A}{1+i}\left(1 + \frac{1}{(1+i)} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} ...\right) = \frac{A}{1+i}\left(\frac{1+i}{i}\right) = \frac{A}{i}$. Given the net present value, we can amortize this net present value to the annual payment with the formula $A = iP$.

results in errors for the wider gaps. The reason is that at wider gaps, there is a possibility for the demand for a home which is smaller that the minimum $\bar{h}$. This integral gives the expected costs of *one* household whose gap is unknown. Multiplied with the total number of households it gives the total welfare loss in the Netherlands. We will solve this integral numerically.

4.2 Policy analysis

In this section we apply our model to examine the effects on moving behaviour and welfare of a reduction in the transfer tax rate. Recently, the rate has been temporarily lowered from 6% to 2%. We examine what the effects are of such a tax rate reduction, but we limit ourselves to a structural (i.e. permanent) reduction. In addition, we examine the effects of abolishing the transfer tax completely. As mentioned before, we will work with three sets of parameters. Then, assuming that the moving costs in 2005 were 10%, 15% or 20%, the model calculates that this leads to a certain behaviour of the household. The behaviour of the household is characterized by the μ and the σ. Since the μ was taken fixed, different initial moving costs lead to three different σ's. When we do policy experiments, we need to do them three times, using the three different types of behaviour characterized by the σ. The results are given in table 4.1, which also show new average times between moves (i.e. the time an average household spends in a particular house. The results show that the welfare loss due to transfer tax is around 1 billion, assuming that the initial moving costs are larger than just the transfer tax, and the number of moves increases between 60% and 20%. These results should be interpreted cautiously as the above model parameters do not result in uniformly acceptable behavioural implications. This may also distort the magnitude of welfare effects and reported effects in moving behaviour.

It would be convenient if we could approximate the welfare costs of moving cost by a simpler formula, than running the model. We therefore plot the welfare loss on several percentages of the total moving costs. This looks very much like a root function. We get a good fit with the following formula:

$$W_{loss,total} = c\sqrt{\gamma_{\%}} \cdot 10^9$$

where c is some constant. These functions are plotted in figure 4.1 . The figure shows the non linearity in the model. The lower the initial moving costs, the higher the effects of transfer tax reduction. However, in the range of 0% $\rightarrow$ 3% the results are fairly close to each other. Also from table 4.1 it can be seen that the welfare gains with the policy experiments are very similar although the resulting moving behaviour is different. This means that using the behaviour of the 10% scenario we get the almost the same welfare gains with reduction in transfer tax, but bigger differences in moving time than using the 20% scenario.

Initial Moving Costs ⇒	2%	3%	6%	10%	15%	20%
Welfare loss	2.5 billion	3.7 billion	7.5 billion	12 billion	19 billion	25 billion
Tax revenue	3.4 billion	3.4 billion	3.4 billion	3.4 billion	3.4 billion	3.4 billion
Case -1% points						
-Net welfare gain, of which:	+0.27 billion	+0.12 billion	+0.09	+0.07	+0.07	+0.07
>Welfare gain households	+0.74 billion	+0.69 billion	+0.66 billion	+0.64 billion	+0.64 billion	+0.64 billion
>Change in tax revenue	-0.57 billion	-0.57 billion	-0.57 billion	-0.57 billion	-0.57 billion	-0.57 billion
-Moving time	12 years	14 years	15 years	16 years	16 years	17 years
-Number of moves	+46%	+24%	+10%	+6%	+4%	+2.8%
Case -4% points						
-Net welfare gain, of which:			+0.9 billion	+0.5 billion	+0.4 billion	+0.3 billion
>Welfare gain households			+3.2 billion	+2.8 billion	+2.7 billion	+2.6 billion
>Change in tax revenue			-2.3 billion	-2.3 billion	-2.3 billion	-2.3 billion
-Moving time			9 years	13 years	14 years	15 years
-Number of moves			+79%	+31%	+18%	+13%
Case -6% points						
-Net welfare gain, of which:			+4.1 billion	+1.2 billion	+0.8 billion	+0.7 billion
>Welfare gain households			+7.5 billion	+4.6 billion	+4.2 billion	+4.1 billion
>Change in tax revenue loss			-3.4 billion	-3.4 billion	-3.4 billion	-3.4 billion
-Moving time				10 years	13 years	14 years
-Number of moves				+62%	+31%	+21%

Table 4.1: Welfare losses due to transfer tax, for five sets of parameters.
Here the moving time is fixed at 17 years and we calculated the behaviour, i.e. calculated the sigma at five different moving costs. For these sets, the total welfare losses to households are calculated and the consequences are evaluated of lowering the transfer tax by 1%, 4% and 6% points. The tax revenue is calculated with $6\% \cdot h_{avg} \cdot NHH/T^*$ and the change in tax revenue as the difference $\left(\gamma_{\%,new} - \gamma_{\%,old}\right) \cdot h_{avg} \cdot NHH/T^*_{old}$. Furthermore, when we have a tax revenue of 3.4 billion euro at 3% moving costs, we assume that the tax is compensated by moving benefits.

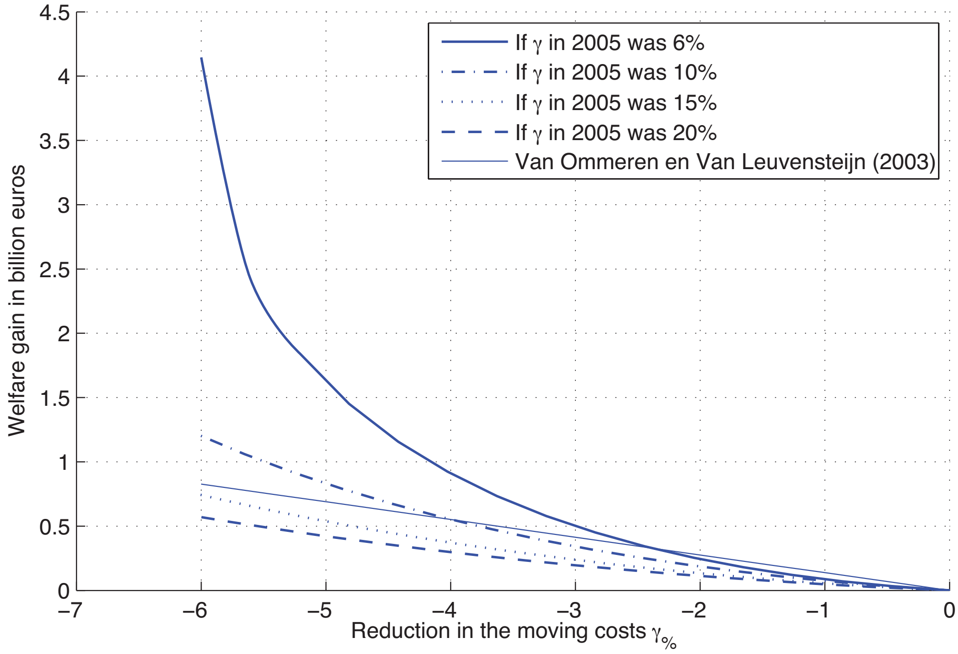

Figure 4.1: Plots of the approximated welfare loss. They show the welfare gains of reducing the moving costs in percent points by lowering the transfer tax. This is of couse only possible up to 6 percent points.

Chapter 5

Discussion

In this research, we used a model based on Harrison et al. (1983), Dixit and Pindyck (1994) and Romijn (2000) to analyse the effects of moving costs. The model is meant as an extension of Donders et al. (2010), that assumed optimal housing consumption of all households. We used the data, a cross section of the housing market in 2005, as well as the utility model of Donders et al. (2010). This was linked with the moving behaviour model of Romijn (2000). The parameter estimation not always produced plausible results, and generated a large bandwidth. We could find no parameter set that produced a plausible combination of moving time, moving costs and jumps. Always at least one of them has an unrealistic value. Another difficulty was to find data about the moving frequency. From WoON2006 we only have the time of the last move that a household has made. We found an estimate of the moving time to validate the results. This estimate was within the bandwidth of Ekamper and Van Huis (2002).

The welfare loss due to transfer tax is approximately 1 billion euro according to our model although this result has to be interpreted consciously since it is at a very wide control band. The only other estimate of the welfare loss is done by Van Ommeren and van Leuvensteijn (2003). They state that lowering the transfer tax by every 1% point will increase the number of moves by 8%. This is broadly in line with our analysis of a 1% point reduction in moving costs. Our model is however not linear, and if we consider the case of lowering the transfer tax by 4% points we observe an increase of the number of moves by 31% in the $\gamma = 10\%$ scenario and an increase of 18% in the $\gamma = 15\%$ scenario. In the scenario with an initial moving costs of 10%, this observation is very close to Van Ommeren and van Leuvensteijn (2005). Van Ommeren and van Leuvensteijn (2005) used this estimate to perform a dead weight lost analysis[1]. They consider a market for moves, with a fully elastic supply and a demand function for moves with an elasticity[2] of 8. Then they calculate a loss of welfare by a 6% transfer tax by $\frac{1}{2} \cdot 8 \cdot 6\% = 0.24$ euro per euro income out of transfer tax. With a revenue of 3.4 billion euro per year[3], the loss of welfare is estimated by 0.82 billion euro, which is close to some of our results. But, Van Ommeren and Van Leuvensteijn (2003) used a linear model, and our model is obviously not. We have a convex demand function, resulting in a lower dead weight loss. Our model describes non linear behaviour, were most welfare losses are caused by the first 2-3% points. This can be seen in figure 4.1 were the line of welfare gain becomes a lot steeper if we arrive at the last percentages that can be reduced. If we construct a demand curve for moves and calculate the dead weight loss, we come to a similar order of welfare losses. However, our model is non linear which gives higher differences in welfare loss when taking higher initial moving costs.

The real option theory is a working approach for modelling moving behaviour but there need to be further research. The model does not give a plausible combination of moving time, moving

[1]For an introduction into dead weight loss analysis, the reader is referred to e.g. Varian (1992)

[2]The elasticity is given by $\frac{(\Delta n/n)}{\tau_\gamma} = \frac{8\%}{1\%}$ where n is the number of moves and τ_γ the amount transfer tax

[3]Van Ommeren and Van Leuvensteijn (2003) considered a revenue of 3 billion euro per year. However, we calculated the revenue of transfer tax by multiplying the average value of a standard house, times 6%, times the number of owner occupied households divided by the average moving time of 17 years, which is 3.4 billion euro.

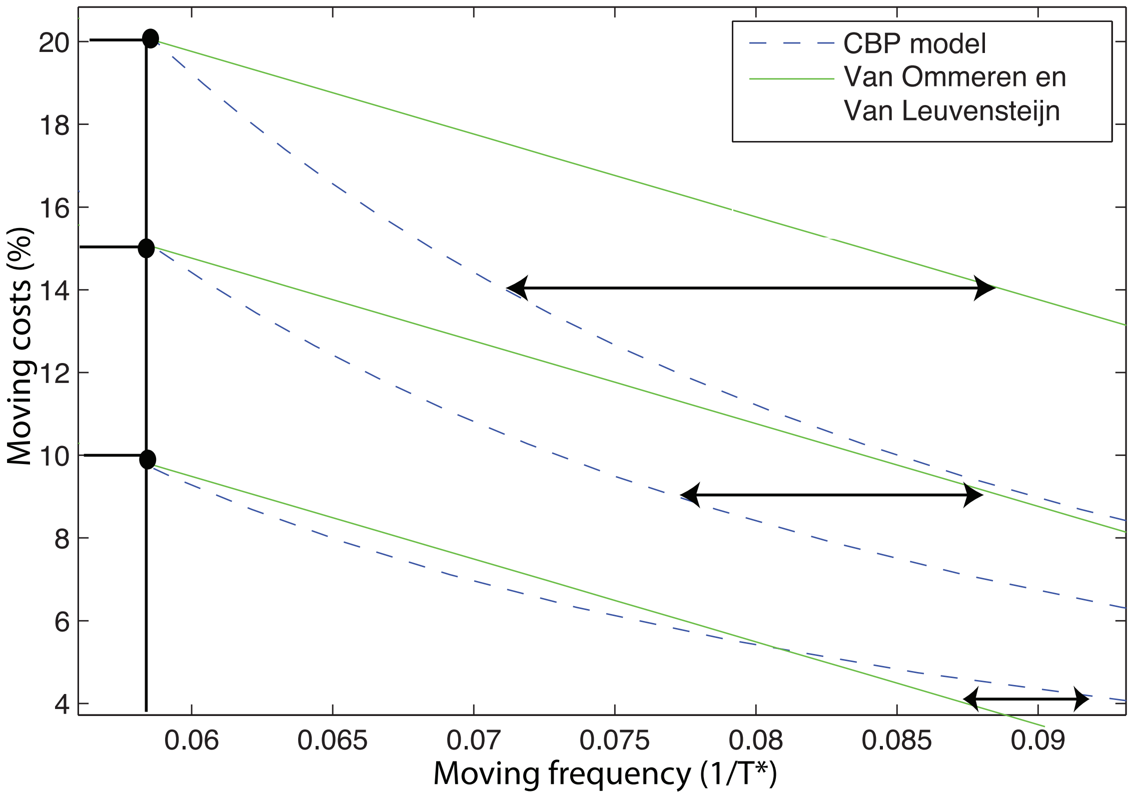

Figure 5.1: Demand curves of the control band policy model and Van Ommeren and Van Leuvensteijn (2003), calibrated at several initial moving costs ($\gamma = 10\%, 15\%$ and 20%). The arrows show the differences in moving times between both methods when removing 6% transfer tax. This is because of the non linearity in our model.

costs and jumps. The 3% moving costs with $T^* = 17$ gave a plausible width of the control band (and jumps) but it implies that households do not experience the transfer tax as a restraint from moving as we would expect the movings costs to be at least 6% but we leave this to further research. This could mean that the moving costs as tranfer tax are not experienced as 'out of pocket' but just as costs that make the house more expensive. Another topic for further research is to find the connection between our measure of loss of utility, and the method of dead weight loss analysis. For this reason, the outcomes of the model has to be considered as indicative.

The specification of the model however is rather simple and also the calibration produced a solution with a large bandwidth. Extensions could be made in the utility function. We assumed a constant preference δ in line with Donders et al. (2010). However, we observed people, on average, do not live in the 'right' house anymore at the higher disposable incomes. They tend to choose homes that are too small. Perhaps this could be the result of changing preferences at higher incomes. A more refined model of preference would result in a better model of choice between housing services and other goods and services. Furthermore, households move because they get offered a job in another city. The reason for moving is not observable in the gap between actual and desired consumption from the model, although the house in the new location yields a lot more utility. This would require extending the model by including location choice.

In this model, we only considered the owner occupied market. We ignored the possibility of entry by rental households or the exit to rental housing. The reason is that the rental market is rationed, and their is a big difference in price for the same amount of housing services. When one moves from the relatively cheap rental housing to owner occupied, the loss of subsidy implicit in the cheap rental rates can be seen as moving costs as well. An extension to the rental market could be a topic for future research as well.

The drift parameter μ was assumed to be constant. However, shocks in the housing market occur, such as the crisis in 2008. This was probably the reason of a decline in the number of moves after 2008. The current model is capable of estimating the long term moving behaviour under the new drift rate, but to model the behaviour during a shock in the drift rate, the model should be extended. Furthermore, different households can be thought of having a different drift rate.

Another extension that can be made is in the dynamic model of desired consumption of housing services. In our model, we considered households with infinite lifetime. From Ekamper and van Huis (2002) we know that 16% of the people face a change in the amount of persons in the household per year, and it gives reason to 50% of the moves. The birth and death of households is therefore very common. Therefore it could be an interesting approach to model the desired consumption of a person instead of households.

A last extension that could be made is related to the fact that our model is not possible to show the effects of shocks. The 1st of July, the Dutch administration temporarily lowered the transfer tax by 4% points for one year. How people will respond to temporal reduction in moving costs, could be an interesting suggestion for new research.

Nevertheless, with extensions this model will be able to evaluate the effects on moving behaviour of moving costs. This can be used to evaluate policies such as transfer tax and calculate the welfare effects.

Appendix A

Symbol Glossary

b disposable income

δ housing preference parameter

$\mathbb{E}$ expectation operator

γ lump sum moving costs

h consumption of housing services of an agent in WoON

h_{avg} average price of a house

h^* optimal consumption of housing services of an agent

$\bar{h}$ minimal consumption of housing services

NHH number of households

p_x price of other goods and services

Q optimal gap $u(t)$, considering uncertainty

r risk-free interest rate

s lower barrier of the policy

S upper barrier of the policy

t time

T_n stopping time

T^* Hitting time of the barrier, or time of occupancy before moving.

$U(h, x)$ utility function

$u(t)$ gap between the logarithm of the desired and actual housing services consumption

p_h price of housing services

p_x price of other goods and services

p_c' price of aggregate consumption or the inverse of the marginal utility of income

x starting value of u

or, the consumption of other goods and services

ξ_n jump in housing services consumption at stopping time

$Z^d(t)$ demand for housing services

$Z(t)$ actual housing services consumption

$z^d(t)$ natural logarithm of the demand for housing services

$z(t)$ natural logarithm of the demand for housing services

Appendix B

Values of parameters

From Donders et al. (2010) we have taken some values for parameters. These are listed here.

Variable	Value	Description
μ_1	0.0065	Steady state growth of housing market.
μ_2	0.0033	As μ_1, excluding growth in the number of households
r_1	2%	risk free interest rate
r_2	5%	risk bearing interest rate
δ	0.145	preference parameter
NHH	$3,644,000$	Number of owner occupied households
$U^{''}(z^d)_{1\text{ std. house}}$	$-2.1 \cdot 10^4$	Factor to convert utility into euros
h_{avg}	$268,094$	Average value of one housing service (WOZ waarde)
p_h	$11,643$	Price of owning one housing service per year
$\underline{h}$	0.3570754	minimal housing consumption for one adult household

Appendix C

The price of welfare loss

Here we give the derivation for the price of welfare loss. It is helpful to look at figure 2.1. As described in subsection 2.1.1, we assume that for a certain supraminimal income $\hat{b} = b - \bar{h}p_h$ and a utility function as given in equation 2.2, the utility maximizing consumption of housing services and other goods and services is given by h^* and x^*. Now, the actual consumption differs from the optimum i.e. h, x. Restricted by its budget, the household has a lower utility. Its loss of utility is given by $U^*(h^*, x^*) - U(h, x)$. If the household could choose its consumption of housing services freely he could reach utility $U(h, x)$ with a lower budget $\hat{b}'$. The difference between $\hat{b}'$ and $\hat{b}$ is the income equivalent of the welfare loss. This is given by minimizing the budget for a given utility $(\underbrace{h - \bar{h}}_{\hat{h}})^{\delta} x^{1-\delta}$.

$$\min_{h', x'} \hat{b}' = p_h \hat{h}' + p_x x' \tag{C.1}$$

$$\text{s.t.} \quad \hat{h}'^{\delta} x'^{1-\delta} = \hat{h}^{\delta} x^{1-\delta} \tag{C.2}$$

The Lagrangian and its partial derivatives, which should be zero at the minimum, are given by:

$$H(\hat{h}', x', \lambda) = p_h \hat{h}' + p_x x' + \lambda \left(\hat{h}^{\delta} x^{1-\delta} - \hat{h}'^{\delta} x'^{1-\delta} \right) \tag{C.3}$$

$$\frac{\partial H}{\partial \hat{h}'} = p_h - \lambda \delta \hat{h}'^{\delta-1} x'^{1-\delta} = 0 \tag{C.4}$$

$$\frac{\partial H}{\partial x'} = p_x - \lambda (1-\delta) \hat{h}'^{\delta-1} x'^{-\delta} = 0 \tag{C.5}$$

$$\frac{\partial H}{\partial \lambda} = \hat{h}^{\delta} x^{1-\delta} - \hat{h}'^{\delta} x'^{1-\delta} = 0 \tag{C.6}$$

Then we multiply $\frac{\partial H}{\partial \hat{h}'}$ with $\hat{h}'$ and $\frac{\partial H}{\partial x'}$ with x' and substitute it in the budget constraint:

$$\hat{b} = \lambda \left(\delta \hat{h}'^{\delta} x'^{1-\delta} + (1-\delta) \hat{h}'^{\delta-1} x'^{1-\delta} \right) \tag{C.7}$$

$$\hat{b} = \lambda \hat{h}'^{\delta} x'^{1-\delta} \tag{C.8}$$

Putting the latter equation back into the partial derivatives of the Hamiltonian gives us $\hat{h'} = {}^{\delta \hat{b}'}/_{w}$ and $x' = {}^{(1-\delta)\hat{b}'}/_{p_x}$. We put these values into the utility function to get an expression for $\hat{b}'$ in terms of the actual consumption of housing services and other goods:

$$\hat{b}' = \left(\frac{p_h \hat{h}}{\delta} \right)^{\delta} \left(\frac{p_x x}{1-\delta} \right)^{1-\delta} = \left(\frac{p_h}{\delta} \right)^{\delta} \left(\frac{p_x}{1-\delta} \right)^{1-\delta} U(h, x) \equiv p_c' U(h, x) \tag{C.9}$$

where p_c is the price of the aggregate consumption (housing services and other goods and services). In the same way, we can derive that

$$\hat{b} = \left(\frac{p_h \hat{h}^*}{\delta}\right)^{\delta} \left(\frac{p_x x^*}{1-\delta}\right)^{1-\delta} = \left(\frac{p_h}{\delta}\right)^{\delta} \left(\frac{p_x}{1-\delta}\right)^{1-\delta} U^*(h,x) \equiv p_c' U^*(h,x) \tag{C.10}$$

Then the income equivalent of welfare loss is $\hat{b}' - \hat{b} = b' - b$. Now we can give a price to the loss of utility:

$$b' - b = p_c' \left(U - U^*\right) \tag{C.11}$$

which gives us the price of a loss in utility.

Appendix D

Derivation of an optimal control band policy

In this subsection, we show how the optimal control band policy is derived using the Bellman equation. Between stopping times, no moving costs are paid. Furthermore, the actual consumption vanishes when solving the integral from $0 \to \infty$, hence can use the process $x(t)$ instead of $u(t)$. Hence, our stochastic optimal control problem reduces to minimizing:

$$
\begin{aligned}
C(x) &= \mathbb{E}\left\{\int_0^\infty {}^1\!/\!{}_2\, x(t)^2 e^{-rt} dt\right\} && \text{(D.1)}\\
\text{s.t.} &\quad dx(t) = \mu dt + \sigma dw(t),\ x(0) = x && \text{(D.2)}
\end{aligned}
$$

for $s \leq x \leq S$. We use the Bellman equation for an optimal stopping problem[1]:

$$
C(x,t) = \min\left\{\gamma, {}^1\!/\!{}_2\, x^2 dt + (1+rdt)^{-1}\mathbb{E}\left[C(x+dx, t+dt)\right]\right\}. \qquad \text{(D.3)}
$$

Now, between $s \leq x \leq Q$ we can reduce the Bellman equation and rewrite it by using Ito's lemma[2]:

$$
\begin{aligned}
C(x,t) &= {}^1\!/\!{}_2\, x^2 dt + (1+rdt)^{-1}\mathbb{E}\left[C(x+dx,t+dt)\right]\\
(1+rdt)\,C(x,t) &= (1+rdt)\,{}^1\!/\!{}_2\, x^2 dt + \mathbb{E}\left[C(x+dx,t+dt)\right] && \text{(D.4)}\\
rC(x,t)dt &= {}^1\!/\!{}_2\, x^2 dt && \text{(D.5)}\\
&\quad + \mathbb{E}\left[\left[C_t(x,t) + \mu C_x(x,t) + {}^1\!/\!{}_2\,\sigma^2 C_{xx}(x,t)\right]dt + \sigma C_x(x,t)dw\right] && \text{(D.6)}\\
&= {}^1\!/\!{}_2\, x^2 dt && \text{(D.7)}\\
&\quad + \left[C_t(x,t) + \mu C_x(x,t) + {}^1\!/\!{}_2\,\sigma^2 C_{xx}(x,t)\right]dt + \mathbb{E}\left[\sigma C_x(x,t)dw\right] && \text{(D.8)}
\end{aligned}
$$

The term dt^2 goes to zero faster than dt and dw and therefore we let $dt^2 \to 0$. Because we assumed that the desired consumption of housing services is going to infinity, we can leave the time out since the costs are independent of where we are in time. Furthermore, the expectation of a Brownian motion is zero. We get:

$$
\begin{aligned}
(1+rdt)\,C(x,t) &= {}^1\!/\!{}_2\, x^2 dt + C(x,t) + \left[\mu C_x(x,t) + {}^1\!/\!{}_2\,\sigma^2 C_{xx}(x,t)\right]dt && \text{(D.9)}\\
rC(x)dt &= {}^1\!/\!{}_2\, x^2 dt + \left[\mu C_x(x,t) + {}^1\!/\!{}_2\,\sigma^2 C_{xx}(x,t)\right]dt\\
rC(x) &= {}^1\!/\!{}_2\, x^2 + \mu C_x(x) + {}^1\!/\!{}_2\,\sigma^2 C_{xx}(x) && \text{(D.10)}
\end{aligned}
$$

which holds for $s \leq x \leq Q$ and we impose the boundary conditions which follow from the Bellman equation:

$$
\begin{aligned}
C(S) &= C(Q) + \gamma && \text{(D.11)}\\
C(s) &= C(Q) + \gamma.
\end{aligned}
$$

[1]See Dixit and Pindyck (1994), pp. 103, equation (6)

[2]See Dixit and Pindyck (1994), pp. 109, 112, equation (13)

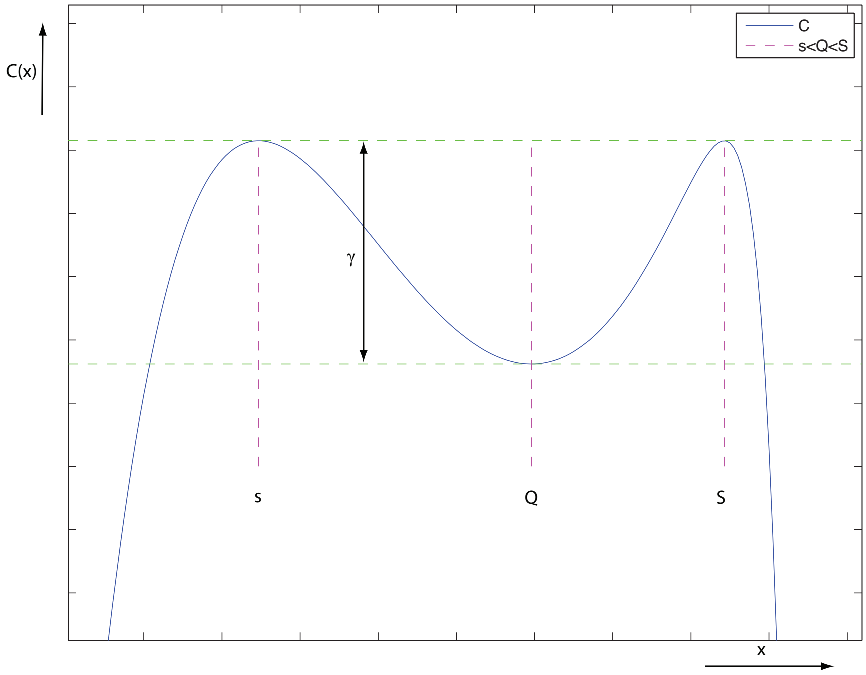

Figure D.1: Cost function

These boundary conditions mean that the household decides to move, i.e. pay the moving cost and change its consumption to Q if and only if the cost function at the barriers have the same value as the cost function at the optimum *plus* the moving cost. The solution to equation D.10 is given by:

$$\begin{aligned} C(x) &= Ae^{\alpha x} + Be^{-\beta x} + v_0 + v_1 x + {}^1\!/_2 v_2 x^2, \\ & \quad \text{for } s \leq x \leq S \end{aligned} \tag{D.12}$$

with by substituting equation D.12 into equation D.10:

$$\alpha = \frac{\sqrt{\mu^2 + 2r\sigma^2} - \mu}{\sigma^2} > 0 \tag{D.13}$$

$$\beta = \frac{\sqrt{\mu^2 + 2r\sigma^2} + \mu}{\sigma^2} > 0 \tag{D.14}$$

and

$$v_0 = {}^1\!/_2 \frac{\sigma^2}{r^2} + \frac{\mu^2}{r^3},\ v_1 = \frac{\mu}{r^2},\ v_2 = \frac{1}{r}. \tag{D.15}$$

Furthermore, the constants $A(\mu, \sigma, \gamma, r, s, S, Q)$ and $B(\mu, \sigma, \gamma, r, s, S, Q)$ can be found by substituting equation D.12 into the boundary conditions in equation D.11. The cost function can be extended for starting values out of the control band by letting the agent immediately relocate and pay the moving costs by:

$$C(x) = C(Q) + \gamma, \text{ for } x \notin [s, S]. \tag{D.16}$$

Before we can solve for optimal values of the control band policy, we need more conditions. At Q, we want the cost function to be minimized. The necessary and sufficient conditions for a minimum at Q are given by:

$$C_x(Q) = 0,\ C_{xx}(Q) > 0. \tag{D.17}$$

A plot of the cost function is shown in figure D.1 . In this plot, it is clear that between s and S, the function is unimodal. Furthermore, we need two other conditions in order to solve the control band policy which are provided by the the smooth pasting conditions[3]:

$$
\begin{aligned}
C_x(s) &= C_x(Q) && \text{(D.18)} \\
C_x(S) &= C_x(Q). && \text{(D.19)}
\end{aligned}
$$

This fully characterises the optimal control band policy (s, S, Q) for a given (μ, σ, γ, r).

[3]See Dixit and Pindyck (1994), pp. 109, equation (15)

Appendix E

Distribution in the gap

The process $u(t)$ is approximated by a discrete time process with time steps Δt and state jumps Δh. From a state u_0 at t_0, it goes up to $u_0 + \Delta h$ with a probability p and down to $u_0 - \Delta h$ with probability q at $t_0 + \Delta t$. This process converges to the continuous time process when $\Delta t \to 0$ if we impose:

$$\Delta h = \sigma\sqrt{\Delta t}, \tag{E.1}$$
$$p = \frac{1}{2}\left(1+\frac{\mu}{\sigma}\sqrt{\Delta t}\right) = \frac{1}{2}\left(1+\frac{\mu}{\sigma^2}\Delta h\right) \tag{E.2}$$
$$q = \frac{1}{2}\left(1-\frac{\mu}{\sigma}\sqrt{\Delta t}\right) = \frac{1}{2}\left(1-\frac{\mu}{\sigma^2}\Delta h\right) \tag{E.3}$$

The process in the control band is shown in figure E.1 . We are looking for the density function $\phi(u)$. At a certain point, where $u \neq Q$, the density is equal to:

$$\phi(u) = p\phi(u - \Delta h) + q\phi(u + \Delta h).$$

We take the Taylor expansion:

$$\phi(u) = \frac{1}{2}\left(1+\frac{\mu}{\sigma^2}\Delta h\right)\left(\phi(u) - \Delta h\phi'(u) + \frac{1}{2}(\Delta h)^2\phi'' + ...\right) + \tag{E.4}$$
$$\frac{1}{2}\left(1-\frac{\mu}{\sigma^2}\Delta h\right)\left(\phi(u) + \Delta h\phi'(u) + \frac{1}{2}(\Delta h)^2\phi'' + ...\right) \tag{E.5}$$
$$= \phi(u) - \left(\mu/\sigma^2\right)(\Delta h)^2\phi'(u) + \frac{1}{2}(\Delta h)^2\phi''(u) + ... \tag{E.6}$$

The omitted terms converge to zero faster than $(\Delta h)^2$ and (Δh). Here we can cancel $\phi(u)$ and divide by $(\Delta h)^2$ and taking the limit when $\Delta h \to 0$. This expression leads to the following differential equation[1]:

$$\phi''(u) = \theta\phi'(u),\ \theta = {}^{2\mu}/_{\sigma^2},\ u \neq Q. \tag{E.7}$$

The general solution of this ordinary differential equation is:

$$\phi(u) = \begin{cases} A_1 e^{\theta u} + B_1, & s < u < Q \\ A_2 e^{\theta u} + B_2, & Q < u < S \end{cases} \tag{E.8}$$

Now we define the boundary equations by looking at what is happening at the control band. At a point just below S, the density is equal to:

$$\phi(S - \Delta h) = p\phi(S - 2\Delta h) \tag{E.9}$$

[1]See Dixit and Pindyck (1994), pp. 83-84

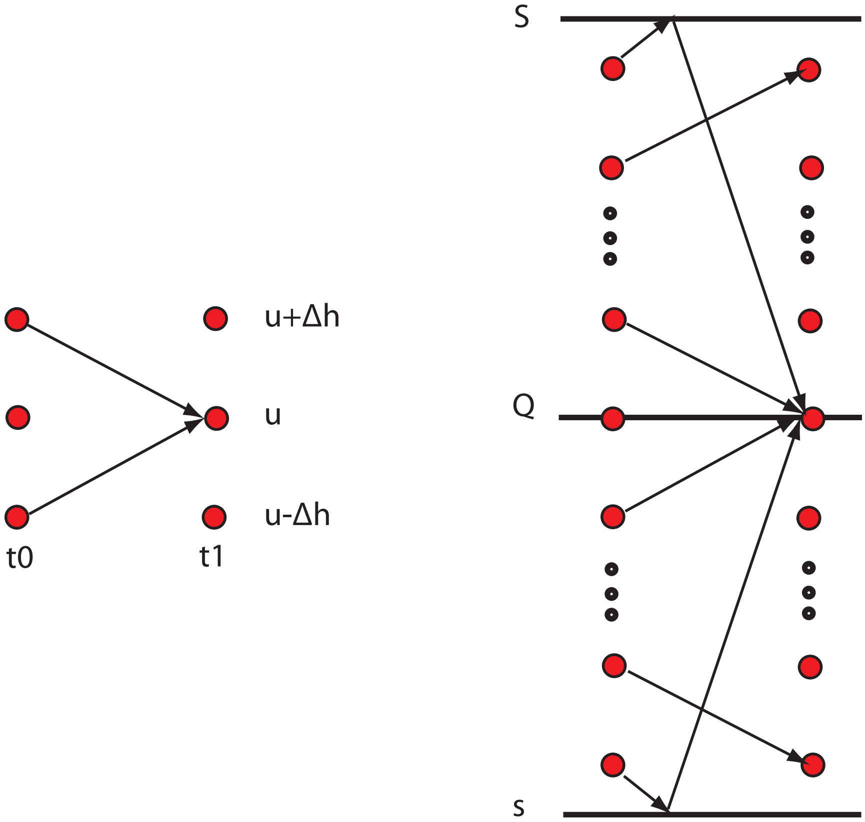

Figure E.1: Discrete time process in the control band

and above s:

$$\phi(s + \Delta h) = q\phi(s + 2\Delta h). \tag{E.10}$$

We also take the Taylor expansion at the boundaries in the same way and obtain the following conditions at the boundaries:

$$\phi(s) = 0 \tag{E.11}$$
$$\phi(S) = 0 \tag{E.12}$$

Then at the optimal level Q, we want $\phi(u)$ to be continuous, but it does not need to be differentiable. Then the third condition is::

$$\lim_{u\uparrow Q}\phi(u) = \lim_{u\downarrow Q}\phi(u) \equiv \phi(Q) \tag{E.13}$$

Now substituting equation E.11 and equation E.12 in equation E.14 we get $B_1 = -A_1 e^{\theta s}$ and $B_2 = -A_2 e^{\theta S}$ which results in:

$$\phi(u) = \begin{cases} A_1 \left[e^{\theta u} - e^{\theta s}\right], & s < u < Q \\ A_2 \left[e^{\theta u} - e^{\theta S}\right], & Q < u < S \end{cases} \tag{E.14}$$

Then using E.13 we obtain a relation between A_1 and A_2:

$$\frac{A_1}{A_2} = \frac{e^{\theta Q} - e^{\theta S}}{e^{\theta Q} - e^{\theta s}} = c < 0$$

Then, because we define a probability density function, the area under the function should equal one:

$$\int_s^S \phi(u)du = \int_s^Q cA_2 \left[e^{\theta u} - e^{\theta s}\right] du + \int_Q^S A_2 \left[e^{\theta u} - e^{\theta S}\right] du = 1$$

and then we solve:

$$A_2 = -\frac{1}{ce^{\theta s}(Q - s) + e^{\theta S}(S - Q)}.$$

E.1 Derivation of the distribution with noise

We assume that we observe the sum of the *independent* processes $u(t)$ and the noise $\varepsilon(t)$. Given the distributions of those variables, we are looking for the distribution of the sum of this process i.e. the observed gap. To show this, consider the cumulative density of $u(t) + \varepsilon(t)$ at x:

$$\hat{u}(t) = u(t) + \varepsilon(t) \tag{E.15}$$

The probability density function of $u(t)$ is given by equation E.8 and the density of the noise $\varepsilon(t)$ is given by:

$$f_\varepsilon(\varepsilon) = \frac{1}{\sqrt{2\pi\sigma_\varepsilon^2}} e^{-\frac{(\varepsilon - \mu_\varepsilon)^2}{2\sigma_\varepsilon^2}}. \tag{E.16}$$

Then the density function $f_{\hat{u}}(\hat{u})$ is given by the convolution of the density functions $\phi(u)$ and $f_\varepsilon(\varepsilon)$:

$$\begin{aligned} \mathbb{P}(u + \varepsilon \leq x) &= \int_{-\infty}^{+\infty} \mathbb{P}(u \leq x - y) f_\varepsilon(y)dy \\ &= \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{x-y} \phi(z)dz\right] f_\varepsilon(y)dy \end{aligned} \tag{E.17}$$

The probability density function is obtained by differentiating the cumulative density function w.r.t. x:

$$
\begin{aligned}
\frac{\partial}{\partial(x-y)}\left[\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{x-y}\phi(z)dz\right]f_\varepsilon(y)dy\right] &= \int_{-\infty}^{+\infty}\phi(x-y)f_\varepsilon(y)dy\,\text{(convolution)} \\
&= \int_{-\infty}^{+\infty}\phi(u)f_\varepsilon(x-u)du \\
&= \int_{s}^{Q}\left[A_1e^{\theta u}+B_1\right] \\
&\quad\times\frac{1}{\sqrt{2\pi}\sigma_\varepsilon}e^{-\frac{1}{2}\left(\frac{x-u-\mu_\varepsilon}{\sigma_\varepsilon}\right)^2}du \qquad \text{(E.18)} \\
&\quad+\int_{Q}^{S}\left[A_2e^{\theta u}+B_2\right] \qquad \text{(E.19)} \\
&\quad\times\frac{1}{\sqrt{2\pi}\sigma_\varepsilon}e^{-\frac{1}{2}\left(\frac{x-u-\mu_\varepsilon}{\sigma_\varepsilon}\right)^2}du \qquad \text{(E.20)}
\end{aligned}
$$

Then evaluating part by part because of the linearity of the integral:

$$
\int_{s}^{Q}A_1e^{\theta u}\frac{1}{\sqrt{2\pi}\sigma_\varepsilon}e^{-\frac{1}{2}\left(\frac{x-u-\mu_\varepsilon}{\sigma_\varepsilon}\right)^2}du = \frac{A_1}{\sqrt{2\pi}\sigma_\varepsilon}\int_{s}^{Q}e^{\theta u-\frac{1}{2}\left(\frac{x-u-\mu_\varepsilon}{\sigma_\varepsilon}\right)^2}du \qquad \text{(E.21)}
$$

$$
\text{define:}\qquad v=\frac{x-u-\mu_\varepsilon}{\sigma_\varepsilon},\ dv=-\frac{1}{\sigma_\varepsilon}du \qquad \text{(E.22)}
$$

$$
\begin{aligned}
\int_{s}^{Q}e^{\theta u-\frac{1}{2}\left(\frac{x-u-\mu_\varepsilon}{\sigma_\varepsilon}\right)^2}du &\rightarrow -\sigma_\varepsilon\int_{\frac{x-s-\mu_\varepsilon}{\sigma_\varepsilon}}^{\frac{x-Q-\mu_\varepsilon}{\sigma_\varepsilon}}e^{-\theta\sigma_\varepsilon v+\theta(x-\mu_\varepsilon)-\frac{1}{2}v^2}dv \\
&= -\sigma_\varepsilon\int_{\frac{x-s-\mu_\varepsilon}{\sigma_\varepsilon}}^{\frac{x-Q-\mu_\varepsilon}{\sigma_\varepsilon}}e^{-\frac{1}{2}(v+\theta\sigma_\varepsilon)^2+\frac{1}{2}\theta^2\sigma_\varepsilon^2+\theta(x-\mu_\varepsilon)}dv \\
&= -\sigma_\varepsilon e^{\frac{1}{2}\theta^2\sigma_\varepsilon^2+\theta(x-\mu_\varepsilon)}\int_{\frac{x-s-\mu_\varepsilon}{\sigma_\varepsilon}}^{\frac{x-Q-\mu_\varepsilon}{\sigma_\varepsilon}}e^{-\frac{1}{2}(v+\theta\sigma_\varepsilon)^2}dv
\end{aligned}
$$

$$
\text{Now:}\qquad \frac{1}{2}(v+\theta\sigma_\varepsilon)^2=\left(\frac{1}{\sqrt{2}}v+\frac{1}{\sqrt{2}}\theta\sigma_\varepsilon\right)^2
$$

$$
\text{define:}\qquad t=\frac{1}{\sqrt{2}}v+\frac{1}{\sqrt{2}}\theta\sigma_\varepsilon,\ dt=\frac{1}{\sqrt{2}}dv
$$

$$
\begin{aligned}
-\sigma_\varepsilon e^{\frac{1}{2}\theta^2\sigma_\varepsilon^2+\theta(x-\mu_\varepsilon)}\int_{\frac{x-s-\mu_\varepsilon}{\sigma_\varepsilon}}^{\frac{x-Q-\mu_\varepsilon}{\sigma_\varepsilon}}e^{-\frac{1}{2}(v+\theta\sigma_\varepsilon)^2}dv &\rightarrow -\sigma_\varepsilon\sqrt{2}e^{\frac{1}{2}\theta^2\sigma_\varepsilon^2+\theta(x-\mu_\varepsilon)}\int_{\frac{1}{\sqrt{2}}\left(\frac{x-s-\mu_\varepsilon}{\sigma_\varepsilon}+\theta\sigma_\varepsilon\right)}^{\frac{1}{\sqrt{2}}\left(\frac{x-Q-\mu_\varepsilon}{\sigma_\varepsilon}+\theta\sigma_\varepsilon\right)}e^{-t^2}dt \\
&= -\sigma_\varepsilon\sqrt{2}e^{\frac{1}{2}\theta^2\sigma_\varepsilon^2+\theta(x-\mu_\varepsilon)}\frac{\sqrt{\pi}}{2} \\
&\times\left[\operatorname{erf}\left(\frac{1}{\sqrt{2}}\left(\frac{x-Q-\mu_\varepsilon}{\sigma_\varepsilon}+\theta\sigma_\varepsilon\right)\right)\right. \\
&- \left.\operatorname{erf}\left(\frac{1}{\sqrt{2}}\left(\frac{x-s-\mu_\varepsilon}{\sigma_\varepsilon}+\theta\sigma_\varepsilon\right)\right)\right]\rightarrow \qquad \text{(E.23)}
\end{aligned}
$$

$$
\begin{aligned}
\int_{s}^{Q}A_1e^{\theta u}\frac{1}{\sqrt{2\pi}\sigma_\varepsilon}e^{-\frac{1}{2}\left(\frac{x-u-\mu_\varepsilon}{\sigma_\varepsilon}\right)^2}du &= -\frac{A_1}{2}e^{\frac{1}{2}\theta^2\sigma_\varepsilon^2+\theta(x-\mu_\varepsilon)} \\
&\times\left[\operatorname{erf}\left(\frac{1}{\sqrt{2}}\left(\frac{x-Q-\mu_\varepsilon}{\sigma_\varepsilon}+\theta\sigma_\varepsilon\right)\right)\right. \\
&- \left.\operatorname{erf}\left(\frac{1}{\sqrt{2}}\left(\frac{x-s-\mu_\varepsilon}{\sigma_\varepsilon}+\theta\sigma_\varepsilon\right)\right)\right] \qquad \text{(E.24)}
\end{aligned}
$$

where $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$. The other parts of the distribution are:

$$\int_s^Q B_1 \frac{1}{\sqrt{2\pi}\sigma_\varepsilon} e^{-\frac{1}{2}\left(\frac{x-u-\mu_\varepsilon}{\sigma_\varepsilon}\right)^2} du = -\frac{B_1}{2} \tag{E.25}$$

$$\times \left[\text{erf}\left(\frac{1}{\sqrt{2}}\left(\frac{x-Q-\mu_\varepsilon}{\sigma_\varepsilon}\right)\right)\right.$$

$$\left. -\text{erf}\left(\frac{1}{\sqrt{2}}\left(\frac{x-s-\mu_\varepsilon}{\sigma_\varepsilon}\right)\right)\right] \tag{E.26}$$

and analogously:

$$\int_Q^S A_2 e^{\theta u} \frac{1}{\sqrt{2\pi}\sigma_\varepsilon} e^{-\frac{1}{2}\left(\frac{x-u-\mu_\varepsilon}{\sigma_\varepsilon}\right)^2} du = -\frac{A_2}{2} e^{\frac{1}{2}\theta^2\sigma_\varepsilon^2 + \theta(x-\mu_\varepsilon)}$$

$$\times \left[\text{erf}\left(\frac{1}{\sqrt{2}}\left(\frac{x-S-\mu_\varepsilon}{\sigma_\varepsilon} + \theta\sigma_\varepsilon\right)\right)\right. \tag{E.27}$$

$$\left. -\text{erf}\left(\frac{1}{\sqrt{2}}\left(\frac{x-Q-\mu_\varepsilon}{\sigma_\varepsilon} + \theta\sigma_\varepsilon\right)\right)\right] \tag{E.28}$$

and:

$$\int_Q^S B_2 \frac{1}{\sqrt{2\pi}\sigma_\varepsilon} e^{-\frac{1}{2}\left(\frac{x-u-\mu_\varepsilon}{\sigma_\varepsilon}\right)^2} du = -\frac{B_2}{2}$$

$$\times \left[\text{erf}\left(\frac{1}{\sqrt{2}}\left(\frac{x-S-\mu_\varepsilon}{\sigma_\varepsilon}\right)\right)\right. \tag{E.29}$$

$$\left. -\text{erf}\left(\frac{1}{\sqrt{2}}\left(\frac{x-Q-\mu_\varepsilon}{\sigma_\varepsilon}\right)\right)\right] \tag{E.30}$$

All together this gives us the density of the sum of the stochastic process u and ε.

Numerically, these formulation of the density is problematic. In the density function, two error functions are subtracted, also for values in the tails of the function. Because the values of the error function are getting to $\inf(-1)$ and $\sup(1)$ we might get to the order of 10^{-16} which is the limit of the computers we use and therefore yield inaccurate results. These are mainly problematic for equations E.24 and E.27 because the first parts multiply these inaccuracies with possibly very large values. This is visible as oscillations in plots of the density and is therefore unsuitable for parameter estimation. We therefore reformulate the problem[2] in order to avoid numerical errors. The problem is the *floating point* notation of the computer. The problematic numbers are $\inf(-1)$ and $\sup(1)$ which are notated by ± 0.99999.... If we add or subtract 1 we can get to a more efficient notation ± 0.000.......something which can be more efficiently written as e.g. $\pm\text{something} \cdot 10^{-16}$. Let us first restate some properties of the error function:

$$\begin{aligned}
\text{erf}(-a) &= -\text{erf}(a) \\
\text{erfc}(a) &= 1 - \text{erf}(a) \\
\text{erfc}(+\infty) &= 1 - \text{erf}(+\infty) = 1 - 1 = 0 \text{ which is convenient for floating point} \\
\text{erfc}(-a) - \text{erfc}(-b) &= -\text{erf}(-a) + \text{erf}(-b) = \text{erf}(a) - \text{erf}(b)
\end{aligned}$$

We use this properties to get to a new formulation which gives results accurate enough for param-

[2]Thanks to Arie ten Cate

eter estimation:

$$
\begin{aligned}
f(x) &= -\frac{A_1}{2} e^{\frac{1}{2}\theta^2\sigma_\varepsilon^2+\theta(x-\mu_\varepsilon)} \\
&\times \left[\operatorname{erfc}\left(-\frac{1}{\sqrt{2}}\left(\frac{x-Q-\mu_\varepsilon}{\sigma_\varepsilon}+\theta\sigma_\varepsilon\right)\right) - \operatorname{erfc}\left(-\frac{1}{\sqrt{2}}\left(\frac{x-s-\mu_\varepsilon}{\sigma_\varepsilon}+\theta\sigma_\varepsilon\right)\right)\right] \\
&+ -\frac{B_1}{2} \\
&\times \left[\operatorname{erfc}\left(-\frac{1}{\sqrt{2}}\left(\frac{x-Q-\mu_\varepsilon}{\sigma_\varepsilon}\right)\right) - \operatorname{erfc}\left(-\frac{1}{\sqrt{2}}\left(\frac{x-s-\mu_\varepsilon}{\sigma_\varepsilon}\right)\right)\right] \\
&+ -\frac{A_2}{2} e^{\frac{1}{2}\theta^2\sigma_\varepsilon^2+\theta(x-\mu_\varepsilon)} \\
&\times \left[\operatorname{erfc}\left(-\frac{1}{\sqrt{2}}\left(\frac{x-S-\mu_\varepsilon}{\sigma_\varepsilon}+\theta\sigma_\varepsilon\right)\right) - \operatorname{erfc}\left(-\frac{1}{\sqrt{2}}\left(\frac{x-Q-\mu_\varepsilon}{\sigma_\varepsilon}+\theta\sigma_\varepsilon\right)\right)\right] \\
&+ -\frac{B_2}{2} \\
&\times \left[\operatorname{erfc}\left(-\frac{1}{\sqrt{2}}\left(\frac{x-S-\mu_\varepsilon}{\sigma_\varepsilon}\right)\right) - \operatorname{erfc}\left(-\frac{1}{\sqrt{2}}\left(\frac{x-Q-\mu_\varepsilon}{\sigma_\varepsilon}\right)\right)\right] \qquad \text{(E.31)}
\end{aligned}
$$

Even now, we get some accuracy problems. Therefore we use in Matlab the scaled complementary error function `erfcx` which is defined by $e^{x^2}\operatorname{erfc}(x)$ which gives us stable solutions.

Appendix F

Density function of a Brownian motion with drift between two barriers

We are interested in the density of the moving times. We modelled this with a Brownian motion with drift and a control band policy. This problem is equivalent to the expected hitting time of a Brownian motion W_t with drift ν in a two-sides barrier. The Brownian motion with drift is defined by $X_t = \nu t + W_t)$ with starting point 0 and the barriers are given by two numbers a and b such that $a < 0 < b$. Equivalence with the moving frequency is obtained by setting:

$$a = s - Q \quad \text{(F.1)}$$

$$b = S - Q \quad \text{(F.2)}$$

$$X_0 = u(0) - Q. \quad \text{(F.3)}$$

We define the stopping or hitting times as:

$$\begin{aligned} T_a &= \inf\{t : X_t = a\} \\ T_b &= \inf\{t : X_t = b\} \\ T^* &= T_a \wedge T_b. \end{aligned}$$

To derive the density we follow Jeanblanc et al. (2009). We start with an explanation of Doob's optional sampling theorem to calculate the expectation and probabilities for a Brownian motion without drift. To derive the density, we can not do this directly hence we use the Laplace transform of the density. Using this concept, we can obtain the Laplace transform of the density for a Brownian motion with drift. In the last part, this Laplace transform is inverted using the Bromwich integral. The explicit solution is obtain by using the Cauchy residue theorem.

F.1 Doob's optional sampling theorem

For the derivation of the moving frequency we need Doob's optional sampling theorem which is defined by[1]:

Theorem F.1.1. *Let $X_1, X_2, X_3, \ldots$ be a martingale and τ a stopping time with respect to $X_1, X_2, X_3, \ldots$. If $\mathbb{E}\tau < \infty$ and there exists a constant c such that for all i, $\mathbb{E}(|X_{i+1} - X_i|) \leq c$ then $\mathbb{E}X_\tau = \mathbb{E}X_1$.*

We will use this theorem to find the distribution of the hitting time between two barriers. First we consider the case where the drift is zero i.e. $\nu = 0$. We first want to find the probability

[1] Wikipedia

that the Brownian motion hits one barrier before the other. Secondly we use this probabilities to calculate the expected hitting time.

Proposition F.1.1. *Let W be a Brownian motion starting from 0 and $a < 0 < b$. Then:*

$$\mathbb{P}_0(T_a < T_b) = \frac{b}{b-a}$$

and $\mathbb{E}_0 T^* = -ab$.

PROOF: Before we can use Doob's optional sampling theorem, we need to proof that the expectation of the hitting time is smaller than infinite. We will do this later. A Brownian motion without drift is a Martingale, then applying Doob's optional sampling theorem we get:

$$\begin{aligned}
\mathbb{E}_0 X_0 &= 0 = \mathbb{E}_0 X_{T^*} = \mathbb{E}_0 W_{T_a \wedge T_b} \\
&= a\mathbb{P}_0(T_a < T_b) + b\mathbb{P}_0(T_b < T_a) \text{ and obviously:} \\
1 &= \mathbb{P}_0(T_a < T_b) + \mathbb{P}_0(T_b < T_a)
\end{aligned}$$

and then it is easily derived that $\mathbb{P}_0(T_a < T_b) = \frac{b}{b-a}$.

To get the expected hitting time, we do a trick. First quadratise the process and obtain $W^2_{t \wedge T_a \wedge T_b}$ with expectation t. This is obviously no martingale anymore so we cannot apply Doob's optional sampling theorem. Let us first proof the following theorem:

Theorem F.1.1. *Let W_t,$t \geq 0$ be a Brownian motion with filtration $\mathcal{F}_t, t \geq 0$. Then $W_t^2 - t$ is a martingale.*

PROOF: For $0 < s < t$ we have:

$$\begin{aligned}
\mathbb{E}\left[W_t^2 | \mathcal{F}_s\right] &= \mathbb{E}\left[(W_s + (W_t - W_s))^2 | \mathcal{F}_s\right] \\
&= \mathbb{E}\left[W_s^2 + 2W_s(W_t - W_s) + (W_t - W_s)^2 | \mathcal{F}_s\right] \\
&= W_s^2 + 2W_s \cdot \mathbb{E}\left[(W_t - W_s) | \mathcal{F}_s\right] + \mathbb{E}\left[(W_t - W_s)^2 | \mathcal{F}_s\right] \\
&= W_s^2 + t - s
\end{aligned}$$

hence the process $W^2_{t \wedge T_a \wedge T_b} - t \wedge T_a \wedge T_b$ is a martingale and we apply Doob's optional sampling theorem:

$$\begin{aligned}
0 &= \mathbb{E}_0\left[W^2_{t \wedge T_a \wedge T_b} - t \wedge T_a \wedge T_b\right] \\
&= a^2 \mathbb{P}_0(T_a < T_b) + b^2 \mathbb{P}_0(T_b < T_a) - \mathbb{E}_0 T^* \\
\mathbb{E}_0 T^* &= -ab
\end{aligned}$$

This provides us with the third equation that proves the proposition. It is easily seen that this expectation is bounded since $W^2_{t \wedge T_a \wedge T_b}$ is always bounded by the barriers and $t \geq 0$ hence the second condition for Doob's optional sampling theorem is satisfied.

F.2 The Laplace transform of the hitting time

A direct computation of the distribution of the hitting time is not possible. Hence we use the Laplace transform to restate the problem. We use the inverse Laplace transform to find the distribution. Following again Jeanblanc et al (2009):

Proposition F.2.1. *Let W_t,$t \geq 0$ be a Brownian motion with filtration $\mathcal{F}_t, t \geq 0$ and starting from 0 and let $a < 0 < b$ be the barriers. Then the Laplace transform of T^* is:*

$$\mathbb{E}_0\left[\exp\left(-\frac{\lambda^2}{2} T^*\right)\right] = \frac{\cosh\left[\lambda(a+b)/2\right]}{\cosh\left[\lambda(a-b)/2\right]}$$

Proof: The Laplace transform of a random variable X with density function f is given by $(\mathcal{L}f)(s) = \mathbb{E}\left[e^{-sX}\right]$. To find the Laplace transform, we would like to write it as an exponential and also that it is a martingale to apply optional sampling. We can proof that $\exp\left(\lambda\left(W_{t\wedge T^*} - \frac{a+b}{2}\right) - \frac{\lambda^2(t\wedge T^*)}{2}\right)$ is a martingale and we can easily see that it is bounded hence this is such a function. This because $W_{t\wedge T^*}$ is always smaller than the barriers and $\exp\left(-\frac{\lambda^2(t\wedge T^*)}{2}\right)$ will go to zero is $t\to\infty$. Now we use again Doob's optional sampling theory for this process:

$$\begin{aligned}\exp\left(-\lambda\left(\frac{a+b}{2}\right)\right) &= \mathbb{E}\left[\exp\left(\lambda\left(W_{T^*} - \frac{a+b}{2}\right) - \frac{\lambda^2 T^*}{2}\right)\right] \\ &= \exp\left(\lambda\frac{b-a}{2}\right)\mathbb{E}\left[\exp\left(-\frac{\lambda^2T^*}{2}\right)\mathbb{I}_{\{T^*=T_b\}}\right] \\ &\quad + \exp\left(\lambda\frac{a-b}{2}\right)\mathbb{E}\left[\exp\left(-\frac{\lambda^2T^*}{2}\right)\mathbb{I}_{\{T^*=T_a\}}\right]\end{aligned} \tag{F.4}$$

we want another equation and by the reflection principle we use $-W$ and get:

$$\begin{aligned}\exp\left(-\lambda\left(\frac{a+b}{2}\right)\right) &= \mathbb{E}\left[\exp\left(\lambda\left(-W_{T^*} - \frac{a+b}{2}\right) - \frac{\lambda^2 T^*}{2}\right)\right] \\ &= \exp\left(\lambda\frac{3b-a}{2}\right)\mathbb{E}\left[\exp\left(-\frac{\lambda^2T^*}{2}\right)\mathbb{I}_{\{T^*=T_b\}}\right] \\ &\quad + \exp\left(\lambda\frac{-b-3a}{2}\right)\mathbb{E}\left[\exp\left(-\frac{\lambda^2T^*}{2}\right)\mathbb{I}_{\{T^*=T_a\}}\right]\end{aligned} \tag{F.5}$$

This is a system of two linear equations with two unknowns, we get:

$$\begin{cases}\mathbb{E}\left[\exp\left(-\frac{\lambda^2T^*}{2}\right)\mathbb{I}_{\{T^*=T_b\}}\right] = \frac{\sinh(-\lambda a)}{\sinh(\lambda(b-a))} \\ \mathbb{E}\left[\exp\left(-\frac{\lambda^2T^*}{2}\right)\mathbb{I}_{\{T^*=T_a\}}\right] = \frac{\sinh(\lambda b)}{\sinh(\lambda(b-a))}\end{cases} \tag{F.6}$$

Now we can derive the Laplace transform of random variable T^* with operator $\lambda^2/2$:

$$\mathbb{E}\left[e^{-\lambda^2T^*/2}\right] = \mathbb{E}\left[e^{-\lambda^2T^*/2}\mathbb{I}_{\{T^*=T_b\}}\right] + \mathbb{E}\left[e^{-\lambda^2T^*/2}\mathbb{I}_{\{T^*=T_a\}}\right] \tag{F.7}$$

and after filling in and rearranging we get the result in the proposition.

F.3 Laplace transform for drifted Brownian motion

To derive the Laplace transform for drifted Brownian motion we need to find a new function that is a martingale. Let $\boldsymbol{W}^\nu$ be the law of a Brownian motion with drift ν. The hitting times depend on the process $X_t = \nu t + W_t$ and are denoted by $T^*(X) = T_a(X)\wedge T_b(X)$. For convenience, we write $T^* = T^*(X)$. Jeanblanc et al. (2009) uses Cameron-Martin's theorem to write the Laplace transform as an exponential martingale:

$$\boldsymbol{W}^\nu\left(\exp\left(-\frac{\lambda^2}{2}T^*\right)\right) = \mathbb{E}\left(\exp\left(\nu W_{T^*} - \frac{\nu^2}{2}T^*\right)\exp\left(-\frac{\lambda^2}{2}T^*\right)\right) \tag{F.8}$$

$$= \mathbb{E}\left[e^{\nu W_{T^*} - (\nu^2+\lambda^2)T^*/2}\mathbb{I}_{\{T^*=T_b\}}\right] \tag{F.9}$$

$$+\mathbb{E}\left[e^{\nu W_{T^*} - (\nu^2+\lambda^2)T^*/2}\mathbb{I}_{\{T^*=T_a\}}\right] \tag{F.10}$$

$$= e^{\nu b}\mathbb{E}\left[e^{-(\nu^2+\lambda^2)T^*/2}\mathbb{I}_{\{T^*=T_b\}}\right] \tag{F.11}$$

$$+e^{\nu a}\mathbb{E}\left[e^{\nu W_{T^*} - (\nu^2+\lambda^2)T^*/2}\mathbb{I}_{\{T^*=T_a\}}\right] \tag{F.12}$$

and with the result from the previous paragraph we obtain:

$$\boldsymbol{W}^\nu \left(\exp\left(-\frac{\lambda^2}{2} T^* \right) \right) = e^{\nu b} \frac{\sinh(-\mu a)}{\sinh(-\mu(b-a))} + e^{\nu a} \frac{\sinh(\mu b)}{\sinh(-\mu(b-a))}$$

with $\mu^2 = \nu^2 + \lambda^2$.

F.4 Density function of the hitting time

The last step to obtain the density function is it inverting the Laplace transform. This is defined by the Bromwich integral, which is the line integral given by:

$$\mathcal{L}^{-1}\{F(s)\} = f(t) = \frac{1}{2\pi i} \lim_{T\to\infty} \int_{\gamma - iT}^{\gamma + iT} e^{st} F(s) ds \tag{F.13}$$

where it is integrated along the vertical line $Re(s) = \gamma$ in the complex plane. To solve it, we use Cauchy's Residue Theorem. Then we obtain following Jeanblanc et al. (2009):

$$\mathbb{P}_x(T^* \in dt) = e^{-\nu^2 t/2} \left(e^{\nu(a-x)} ss_t(b-x, b-a) + e^{\nu(b-x)} ss_t(x-a, b-a) \right) dt, \tag{F.14}$$

$$\text{with } ss_t(u,v) = \frac{1}{\sqrt{2\pi t^3}} \sum_{k=-\infty}^{\infty} (v-u+2kv) e^{-(v-u+2kv)^2/2t}. \tag{F.15}$$

F.5 Expectation of the hitting time

The expected value is obtained by integrating the density function of the hitting time. Since integrating the above p.d.f. is rather difficult, they use a different approach. We use the probabilities that the process hits either one of the barriers as derived by Garmanik (2005). Consider de process $X_t = \nu t + W_t$ and let $T_a(X)$ be the time the process hits the lower barrier a and $T_b(X)$ be the time the process hits the upper barrier b and let $T^*(X) = T_a(X) \wedge T_b(X)$. Then, using the theorem of martingales, we have:

$$\mathbb{E}X_{T^*} = \nu \mathbb{E}T^* = a \cdot \mathbb{P}(T_a < T_b) + b \cdot \mathbb{P}(T_b < T_a). \tag{F.16}$$

To derive the probabilities we follows Garmanik (2005). They assume that the drift $\nu < 0$. In that case $\lim_{t\to\infty} X_t = -\infty$ almost surely. Then they state that $T_{ab} \leq T_a < \infty$ almost surely. They want to compute $\mathbb{P}(T_{ab} = T_a)$. With $\nu < 0$ they consider $q(\beta) = \nu\beta + \frac{1}{2}\beta^2$.

Then they proof that the process $V(t) = e^{\beta X_t - q(\beta)t}$ is a martingale for every β. Because $V(t)$ is a martingale, they apply Doob's optional stopping theorem. They suppose that β is such that $q(\beta) \geq 0$. Then, almost surely $0 \leq V(t \wedge T^*) \leq e^{\beta b}$ where b is the upper barrier of the control band. For this reason it can be concluded that this martingale is bounded hence we can apply Doob's optional sampling theorem:

$$\mathbb{E}[V(T^*)] = V(0) = 1. \tag{F.17}$$

Then they set $\beta = -2\nu$. This results in $q(\beta) = 0$, then:

$$V(T^*)\mathbb{I}_{\{T^* = T_a\}} = e^{-2\nu a}\mathbb{I}_{\{T^* = T_a\}} \tag{F.18}$$

$$V(T^*)\mathbb{I}_{\{T^* = T_b\}} = e^{-2\nu b}\mathbb{I}_{\{T^* = T_b\}} \tag{F.19}$$

Using now Doob's optional sampling theorem in equation F.17:

$$1 = e^{-2\nu a}\mathbb{P}(T^* = T_a) + e^{-2\nu b}\mathbb{P}(T^* = T_b) = \mathbb{E}[V(T^*)].$$

Using $\mathbb{P}(T^* = T_a) + \mathbb{P}(T^* = T_b) = 1$ we can calculate the probabilities:

$$\mathbb{P}(T^* = T_a) = \frac{1 - e^{-2\nu b}}{e^{-2\nu a} - e^{-2\nu b}} \tag{F.20}$$

$$\mathbb{P}(T^* = T_b) = \frac{1 - e^{-2\nu a}}{e^{-2\nu b} - e^{-2\nu a}} \tag{F.21}$$

and the associated expectation of the hitting time is then calculated using equation F.16 by:

$$\mathbb{E}T^* = \frac{a \cdot \mathbb{P}\left(T^* = T_a\right) + b \cdot \mathbb{P}\left(T^* = T_b\right)}{\nu} \tag{F.22}$$

$$= \frac{a \cdot \left(1 - e^{-2\nu b}\right) - b \cdot \left(1 - e^{-2\nu a}\right)}{\left(e^{-2\nu a} - e^{-2\nu b}\right)\nu} \tag{F.23}$$

Appendix G

Matlab Code

G.1 Simulation of moving behaviour

G.1.1 Simulation

```
clc;clear;clf;

global mu sigma r gamma
mu=-0.0032;
% Given T*=17
sigma=0.135;gamma=0.3796;  %3%
%sigma=0.192;gamma=0.7592;  %6%
%sigma=0.248;gamma=1.2653;  %10%
%sigma=0.303;gamma=1.8980;  %15%
%sigma=0.350;gamma=2.5306;  %20%
r=0.05;

t0=0;dt=0.01;T=100;t=t0:dt:T;

N=1;
zd=zeros(N,(T-t0)/dt+1);
zd(:,1)=sigma*sqrt(t0)*randn(N,1)-mu*t0;
z=zeros(N,(T-t0)/dt+1);
jrkomwon=zeros(N,1)*t0;
phi=zeros(N,(T-t0)/dt+1);
aantal=zeros(N,1);

for i=1:(T-t0)/dt                    % demand: BM, N times, length T
    zd(:,i+1)=zd(:,i)-mu*dt+sigma*randn(N,1)*sqrt(dt);
end

plot(t,zd)
x=-zd(1)+z(1);
%% solving
cbp0=[-0.1;0.1;0];
c=[mu,sigma,r,gamma];
[cbp,fval]=fsolve(@(cbp) valuefun(cbp,c),cbp0);
s=cbp(1);S=cbp(2);Q=cbp(3);                % Control band policy
jumps=exp(cbp)
%%
u=zeros(N,(T-t0)/dt+1);
xi=zeros(N,(T-t0)/dt+1);
for i=1:(T-t0)/dt+1                  % actual space use
```

```
    for j=1:N
        if z(j,i)-zd(j,i)<s
            if i==1
                xi(j,i)=Q-(-zd(1)+z(1));
            else
                xi(j,i)=Q-s;
            end
            z(j,i:(T-t0)/dt+1)=Q+zd(j,i);
            phi(j,i)=gamma;
            jrkomwon(j,1)=t(i);
            aantal(j,1)=aantal(j,1)+1;
        elseif z(j,i)-zd(j,i)>S
            if i==1
                xi(j,i)=Q-(-zd(1)+z(1));
            else
                xi(j,i)=Q-S;
            end
            z(j,i:(T-t0)/dt+1)=Q+zd(j,i);;
            phi(j,i)=gamma;
            jrkomwon(j,1)=t(i);
            aantal(j,1)=aantal(j,1)+1;
        else
            z(j,i:(T-t0)/dt+1)=z(j,i);
            phi(j,i)=0;
        end
    end
end

u=z-zd;                        % gap

Laatste_verhuizing=mean(T-jrkomwon)
T/mean(aantal)

plot(t,zd)
hold on
stairs(t,z','g')
stairs(t,u','r')
stairs(t,0.5*u.^2,'c')
plot(t,[s S Q]'*ones(1,(T-t0)/dt+1),'-. m');
hold off
```

G.2 Parameter estimation

G.2.1 Calibration

```
clc;clear;
load /Users/paulwigt/Documents/Thesis/DataWoON2006/data
%% Remove outliers
data=data(data.b_org<150000,:);
data=data(data.gap>-100,:);
data=data(data.b_org./data.brutohh<300,:);
data=data(data.b_org./data.brutohh>-100,:);
weights=data.hweegwon./sum(data.hweegwon); % schalen zodat som gewichten 1 is
%% Histogram relative gap
data.rel_gap=log(data.volume_act./data.volume_opt);

f=round(data.hweegwon); % Mag dat(op 4 miljoen huizen)? ja, dat mag!
```

```
edges=linspace(min(data.rel_gap),max(data.rel_gap),100);
[count,bin] = histc(data.rel_gap,edges);
totfreq = accumarray(bin,f,[length(edges) 1]);
totfreq=totfreq/sum(totfreq)/(edges(2)-edges(1));
bar(edges,totfreq,0.5)
xlabel('Difference between the actual and desired consumption of housing
    services: ln(h/h^{*})');
ylabel('Relative frequency');
U=[edges' totfreq];U=double(U);
%% Opt without noise.
var0=[0.4,1];
var0=[0.08,3.1];
var0=[0.1667 1.5046];
[F RESNORM]=lsqcurvefit(@lsqDist,var0,U(:,1),U(:,2));
F(3)=1;
%% Opt. with noise
var0=[0.05,0.3,0.4];     % gives 2%, 43 years -16144

q1=data.gap;
w1=round(data.hweegwon);

% Free variables: sigma gamma noise
[F ConfInt]=mle(q1,'pdf',@lsqDist4,'start',var0,'lowerbound'
    ,[0.03,0.1,0.01],'frequency',w1)
%%
Likelihood=lsqDist4(q1,F(1),F(2),F(3));
Likelihood=sum(log(Likelihood));
%% Report
disp('# Callibrated Variables:')
disp(['Given mu = ' num2str(mu) ' and r = ' num2str(r) ' we find:'])
disp(['sigma = ' num2str(sigma) ' and the moving costs gamma = ' num2str(
    gamma)])
disp(['and the noise has a variance of ' num2str(se) '.'])
disp(' ')
disp(['# Now we can see that optimally, a houshold moves when he lives:'])
disp([' '  num2str(exp(s)) ' times too small or ' num2str(exp(S)) ' times
    too big to optimal ' num2str(exp(Q)) '.'])
disp(' ')
pc=1;Upp1=-2.1e4;
disp(['Gamma = ' num2str(-pc*Upp1*gamma/2680) ' %'])
a=(s-Q);b=(S-Q);
ea=exp(-2*mu*a/(sigma^2));
eb=exp(-2*mu*b/(sigma^2));
Ts=((ea-1)*b+(1-eb)*a);
Ts=Ts/(mu*(ea-eb));
disp(['   T* = ' num2str(Ts) ' years.'])
disp(['The Log-likelihood is: ' num2str(Likelihood)])
```

G.2.2 Function file of density without noise

```
function [F J]=lsqDist(var,edges)
mu=-0.0032;
sigma=var(1);
r=0.05;
gamma=var(2);

c=[mu,sigma,r,gamma];
cbp0=[-1;1;0.1];
[cbp,fval]=fsolve(@(cbp) valuefun(cbp,c),cbp0);
```

```

s=cbp(1);S=cbp(2);Q=cbp(3);
g=2*mu/(sigma^2);

x1=edges(edges<=Q);
x2=edges(edges>Q);

c=(exp(g*Q)-exp(g*S))/(exp(g*Q)-exp(g*s));
A2=-1/(c*exp(s*g)*(Q-s)+exp(g*S)*(S-Q));
A1=c*A2;

phi1=A1*(exp(g*x1)-exp(g*s));
phi2=A2*(exp(g*x2)-exp(g*S));

phi=[phi1; phi2];
phi(edges<s)=0;
phi(edges>S)=0;

F=phi;
```

G.2.3 Function file of density with noise

```
function [F]=lsqDist4(data,sigma,gamma,se)
%second adaption for MLE: From erfc --> erfcx
% Model
mu=-0.0032;
r=0.05;
% Noise (unbiased)
me=0;
% Control Band
c=[mu,sigma,r,gamma];
cbp0=[-0.4;0.5;0.1];
[cbp]=fsolve(@(cbp) valuefun(cbp,c),cbp0);
%cpb=cbp0;
s=cbp(1);S=cbp(2);Q=cbp(3);
g=2*mu/(sigma^2);

x=data;
c=(exp(g*Q)-exp(g*S))/(exp(g*Q)-exp(g*s));
A2=-1/(c*exp(s*g)*(Q-s)+exp(g*S)*(S-Q));
A1=c*A2;
B1=-A1*exp(g*s);
B2=-A2*exp(g*S);

s2=sqrt(2);

argQe=-((x-Q-me)/se+g*se)/s2;
argse=-((x-s-me)/se+g*se)/s2;
argQ=-((x-Q-me)/se)/s2;
args=-((x-s-me)/se)/s2;
argSe=-((x-S-me)/se+g*se)/s2;
argS=-((x-S-me)/se)/s2;

f1=(-A1/2)*(exp(0.5*(g*se)^2+g*(x-me)-argQe.^2).*erfcx(argQe)...
    -exp(0.5*(g*se)^2+g*(x-me)-argse.^2).*erfcx(argse));
f2=(-B1/2)                          .*(erfc(argQ) -erfc(args));
f3=(-A2/2)*(exp(0.5*(g*se)^2+g*(x-me)-argSe.^2).*erfcx(argSe)...
    -exp(0.5*(g*se)^2+g*(x-me)-argQe.^2).*erfcx(argQe));
f4=(-B2/2)                          .*(erfc(argS) -erfc(argQ));
```

```
vars=[sigma,gamma,se];
F=[f1 f2 f3 f4 f1+f2+f3+f4];
F=(f1+f2+f3+f4);
%% Filtering
F(F<=0)=1e-16; % Avoid values equal to zero or negative(considering 1e-16 sufficiently small):
%Filter from NaN values: Consequence is that these MLE's are zero.
a=isnan(F);F(a==1)=1e-16;
```

G.3 Calculation of the welfare loss

```
clc;clear;
% Input
mu=-0.0032;
r=0.05;

% 3% T* = 17
gamma=0.38;sigma=0.135;
% 6% T* = 17
gamma=0.76;sigma=0.192;
% 10% T* = 17
gamma=1.27;sigma=0.248;
% 15% T* = 17
gamma=1.90;sigma=0.303;
% 20% T* = 17
gamma=2.53;sigma=0.350;

gamma=[gamma gamma-1*0.1267 gamma-4*0.1267 gamma-6*0.1267];
%% Berekening voor andere verhuiskosten
for i=1:length(gamma)

NHH=3644000;     % No of households

% Berekening van de Control Band
[cbp,fval]=fsolve(@(cbp) valuefun(cbp,[mu sigma r gamma(i)]),[-0.1 0.1 0]);
s=cbp(1);S=cbp(2);Q=cbp(3);

% Create a vector to integrate along
dx=(S-s)/99;
x=s:dx:S;x=x';

% Evaluate the densities
phi=lsqDist([sigma;gamma(i)],x);

% Evaluate the cost
%C=CostFunction(x,mu,sigma,r,gamma);     % Expected till infinite
C=0.5*x.^2;                             % Actual Loss of Utility in the base year

% Calculate wellfare loss

Weuro2(i)=NHH*(r*CostFunction(x,mu,sigma,r,gamma(i)).*pcpUpp(0))'*phi*dx;

a=(s-Q);b=(S-Q);
ea=exp(-2*mu*a/(sigma^2));
eb=exp(-2*mu*b/(sigma^2));
Ts(i)=((ea-1)*b+(1-eb)*a);
```

```
Ts(i)=Ts(i)/(mu*(ea-eb));

end
Weuro2
[Weuro2(1) Weuro2(1)-Weuro2(2) Weuro2(1)-Weuro2(3) Weuro2(1)-Weuro2(4)]
Ts
Tprocent=[((1/Ts(2))-(1/Ts(1)))/(1/Ts(1)) ((1/Ts(3))-(1/Ts(1)))/(1/Ts(1))
    ((1/Ts(4))-(1/Ts(1)))/(1/Ts(1))]
%% Graphs
clf
hold on
plot(gamma*pcpUpp(0)/2680-6,[7.5934-3.1*sqrt(gamma*pcpUpp(0)/2680)+0.5747*(
    gamma*pcpUpp(0)/2680-6)],'-','Linewidth',1.1);
plot(gamma*pcpUpp(0)/2680-10,[12.6491-4.0*sqrt(gamma*pcpUpp(0)/2680)
    +0.5747*(gamma*pcpUpp(0)/2680-10)],'-.','Linewidth',1.1);
plot(gamma*pcpUpp(0)/2680-15,[18.5903-4.8*sqrt(gamma*pcpUpp(0)/2680)
    +0.5747*(gamma*pcpUpp(0)/2680-15)],':','Linewidth',1.1);
plot(gamma*pcpUpp(0)/2680-20,[24.5967-5.5*sqrt(gamma*pcpUpp(0)/2680)
    +0.5747*(gamma*pcpUpp(0)/2680-20)],'--','Linewidth',1.1);
plot(gamma*pcpUpp(0)/2680-20,[-0.1379*(gamma*pcpUpp(0)/2680-20)],'-','
    Linewidth',0.5);
legend('If \gamma in 2005 was 6%','If \gamma in 2005 was 10%','If \gamma in
    2005 was 15%','If \gamma in 2005 was 20%','Van Ommeren en Van
    Leuvensteijn (2003)')
xlabel('Reduction or increase in the moving costs \gamma_%')
ylabel('Welfare gain in billion euros')
axis([-7 0 0 4.5])
grid
hold off
```

```
function F=pcpUpp(zd)
ph=11643;
hb=0.3570754;
d=0.145;

F=(ph/d)^d;

F=F*ph*exp(2*zd).*(exp(zd)-hb).^(d-1);

F=F/(1-d);

F=F./((-ph*(hb-exp(zd))/d).^d);
```

Bibliography

[1] Centraal Bureau voor Statistiek, *Woononderzoek Nederland(WoON) 2006*, Versie 1.2, January 2006.

[2] Dixit, Avinash K. and Pindyck Robert S., (1994), *Investment under Uncertainty,* Princeton University Press, Princeton NJ.

[3] Donders, Jan, van Dijk, Machiel F., and Romijn, Gerbert, (2010), *De hervorming van het Nederlandse Woonbeleid,* CPB Bijzondere Publicatie 84.

[4] Peter Ekamper and Mila van Huis, (2002), "Verhuisredenen", *DEMOS, Bulletin over bevolking en samenstelling*, Nederlands Interdiciplinair demografisch instituut, Jaargang 18, nov/dec 2002.

[5] Gamarnik, David, (2005), *Lecture notes on advanced stochastic processes*, Fall 2005, Sloan School of Management, Massachusets Institute of Technology.

[6] Geering, H. P., Dondi, G., Herzog, F., Keel, S., (2007), *Stochastic Systems*, IMRT Eidgenössische Technische Hochschule Zürich.

[7] Harrison, J. Michael, Thomas M. Sellke and Allison J. Taylor, (1983), "Impulse Control of Brownian Motion", *Mathematics of Operations Research*, 8:3, pp. 454-66.

[8] Heij, Christiaan, Boer, Paul de, Franses, Philip Hans, Kloek, Teun, Dijk, Herman K. van, (2002), *Econometric methods with applications in business and economics,* Oxford University Press.

[9] Susan Himmelweit, Roberto Simonetti and Andrew Trigg, (2001), *Microeconomics. Neoclassical and Institutionalist Perspectives on Economic Behaviour,* Cengage Learning (Jul. 31st, 2001).

[10] Jeanblanc, Monique, Yor, Marc, Chesney, Marc, (2009), *Mathematical Methods for Financial Markets*, Springer Verlag, London.

[11] Lalley, Steve, (2001), *Lecture notes on Mathematical Finance,* Fall 2001, University of Chicago, http://galton.uchicago.edu/~lalley/Courses/390/

[12] Ommeren, Jos van, Leuvensteijn, Michiel van, (2003), "New Evidence of the effect of transaction costs on residential mobility", *CPB Discussion Paper,* No 18, May 2003.

[13] Romijn, Gerbert, (2000), *Economic Dynamics in Dutch Construction*, Dissertation Erasmus University Rotterdam.

[14] Shreve, Steven E., (2003), *Stochastic Calculus for Finance I, The Binomial Asset Pricing Model,* Springer Verlag, New York.

[15] Shreve, Steven E., (2004), *Stochastic Calculus for Finance II, Continuous Time Models,* Springer Verlag, New York.

[16] Varian, Hal R. (1992), *Microeconomic Analysis,* W.W. Norton & Company, New York, London.