CPB Netherlands Bureau for Economic Policy Analysis

CPB Discussion Paper | 257

Optimal Discount Rates for Investments in Mitigation and Adaptation

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September 5, 2013

Abstract

This paper develops a theory of asset pricing in which discount rates for investments in all assets, including adaptation and mitigation, are endogenously determined. Exploiting the characteristics of adaptation and mitigation in terms of climatic risk, I show that adaptation requires a lower discount rate, whereas mitigation does not. Inspection of the Ramsey rule reveals that the social discount rate equals the social rate of return on optimally-invested aggregate wealth minus the risk premium on that wealth. This risk premium compensates investors for bearing market risk and the risk of unfavorable changes in the economy resulting from climate change.

Keywords: discounting; investments; mitigation; adaptation; endogenous climate risk; risk premium.

JEL Codes: G11; G12; H43; Q51; Q54.

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1 Introduction

Uncertainty abounds in projections of future climate change. We do not know with any reasonable amount of precision the relationship between emissions, the (ultimate) concentration of carbon dioxide in the atmosphere and average global temperature. For example, Meinshausen et al. (2009) show that, under the IPCC A1FI scenario, the 95% confidence interval for average global temperature in 2100 ranges from 3.0 °C to 8.4 °C. More recently, Roe and Bauman (2012) consider the temperature uncertainty belonging to the IPCC A2 scenario for the period 2010 to 2300. They show that the 95% confidence interval for the average global temperature in 2100 ranges from 3.0 °C to 6.4 °C, which widens to 3.0 °C to 9.3 °C in 2200 and 3.0 °C to 10.6 °C in 2300.

Uncertainty abounds in projections of the economic impact of climate change as well. In his comprehensive literature review, Tol (2009) lists fourteen estimates of this so-called output effect for selected benchmark warmings. For the ten estimates that consider a benchmark warming of 2.5 °C, he finds an average output effect of -0.7% of GDP with a standard deviation of 1.2% of GDP. More importantly, none of these estimates has considered benchmark warmings of more than 3.0 °C.[1] Hence, our 'knowledge' on the economic impact of temperature rises beyond 3.0 °C is entirely based on extrapolation.

This paper theoretically investigates how these temperature-related uncertainties affect the discount rates of mitigation and adaptation investments. To that extent, I develop a theory of asset pricing in which productivity and temperature are uncertain, interdependent and endogenously determined. Consumers determine, through their consumption and investment decisions, both the expected rate of return on invested wealth and the flow of pollution, adjusting their decisions as new information on the rate of return and temperature becomes available. In equilibrium, asset markets clear and determine the equilibrium rates of return or the 'optimal' discount rates for all assets, including investments in adaptation and mitigation and the risk-free asset.

The main contributions of this paper are fourfold. The first is to formally characterize mitigation and adaptation in terms of the underlying model parameters by making explicit how the rate of return for mitigation and adaptation depend on temperature: investments in adaptation reduce the impact of unexpected changes in temperature on the rate of return, at least for sufficiently high temperatures; investments in mitigation reduce pollution. The second is to use this characterization to show that the discount rate for adaptation investments is lower than that of 'comparable' other, i.e., non-adaptation, investments *provided* that the risk characteristics of adaptation investments somehow 'counterbalance' the risk characteristics of the economy at large. Formally, this intuitive idea of counterbalancing not only requires that adap-

[1]Of the other four estimates, two consider a benchmark warming of 1.0 °C, and two others a benchmarking warming of 3.0 °C.

tation investments are less vulnerable to temperature shocks, but also that unexpected increases in temperature decrease the rate of return on optimally-invested aggregate wealth *and* that increases in temperature increase the marginal value of wealth. I also conclude that the optimal discount rate for mitigation investments may be either lower or higher than the optimal discount rate of 'comparable' investments, because the characterization of mitigation does not allow for stronger conclusions regarding its discount rate. The third comtribution is to give a full characterization of the social discount rate in a model in which productivity and temperature are uncertain, interdependent and endogenously determined. The fourth is that the risk premium on aggregate wealth compensates consumers not only for bearing market risk, but also for bearing the risk attached to any unfavorable changes in the investment opportunity set brought about by rising temperatures. When higher temperatures both raise the marginal value of wealth and are associated with lower wealth levels, the risk premium on the second risk is positive: ceteris paribus, it raises the risk premium on optimally-invested aggregate wealth.

The model used here is related to the general equilibrium theory of asset pricing as developed by Cox et al. (1985). They characterize discount rates for the case where the probability distribution of current output depends on a number of exogenous state variables, which are themselves randomly changing over time. Special cases of the Cox et al. (1985) model have been used to investigate a number of widely diverging issues. For example, Obstfeld (1995) studies how global portfolio diversification affects expected consumption growth and national welfare. Detemple (1986) considers the effect of partial information on asset prices and concludes that the classic asset-pricing relationships continue to hold under an appropriate reinterpretation of the state variables: asset prices are not determined by the underlying unobservable state variables, but by their conditional expectation. Duffie and Zame (1989) extend the model by Cox et al. (1985) to multi-agent economies. None of these papers considered endogenous (environmental) risks.

In addition, this paper is related to the growing literature on the term structure of the risk-free rate or social discount rate. Gollier (2007) shows that the shape of the term structure depends on our view of how the uncertainty on future aggregate consumption evolves with the time horizon. In particular, he shows that a downward-sloping term structure is justified whenever the uncertainty about aggregate consumption increases at a rate larger than what would be obtained by a pure random walk. Gollier (2010) extends this result to the case of a multi-attribute utility function when the sensitiveness of the environmental quality to changes in GDP per capita is uncertain. Weitzman (2010) studies the term structure of the risk-free rate by using a Ramsey optimal growth model in which future productivity is both uncertain and persistent. He shows that the risk-free rate is declining over time, because of a 'fear factor' associated with catastrophic low-productivity states. All these works consider optimal discount rates for investments with *certain* payoffs in a world with *exogenous* risks.

Finally, this paper relates to a small, but growing, literature that considers discount rates for climate investments. Using a portfolio approach in a two-period model in which the risk-free rate is exogenous, Sandsmark and Vennemo (2007) show that the rate of return on investments that reduce the likelihood of damage must be lower than the risk-free rate. Aase (2011) considers asset pricing in a production economy and discusses possible risk adjustments of the discount factor for mitigation projects. Finally, Gollier (2012) and Weitzman (2012) investigate the term structure of discount rates for mitigation investments in the presence of catastrophes, where mitigation investments are defined as low-beta hedge assets for unexpected changes in consumption. None of these papers provides a systematic framework for analyzing optimal discount rates when productivity and temperature are uncertain, interdependent and endogenously determined.

The setup of this paper is as follows. Section 2 presents the model and the characterization of mitigation and adaptation investments, in terms of the model's parameters. Section 3 derives the optimal discount rates for all investments and discusses the relationship of these results to the literature. Section 4 concludes.

2 The Model

I propose a simple general equilibrium model with two aggregate consumption goods. The first, produced consumption, is denoted by $C(t)$, where the index t denotes continuous time. The second, a non-produced consumption good, is denoted by $T(t)$. In the context of climate change, it is natural to think of temperature, in which case non-produced consumption is a bad. The representative consumer maximizes discounted expected utility over these consumption goods according to

$$E\int_0^\infty e^{-\delta t}U[C(t),T(t)]\mathrm{d}t, \tag{1}$$

where $U(.)$ denotes the utility function of the representative consumer and δ is the utility discount rate or pure rate of time preference. Utility is increasing in consumption ($U_C > 0$), decreasing in temperature ($U_T < 0$), and concave in both consumption and temperature ($U_{CC}, U_{TT} < 0$). Temperature $T(t)$ evolves according to the following stochastic differential equation:

$$\mathrm{d}T(t) = \theta(T(t))P(t)\mathrm{d}t + s(T(t))'\mathrm{d}\omega(t). \tag{2}$$

In this equation, $P(t)$ is the economy's pollution flow at time t, $\omega(t)$ is an $n+1$ dimensional Wiener process in $\mathbb{R}^{n+1}$, $s(T)$ is the $n+1$ dimensional temperature-volatility vector and $s(T)'s(T)$ is the variance of temperature.[2] According to (2), the expected change in temperature $\theta P\mathrm{d}t$ over the small interval $(t,t+dt)$ is proportional to the pollution flow P and depends on temperature T through θ. The latter may be roughly thought of as a short-term 'climate

[2]From now on, I suppress subscripts where possible.

sensitivity' parameter capturing the expected impact of pollution on temperature in the interval $(t, t+dt)$.

Production of the physical consumption good may take place in any of n different sectors. I assume there is free entry into all sectors and that consumers invest in physical production directly, in effect creating their own firms. Consumers and firms are competitive and act as price takers in all markets. Let $K_i(t)$ denote the physical investment in sector i. When the output of each sector is continuously reinvested, $K(t) = (K_1(t), K_2(t), \ldots, K_n(t))'$ obeys the following system of stochastic differential equations:[3]

$$\mathrm{d}K(t) = I_K \alpha(T(t))\mathrm{d}t + I_K G(T(t))\mathrm{d}\omega(t), \tag{3}$$

where $\alpha(T)$ is the corresponding vector of the expected private rates of return in these sectors, $G(T)$ is the return-volatility matrix and I_K is an $n \times n$ diagonal matrix whose ith diagonal element is the ith component of $K(t)$. The expected private rate of return refers here to the rate of return that would apply if the temperature process in (2) were exogenous. I assume that both α and G are continuous, twice differentiable functions of temperature T, which implies that the investment-opportunity set is nonconstant – it changes with temperature through the expected private rates of return α and the return-volatility matrix G.[4] The matrix GG' denotes the covariance matrix of the rates of return, while Gs denotes the covariance between incremental returns on the production processes and unanticipated changes in temperature $T(t)$. Depending on the sector, this covariance may be either positive, negative or zero. Normalizing the Wiener process such that a positive number is associated with an increase in temperature gives $s(T) > 0$. Consequently, negative return volatilities indicate that higher temperatures are associated with lower rates of return. Notice that the production processes have constant returns to scale in the sense that the distribution of an investment's rate of return does not depend on its size. Finally, let γ_i denote sector i's pollution intensity, which is measured per unit of physical investment. Sector i's pollution flow at time t, $P_i(t)$, is then equal to $\gamma_i K_i(t)$.[5]

Using the basic structure laid down by equations (2) and (3), adaptation and mitigation are now naturally introduced by slightly tailoring the sector definition as follows. Instead of defining sectors according to the *type of good* they produce, I define sectors according to *the way they produce* the consumption good. There are three of these sectors: the first is the regular sector, the second the adaptation sector, and the third the mitigation sector. One way to

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[3]To facilitate interpretation, the stochastic differential equation for sector i is given by $\mathrm{d}K_i(t)/K_i(t) = \alpha_i(T)\mathrm{d}t + g_i(T)\mathrm{d}\omega(t)$, where g_i denotes the ith row vector of G.

[4]The investment-opportunity set is described by the transition probabilities for the rate of return on each investment over the next trading interval $(t, t+\mathrm{d}t)$. See Merton (1992) for a discussion of this concept.

[5]With minor modifications, the framework can be expanded to include a more comprehensive description of the environment by including additional state variables – such as oceanic and atmospheric carbon stocks, the sea level and biodiversity – and their relations (cf. Cox et al. (1985)).

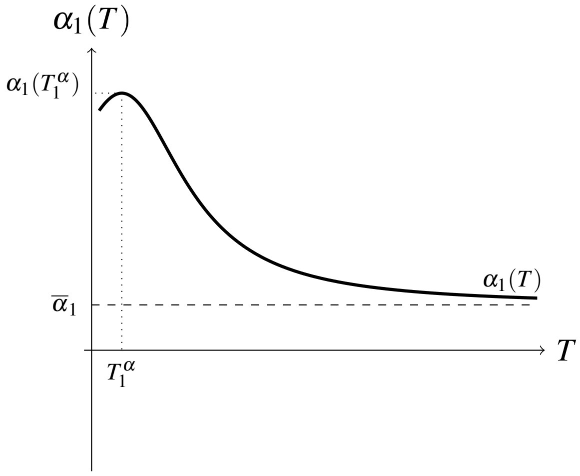
think about this sector definition is that sectors combine a common technology to produce the consumption good with a sector-specific technology. Examples of such sector-specific technologies are coal-generated electricity for the regular sector, dikes or heat-resistant crops for the adaptation sector and wind energy or concentrated solar power for the mitigation sector. The key question now is how both adaptation and mitigation investments differ from regular investments in terms of their expected private rates of return $\alpha_i(T)$ and their return-volatility $g_i(T)$. Surprisingly, this question has received virtually no discussion in the literature. A notable exception is Sandsmark and Vennemo (2007), but in their theoretical model the treatment of climate change is only implicit as the properties of climate investments are stated in terms of consumption instead of temperature or environmental quality. Instead, in this paper I will make explicit the way in which the rate of return on adaptation and mitigation investments depends on temperature. For the sake of reference, however, I start with the more common assumptions regarding the rate of return on regular investments, which are commonly stated by means of the damage function. For example, Nordhaus (2008) assumes that the damage function is of the quadratic form $1/[1+(T/\psi)^2]$, which – ignoring depreciation – implies that the expected private rate of return is also of the quadratic form and is given by $\alpha_0/[1+(T/\psi)^2]$.[6] Here, α_0 is the expected private rate of return at $T=0$, whereas ψ is an exogenously chosen parameter. Two characteristics stand out. The first is that the expected private rate of return has a unique maximum at $T=0$. The second is that the private expected rate of return approaches zero as $T\to\infty$. I slightly generalize these assumptions and assume that the expected private rate of return in the regular sector $\alpha_1(T)$ is single peaked at $0\le T=T_1^\alpha<\infty$; that it is strictly increasing in T for $T<T_1^\alpha$ and strictly decreasing in T for $T>T_1^\alpha$;[7] and, finally, that it approaches $\overline{\alpha}_1$ when $T\to\infty$. Figure 1 visualizes these assumptions.

For reasons that will become clear later, I will refrain from making assumptions on both the expected private rate of return for adaptation ($\alpha_2(T)$) and mitigation investments ($\alpha_3(T)$). Turning next to the assumptions on sector i's return volatility g_i, I will differentiate 'shocks' *according to their origin* into climatic and economic 'shocks'. Climatic shocks originate in the climate system; economic shocks originate in the economy. Examples of the former are the unexpected release of methane from sinks, unexpected changes in the atmospheric water-vapor concentration and unexpected changes in solar intensity. Examples of the latter are unexpected changes in productivity, technology or demand. Both types of shocks may, and in general will, affect both temperature and productivity. To formalize, define $\omega(t)\equiv(\omega_e(t),\omega_c(t))'$. Here, $\omega_e(t)$ and $\omega_c(t)$ are Wiener processes of dimension n_e and n_c, representing economic and

[6] Take $\mathrm{d}K(t)=[1-D(T)]\alpha_0K(t)\mathrm{d}t$, where $D(T)$, the damage function as a fraction of output, is given by $1-1/[1+(T/\psi)^2]$. Dividing both sides by $K(t)$ gives $\mathrm{d}K(t)/K(t)=\alpha_0/[1+(T/\psi)^2]\mathrm{d}t$, implying that the (expected) rate of return equals $\alpha_0/[1+(T/\psi)^2]$.

[7] These assumptions are in line with observations by Tol (2009, p. 34): 'some estimates [...] point to initial *benefits* of a modest increase in temperature, followed by losses as temperatures increase further.'

Figure 1: Private rate of return in the regular sector



climatic shocks, respectively. Correspondingly, define the return volatility $g_i \equiv (g_{ei}, g_{ci})$ and the temperature volatility $s \equiv (s_e, s_c)$. The following assumption formalizes the idea that adaptation alleviates the impact of temperature shocks, at least for sufficiently high temperatures.

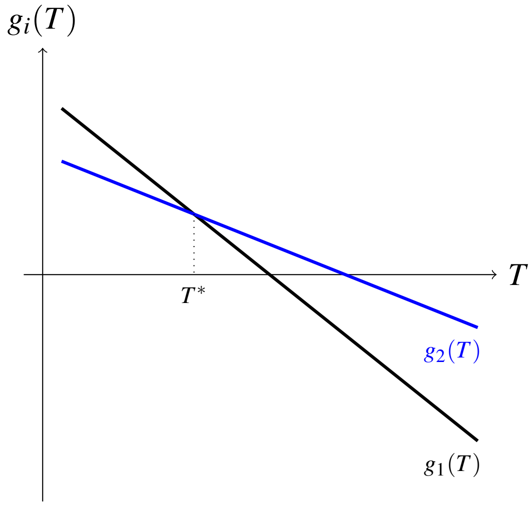
To make the assumptions on the return volatility g_i in each sector explicit differentiate between *climatic* and *economic* 'shocks'. Climatic shocks originate in the climate system, change temperature and affect the economy. Examples include the unexpected release of methane from sinks, unexpected changes in the atmospheric water-vapor concentration or unexpected changes in solar intensity. Economic shocks originate in the economy, affecting temperature through their (possible) impact on pollution. Examples are unexpected changes in productivity, technology or demand. To formalize, define $\omega(t) \equiv (\omega_e(t), \omega_c(t))'$. Here, $\omega_e(t)$ and $\omega_c(t)$ are Wiener processes of dimension n_e and n_c, representing economic and climatic shocks, respectively. Correspondingly, I define $g_i \equiv (g_{ei}, g_{ci})$ and $s \equiv (s_e, s_c)$. To capture that adaptation alleviates - for sufficiently high temperatures - the impact of temperature shocks, I assume that, compared to the regular sector, the adaptation sector is associated with a higher return volatility from climatic shocks. The following assumption formalizes this idea.

Assumption 1. *There exists a T^* such that for $T < T^*$, we have $g_{c1}(T) > g_{c2}(T)$, while for $T > T^*$, we have $g_{c1}(T) < g_{c2}(T)$,*

where the comparison of the vectors $g_{c1}(T)$ and $g_{c2}(T)$ is element wise. Figure 2 illustrates assumption 1 for the case of a single climatic shock. Let $g_{cij}(T)$ denote the jth element in the vector $g_{ci}(T)$. Temperatures below T^* are then associated with $g_{c1j}(T) > g_{c2j}(T)$, whereas temperatures above T^* are associated with $g_{c1j}(T) < g_{c2j}(T)$. For example, compared with traditional crops, heat-resistant crops may respond less favorably to a climatic shock for temperatures below T^* and vice versa.

Remark 1. Surprisingly, assumption 1 cannot be extended to the mitigation sector, because

Figure 2: Return volatility from a climatic shock in the regular and adaptation sectors



there seems to be no compelling reason to assume that a sector's return volatility from climatic shocks g_{ci} and its pollution intensity γ_i are systematically related. That is, two sectors that happen to have the same pollution intensity may very well have completely different return volatilities from climatic shocks, whereas two sectors that happen to have different pollution intensities may very well have equal return volatilities from climatic shocks. For example, the rates of return on concentrated solar power and wind energy will respond differently to unexpected changes in either wind speed or sun hours, *despite* the fact that both technologies have a pollution intensity of zero. Moreover, it seems not unreasonable to assume that unexpected changes in wind speed will have no effect on either the rate of return on coal-fired power or that on nuclear power, notwithstanding their very different pollution intensities. In addition, unexpected increases in temperature may very well affect the rate of return on both of these technologies in similar ways through the availability of cooling water. Therefore, any systematic relationship between a sector's return volatility from climatic shocks and its pollution intensity appears to be nonexistent. Finally, notice that regular, adaptation and mitigation investments may also differ in terms of their return volatility from economic shocks g_{ei}. However, there seems to be no truly compelling reason for adaptation or mitigation investments to have a systematically different return volatility from economic shocks than regular investments.[8]

Finally, the pollution intensity in the mitigation sector is zero as renewables are used to pro-

[8]The available literature is almost silent on the properties of adaptation and mitigation regarding risk. A notable exception is Sandsmark and Vennemo (2007), who assume that 'climate' investments will both reduce portfolio risk and alter the future distribution function for returns. Since the only source of aggregate fluctuations in their model originates from climate change, these 'climate' investments are in fact a combination of adaptation investments, which reduce portfolio risk, and mitigation investments, which alter the future distribution function of returns through temperature.

duce the aggregate consumption good. The pollution intensities in the regular and adaptation sectors are assumed to be equal and strictly positive.

Assumption 2. $0 = \gamma_3 < \gamma_2 = \gamma_1$.

Besides investing in the regular, adaptation or mitigation sector, the representative consumer can also lend and borrow at an endogenously determined risk-free rate r and invest in at least one contingent claim to amounts of the consumption good. The value of this claim is governed by the following stochastic differential equation:

$$\mathrm{d}F = (F\beta - \zeta)\mathrm{d}t + Fh'\mathrm{d}\omega(t) \tag{4}$$

and will in general depend on all variables necessary to describe the state of the economy. In (4), $F\beta$ denotes the total mean return on the contingent claim, while ζ denotes the payout received. Notice that (4) can be derived within the context of the model (see Appendix A). The variance of the rate of return on this claim is given by $h'h$.

Having defined the investment-opportunity set in this way, the representative consumer will allocate his available wealth among the investment opportunities in the basis and the riskless opportunity, borrowing or lending.[9] Let a be a vector whose ith element denotes the share of wealth invested in sector i and let b be a scalar denoting the share of wealth invested in the contingent claim. The budget constraint of the representative consumer is described by[10]

$$\mathrm{d}W = \left(a'(\alpha - r1)W + b(\beta - r1)W + rW - C\right)\mathrm{d}t + W\left(a'G + bh'\right)\mathrm{d}\omega(t), \tag{5}$$

where aW is a vector whose elements denote the amount of wealth invested in each of the production processes, bW denotes the amount of wealth invested in the contingent claim, and 1 is a 3×1 unit vector. According to (5), expected changes in wealth are determined by the expected private excess return on wealth, i.e., the expected private return on wealth in excess of the risk-free rate, plus the risk-free return on total wealth minus the flow of consumption. The representative consumer maximizes his lifetime utility (1) over consumption and the investment strategy, a and b, subject to the budget constraint (5) and the temperature equation (2). Setting up the Bellman equation gives[11]

$$\delta J(W,T) = \max_{C,a,b}\left\{U(C,T) + \frac{E\left(\mathrm{d}J(W,T)\right)}{\mathrm{d}t}\right\}, \tag{6}$$

[9]The basis is defined as the set of production processes and contingent claims such that any other contingent claim can be written as a linear combination of the assets in the basis. See Merton (1977) for a complete description of this concept.

[10]See Merton (1971) for a detailed explanation of the form of the budget constraint. Notice that by definition, we have $K_i(t) = a_i(t)W(t)$.

[11]Throughout the paper, I assume that the value function is well defined and that an equilibrium to (6) exists.

where the maximum is subject to $C \geq 0$ and $a > 0$.[12] Using (2) and (5), $P = \sum \gamma_i K_i(t) = \gamma' K(t)$, and the fact that, in equilibrium, firms invest all attracted capital $K = aW$(6) can be written as

$$\begin{aligned}\delta J(W,T) = \max_{C,a,b} \Bigg\{ & U(C,T) + W\mu(W)J_W + \theta a'\gamma W J_T \\ & + \frac{1}{2}(W^2 a'GG'a + 2W^2 a'Ghb + W^2 bh'hb)J_{WW} \\ & + (Wa'Gs + Wbh's)J_{WT} + \frac{1}{2}s'sJ_{TT} \Bigg\},\end{aligned} \tag{7}$$

where $W\mu(W) = E(\mathrm{d}W)/\mathrm{d}t$. For the value function, subscripts denote derivatives with respect to the states W and T. Differentiating (7) with respect to the controls C, a and b gives the first-order conditions

$$0 = U_C - J_W, \tag{8a}$$
$$0 = (\alpha - r1)WJ_W + \theta\gamma W J_T + (GG'a + Ghb)W^2 J_{WW} + GsWJ_{WT}, \tag{8b}$$
$$0 = (\beta - r)WJ_W + (h'G'a + h'hb)W^2 J_{WW} + h'sWJ_{WT}. \tag{8c}$$

An equilibrium is defined as a set of stochastic processes $(r, \beta; a, C)$ satisfying (8) and the market-clearing conditions $\sum a_i = 1$ and $b = 0$. Before stating the main results, I characterize the behavior of the economy in response to increases in temperature. To this end, let $\bar{g}_c(T) = a' G_c(T)$ represent the return volatility on aggregate wealth with respect to climatic shocks. Here $G_c(T)$ is defined as the matrix whose ith row is given by $g_{ci}(T)$. A climatic shock is called *unfavorable* when the return volatility on aggregate wealth with respect to climatic shocks is negative, i.e., $\bar{g}_c(T) < 0$. The scarce empirical evidence suggests that the contemporaneous covariance between the rate of return on aggregate wealth and temperature increases may indeed be negative: using an unbalanced panel data set of 38 countries and an average coverage of 20 years, Bansal and Ochoa (2011) show that countries close enough to the Equator have a negative contemporaneous covariance between the return on equity and temperature increases. They also show that this covariance is positive for a number of European countries, which are in the group of countries furthest away from the Equator. This suggests that, at least for European countries and for historical temperature levels, an unexpected increase in temperature may also be *favorable*. Of course, both the sign and magnitude of this covariance might change when climate change unfolds and temperature rises.

Next, and following Merton (1992), I define an increase in the state variable T to be *unfavorable* when $J_{WT} > 0$. In that case, higher temperatures are associated with a higher marginal

[12] The case of strictly positive investment provides for a more streamlined setting. The analysis can be extended to allow for zero investment in some sectors (Cox et al., 1985).

value of wealth and, by (8a), with a higher marginal value of consumption. In case utility is not state dependent, we have $U_T = 0$, which implies again by (8a) that $J_{WT} = U_{CC}\hat{C}_T$. Hence, we must have that $\text{sgn}(J_{WT}) = -\text{sgn}(\hat{C}_T)$, i.e., unfavorable increases in temperature are associated with decreases in consumption. Again, at modest temperatures, it might well be the case that increases in the state variable temperature are in fact favorable, i.e., $J_{WT} < 0$.

3 Optimal discount rates

The model proposed in the previous section allows the characterization of the equilibrium expected rates of return for all investments, including investments in adaptation, mitigation and the risk-free asset. These rates of return can be interpreted as the *optimal* discount rates associated with these investments: they not only recognize that both the rate of return on invested wealth and changes in temperature are uncertain; they also take account of the fact that wealth and temperature are interdependent, as temperature affects wealth and wealth – through the level of pollution – affects temperature; and finally, they take into account that (the level of) risk is endogenous and determined by the consumer's consumption and investment decisions. I start by characterizing the risk premium on optimally-invested wealth.[13]

Proposition 1. *The risk premium on optimally-invested wealth, ϕ_W, is equal to*

$$\left(\frac{-J_{WW}W}{J_W}\right)\left(\frac{var(W)}{W^2}\right)+\left(\frac{-J_{WT}}{J_W}\right)\left(\frac{cov(W,T)}{W}\right) \tag{9}$$

PROOF: See Appendix A.

Proposition 1 states that, in equilibrium, consumers are compensated for bearing market risk and for bearing the risk attached to any unfavorable changes in the investment-opportunity set brought about by temperature. The compensation for market risk is the familiar compensation required by a one-period risk-averse mean-variance investor (cf. Merton (1992)). It is given by the product of the coefficient of relative risk aversion of the value function $\frac{-J_{WW}W}{J_W}$ and the variance of the rate of return on optimally-invested wealth $\frac{var(W)}{W^2}$. The compensation for bearing risk attached to unfavorable changes in the investment-opportunity set $\frac{-J_{WT}}{J_W}\frac{cov(W,T)}{W}$ arises because the investment-opportunity set depends on temperature and is therefore stochastic. The risk premium (9) is a function of aggregate wealth W and temperature T. Ceteris paribus, the uncertainty associated with changes in the investment-opportunity set increases this risk premium when increases in the state variable T are unfavorable and $cov(W,T)$ is negative. In that case, higher temperatures are simultaneously associated with lower wealth and a higher marginal value of wealth. Using (9), I now characterize the Ramsey rule in a dynamic stochastic general equilibrium model of climate change.

[13] In what follows, $cov(W,T)$ denotes the covariance of changes in optimally-invested wealth with changes in the state variable temperature, while $var(W)$ denotes the variance of changes in optimally-invested wealth.

Proposition 2. *The Ramsey rule is given by*[14]

$$\hat{a}'\alpha + \hat{a}'\theta\gamma\frac{J_T}{J_W} - \left(\frac{-J_{WW}W}{J_W}\frac{var(W)}{W^2} + \frac{-J_{WT}}{J_W}\frac{cov(W,T)}{W}\right) = r(W,T) =$$
$$\delta - \frac{U_{CC}}{U_C}E(\mathrm{d}\hat{C}) - \frac{U_{CT}}{U_C}E(\mathrm{d}T)$$
$$- \frac{U_{CCC}}{U_C}\frac{1}{2}var(\hat{C}) - \frac{U_{CCT}}{U_C}cov(\hat{C},T) - \frac{U_{CTT}}{U_C}\frac{1}{2}var(T). \quad (10)$$

PROOF: See Appendix A.

Before turning to the intuition behind (10), notice that the Ramsey rule in a dynamic stochastic general equilibrium model is, in general, truly stochastic, thereby precluding any comparative static analysis (Breeden, 1986). A common way to overcome this disadvantage is to make sufficiently specific assumptions on either preferences or stochastic processes, which allows an analytical solution to the model. For example, Heal (2009) assumes that the development of climate change is certain, Gollier (2010) assumes that consumption and climate change are exogenously determined and Weitzman (2010) assumes that there exists some unspecified and catastrophic productivity risk. In doing so, however, the ability to derive *optimal* discount rates for investments in adaptation and mitigation for the case in which temperature is both uncertain and endogenous is lost.[15] As it is this paper's objective to derive optimal discount rates for investments in adaptation and mitigation, I will refrain from making more specific assumptions on either the utility function or the stochastic process of wealth and temperature.

Turning to the interpretation of proposition 2, the LHS of (10) represents the rate of return on savings. An increase in savings earns a rate of return equal to the sum of the expected social rate of return on wealth $\hat{a}'(\alpha + \theta\gamma\frac{J_T}{J_W})$ minus the risk premium on optimally-invested wealth ϕ_W. The expected social rate of return consists of the expected private rate of return on wealth $\hat{a}'\alpha$ minus a compensating payment of $-\hat{a}'\gamma\theta J_T/J_W$ to keep expected lifetime utility constant. Here, $\hat{a}'\gamma\theta$ is the expected increase in temperature resulting from an increase in savings and $-J_T/J_W$ is the compensating variation in wealth required to offset changes in temperature and keep expected lifetime utility constant (Breeden, 1986).

The RHS of (10) describes how the risk-free rate required by the consumer to increase the supply of savings depends on preferences, the stochastic processes of consumption and

[14]Throughout the paper, a hat ˆ above a variable denotes an optimal value.

[15]This can be seen from (7), where the term $\theta\gamma' aWJ_T$ prevents the model's analytical solution, even in the case of logarithmic utility. To see this, take $U(C) = \ln C$ and guess that $J(W,T) = M(T)\ln W + N(T)$. From (7), we obtain $\delta(M\ln W + N) = -\ln M + \ln W + (\hat{a}'\alpha M - 1) + \theta\gamma'\hat{a}M_T W\ln W + \theta\gamma'\hat{a}N_T - \frac{1}{2}\hat{a}'GG'\hat{a}M + \hat{a}'GsM_T + \frac{1}{2}s'sM_{TT}\ln W + \frac{1}{2}s'sN_{TT}$. Collecting the coefficients of $W\ln W$ and W respectively gives $M_T = N_T = 0$. Hence, $M_{TT} = N_{TT} = 0$. Substituting this into (7) gives $\delta M = 1$ and $N = 1 + (\frac{\hat{a}'\alpha}{\delta} - \frac{1}{\delta}) - \frac{1}{2}\frac{\hat{a}'GG'\hat{a}}{\delta^2}$. Substituting these results into (8b) gives $\hat{a} = (GG')^{-1}(\alpha - r1)/W$. Hence, unless both G and α are independent of T, optimal investment will be a function of both T and W, thereby contradicting that $N_T = 0$.

the stochastic process of temperature. First, the risk-free rate is positively related to the utility discount rate or pure rate of time preference, ceteris paribus.[16] Second, the risk-free rate is positively related to expected consumption growth and – when consumers are correlation averse, i.e., $U_{CT} > 0$ – negatively related to the expected increase in temperature. Intuitively, correlation-averse consumers want to increase future consumption to mitigate the detrimental effect of an expected increase in temperature (Eeckhoudt et al., 2007). This can be accomplished by increasing savings, which decreases the risk-free rate. Third, for a prudent consumer, i.e., $U_{CCC} > 0$, the risk-free rate is negatively related to the variance of consumption: increases in consumption risk can be tempered by a higher future consumption level, which increases savings and reduces the risk-free rate. Fourth, when a consumer is cross prudent in temperature, i.e., $U_{CCT} < 0$, lower temperatures mitigate consumption risk. Inspection of (10) reveals that an increase in consumption risk will decrease the risk-free rate if the covariance between consumption and temperature is negative and vice versa. Intuitively, higher consumption risk increases the risk of bad outcomes if the covariance between consumption and temperature is negative. In that case, low consumption will tend to go together with high temperature, which will increase the desire to save and decrease the risk-free rate for a consumer who is cross prudent in temperature. Finally, for a consumer who is cross prudent in consumption, i.e., $U_{CTT} > 0$, the risk-free rate will be negatively related to the variance of temperature. For such a consumer, an increase in temperature risk can be tempered by a higher level of future consumption, which increases savings and decreases the risk-free rate.

Remark 2*.* Proposition 2 generalizes several well-known analytical results in the literature. Under certainty, Heal (2009) shows that an increase in temperature increases the consumption discount rate if $U_{CT} > 0$, that is, if consumption and temperature are substitutes in the Edgeworth-Pareto sense.[17] In their theorem 1, Cox et al. (1985) derive the Ramsey rule, when all state variables except wealth are exogenous. Their result mirrors proposition 2 except that in their case the risk-free rate is determined by the expected private rate of return, instead of the expected social rate of return. Menegatti (2009) shows that the last three terms on the RHS of (10) are a necessary and sufficient condition for positive precautionary savings in the presence of two small, interdependent risks. Equation (10) shows that his result generalizes locally to a general equilibrium framework in which the distribution of the interdependent risks is endogenously determined. Finally, Gollier (2010) extends the Ramsey rule to a consumption economy when consumption and environmental quality follow a bivariate geometric Brownian motion. He obtains a condition similar to the RHS of (10).

[16] Since all inferences on the Ramsey rule in this paper are ceteris paribus, I refrain from repeating this henceforth.

[17] Heal (2009) uses environmental quality instead of temperature as an argument in the utility function. As increases in temperature are associated with lower environmental quality, his assumption of a negative cross derivative of the utility function is equivalent to assuming that consumers are correlation averse.

Remark 3. Proposition 2 highlights the options for balancing the demand and supply of savings in a climate-change economy. The conventional way is to change the consumption-savings decision (Ramsey, 1928). An alternative way is to shift investment between the regular and mitigation sectors, while keeping the share of adaptation in invested wealth constant. By assumption 2, such an investment shift will lower the flow of pollution $\theta\hat{a}'\gamma$, which ceteris paribus increases the social rate of return in the LHS of (10).

Remark 4. Proposition 2 extends the claim by Stern (2008) that the social discount rate, the expected social rate of return on investments and the expected private rate of return on investments are wholly different concepts, from imperfect to perfect economies. To see this, notice that the social discount rate is by definition equal to the risk-free rate, since both are equal to the expected rate of change in the marginal utility of consumption.[18] Using (10), this gives

$$\begin{aligned} r(W,T) &= \underbrace{-\frac{E(\mathrm{d}e^{-\delta t}J_W)}{e^{-\delta t}J_W}}_{\text{SDR}} \\ &= \underbrace{\underbrace{\hat{a}'\alpha}_{\text{PRI}} + \underbrace{\hat{a}'\theta\gamma\frac{J_T}{J_W}}_{\text{compensation}}}_{\text{SRI}} - \left(\underbrace{\underbrace{\frac{-J_{WW}W}{J_W}\frac{var(W)}{W^2}}_{\text{market risk}} + \underbrace{\frac{-J_{WT}}{J_W}\frac{cov(W,T)}{W}}_{\text{unfavorable changes}}}_{\text{risk premium on optimally-invested aggregate wealth}}\right). \end{aligned} \tag{11}$$

Hence, the social discount rate is equal to the expected social rate of return on optimally-invested aggregate wealth less the risk premium on that wealth, ϕ_W, implying that the social discount rate and the social rate of return on investments are different concepts. Moreover, the expected social rate of return on optimally-invested wealth differs from the expected private rate of return on optimally invested wealth by a compensation payment to compensate the consumer for the expected change in lifetime utility.

The next proposition characterizes the optimal excess discount rate, i.e., the optimal discount rate in excess of the risk-free rate, for investments with uncertain payoffs in all sectors.

Proposition 3. *In sector i, the optimal excess discount rate, $\beta_i - r$, is given by*

$$-\frac{J_{WW}}{J_W}\frac{cov(K_i,W)}{K_i} - \frac{J_{WT}}{J_W}\frac{cov(K_i,T)}{K_i}. \tag{12}$$

PROOF: See Appendix A.

[18] This holds for the social discount rate by definition (Heal, 2005) To see that the risk-free rate equals the expected rate of change in the marginal utility of consumption, apply Ito's lemma to $\mathrm{d}J_W$, rewrite (A.9) as $r = -\frac{E(\mathrm{d}e^{-\delta t}J_W)}{e^{-\delta t}J_W}$ and notice that, by (8a), $J_W = U_C$.

This proposition states that the optimal excess discount rate in sector i is proportional to both the covariance of its rate of return with changes in wealth and the covariance of its rate of return with changes in temperature, extending theorem 2 of Cox et al. (1985) to endogenous state variables, such as temperature. Intuitively, consumers are willing to accept a lower rate of return on assets that tend to pay off more when wealth is lower; moreover, when increases in temperature are unfavorable, they are also willing to accept a lower rate of return on assets that pay off more when temperature is higher. Notice that (12) can be rewritten into two different and familiar ways. First, applying Ito's lemma to $\mathrm{d}J_W$ gives that the optimal excess discount rate in sector i is equal to the covariance of that sector's rate of return with changes in the marginal value of wealth, i.e., $-\frac{cov(K_i,J_W)}{K_iJ_W}$. Intuitively, consumers are willing to accept a lower rate of return on assets that tend to pay off more when the marginal value of wealth is higher. Second, for the case of non-state-dependent utility, i.e., $U(C,T) = U(C)$, the optimal excess discount rate in sector i is proportional to the covariance of that sector's rate of return with optimal consumption, i.e., $-\frac{U_{CC}}{U_C}cov(\hat{C},K_i)$.

Building on proposition 3, the following proposition formalizes the conditions under which adaptation investments require a lower discount rate than regular investments.

Proposition 4. *Suppose that adaptation and regular investments have similar return volatilities from economic shocks, i.e., $g_{e2}(T) = g_{e1}(T)$. Moreover, suppose that both climatic shocks and increases in the state variable temperature are unfavorable, i.e., $\bar{g}_c(T) < 0$ and $J_{WT} > 0$. The difference in discount rates for adaptation and regular investments is then given by*

$$\beta_2 - \beta_1 = -\frac{WJ_{WW}}{J_W}(g_{c2} - g_{c1})\bar{g}_c' - \frac{J_{WT}}{J_W}(g_{c2} - g_{c1})s_c. \tag{13}$$

Under assumption 1, this gives that the optimal discount rate for adaptation investments is smaller than the optimal discount rate for regular investments if and only if $T > T^*$.

PROOF: See Appendix A.

The intuition of this result is that adaptation investments are – under certain conditions – a hedge against unexpected changes in the state variables wealth and temperature, provided that these changes result from climatic shocks. To see why, suppose that $T > T^*$, which implies by assumption 1 that $g_{c2}(T) > g_{c1}(T)$: adaptation investments have a higher return volatility than regular investments. Under the conditions stated in proposition 4, this higher return volatility of adaptation investments decreases its discount rate for two reasons. The first reason is that adaptation investments tend to pay off more than regular investments in cases where climatic shocks decrease aggregate wealth, i.e., $\bar{g}_c < 0$. Notice that this particular argument for a lower discount rate can be traced back to higher demand of less risky assets by a single-period mean-variance maximizer (Merton, 1992, p. 384). The second reason is that adaptation investments are a hedge against unfavorable shifts in the investment-opportunity set resulting from climatic shocks (Merton, 1992, p. 384). Of course, this requires that the temperature increase as a result of the climatic shock is unfavorable, i.e., $J_{WT} > 0$. Notice that differences in the return volatility of economic shocks may still result in overall higher discount rates for adaptation investments. For example, investments in heat-resistant crops or innovative coast protection may be subject to specific productivity, demand or technology risks, implying that they are – all things considered – more risky than regular investments.

Remark 5. With free entry and exit and constant returns to scale, there will be no incentive for firms to enter or leave a sector if and only if the terms on which it can acquire capital (the discount rate) are identical to the technologically determined physical returns in that sector (cf. Cox et al. (1985)). Hence, in equilibrium, the optimal discount rate for the ith sector, β_i, will be equal to the technologically determined α_i of that sector. From theorem 4 and assumption 1, it then follows that $\alpha_2(T) < \alpha_1(T)$ if and only if $g_2(T) < g_1(T)$. Hence, in equilibrium, the assumptions on the technologically determined parameters $g_i(T)$ and $\alpha_i(T)$ must in some sense 'match', i.e., whenever $0 < \hat{a}_i < 1$ (which is the assumption in this paper), we must have that $\alpha_i(T) = \beta_i = r - \frac{J_{WW}}{J_W} g_i G' \hat{a} W - \frac{J_{WT}}{J_W} g_i s$. Notice that – under constant returns to scale – we will have either $\hat{a}_i = 1$ or $\hat{a}_i = 0$ if the assumptions on $g_i(T)$ and $\alpha_i(T)$ do not 'match'. For example, consider a sector in which the expected rate of return is below the risk-free rate, i.e., $\alpha_i(T) < r$, while its return volatility is such that it warrants a positive risk premium, i.e., $-\frac{J_{WW}}{J_W} g_i G' \hat{a} W - \frac{J_{WT}}{J_W} g_i s > 0$. Investment in this sector is then dominated by investment in the risk-free opportunity. Hence, $\hat{a}_i = 0$.[19]

Remark 6. Proposition 4 cannot be extended to the mitigation sector. To see why, suppose that regular and mitigation investments are characterized by equal return volatilities from economic shocks, i.e., $g_{e1} = g_{e3}$, which implies that any remaining differences in discount rates can be fully attributed to differences in return volatilities from climatic shocks. In addition, suppose – exploiting the fact that the return volatility of mitigation investments is not systematically related to that sector's pollution intensity – that regular and mitigation investments also have equal return volatility from climatic shocks, i.e., $g_{c1} = g_{c3}$. It then immediately follows that the optimal discount rates for regular and mitigation investments will be equal, i.e., $\beta_1 = \beta_3$. Moreover, notice that the pollution intensities γ_i have no effect on the optimal discount rate for mitigation investments. The key to understanding this surprising result is that within the consumption capital asset pricing model (CCAPM), *time-t* expected excess returns are proportional to the *time-t* covariance of their return with consumption. Subsequently, any two assets that have identical return volatilities must have identical discount rates. This result stands in contrast to the claim of Sandsmark and Vennemo (2007) that mitigation investments *must* have lower discount rates because they reduce future (*non time-t*) risk.[20] Within their model, this

[19] Notice that is is straightforward to allow for the possibility of zero investment in some sectors, see Cox et al. (1985) for further details.

[20] Another reason why climate investments in Sandsmark and Vennemo (2007) are associated with lower discount rates is that these investments are assumed to have a negative beta: they tend to pay off more when con-

claim originates from the assumption that the risk-free rate is exogenous: increases in climate investments cannot change the risk-free rate despite the fact that they increase expected (future) consumption.[21] However, it is well known that when consumers expect consumption to rise, they would like to reduce their savings, which raises the risk-free rate, see, for example, Gollier (2007). Hence, when determining optimal discount rates for mitigation investments, it is crucial to recognize that the risk-free rate is endogenous.

4 Conclusion

This paper provides a framework for understanding discount rates of adaptation and mitigation investments as their equilibrium expected rates of return, taking into account both economic and climatic risk. Previous literature has either focused on explaining the risk-free rate in a partial equilibrium context with climatic risk or a general equilibrium context with economic risk only or resorted to outside-the-model reasons, such as ethics or the presence of long-term climatic uncertainty. While this literature has provided useful insights into the economics of discounting, it cannot explain if, and why, the discount rates for adaptation and mitigation *investments* should differ from the discount rate for regular investments.

Using a simple stochastic model of climate change, I have shown that the optimal excess discount rate of an investment must be proportional to the covariance of its rate of return with the state variables wealth and temperature. The insight that state variables other than wealth may explain differences in optimal excess discount rates is consistent with the consumption capital asset pricing model. According to the CCAPM, investors care not only about high expected return and low return variance, but also about the covariances of their portfolio returns with state variables. Interestingly, climate science provides us with a whole list of candidate state variables, such as temperature, the atmospheric stock of carbon dioxide, the sea level, the worldwide population and our knowledge of the climate system. All of these state variables could easily be incorporated in the framework developed here.

Although my model is highly stylized and does not have a known analytical solution, it permits a clear conclusion. Given equal return volatility from economic shocks, adaptation investments will earn a lower discount rate provided that both climatic shocks and increases in temperature are unfavorable (and temperatures are sufficiently high). Moreover, I have argued that a similar conclusion cannot be drawn for mitigation investments. Unfortunately, any realistic implementation of the model is beyond the scope of this paper as it would require a

sumption is low. However, their climate investment combines characteristics of both adaptation and mitigation. See footnote 8.

[21] Technically, Sandsmark and Vennemo (2007) assume that the partial derivative of the distribution function of consumption with respect to climate investments is positive, i.e., it satisfies the condition of first order stochastic dominance. Thus, a rise in climate investments will increase the expected consumption level in the next period.

considerable extension of the model both in terms of the number of state variables and in its setup. For example, the climate module in the Nordhaus DICE model consists of five state variables compared with only one state variable, temperature, in my model. In addition, successful implementation of the model might require either the use of Epstein-Zin-Weil preferences or the modeling of economic shocks through a Poisson process in order to avoid the well-known equity premium and risk-free rate puzzle (Barro, 2009). There is therefore much scope for further research along these lines.

A Appendix

DERIVATION OF EQUATION 4

Recognizing that the value of a contingent claim is a function of the state variables W and T, we get, applying Ito's lemma:

$$\mathrm{d}F(W,T) = [F_W \mathrm{d}W + F_T \mathrm{d}T] + \frac{1}{2}\left[F_{WW}(\mathrm{d}W)^2 + 2F_{WT}(\mathrm{d}W)(\mathrm{d}T) + F_{TT}(\mathrm{d}T)^2\right]$$

Using equations (2) and (5) to substitute out $\mathrm{d}W$ and $\mathrm{d}T$, we get after some rearrangements:

$$\mathrm{d}F = (F\beta - \zeta)\mathrm{d}t + Fh'\mathrm{d}\omega(t), \tag{A.1}$$

where

$$Fh' = F_W W a' G + F_W W b h' + F_T s' \tag{A.2}$$

and

$$\begin{aligned} F\beta - \zeta = & F_W(a'\alpha W + b\beta W - C) + F_T \theta \gamma' a W \\ & + \frac{1}{2}(F_{WW} W^2 (a'GG'a + 2a'Ghb + b^2 h'h) \\ & + 2F_{WT} W (a'G + bh')s + F_{TT} s's). \end{aligned} \tag{A.3}$$

PROOF OF PROPOSITION 1

Rearranging (8c), multiplying by F and using the market-clearing condition $b = 0$ gives

$$\begin{aligned} (\beta - r)F &= -\frac{WJ_{WW}}{J_W} Fh'G'\hat{a} - \frac{J_{WT}}{J_W} Fh's \\ &= -\frac{WJ_{WW}}{J_W}(F_W W \hat{a}' G + F_T s')G'\hat{a} - \frac{J_{WT}}{J_W}(F_W W \hat{a}' G + F_T s')s \\ &= W\phi_W F_W + W\phi_T F_T, \end{aligned} \tag{A.4}$$

where the second equality follows by substitution of Fh' from (A.2) – using $b = 0$ – and the third equality by rearranging terms. Here, ϕ_W and ϕ_T have been implicitly defined and are equal to

$$\phi_W = \frac{-J_{WW}W}{J_W}\frac{var(W)}{W^2} + \frac{-J_{WT}}{J_W}\frac{cov(W,T)}{W}, \tag{A.5}$$

$$\phi_T = \frac{-J_{WW}}{J_W}\frac{cov(W,T)}{W} + \frac{-J_{WT}}{J_W}\frac{var(T)}{W}. \tag{A.6}$$

To see that ϕ_W is indeed the risk premium on optimally-invested aggregate wealth, take a contingent claim whose value is always equal to aggregate wealth, i.e., $F = W$. Its derivatives are $F_W = 1$ and $F_T = 0$. Substituting this into (A.4) gives that the rate of return on aggregate wealth β_W equals $r + \phi_W$. Hence, the risk premium on optimally-invested aggregate wealth is equal to ϕ_W.

PROOF OF PROPOSITION 2

First, I derive the LHS of (10). Notice that from (8) the equilibrium solution for a, r and β is partially separable in terms of the (derivatives of the) value function, J. When $b = 0$, (8b) determines a and r. Given a and r, (8c) determines β. Denote the equilibrium value of the control variables in (8) by $\hat{C}, \hat{a}$ and $\hat{b}$. As in Cox et al. (1985), the optimal value of the equilibrium interest rate can be determined by examining two related planning problems. The first planning problem has the same physical production opportunities and interaction between the economy and the environment, but has no borrowing, lending and contingent claims. The second planning problem is identical to the first planning problem with borrowing and lending allowed (but contingent claims not).

Let $\tilde{a}$ and $\tilde{C}$ denote the optimal investment and consumption strategy for the first planning problem and $\tilde{J}$ the corresponding value function. The portfolio allocation of the first planning problem can now be written as the following quadratic programming problem:

$$\max_a \; a'\phi + a'Da \tag{A.7}$$

$$\text{s.t. } a'1 = 1$$

where $\phi = \alpha W\tilde{J}_W + \theta\gamma W\tilde{J}_T + GsW\tilde{J}_{WT}$, $D = \frac{1}{2}GG'W^2\tilde{J}_{WW}$, and 1 denotes the unit vector. Solving (A.7) gives (8b) for the case without borrowing or lending, $r = 0$, and without contingent claims, $b = 0$. Let λ be the shadow price corresponding to the market-clearing condition, $a'1 = 1$. In the optimum

$$\phi - \tilde{\lambda} * 1 + 2D\tilde{a} = 0.$$

Consider the second planning problem with borrowing and lending at $\tilde{r}$ and indirect utility at $\tilde{\tilde{J}}$. Inspection shows that if $\tilde{\tilde{J}} = \tilde{J}$ and $\tilde{r} = \tilde{\lambda}/W\tilde{J}_W$, then $(\tilde{r}, \tilde{a}, \tilde{C})$ is the equilibrium for the second planning problem. The equilibrium interest rate, $\tilde{r}$, is proportional to the shadow price, $\tilde{\lambda}$. Hence, in equilibrium the economy characterized by (1)-(6) has $\hat{C} = \tilde{C}$, $\hat{a} = \tilde{a}$, $r = \tilde{r}$ and $J = \tilde{J} = \tilde{\tilde{J}}$. This gives

$$\begin{aligned} r(W,T) = \frac{\tilde{\lambda}}{WJ_W} &= \frac{\hat{a}'\left(\alpha WJ_W + \theta\gamma WJ_T + GsWJ_{WT} + GG'\hat{a}W^2J_{WW}\right)}{WJ_W} \\ &= \hat{a}'(\alpha + \theta\gamma\frac{J_T}{J_W}) - \left(\frac{-J_{WW}W}{J_W}\right)\left(\frac{var(W)}{W^2}\right) - \left(\frac{-J_{WT}}{J_W}\right)\left(\frac{cov(W,T)}{W}\right), \end{aligned} \tag{A.8}$$

which is the LHS of (10). The final step uses the expressions for the variance of wealth and the covariance between wealth and temperature evaluated at $a = \hat{a}$ and $b = 0$, which follow from (2) and (5).

Second, I derive the RHS of (10). Differentiate (7) with respect to W recognizing that both C and a are functions of W. Using the first-order conditions (8a) and (8b), we get after rearrangement of terms:

$$\begin{aligned} r(W,T) = -\Bigg(&\frac{1}{2}var(W)J_{WWW} + cov(W,T)J_{WWT} + (\hat{a}'\alpha W - \hat{C})J_{WW} \\ &+ \theta\hat{a}'\gamma WJ_{WT} + \frac{1}{2}s's J_{WTT} - \delta J_W\Bigg)/J_W. \end{aligned} \tag{A.9}$$

Differentiation of (8a) with respect to the states W and T gives $J_{WW} = U_{CC}\hat{C}_W$ and $J_{WT} = U_{CC}\hat{C}_T + U_{CT}$. Differentiating again gives $J_{WWW} = U_{CCC}\hat{C}_W^2 + U_{CC}\hat{C}_{WW}$, $J_{WWT} = U_{CCC}\hat{C}_W\hat{C}_T + U_{CCT}\hat{C}_W + U_{CC}\hat{C}_{WT}$ and $J_{WTT} = U_{CCC}\hat{C}_T^2 + U_{CCT}\hat{C}_T + U_{CC}\hat{C}_{TT} + U_{CCT}\hat{C}_T + U_{CTT}$. Applying Ito's lemma to $\mathrm{d}\hat{C}(W,T)$ gives $E(\mathrm{d}\hat{C})/\mathrm{d}t = \hat{C}_W(\hat{a}'\alpha W - \hat{C}) + \hat{C}_T\theta\hat{a}'\gamma W + \frac{1}{2}\hat{C}_{WW}var(W) + \hat{C}_{WT}cov(W,T) + \frac{1}{2}\hat{C}_{TT}var(T)$, $var(\hat{C}) = \hat{C}_W^2 var(W) + 2\hat{C}_W\hat{C}_T cov(W,T) + \hat{C}_T^2 var(T)$ and $cov(\hat{C},T) = \hat{C}_W cov(W,T) + \hat{C}_T var(T)$, where I have used (2) to obtain the last expression. Substituting these expressions into (A.9) and collecting terms gives the desired result.

PROOF OF PROPOSITION 3

The required rate of return in each sector can be valued in the same way as any other contingent claim. For sector i, this implies $F = K_i$ and $h' = g_i$. Substituting this into (8c) gives after some rearrangement:

$$\begin{aligned} \beta_i =& r - \frac{J_{WW}}{J_W}g_iG'\hat{a}W - \frac{J_{WT}}{J_W}g_is \\ =& r - \frac{J_{WW}}{J_W}\frac{cov(K_i,W)}{K_i} - \frac{J_{WT}}{J_W}\frac{cov(K_i,T)}{K_i}, \end{aligned} \tag{A.10}$$

where the second expression follows from (2), (3) and (5).

PROOF OF PROPOSITION 4
Using (A.10) and subtracting β_2 from β_1, we get, using the fact that by assumption $g_{e1} = g_{e2}$,

$$\beta_2 - \beta_1 = -\frac{WJ_{WW}}{J_W}(g_{c2} - g_{c1})\bar{g}_c' - \frac{J_{WT}}{J_W}(g_{c2} - g_{c1})s_c. \tag{A.11}$$

Since $s_c > 0$ and by assumption $J_{WT} > 0$, $\bar{g}_c < 0$, the sign of $\beta_2 - \beta_1$ will be opposite to the sign of $g_{c2} - g_{c1}$. By assumption 1, it follows that $\beta_2 < \beta_1$ if $T > T^*$ and $\beta_2 > \beta_1$ if $T < T^*$, because $g_{c2} - g_{c1} < 0$ for $T < T^*$ and $g_{c2} - g_{c1} > 0$ for $T > T^*$.

References

Aase, K. K. (2011), "The Long Term Equilibrium Interest Rate and Risk Premiums under Uncertainty," The Norwegian School of Economics and Business Administration Discussion Paper 4, downloadable at `http://brage.bibsys.no/nhh/bitstream/URN:NBN:no-bibsys_brage_24266/1/dpfor2011-4.pdf`.

Bansal, R. and M. Ochoa (2011), "Temperature, Aggregate Risk, and Expected Returns," NBER Working Paper 17575, downloadable at `http://www.nber.org/papers/w17575`.

Barro, R. (2009), "Rare Disasters, Asset Prices, and Welfare Costs," *American Economic Review*, **99**, 243–264.

Breeden, D. (1986), "Consumption, Production, Inflation and Interest Rates," *Journal of Financial Economics*, **16**, 3–39.

Cox, J. C., J. Ingersoll, and S. Ross (1985), "An Intertemporal General Equilibrium Model of Asset Prices," *Econometrica*, **53**, 363–384.

Detemple, J. (1986), "Asset Pricing in a Production Economy with Incomplete Information," *The Journal of Finance*, **41**, 383–391.

Duffie, D. and W. Zame (1989), "The Consumption-Based Capital Asset Pricing Model," *Econometrica*, **57**, 1279–1297.

Eeckhoudt, L., B. Rey, and H. Schlesinger (2007), "A Good Sign for Multivariate Risk Taking," *Management Science*, **3**, 117–124.

Gollier, C. (2007), "The Consumption-Based Determinants of the Term Structure of Discount Rates," *Mathematics and Financial Economics*, **1**, 81–101.

Gollier, C. (2010), "Ecological Discounting," *Journal of Economic Theory*, **145**, 812–829.

Gollier, C. (2012), "Evaluation of Long-Dated Investments under Uncertain Growth Trend, Volatility and Catastrophes," *TSE Working Paper Series 12-361*.

Heal, G. (2005), "Intertemporal Welfare Economics and the Environment," in K.-G. Mäler and J. Vincent (eds.), *Handbook of Environmental Economics*, volume 3, chapter 21, Elsevier, pp. 1105–1145.

Heal, G. (2009), "Climate Economics: A Meta-Review and Some Suggestions for Future Research," *Review of Environmental Economics and Policy*, **3**, 4–21.

Meinshausen, M., N. Meinshausen, W. H. S. B. Raper, K. Frieler, R. Knutti, D. Frame, and M. Allen (2009), "Greenhouse-Gas Emission Targets for Limiting Global Warming to 2 °C," *Nature*, **458**, 1158–1163.

Menegatti, M. (2009), "Optimal Saving in the Presence of Two Risks," *Journal of Economics*, **96**, 277–288.

Merton, R. (1971), "Optimium Consumption and Portfolio Rules in a Continuous-Time Mod," *Journal of Economic Theory*, **3**, 373–413.

Merton, R. (1977), "On the Pricing of Contingent Claims and the Modigliani-Miller Theorem," *Journal of Financial Economics*, **5**, 241–249.

Merton, R. C. (1992), *Continuous-Time Finance*, Basil Blackwell.

Nordhaus, W. (2008), *A Question of Balance: Weighing the Options on Global Warming Policies*, Yale University Press.

Obstfeld, M. (1995), "Risk-Taking, Global Diversification, and Growth," *American Economic Review*, **84**, 1310–1329.

Ramsey, F. (1928), "A Mathematical Theory of Saving," *Economic Journal*, **38**, 543–559.

Roe, G. and Y. Bauman (2012), "Climate Sensitivity: Should the Climate Tail Wag the Policy Dog?" *Climate Change*, 1–16.

Sandsmark, M. and H. Vennemo (2007), "A Portfolio Approach to Climate Investments: CAPM and Endogenous Risk," *Environmental and Resource Economics*, **37**, 681–695.

Stern, N. (2008), "The Economics of Climate Change," *American Economic Review*, **98**, 1–37.

Tol, R. (2009), "The Economic Effects of Climate Change," *Journal of Economic Perspectives*, **23**, 29–51.

Weitzman, M. (2010), "Risk-Adjusted Gamma Discounting," *Journal of Environmental Economics and Management*, **60**, 1–13.

Weitzman, M. (2012), "Rare Disasters, Tail-Hedged Investments, and Risk-Adjusted Discount Rates," *NBER Working Paper 18496*.

Publisher:

CPB Netherlands Bureau for Economic Policy Analysis
P.O. Box 80510 | 2508 GM The Hague
T (070) 3383 380

September 2013 | ISBN 978-90-5833-617-0