CPB Netherlands Bureau for Economic Policy Analysis

CPB Discussion Paper | 192

Optimal regulation under unknown supply of distributed generation

Rob Aalbers
Viktória Kocsis
Victoria Shestalova

Optimal regulation under unknown supply of distributed generation*

Rob Aalbers† Viktória Kocsis‡ Victoria Shestalova§
CPB Netherlands Bureau for Economic Policy Analysis

November 9, 2011

Abstract

As distributed generation (DG) continues to expand, larger low-voltage networks will be required in the future. However, regulated distribution network operators (DNOs) need to invest in new infrastructure without knowing a relevant determinant of network costs, the future amount of DG. Due to uncertainty, optimal network capacity needs to reflect the expected demand for capacity over all possible DG states. Therefore, not all capacity will be used if a low level of DG occurs. Optimal regulation that is set under asymmetric information about future DG needs to create incentives for the DNO to invest in this 'excess capacity' and also encourage optimal network utilization. In this case, an option menu that includes fixed fees and positive network charges on DG-producers fulfills these requirements and implements the first-best optimum. On the contrary, price-cap and revenue-cap regulation lead to either underinvestment or high information rents to the DNO.

Key words: distributed generation, low-voltage network, investment, regulation, option menu

JEL codes: L12, L51, Q42, Q48

*We would like to thank Paul de Bijl, Jan Boone, Marga Buys-Trimp, Arie ten Cate, Jelte de Jong, Machiel Mulder, and Gijsbert Zwart and the participants at TREIN meetings, EARIE 2011, and NMa Economenluch for helpful comments. We gratefully acknowledge the financial support of Agentschap NL. The responsibility for the contents of this Discussion Paper remains with the authors.

†R.F.T.Aalbers@cpb.nl

‡Corresponding author. V.Kocsis@cpb.nl, address: P.O.Box 80510, 2508 GM The Hague, The Netherlands, tel: +31 70 338 3332

§V.Shestalova@cpb.nl

1 Introduction

In Europe, many low-voltage networks will become obsolete within ten to fifteen years. The imminent depreciation of existing networks requires that distribution network operators (DNOs) invest in new network infrastructure. Concerns arise whether DNOs will invest optimally, particularly in the optimal network size when replacing their old infrastructure with a new one. These concerns arise for two reasons.

First, DNOs face large uncertainty about the diffusion of new technologies. Uncertainty concerns the propagation of small-scale distributed generation (DG) that may be an important driver of network costs. Depending on the cost and the efficiency of DG technologies, such as solar photovoltaic (PV), micro-CHP, and heat pumps,[1] electricity production by households may become a driver of network capacity. However, the replacement of polluting cars by new clean electric cars may also substantially raise households' consumption of electricity. The uncertainty about future demand for network capacity may affect investment into the network because network investments are lumpy, with high investment costs, and have the economic lifetime of 40 to 50 years. This implies that if the invested capacity becomes insufficient in the meantime, extending the network with additional cables is extremely expensive.[2]

Second, regulation influences the DNOs' investment incentives. Regulation can improve incentives by adequately taking into account the effects of new technological developments and the relating uncertainty. The current regulatory practices with respect to DG vary over Europe. For instance, energy regulators in the Netherlands and the UK, have already been considering DG as a potential cost driver. They allow higher revenues to DNOs that have a larger amount of DG connected to the network (see NMa 2010 and Jamasb and Marantes 2011).[3] Furthermore, DNOs in the UK may also impose user tariffs on DG, while in the Netherlands this tariff is set at zero by law (Niesten 2010 and De Joode et al. 2010). In most other EU countries, the regulation does not account for DG and there are no tariffs for DG producers (Nieuwenhout et al. 2010).

Taking both these factors into consideration, we address three questions. First, what is the socially optimal level of investment under uncertainty about the development of DG technologies? Second, how can it be implemented by ex-ante regulation? Finally, what are the effects of other commonly used regulatory regimes, and why are these regimes suboptimal under uncertainty?

We analyze these questions in a one-shot sequential game theoretical model with households, a DNO, and the regulator. Households consume and produce electricity by employing DG devices at home. Both consumption and production fluctuate over time, and the maximum of the peak electricity inflow (peak-consumption) and outflow (peak-production) determines the amount of network

[1] Strictly speaking, heat pumps are not considered as DG-technologies, but they produce electricity as well, similarly to DG.

[2] This is especially true for underground cables because the costs of digging into the ground to lay cables down are substantial.

[3] In the Netherlands, only mid- and large-scale DG producers are considered.

capacity a household needs to buy (measured in kW). Ex ante, the households' peak-consumption is known to all players, but there is uncertainty about DG. In particular, the DG state may turn out to be either low or high. The DNO invests under uncertainty, after which it observes the realized state and sets network tariffs for households. We allow for a three-part tariff, consisting of two separate linear network-capacity tariffs on peak-consumption and peak-production and a fixed fee.[4] Similarly to other theoretical papers on incentive regulation (e.g., Lewis and Sappington 1988), we assume that the information about the realized state of DG is private to the DNO and cannot be verified by the regulator. In practice, the regulator observes only aggregated information about households' peaks, since collecting detailed information on the allocation of DG and consumption peaks over the entire network would involve substantial costs.

Therefore, the regulator cannot write a contract conditional on the DG state.

We find that if a high future DG production is likely – for instance because DG technologies will become cheaper or DG will generate high revenues – then the optimal network capacity is fully determined by DG peak and exceeds the capacity that is needed for peak-consumption.[5] We distinguish this situation as a DG-driven network and further focus on it as it represents the most relevant case for us.[6] Due to uncertainty, it is optimal in this case to install the amount of network capacity that is not fully used in the low DG state in order to be able to accommodate more DG in the high state. In the high state, the DG peak is higher than the consumption peak and there is no excess capacity. This optimum represents the first-best solution. The relating optimal linear tariff on DG capacity - in contrast with the most common EU practice, which is zero - is positive while the linear tariff on peak-consumption is zero. These tariffs encourage optimal network usage and can be seen as an alternative to physical demand rationing, such as network service interruptions. Furthermore, the DG tariff contains a mark-up due to uncertainty; therefore, the fixed charge is negative in order to reduce the relating rents of the DNO. In the low-production state, the network capacity exceeds households' peak-consumption and -production. Therefore, both linear tariffs become zero, and the positive fixed fee compensates for the optimal excess network capacity installed under uncertainty.

We also argue that since the DNO has superior information on the allocation of households' peaks, the regulator can make use of this information by offering an option menu. An option menu contains three-part tariffs for each potential state of the world, which tariffs correspond to optimal prices. In this way, the option menu can implement the first-best optimum. Our results also show that current regulatory practices differ from the optimal pricing scheme. With no

[4] This tariff structure is flexible and is often used in network industries (see e.g., Laffont and Tirole 1991, Lewis and Sappington 1988).

[5] Chen et al. (2006) have put a similar argument forward.

[6] If consumption determines network capacity, then the DNO invests under certainty and sets a single positive consumption and zero DG tariff. The regulator can also observe demand and determine the respective single price cap.

price discretion, as in the case of price-cap regulation, the DNO earns a high information rent. At the other extreme, full price discretion, such as under revenue-cap regulation, ruins investment incentives by allowing the DNO to increase profits by simply rationing household demand for network capacity by means of high tariffs.

Relating literature

Due to the novelty of the problem, no economic literature exists that analyzes the effects of regulation on investment in distributed generation by households and in the optimal network size by the DNO under uncertain demand. Nonetheless, our results relate to the literature on incentive regulation, particularly about information problems on the demand side of the market.

First of all, the literature on optimal investments of regulated monopolies under uncertain demand is limited. Dobbs (2004) shows that intertemporal price-cap regulation provides little investment incentives for the DNO. Since Dobbs analyzes the optimal timing of investments, his results about delayed investments can be translated as underinvestment for our case. However, his paper does not consider asymmetric information about demand and does not specify optimal regulation.

Second, a somewhat more extensive literature exists that analyzes the effects of asymmetric information about demand on pricing decisions and determines the relating optimal regulation. Lewis and Sappington (1988) analyze a single-product monopoly when fixed fees are possible. They recommend an option menu and thus price discretion if the firm's marginal costs are non-decreasing because then the monopoly can employ its superior knowledge about demand when setting prices and achieve the first-best outcome. The most important differences between Lewis and Sappington (1988) and our paper are that we consider multiple products - peak-consumption and -production - and the investment decisions of regulated monopolies

Regarding the first difference, the two-product model of Armstrong and Vickers (2000) is more closely related to our analysis. They find that whether price discretion is necessary depends on the nature of demand shocks, and how shocks influence price elasticities. However, compared to our paper, they assume information asymmetry relating to both products and no fixed fees (i.e., only second-best optimum is possible). Because of these differences, we find unambiguously that to achieve the first-best optimum, the DNO should be offered price discretion in the form of an option menu consisting of two sets of three-part tariffs.

In addition, Armstrong and Vickers also do not consider incentives for investment, particularly under uncertainty about demand, which makes our result different from the standard literature on incentive regulation under unknown demand.

Finally, our results about offering multiple products resembles the peak-load pricing literature (see e.g., Crew et al. 1995). According to this literature, the optimal peak-period tariff is higher than the off-peak tariff because peak drives network costs. In our case of a DG-driven network, DG determines network capacity, and therefore, the linear tariff on DG should be higher than the linear tariff on peak-consumption.

The paper proceeds as follows. In Section 2, we describe our model. We find the social optimum in Section 3 and the relating optimal regulation in section 4. We evaluate several commonly used regulatory regimes in Section 5. In Section 6, we discuss policy implications. In Section 7, we draw conclusions.

2 Model

A monopoly distribution network operator (DNO) provides network infrastructure in a local area, through which households can transport electricity. We consider them as homogenous and therefore from now on focus on a representative household. A household is a consumer and a producer of electricity. We refer to the household's electricity production as distributed generation (DG), for which it needs to install one type of DG-devices, such as solar PVs or micro-CHP.[7] The household has to buy also network capacity to be able to flow electricity out of and into the network at any moment in time. This capacity can be seen as the size or the number of cables laid down between the transformer and the household, and is measured in kW. Peaks in consumption and production over the entire period determine this capacity. We assume that the future cost of installing DG devices can be high or low, thus reducing or increasing peak DG production, respectively. The future development of DG technologies and so the costs are unknown the DNO at the time of investment in a new network. Therefore, to determine the network size, the DNO maximizes its expected profit.

The DG cost is revealed to the DNO and the household only after the network has been built. Then the DNO offers a take-it-or-leave-it contract to the household, who in turn either accepts or rejects it. The contract specifies linear tariffs for peak-consumption and peak-production, and a fixed fee. As we mentioned in the introduction, this type of non-linear pricing is commonly used in network industries. The linear tariffs control network usage and, therefore, represent an alternative to other forms of rationing, such as licenses or bans. At the same time, a fixed fee allows for the efficient marginal-cost pricing and can be used as an instrument to redistribute welfare between the DNO and households.

Furthermore, the DNO's tariffs are subject to regulation. Similarly to the analyses of monopoly regulation with unknown demands by Lewis and Sappington (1988) and Armstrong and Vickers (2000), we assume that actual demands are not observable by the regulator, although it can be verified that the firm is serving all demand at its prices. The regulator has to face high costs in order to observe the actual allocation of peak-consumption and -production over the entire network. This means that regulatory contracts that use ex post information on realized demand are infeasible. In particular, global price-cap regulation

[7] In this model, we only focus on the presence of new technologies in the production side. We could model consumption-side developments, such as electric cars, in a similar way.

whereby the regulator caps the average price offered by the firm using the realized outputs as weights is ruled out. Therefore, we restrict our attention to ex ante regulation regimes. Optimal regulation should be such that it motivates the DNO to install the welfare-maximizing amount of capacity and to achieve its optimal utilization.

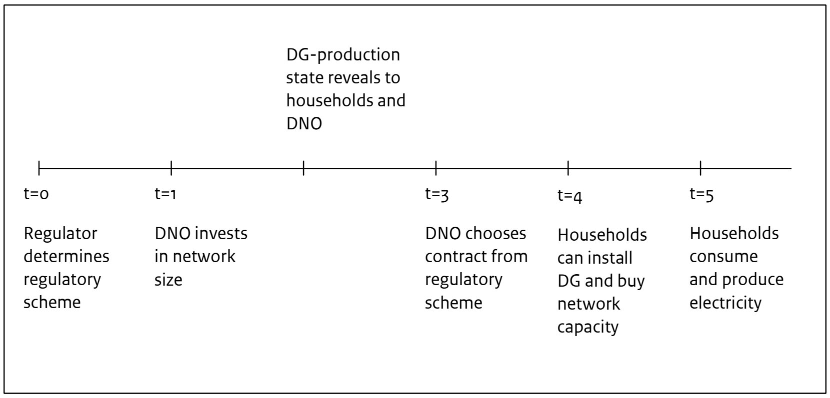

Figure 1. Timing of decisions

We solve this model (illustrated also in Figure 1) by backward induction for two cases. First, we calculate the social optimum as if the regulator operated the DNO and could choose the network capacity and set prices conditional on the DG cost (Section 3). After that, we determine the optimal regulatory contract (Section 4).

2.1 Representative household

We assume that households are homogeneous, therefore we consider a single representative household. This household consumes and produces electricity. Since electricity consumption and production patterns are stochastic and their peak and off-peak moments vary in time, consumption may reach its peak at the moment when there is no production, and vice versa. For example, a solar panel does not produce electricity in the dark evening hours, while consumption may reach its maximum in these hours of the day. On the contrary, at the production peak, which is at daylight, the household may consume very little electricity. A reliable connection to the network has to be such that it accommodates peak-consumption and -production at any moment in time, therefore even in cases when only consumption or production occurs and at the same time peaks. To take this worst-case scenario into consideration, we argue that the household's maximum load approximates peak-consumption q or peak-production z, both measured in kW.[8]

[8] The following mathematical formulation underlies this argument. Let $Q(t)$ and $Z(t)$ denote consumption and production load at time t, respectively. The network capacity a household buys should be as large as it allows for the maximum difference between consumption and production loads over time, that is, $\max_t |Q(t) - Z(t)|$. Using our notations, we know

We assume that households derive utility from both products q and z, and their preferences are separable in them. On top of that, the household has to pay to the DNO to be able to transport electricity into and out of the network. Demand for peak-consumption and -production will therefore depend on these utilities and the capacity tariffs.

We denote the household's preferences for q by the net utility function $v(q)$. Since the household can only increase peak-consumption by having more electric equipment at home, which is increasingly expensive, we assume that the net utility function has an inverted-U shape with a global maximum at $q_{max} = \arg\max v(q) > 0$.[9]

Similarly, we assume that the household's net utility of z is expressed by the function $p(z)$. Since the produced electricity can be sold in the market,[10] this net utility is simply the profit that a household can make at a given peak-production. For simplicity, we assume that the revenue is expressed by a linear function rz. In addition, costs can be seen as relating to a single fixed investment, however, z determines the size of investment. Therefore, DG costs depend on z. For simple exposition, we express DG costs by the convex function $\frac{cz(1+z)}{2}$, where $r, c > 0$. Consequently, we obtain the following quadratic functional form: $p(z) = rz - \frac{cz(1+z)}{2}$.[11] This function has a global maximum at the point $z_{\max} = (r - c/2)/c$.

Due to the uncertainty about the future development of DG technologies, the DG-cost parameter c is initially unknown, only its distribution is common knowledge. This parameter can take two values: c_H corresponds to low DG costs and consequently high DG state (H) and c_L corresponds to high DG costs and so low DG state (L). The probability of c_H and c_L is β and $1 - \beta$, respectively. For the sake of simplicity, we also assume that $c_L = 2r$, implying that DG is not profitable in the low state: $z_L = 0$. Furthermore, we assume that $c_H < 2r$, so that $z_{max,H} > 0$. Because no DG production occurs in state L, we drop the indices H, L next to c and z. Therefore, state H is characterized by the low cost parameter c and the relating peak-production z and net profit $p(z) \geq 0$; and state L is characterized by no production.

The household decides how much q and z to buy from the DNO. We assume that the DNO charges the household three-part tariffs, comprising linear tariffs

---

that $q = \max_t Q(t)$ and $z = \max_t Z(t)$. It is also plausible to assume that there are moments when consumption and production is zero: $\min_t Q(t) = \min_t Z(t) = 0$. As a consequence, it may occur that for $\hat{t} = \arg\max Q(t) : Z(\hat{t}) = 0$ and for $\tilde{t} = \arg\max Z(t) : Q(\tilde{t}) = 0$, where $\hat{t} \neq \tilde{t}$. It then follows that $\max_t |Q(t) - Z(t)| = \max_t |Q(t), Z(t)| = \max(q, z)$.

[9] The function $v(q)$ satisfies the following conditions: $v(0) = 0$; $v' \geq 0$ for $q \leq q_{max}$ and $v' < 0$ otherwise; $v'' < 0$.

[10] We assume that the household is price taker, and therefore, the electricity price is exogenously given. We also assume that the household's total electricity production (kWh) is proportional to its peak-production (kW). However, we ignore the possibility that the household may optimize its peak-production by switching DG facilities on and off. Therefore we do not model the actual electricity production decisions of households.

[11] The function $p(z)$ satisfies: $p(0) = 0$; $p' \geq 0$ for $z \leq z_{max}$ and $p' < 0$ otherwise; $p'' < 0$.

on peak-consumption and peak-production, and a fixed fee:[12,13]

$$\begin{aligned} &(t_{qL}, 0, t_L) \text{ in the low-production state,} \\ &(t_{qH}, t_z, t_H) \text{ in the high-production state.} \end{aligned} \tag{1}$$

While linear tariffs are assumed to be non-negative, we allow the fixed fee to take both positive and negative values. A negative fixed charge is in fact a lump-sum transfer that the network owner makes to households. Such transfers are feasible in reality and can be imposed by the regulator.[14] We can now express the household's surplus in each state as follows:

$$\begin{aligned} H \quad &: \quad S(t_{qH}, t_z, t_H) = v(q_H) + p(z) - t_{qH}q_H - t_z z - t_H, \\ L \quad &: \quad S(t_{qL}, 0, t_L) = v(q_L) - t_{qL}q_L - t_L. \end{aligned}$$

We can determine the household's demand for q and z by maximizing these surplus functions. Maximization yields the following first order conditions (FOCs): $v_i' = v'(q_i) = t_{qi}$ for $i = L, H$ and $p' = p'(z) = t_z$. By inversion, we obtain the respective demand functions: $q_i = q(t_{qi})$ for $i = L, H$ and $z = z(t_z)$. Furthermore, the linear tariffs need to satisfy the conditions $t_{qi} < v'(0)$ and $t_z < p'(0)$ so that the household has a positive net-utility from buying these products. In addition, for q and z always satisfies that $q \leq q_{max}$ and $z \leq z_{max}$. We assume that the household rejects the contract that the DNO offers and buys no network capacity if its surplus is negative $(S(t_{qH}, t_z, t_H) < 0, S(t_{qL}, 0, t_L) < 0)$.

2.2 DNO

The monopoly DNO invests in a new local network and delivers network services to households. Even though households are homogenous with respect to their preferences, they may have different consumption and production patterns that affect the aggregated demand for network services, called the network load. We first derive the maximum load, and then determine the DNO's optimization problem.

Without loss of generality, we assume that the households are also homogenous with respect to their electricity consumption profile. Therefore, the aggregated peak-consumption on the network projected to a representative consumer is also q. With respect to DG, we assume that the households' DG peaks do not necessarily coincide, which is why the total production peak per household may be less than z. We express that by ρz, where ρ $(0 < \rho \leq 1)$ is a parameter

[12] By setting separate prices on q and z, we assume that the DNO is able to measure consumption and production peaks separately. In practice, this is the case with smart meters.

[13] We set the linear tariffs for peak-production in the low state as zero because the maximum peak-production in that state is zero, which will not be affected by any other value of the DG tariff.

[14] Also, in unregulated businesses, companies sometimes make such transfers to consumers: think of free phones provided by telecom operators or presents to new subscribers.

reflecting the degree of simultaneity in DG production.[15,16] This implies that the maximum load on the entire network is equal to $\max(q, \rho z)$. We assume that providing q and z on the network has no cost.

We denote the total network capacity installed by the DNO by k. We assume that the investment costs are $C(k) = Ck$, where C is the marginal cost of building network capacity. For simplicity, we do not include fixed investment costs. If this cost is positive, it is simply covered by the fixed fee. We additionally introduce the technical assumption that $C < v\prime(0)$ to guarantee that the problem has a non-trivial solution.

The DG state becomes known to the DNO before it sets prices to the household. However, the decision on k is taken under uncertainty. Therefore, the DNO maximizes profits in both production states with respect to three-part tariffs (1):

$$\begin{aligned} H &: \quad \pi_H \equiv \pi(t_{qH}, t_z, t_H, k) = t_{qH} q_H + t_z z + t_H - Ck, \\ L &: \quad \pi_L \equiv \pi(t_{qL}, 0, t_L, k) = t_{qL} q_L + t_L - Ck. \end{aligned}$$

To determine network capacity, the DNO maximizes its expected profit:

$$E\pi(.) = \beta\pi(t_{qH}, t_z, t_H, k) + (1-\beta)\,\pi(t_{qL}, 0, t_L, k).$$

where β is the probability of high DG production.

We assume that outages and non-price rationing of demand for capacity are not allowed in any state of the world. Therefore, the network capacity should be sufficient for peak-consumption and peak-production in both states:

$$\begin{aligned} k - q_H &\geq 0, \\ k - q_L &\geq 0, \\ k - \rho z &\geq 0. \end{aligned} \tag{2}$$

2.3 Regulator

The regulator maximizes the weighted sum of the households' expected surplus and the DNO's expected profit:

$$EW(RC) = \beta\left(S_H(RC) + \alpha\pi_H(RC)\right) + (1-\beta)\left(S_L(RC) + \alpha\pi_L(RC)\right), \tag{3}$$

where β is the probability of high DG production, RC stands for the regulatory contract offered to the firm, and α denotes the weight on profit, $0 \leq \alpha \leq 1$. If

[15] Here the simultaneity coefficients are constant and do not depend on the number of network users. This is a reasonable assumption, since in practice, the ratio of per household network capacity and the respective individual peaks converges to a constant value as the number of households increases.

[16] For example, if individual DG peaks of households are fully correlated ($\rho = 1$), such as the case for solar PV, each household has its peak at the same time. Then the effective contribution of each household to the peak-load on the network coincides with the individual production peak: $\rho z = z$. In contrast, when individual peaks are spread over time ($\rho < 1$), e.g., in the case of micro-CHP or heat pumps, then the household's effective contribution to the peak-load becomes smaller: $\rho z < z$.

$\alpha = 0$, the regulator only cares about households and is indifferent about how much profit the DNO obtains. If $\alpha = 1$, the regulator equally weighs household surplus and the DNO's profit, as if households fully owned the firm.

We consider two cases: first, we find the social optimum as if the regulator operated the DNO and could choose the network capacity and set prices conditional on the DG cost; and second, we determine the optimal regulatory contract. In each case, the regulator has to consider several constraints to guarantee the physical and financial feasibility of the DNO's operation in every production state.

To determine the social optimum, the regulator needs to meet *capacity constraints* (2).

Besides, we assume that the DNO has limited liability, implying that profits should be non-negative in any state of the world. The reason for using this assumption is as follows. Even though the firm signs the contract with the regulator before knowing which production state will occur, it sets prices after learning the realized production state, such as the case in Armstrong and Vickers (2000) and Laffont and Martimort (2002). This means that the regulatory contract should satisfy the following *participation constraints*:

$$\pi(t_{qL}, 0, t_L, k) \geq 0, \tag{4}$$

$$\pi(t_{qH}, t_z, t_L, k) \geq 0. \tag{5}$$

When determining the optimal contract, the regulator also needs to take the above mentioned constraints into consideration. In addition, the regulator wants to make use of the DNO's private information. Therefore, the regulatory contract has to provide sufficient incentives for the DNO to set tariffs according to the realize state of the world. An option menu fulfills this goal (see, Joskow 2008). In the offered option menu, each three-part tariff is designated for one production state. Hence, it has to be designed in a way that the DNO has no incentive to choose the other tariff. Let $\left(T_q^L, T_z^L, T^L\right)$ denote the three-part tariff that is intended to be set in the low-production state and $\left(T_q^H, T_z^H, T^H\right)$ denote the tariffs intended for the high-production state. Therefore, tariffs must satisfy the following *incentive compatibility constraints* of the DNO:[17]

$$\begin{aligned} H &: T_q^H q^H + T_z^H z^H + T^H > T_q^L q^L + T_z^L z^L + T^L \\ L &: T_q^L q^L + T^L > T_q^H q^H + T^H. \end{aligned} \tag{6}$$

3 First-best optimum

In this section, we calculate the social optimum as if the DNO had no information advantage and the regulator could choose the amount of capacity and set prices. First, we determine the equilibrium and then we analyze its characteristics.

[17] Network costs also affect the profits of the DNO, however they are the same in every state (Ck) and therefore fall out of the inequalities.

3.1 Equilibrium

Similarly to tariffs (1), we denote the regulated prices as (T_{qH}, T_z, T_H) and $(T_{qL}, 0, T_L)$ in the high- and low-production states, respectively. The regulator's objective function is:

$$\begin{aligned} & EW(T_{qH}, T_{qL}, T_z, T_L, T_H, k) = \\ = \quad & \beta\left(S(T_{qH}, T_z, T_H) + \alpha\pi(T_{qH}, T_z, T_H, k)\right) \\ & + (1-\beta)\left(S(T_{qL}, 0, T_L) + \alpha\pi(T_{qL}, 0, T_L, k)\right) \end{aligned}$$

The regulator maximizes social welfare subject to capacity constraints (2) and participation constraints (4) and (5). The DNO's non-negative profit conditions always bind, thus we obtain that the optimal fixed charges exactly compensate for the part of network costs that are not covered by the linear prices: $T_L = Ck - T_{qL}q_L$, $T_H = Ck - T_{qH}q_H - T_z z$. The optimization problem then simplifies to:

$$\begin{aligned} & \max_{q_L, q_H, z, k} \{\beta\left(v(q_H) + p(z) - Ck\right) + (1-\beta)\left(v(q_L) - Ck\right) \\ & + \lambda_{qH}\left(k - q_H\right) + \lambda_{qL}\left(k - q_L\right) + \lambda_z\left(k - \rho z\right)\}. \end{aligned} \tag{7}$$

The first observation from this expression is that the equilibrium value of the expected welfare does not depend on α, that is, how the regulator weighs profits. This occurs because the DNO makes zero profit in every state.

From (7), the first order conditions (FOCs) are:

$$\frac{\partial}{\partial q_L} \quad : \quad (1-\beta)v'(q_L) = \lambda_{qL} \tag{8}$$

$$\frac{\partial}{\partial q_H} \quad : \quad \beta v'(q_H) = \lambda_{qH} \tag{9}$$

$$\frac{\partial}{\partial z} \quad : \quad \beta p'(z) = \lambda_z \rho \tag{10}$$

$$\frac{\partial}{\partial k} \quad : \quad \lambda_{qH} + \lambda_{qL} + \lambda_z = C \tag{11}$$

By using equations (8) and (9), we can show that $q_L = q_H$. We prove this by showing that $q_L \neq q_H$ is impossible. Suppose $q_L < q_H$. Then the capacity restriction on q_L cannot be binding: $k - q_L > 0$, and therefore, the shadow price of this capacity restriction must be zero, $\lambda_{qL} = 0$. Substituting this in the first FOC, we obtain that $v'(q_L) = 0$. Therefore, $q_L = q_{max}$, which contradicts to our presumption that $q_L < q_H$. Similarly, $q_L > q_H$ is also impossible. Therefore, they must be equal: $q_L = q_H = q$. As a consequence, the social planner sets the same linear charge on peak-consumption in both states: $T_{qL} = T_{qH} = T_q$. This result arises since the prices are set after the network has been laid down. Consequently, from a social perspective, it is better to increase network utilization as much as possible, rather than ration the network load by setting a higher price in some state.

By combining and simplifying (8)-(11), we obtain the following set of equations determining the equilibrium quantities of both products, q^*, z^* and the required network capacity, k^*:

$$v'(q^*) + \frac{\beta}{\rho} p'(z^*) = C \tag{12}$$
$$k^* = \max(q^*, \rho z^*)$$
$$q^* \leq q_{max}$$
$$z^* \leq z_{max}$$

Based on (12), we formulate the following proposition.

Proposition 1 *Let k^* denote the solution of (12). There may be three types of optimal networks in equilibrium, which are characterized by the following necessary and sufficient conditions: (i) DG-driven network: $q^* = q_{\max} < k^* = \rho z^*$ if $\frac{\beta}{\rho} p'(\frac{q_{\max}}{\rho}) > C$; (ii) consumption-driven network: $\rho z^* = \rho z_{\max} < k^* = q^*$ if $v'(\rho z_{\max}) > C$; (iii) network-cost-driven network: $q^* = \rho z^* = k^*$ for all other specifications.*

As capacity increases, both term in the left-hand side (LHS) of expression (12) decreases. It is, therefore, optimal to increase network capacity as long as the total marginal benefits from an additional capacity unit are still above the marginal cost, up to the point when they become equal to each other. At some level of network capacity, the net utility from one product may reach satiation (i.e., one term in the LHS of expression (12) turns to zero), while the marginal utility of the other product (i.e., the other term in the LHS of expression (12)) still exceeds the marginal network cost C. In such a case, any further increase in network capacity is purely driven by the second product.

A similar result is known in the literature on peak-load pricing (see Crew et al. 1995). In this literature, the same physical facility is used to produce at two periods: 'peak' and 'off-peak', which are treated as two different products. It is namely the 'peak' period that determines the capacity in equilibrium and bears the infrastructure costs. Our result resembles the same principle. In a DG-driven network, z drives network capacity and hence needs to bear network costs. In a consumption-driven network, q corresponds to the 'peak' period. A network-cost-driven network represents a special case, in which both products drive network capacity.

As a consequence of Proposition 1, in DG-driven networks, the optimal network size exceeds the peak-load in the low-production state. This excess capacity ($\rho z^* - q_{\max}$) is necessary because of the uncertain DG production. On the contrary, in consumption- and network-cost-driven networks no excess capacity is necessary in any state of the world because peak-consumption, which is (also) the determinant of network size, is certain.

Given that $v'(q) = T_q, p'(z) = T_z$ and $T_L = Ck - T_{qL} q_L, T_H = Ck - T_{qH} q_H - T_z z$, the optimal tariffs in the case of different network types write:

Network type	*State*	*Tariffs* (T_q, T_z, T)
Consumption-driven	L, H	$(C, 0, 0)$
DG-driven	L	$(0, 0, C\rho z^*)$
	H	$(0, \frac{1}{\beta}C\rho, -\frac{1-\beta}{\beta}C\rho z^*)$
Network-cost-driven	L	$(v'(q^*), 0, (C - v'(q^*))q^*)$
	H	$(v'(q^*), p'(z^*), (C - v'(q^*))q^* - p'(z^*)z^*)$

(13)

In the case of a *consumption-driven network*, optimal tariffs reduce to a single linear tariff on q, because the network size and the load is determined by peak-consumption, which is common knowledge and so uncertainty plays no role in the optimal decision. Consequently, T_q exactly covers the marginal cost and $T_z = 0$. In addition, the transfer is also zero because of certainty: the DNO makes no profit or loss that has to be compensated for. This case can be also seen as a benchmark under certainty about demand.

In the case of a *DG-driven network* in the high state, the linear fee on DG contains a mark-up, which is due to uncertainty. The more likely it is that DG becomes cheap, the smaller the mark-up will be. As a consequence of a mark-up, in the high state the fixed-fee is negative: it distributes the excess profit due to this mark-up back to the household. In the low state, the household simply pays a positive transfer that covers network costs. Tariffs on peak-consumption can be set at zero because this capacity never exceeds peak-production and thus the network size.

In a *network-cost-driven network*, tariffs satisfy $T_q + \frac{\beta T_z}{\rho} = C$. This expression shows that $T_q < C$, implying that $T_L > 0$. Yet, $T_H \gtrless 0$ depending on the magnitude of the marginal network cost C.

Our results correspond to the first-best social optimum. The maximum expected social welfare is:

$$EW^* = v(q^*) + \beta p(z^*) - Ck^*. \tag{14}$$

3.2 Comparative statics

Let us now analyze how the values of certain parameters affect the optimal network size k^* and the occurrence of different network types. We are particularly interested in the effects of DG-cost parameter c, its probability β, the marginal network cost C, and the DG technology dependent simultaneity of peak-production ρ.

Taking derivatives of the first implicit function in (12) with respect to these parameters allows us to evaluate these effects, which are summarized in the following proposition (for the proof see Appendix 8.1):

Proposition 2 *The optimal network capacity k^* is non-decreasing in β and non-increasing in C and c. The effect of a marginal change in parameter ρ on k is generally not monotonous and depends on the relative coefficients of risk aversion of function p in equilibrium. In particular, k^* is non-decreasing in ρ as long as the relative coefficient of risk aversion of function p is smaller than 1.*

To interpret the proposition, let us focus our attention on a DG-driven network, which is the most relevant case in our analysis. There, in the social optimum $k^* = \rho z^*$. It implies that the network capacity and peak-production are proportional to each other. First, we analyze the effects of β and C. We can easily see that the optimal tariff households pay for peak-production, $T_z = \frac{\rho C}{\beta}$ is ceteris paribus decreasing in the probability of low DG cost and increasing in the network cost. The smaller the β, the larger the expected marginal network cost is. It implies a higher mark-up and a lower optimal peak-production and network size. Similarly, a larger marginal network cost has to be covered by a larger linear tariff that reduces z and k.

Second, DG-cost parameter c only indirectly influences peak-production. We know that the marginal net utility p' is monotonously decreasing in c, that is, the larger the c, the less marginal net utility a household obtains for a given peak-production. As a consequence, for a given tariff, a larger DG cost implies a smaller optimal peak-production and network size.

Finally, effect of the simultaneity factor ρ is not monotonous. On the one hand, the more production peaks coincide, the larger the effective marginal network cost (ρC) is, lowering peak-production in equilibrium. On the other hand, for peak-production that largely correlate, for instance as in the case of solar PV, a larger network capacity is required. Therefore, whether network capacity increases or decreases depends on the relative magnitude of these two opposite effects. For peak-production substantially lower than at satiation, the network capacity is non-decreasing in ρ. In practice, it indicates a larger network in the case of solar PV, where peaks coincide (high ρ), than a network mainly with micro-CHP, where peaks distribute more evenly during the day (lower ρ). For peak-production close to satiation, the opposite result holds.

4 Optimal regulation: an option menu

It is a well-known result in the literature that in the case of homogenous consumers and the firm having complete information about the demand, an unregulated monopoly sets total network capacity and linear tariffs at the efficient level if it is allowed to charge a fixed fee (see e.g., Lewis and Sappington 1988 and also Appendix 8.3 for our model). Then the firm appropriates household surplus by this fixed fee and achieves a positive profit. If the regulator values consumer surplus just somewhat more than the firm's profit ($\alpha < 1$ in expression (3)), a positive profit creates an expected welfare loss compared to our social optimum (14):

$$\Delta EW^{UR} = (1-\alpha)\left(v(q^*) + \beta p(z^*) - Ck^*\right) > 0, \tag{15}$$

where UR stands for the unregulated monopoly case. This welfare loss necessitates regulation.

However, as we discussed in Section 2.3, the regulator has an information disadvantage compared to the DNO. The regulator can reduce the information rent to the minimum by offering an option menu because it allows the monopoly some level of price discretion (see also Lewis and Sappington 1988 and Armstrong and Vickers 2000).

The menu specifies two three-part tariffs, from which the DNO can choose: $\left(T_q^L, T_z^L, T^L\right)$ denotes the three-part tariff that is intended to be chosen in the low-production state and $\left(T_q^H, T_z^H, T^H\right)$ denotes the tariffs intended for the high-production state. For this contract, several constraints need to satisfy: *capacity constraints* (2), *participation constraints* (4) and (5), and *incentive compatibility constraints* (6).

In the proposition, we show that given these constraints the regulator can design an option menu for the DNO in such a way that in achieves the first-best optimum. In other words, the firm will accept this offer, install the optimal amount of capacity k^*, and, depending on the production state realized, picks the three-part tariff within the menu that leads to maximum welfare (see the proof in Appendix 8.2).

Proposition 3 *The following menu implements the social optimum:*

$$\begin{aligned}\left(T_q^L, T_z^L, T^L\right) &= (T_q, 0, T)\\ \left(T_q^H, T_z^H, T^H\right) &= (T_q, T_z, T - T_z z^*),\end{aligned}$$

where $T_q = v'(q^*)$, $T_z = p'(z^*)$, and $T = Ck^* - T_q q^*$ are the socially optimal tariffs based on quantities computed from (12).

As a consequence, tariffs in the option menu correspond to the tariffs in the social planner's case (13).

This means that the regulation will be able to achieve the first-best optimum: the socially optimal level of capacity and its utilization despite its information disadvantage about the demand. Compared to the standard literature on incentive regulation, which most commonly receives second-best optimum, we obtain the first-best outcome for two reasons. First, the DNO is allowed to set a fixed fee that enables efficient (marginal cost) pricing for both products and achieves optimal utilization and investment incentives. Second, the investment costs are known by the regulator (the information asymmetry is present on the demand side), therefore the regulator does not need to face efficiency losses. Since two states are possible, the DNO has some level of price discretion and can choose a contract according to the realized demand state. Consequently, the DNO's information rent reduces to zero. Note also that the tariffs in the menu depend only on model parameters, not on other values (e.g., not on the DNO's realized revenue, which may be observable by the regulator).

5 Evaluating common regulatory schemes

In this section, we consider two commonly used ex ante regulatory schemes: three-part-price-cap and revenue-cap regulation. By analyzing these regulatory schemes, we can explain why 'no' or 'full' price discretion is suboptimal in the presence of uncertainty about DG.

5.1 Three-part price cap

From Section 3.1, we know that in the case of a consumption-driven network, the presence of uncertainty does not influence the equilibrium outcome and therefore, a single three-part tariff can implement the social optimum without any welfare loss: $\Delta EW_{cons}^{PC} = 0$, where PC stands for price cap and $cons$ refers to a consumption-driven network.

For the other two network types, a single three-part-tariff cap cannot achieve the social optimum. In order to obtain optimal network capacity and its efficient utilization in the high state, the regulator can set the linear tariffs on q and z equal to the efficient linear tariffs: $T_q = v'(q^*)$ and $T_z = p'(z^*)$, where $v'(q^*) + \frac{\beta}{\rho}p'(z^*) = C$. Households will then demand the optimal amounts of both products (q^*, z^*), which forces the firm to install the optimal amount of network capacity k^*. However, the firm will only accept such a contract if the single lump-sum transfer provides non-negative profits in both states. This fixed fee needs to be positive in order to guarantee that, the firm breaks even in the low-production state. With this fee the DNO earns a positive profit in the high state. This positive profit is due to the information advantage of the DNO. Consequently, the expected welfare loss corresponds to the information rent of the DNO and thus depends on α:

$$\Delta EW_{DG/cost}^{PC} = (1 - \alpha)\beta T_z z^* > 0 \tag{16}$$

where $DG/cost$ refers to a DG- or a network-cost-driven network.

Let us now consider how $T_z = 0$, as commonly used in the EU, affects network capacity and social welfare. If the regulator sets $T_z = 0$, then the demand for product z in the high-production state is equal to z_{max}. This exceeds the optimal quantity in both DG-driven and network-cost-driven networks. To meet this demand, the DNO would need to install $k = z_{max}$, which implies an overinvestment. The firm will only be willing to accept a regulatory contract (i.e., to break even and deliver reliable service) if the transfer covers the cost Cz_{max}. In the high state, it again implies a positive information rent. To sum up, zero linear tariff on peak-production leads to overinvestment and welfare loss due to extra rents to the firm. Consequently, the welfare loss is even higher than (16).

5.2 Revenue-cap regulation

Suppose that the DNO is free to set its tariffs, but the regulator caps the revenue it makes. Let R denote this revenue cap. If the revenue cap is not binding, the DNO acts as in the unregulated case, that is, it sets efficient linear prices that induce the socially optimal peak-consumption and -production, and appropriates total consumer surplus by the fixed fee (see Appendix 8.3). If household and the DNO's surpluses are valued equally ($\alpha = 1$), this solution is socially optimal. However, if household surplus has a higher weight in total welfare ($\alpha < 1$), the welfare loss equals (15):

$$\Delta EW^{RC} = \Delta EW^{UR}.$$

If the revenue cap is binding, the DNO's profit can be expressed as $R - Ck$. The DNO will still extract maximum surplus but now it will also minimize its investment costs. Because it cannot make as large profit as in the unregulated case, it will invest less. Furthermore, the fixed fee will leave the household without any surplus. Consequently, the regulator is better off by not imposing a more stringent revenue cap on the firm.[18]

5.3 Summary

In this section, we analyzed the effects of two different ex ante regulatory regimes on the network size and social welfare. We have shown that each of these regimes is suboptimal in comparison to the option menu proposed in Section 4. A single three-part tariff cap can implement the social optimum only in the case of a consumption-driven network because there the information problem is not present. Otherwise allowing no price discretion leads to a welfare loss due to the information rent of the DNO: the optimal network size can only be achieved by allowing the firm to earn a positive profit in the high-production state. Revenue-cap regulation achieves the socially optimal network size only if the revenue cap is not binding, but this case is equivalent to the unregulated monopoly case; and therefore, as long as the regulator attaches some value to consumer surplus, there is a welfare loss. Under a stricter revenue cap that is set below the monopoly revenue, the DNO will underinvest and still abstract the remaining surplus away from households. As a consequence, a high level of price discretion leaves the DNO with its monopoly profit.

6 Policy implications and further discussion

In our analysis, we showed that the regulator, when setting regulatory constraints, needs to take into account the expected developments in the electricity sector. The major policy implications from our result are the following. First,

[18] The regulator may also restrict tariffs to linear only, that is, the fixed fee $T_i = 0, i = H, L$. It can be easily shown that again a non-binding revenue constraint, i.e., the unregulated case leads to the highest network capacity. This monopoly k is, however, smaller than in the presence of fixed charges. Even though this solution implies underinvestment compared to the social optimum, if α is (very close to) zero, an unregulated monopoly without fixed fees is socially more desirable than one with fixed fees. With fixed fees the monopoly extracts total household surplus, while with linear tariffs only the household is left with some positive surplus.

with the expected shift from consumption-driven towards DG-driven networks, DNOs' costs can no longer be purely born by electricity users, but the burden needs to be shared with DG producers. Secondly, since the network costs depend on network load, rather than on the amount of electricity that flows through the network, the tariffs should be also set on loads (kW), rather than on electricity consumption or production flows (kWh). Finally, the DNO has more detailed information on the distribution of production and consumption peaks over the network. Therefore, it has an information advantage over the regulator about the households' demand for network services, as uncertainty resolves. Regulation by an option menu provides the DNO with incentive to use this information optimally, and works more efficiently than 'traditional' regulation regimes, such as pure price-cap regulation, which does not give the possibility to rebalance the tariffs, or revenue-cap regulation, which gives too much freedom to reallocate costs towards one or another tariff.

Note that the current regulatory practices are not fully in line with these implications. Nieuwenhout et al. (2010) describe currently applied regulatory strategies in several EU-member states and stress that revising regulation is necessary to address this issue. Many countries do not impose network charges on distributed generators (see also De Joode et al. 2010 and Niesten 2010) and base their regulation on electricity flow (kWh). This distortion affects incentives with respect to investment and network utilization for both DG producers and DNOs, and may potentially cause problems in these countries. However, some countries have already made adjustments towards including DG in their regulatory practices. For instance, in the Netherlands, where sliding-scale regulation is applied, medium-scale DG (such as greenhouses) has already been included in the benchmarking that determines an upper limit for the allowed revenues. Furthermore, the tariff structure was adjusted to include prices on load (per kW) rather than traditional consumption charges (per kWh). Yet, prices on DG are still set at zero by law. The UK was ahead of other European countries with introducing new types of regulation. Since 2005, the traditional price-cap regulation has been extended with new elements, including network charges (both fixed charges and user charges) on generators. Although the idea of using option menus for resolving information asymmetry is not new (Laffont and Tirole 1993), very few examples in regulatory practice apply this approach. For instance, in early 1990s in the US, the Federal Communication Commission tried to introduce a menu for regulating the Bell companies, but this practice was abandoned after a few years (Vogelsang 2006).

We close this section with a brief discussion of potential further applications of our results. First, because we model households' utility on the consumption and production side symmetrically, we predict similar results in the case of uncertainty relating to increasing consumption, e.g., due to the future expansion of electric cars. In this case, the focus would shift to consumption-driven networks.[19]

[19] Note, however, that assuming uncertainty about peak-production and peak-consumption will make the problem much more complicated and may change results. See Armstrong and Vickers (2000).

Second, although we assumed homogeneous households in our model, our conclusions are also valid if DG producers may differ in scale, such as households and greenhouses. Furthermore, the model can be extended to incorporate different DG technologies. In that case, the tariffs should be differentiated by technology type, so that users of a technology type with a higher load on the network pay a higher linear charge. Higher linear charges provide an alternative to other forms of non-price rationing, such as disconnecting households that cause high network costs. Another example is solar PVs, which have large and simultaneous production during sunny summer days, loading the network heavily in this period. Instead of limiting the number of solar PVs in a region, the DNO can also reduce DG production by setting a higher linear charge on peak-production from solar PVs.

Finally, even though our model is a one-shot regulation game, we expect our results to provide guidance also in a dynamic setting, when the DNO invests in the first period and uncertainty resolves gradually, during which the DNO can constantly adjust tariffs. However, if the DNO may invest at any moment in time, it may delay investments until it knows demand with more certainty (see e.g., Dobbs 2004).

7 Conclusions

In this paper, we analyzed optimal investment in DG and network capacity and optimal regulation under uncertainty about peak DG production. We focused on the following questions: What is the socially optimal level of investment? Can it be implemented by ex ante regulation? And, what are the effects of other commonly used regulatory regimes on the network size, network utilization, and social welfare?

First, we find that if the diffusion of DG technologies is very likely, for instance, because they become cheap or highly profitable, peak-production, rather than peak-consumption, will determine the optimal network size. In other words, the optimal network capacity is larger than needed for consumption only. However, due to uncertainty, this network capacity is not fully used in the low DG state. Second, since the DNO has better information about the future DG production than the regulator, optimal regulation needs to leave the DNO with some level of price discretion. Consequently, the regulator can reduce the information rent of the DNO by offering an option menu. In the optimal menu, linear charges on peak-consumption and peak-production provide optimal investment incentives and secure efficient network usage in each potential DG state, and the fixed fees compensate for the social costs of uncertainty by redistributing welfare between the firm and households. We stress that allowing a positive linear charge on DG production is crucial for creating right investment and utilization incentives. Finally, the most commonly used ex ante regulatory regimes, such as a simple price cap or revenue cap, are suboptimal compared to the optimal option menu because they allow the DNO to earn high profits, which may be detrimental to welfare.

References

[1] Armstrong, M and Vickers, J, 2000, Multiproduct Price Regulation Under Asymmetric Information, *Journal of Industrial Economics*, 48, 137-160.

[2] Chen, M, Cho, I-K, and Meyn, S P, 2006, Reliability by design in distributed power transmission networks, *Automatica*, 42(8), 1267-1281.

[3] Crew, M A, Fernando, C S, and Kleindorfer, P R, 1995, The theory of peak-load pricing: A survey, *Journal of Regulatory Economics*, 8, 215-248.

[4] De Joode, J, van der Welle, A, and Jansen, J, 2010, Distributed generation and the regulation of distribution networks, In: D N Gaonkar (Eds), Distributed Generation, InTech.

[5] Dobbs, A M, 2004, Intertemporal Price Cap Regulation Under Uncertainty, *The Economic Journal*, 114, 421–440.

[6] Jamasb, T and Marantes, C, 2011, Electricity Distribution Networks: Investment and Regulation, and Uncertain Demand, CWPE 1115 and EPRG 1104.

[7] Joskow, P, 2008, Incentive Regulation and Its Application to Electricity Networks, *Review of Network Economics*, 7(4), 547-560.

[8] Laffont, J-J and Martimort, D, 2002, *The theory of incentives: the principal-agent model*, Princeton University Press, Princeton, N.J., The USA.

[9] Laffont, J-J and Tirole, J, 1991, *Competition in Telecommunications*, MIT Press: Cambridge, MA.

[10] Laffont, J-J and Tirole, J, 1993, *A Theory of Incentives in Regulation and Procurement*, MIT Press: Cambridge, MA.

[11] Lewis, T and Sappington, D, 1988, Regulating a Monopolist with Unknown Demand, *The American Economic Review*, 78(5), 986-998.

[12] NMa, 2010, Methodebesluiten, Accessible at http://www.nma.nl/regulering/energie/elektriciteit/regulering_regionale_netbeheerders/methodebesluiten/default.aspx.

[13] Niesten, E, 2010, Network investments and the integration of distributed generation: Regulatory recommendations for the Dutch electricity industry, *Energy Policy*, 38(8), 4355-4362.

[14] Nieuwenhout, F, Jansen, J, van der Welle, A, Olmos, L, Cossent, L, Gómez, T, Poot, J, Bongaerts, M, Trebolle, D, Doersam, B, Bofinger, S, Lichtner, P, Gerhardt, N, Jacobsen, H, Ropenus, S, Schröder, S, Auer, H,

Weissensteiner, L, Prüggler, W, Obersteiner, C, Zach, K, 2010, Regulatory strategies for selected Member States (Denmark, Germany, Netherlands, Spain, the UK), WP6, http://www.improgres.org/fileadmin/ improgres/user/docs/D8_Final.pdf

[15] Vogelsang, I, 2006, Electricity transmission pricing and performance-based regulation, *The Energy Journal*, 27, 97-126.

8 Appendix

8.1 Social optimum

Proof. (Proposition 2) *For a DG-driven network*: Recall that the function $p(z) = rz - \frac{cz(1+z)}{2}$ is increasing at a decreasing rate up to the point $z_{\max} = (r - c/2)/c$, and that $\rho C < p'(0) = r - \frac{c}{2} > 0$, implying that the problem always results in $k > 0$.

The FOC of the original problem w.r.t. z writes $\frac{\beta p'(k/\rho)}{\rho} - C = 0$. We denote the left-hand side of this FOC by F. Using the rules of implicit function differentiation, we obtain $\frac{dk}{dx} = -\frac{\partial F_z/\partial x}{\partial F_z/\partial k}$ for any parameter of interest $x \in \{C, c, \beta, \rho\}$. From our assumptions, it follows that $\frac{\partial F_z}{\partial k} = \frac{\beta p''(k/\rho)}{\rho^2} = -\frac{\beta c}{\rho^2} < 0$. Therefore, for any x, the sign of the derivative $\frac{dk}{dx}$ coincides with the sign of $\frac{\partial F_z}{\partial x}$:

$$
\begin{aligned}
\frac{\partial F}{\partial C} &= -1 < 0, \\
\frac{\partial F}{\partial c} &= -\frac{\beta}{\rho}\left(\frac{1}{2} + \frac{k}{\rho}\right) < 0, \\
\frac{dF}{d\beta} &= \frac{p'(k/\rho)}{\rho} > 0, \text{ since } k < \rho z_{\max} \\
\frac{\partial F}{\partial \rho} &= \beta \frac{(-k/\rho)p''(k/\rho) - p'(k/\rho)}{\rho^2} = \frac{p'(k/\rho)}{\rho^2}(\sigma_p(k/\rho) - 1) \gtrless 0,
\end{aligned}
$$

where $\sigma_p(z) = \frac{-zp''(z)}{p'(z)} = \frac{zc}{r - c/2 - cz} = \frac{zc}{cz_{\max} - cz} = \frac{z}{z_{\max} - z} \gtrless 1$ is the coefficient of relative risk aversion of the net utility function p. While the effects of C, c and β are monotonous, the effects of ρ depend on $\sigma_p(z)$. If $\sigma_p(z) < 1$, then $\frac{\partial F}{\partial \rho} > 0$. The proposition for the other network types can be proved similarly. ■

8.2 Optimal regulation

Proof. (Proposition 3) The prices in this contract mimic the prices that the social planner would choose in the low- and high-production states respectively, and we have shown that those prices correspond to the optimal quantities (q^*, z^*). Therefore, the DNO must invest in the network capacity that satisfies $k^* \geq \max(q^*, \rho z^*)$. Because the DNO does not gain anything by installing more or less capacity than needed, it will install exactly the optimal amount: $k^* = \max(q^*, \rho z^*)$.

What remains to be shown is that the incentive compatibility conditions (6) and the capacity constraints (2) hold. In the case of $z^* = 0$, which corresponds to the consumption-driven network type, both options in the menu become identical.

Therefore, we only need to check the incentive compatibility constraints for $z^* > 0$. Because $T_q^L = T_q^H = T_q$, in the low-production state, the ICC simplifies to $T^L > T^H$. In the equilibrium it writes as $T > T - T_z z^*$ and therefore $0 > -T_z z^*$. As a consequence, the DNO chooses the tariffs intended for the low state. Peak-consumption given these tariffs, that is q^*, will never exceed the network capacity: $k^* = \rho z^* \geq q^*$. Therefore the capacity constraints satisfy.

In the high-production state, if the DNO chooses the contract intended for the low state with $T_z = 0$, then the demand for z would increase to $z_{\max}$. This network load would exceed the network capacity: $k^* = \rho z^* < \rho z_{\max}$. Since non-price rationing is ruled out (by assumption), the firm has to choose tariffs for the high state: $\left(T_q^H, T_z^H, T^H\right)$. ■

8.3 Unregulated DNO

Because an unregulated DNO can extract consumer surplus by a fixed fee, it will charge the efficient linear tariffs. As a consequence, it will invest in the optimal network size. However, the profits are higher than in the social optimum due to the fixed fees. The proof is as follows. The DNO solves the following problem:

$$\begin{aligned}&\max_{t_{qL},t_{qH},t_z,t_L,t_H,k}\{\beta(t_{qH}q_H + t_z z + t_H) + (1-\beta)\left(t_{qL}q_L + t_L\right) - Ck\\&+\lambda_{qH}\left(k - q_H\right) + \lambda_{qL}\left(k - q_L\right) + \lambda_z\left(k - \rho z\right)\\&+\eta_H\left(v(q_H) + p(z) - t_{qH}q_H - t_z z - t_H\right) + \eta_L\left(v(q_L) - t_{qL}q_L - t_L\right)\}\end{aligned}$$

The two constraints included in the last row require that consumer surplus should be positive in every state, since the profit maximizing DNO must ensure that households are willing to accept the contracts. Because the DNO can set the fixed fee such that it fully extracts consumer surplus, we can simplify the optimization:

$$\begin{aligned}&\max_{q_L,q_H,z,k}\{\beta(v(q_H) + p(z)) + (1-\beta)\,v(q_L) - Ck\\&+\lambda_{qH}\left(k - q_H\right) + \lambda_{qL}\left(k - q_L\right) + \lambda_z\left(k - \rho z\right)\}.\end{aligned}$$

This optimization problem is then exactly the same as for the social optimum. For similar reasons as for the social optimum, $q_H = q_L$: it is not reasonable for the DNO to ration demand for q as long as it has already installed this capacity and is able to receive positive marginal profits from selling these capacities to households. With these simplifications, we get exactly the same expressions as (12). As a result, the firm is willing to invest in the socially optimal network size as long as the sum of its expected marginal revenues from both products (which is also equal to the sum of households' marginal benefits in this case) exceeds marginal costs. Similarly to the social optimum, three types of networks may emerge according to the conditions described in Proposition 1. The unregulated DNO charges marginal cost prices and extracts total consumer surplus by fixed fees. In a general form, the tariffs and the expected social welfare write:

$$\begin{aligned}(t_{qL}, 0, t_L) &= (v'(q^*), 0, v(q^*) - v'(q^*)q^*)\\(t_{qH}, t_{zH}, t_H) &= (v'(q^*), p'(z^*), v(q^*) + p(z^*) - v'(q^*)q^* - p'(z^*)z^*)\\EW^{UR} &= \alpha\left(v(q^*) + \beta p(z^*) - Ck^*\right),\end{aligned}$$

where $v'(q^*) + \frac{\beta}{\rho}p'(z^*) = C$ and $k^* = \max(q^*, \rho z^*)$.

Publisher:

CPB Netherlands Bureau for Economic Policy Analysis
P.O. Box 80510 | 2508 GM The Hague
T (070) 3383 380

November 2011 | ISBN 978-90-5833-531-9