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The Design of Pension Schemes

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Abstract in English

The ageing of the Dutch population, resulting in an increase in the number of retirees relative to the working population, has induced a debate about the sustainability of the Dutch first pillar pension scheme (AOW). The system is financed as a pay-as-you-go system. This paper explores possible alternatives for the AOW. It does so by setting up a stochastic partial equilibrium model to study intragenerational insurance, which inlcudes longevity and productivity risk. The model shows the welfare, labour-market, saving and unintended-bequest effects of a shift from a Beveridge towards a Bismarck system in which pension rights depend on labour-market history. The main conclusion is that a shift of the first pillar pensions from a Beveridge towards a Bismarck system is not necessarily welfare improving from an ex-ante insurance perspective, i.e. before the veil of ignorance is lifted. Moreover, a means test of the first pillar against wealth income, which implies a lower AOW when an individual has wealth income and a lower pension premium for everyone, does not improve welfare in the setting of the model considered in this paper.

Abstract in Dutch

De vergrijzing van de Nederlandse bevolking, met als gevolg een relatieve toename van het aantal gepensioneerden ten opzichte van het aantal werkenden, heeft geleid tot een discussie over de houdbaarheid van de AOW, de eerst pijler van het Nederlandse pensioensysteem. De AOW is gefinancierd volgens een omslagstelsel, de werkenden betalen direct de uitkeringen aan de op dat moment gepensioneerden. Dit rapport verkent mogelijke alternatieven voor de AOW. Afwezigheid van een goede pensioenvoorziening blijkt tot relatieve armoede te lijden. Het onderhavige discussion paper geeft twee belangrijke verklaringen: langer leven dan verwacht en een geringe verdiencapaciteit. In beide gevallen wordt er te weinig gespaard. De beide risico's worden op dit moment verzekerd middels de AOW. De AOW geeft aan alle ingezeten, die permanent in Nederland hebben gewoond een gelijk basis pensioen. De AOW premie staat dus los van de uitkering en leidt daardoor tot minder arbeidsaanbod. Door de AOW uitkering afhankelijk te stellen van het arbeidsmarktverleden wordt dit tegengegaan. Dit onderzoek laat zien dat dit alternatieve systeem niet noodzakelijk welvaartsverhogend is: het leidt weliswaar tot minder arbeidsmarktverstoring, maar de verzekeringswaarde is ook minder. Verder wordt onderzocht of de AOW uitkering ook niet afhankelijk zou moeten zijn van andere inkomsten. Een vermindering van de AOW, als men vermogensinkomsten heeft waardoor iedereen een lagere premie kan betalen, blijkt niet welvaartsverhogend.

Contents

1 Introduction

This paper contributes to the current discussion[1] about the Dutch first pillar pension system (AOW). The AOW's objective is poverty prevention amongst the elderly. For this purpose, the AOW gives an equal basic pension above a certain age to citizens who did permanently live in the Netherlands. Such a system is known in the literature as a Beveridge system. This AOW system is an old system; it came into force in 1957. One can question whether this type of insurance against poverty is still necessary because many of the entitled individuals have been forced to save money for retirement while working (second pillar). In this respect, it is interesting to study if social assistance would be a more appropriate goal of the first pillar pension system in the Netherlands.

The discussion about the AOW is triggered by the sustainability problems of the government's budget due to population ageing and by the relatively favourable income and wealth development of retirees up to 2010. The AOW is pay-as-you-go (PAYG) financed, i.e. the benefits of retirees are paid by the workers. As in the coming decades the number of workers will decline relative to the number of retirees, this financing structure will become more costly and possibly less appropriate because of its goal to prevent poverty. Moreover this relative increase of the number of retirees in the Dutch population also leads to a discussion about shifting up the retirement age.

Studying alternatives for the AOW requires an understanding of the distortions involved in the current set up. There can be distinguished two major distortions. First, the AOW distorts labour supply, because there is no direct link between the premium payments and the AOW entitlement. All Dutch citizens are entitled the same amount of money when they retire. Second, it distorts consumption because premium payments of the working age population to the retirees lead to a tighter budget constraint in the case of liquidity constraints. These distortions diminish the efficiency of the Dutch economy.

Another aspect in the discussion concerns the redistributional effects of the current AOW system. The AOW not only redistributes from workers to pensioners but also from those who live shorter to those who live longer. That is, from for example men to women and from lower to higher skilled workers, because these groups have different life expectancies. This effect seems to be cancelled out by the fact that higher skilled workers have contributed more to the premium pool than lower skilled workers. On balance, the AOW seems to redistribute from higher to lower skilled people and from men to women.

A sound study about the alternatives requires a weighting of all pros and cons. This is extremely difficult and that is why the present paper will take a specific focus. In particular, intergenerational redistribution (between generations) due to the AOW and the differences

[1] See for instance Sap et al. (2009) and Financiën (2013).

between funding or PAYG financing are beyond the scope of this paper. We will focus on the AOW as an intragenerational insurance instrument, i.e. redistribution within an age cohort. The most important life cycle risks, which could lead to relative poverty in retirement and which could benefit from insurance, are longevity and productivity risk. These micro-economic risks are likely to hit individuals differently.[2] Insurance can take place within a cohort.

The analysis of intragenerational redistribution in the current system and its potential alternatives requires a rich stochastic partial equilibrium modelling approach. This paper describes such a stochastic partial equilibrium model and studies its implications. The model incorporates longevity risk and productivity risk, conditional on the skill level of workers. The model includes a first pillar pension scheme, which is defined as the Beveridge part of the pension system. Moreover, it includes a second pension pillar scheme, which links the second pillar pensions to the labour-market history of an individual. Such a second pillar scheme is known as a Bismarck system in the literature. The model also describes that the second pillar comes into force when labour market effort exceeds some franchise level. The model further incorporates private savings, the third pillar of the pension system and labour supply. It also includes liquidity constraints. This model is used to determine the welfare, labour market, saving and unintended bequest effects of the different systems. Moreover, to gain insight in the relative poverty of individuals, consumption distributions at different ages will be determined.

The analysis starts with considering individuals who have just finished education and enter the labour market. These workers can be high, medium or low-skilled workers upon entry and remain of this type throughout there life. Workers are to some extent informed about their future earning opportunities, but unaware of the development of their careers and their time of death. Individuals consider both the distortions they are confronted with as a result of the presence of the AOW system and the intragenerational redistribution of the AOW system. The AOW system provides insurance against the financial effects of the life-cycle uncertainties due to its redistributional features. Intragenerational redistribution takes place within an age cohort, which implies capital funding. That is why the focus of the analysis uses the perspective of an age cohort, which faces the challenge to choose its mutual pension scheme at the start of their working life.

With the model the following three policy questions are addressed: First, what are the effects of a shift from a Beveridge towards a Bismarck first pillar system? In the model this boils down to the elimination of the Beveridge system and the franchise in the second pillar. Second, what are the welfare, labour market and saving effects of an extension of means testing of the first pillar with wealth? Third, which roles do the second and third pillar in the Dutch pension system play when considering alternatives for first pillar pensions?

[2] Macro economic risks hit the whole society and can be insured by intergenerational redistribution. A recent study about social security and macro risk is Broer (2012).

The first conclusion of this paper is that a shift from a Beveridge towards a Bismarck system is not necessarily welfare improving from an ex-ante insurance perspective, *i.e.* before the veil of ignorance is lifted. Indeed, the Bismarck system yields less distortions but also less insurance against lifetime uncertainties. The second conclusion is that a means test of the first pillar against wealth income is not welfare improving. The reason for this is that it distorts the consumption decisions. The third conclusion is that self insurance is insufficient, i.e. it leads to relative poverty among the retirees. This outcome is obtained because lack of perfect foresight leads to a consumption plan mainly driven by life expectancy. Growing older, life expectancy decreases less than age increases. This implies that every year the consumption plan takes this new information into account.

This paper contributes to the literature in several ways. First, this is the first paper for the Netherlands which explores insurance for micro-economic life-cycle risks in the Dutch three pillar pension scheme and its possible alternatives in an integrated framework. Second, it contributes to the literature which uses partial equilibrium models to study pension reforms. Other examples of this type of models, which investigate pension systems, are French (2005) and Sefton et al. (2008). French (2005) explores early retirement due to the tax structure of a social security system and pensions. Sefton et al. (2008) investigates the influence of means testing on retirement behaviour and saving. This paper inquires the pros and cons of intragenerational risk sharing for different pension schemes. It should be noted that the partial equilibrium analysis in these papers has the inherent drawback that not all feedback loops are modelled. In particular, the feedback of unintended bequests on welfare and of aggregated savings on the rate of return are not taken into account. Modelling these feedback loops is complex and beyond the scope of the present paper and of most of the previous studies in this field. Third, by its focus on intragenerational risk sharing this paper falls into the literature which uses stochastic computable general equilibrium models in which both intra- and intergenerational are investigated. Fehr (2008) surveys this literature. Nishiyama and Smetters (2007), Fehr and Habermann (2008) and Fehr and Uhde (2013) present models with intra- and intergenerational transfers, with general equilibrium feedbacks and with institutional features. The present paper is able to provide more insight in some of their main results by focussing on only one aspect of these general equilibrium models with maintaining the institutional features of the pension schemes. Fourth, and at a more technical level, we apply a new algorithm introduced by Judd et al. (2011) and Judd et al. (2012). Hasanhodzic and Kotlikoff (2013) show that this method is able to deal with large-scale overlapping generations (OLG) models with macro economic risks. This was impossible with previous methods because of the curse of dimensionality. This paper shows that the method is suitable for modelling micro-economic risks too. For future work it implies that models with both types of risk are within reach but still challenging to be put forward. The only example of such a model has recently been presented by Harenberg and Ludwig (2013).

The set-up of this paper is as follows. The model is presented in the next Section. Then we discuss the parameterization in Section 3. Section 4 investigates the consequences of longevity risk. Productivity risk is included in Section 5. In this section all policy questions are addressed. Section 6 concludes. The appendices provide details on the parameterized expectation solution method, the main text only offers the most salient details of the model.

2 The model

This section describes the model. It is a dynamic partial equilibrium model which distinguishes one cohort with many different individuals and different pension schemes. The model bases consumption-saving and labour supply of individuals on life-cycle theory. The model distinguishes three skill classes: lower, middle and higher skilled labour. Individuals in a skill class are heterogeneous with respect to their lifetime and productivity. The lifetime and future development of productivity are uncertain. These two sources of uncertainty imply imperfect foresight. By assumption individuals have rational expectations, *i.e.* they form expectations at each point in time conditional on their state at that moment and make no systematic error.

Apart from self insurance the model distinguishes the following pension schemes: an annuity scheme, a pure Beveridge, a pure Bismarck scheme and a means tested Beveridge scheme. The current Dutch first and second pillar pension scheme is approximated by a mixture of a Bismarck scheme and a means tested Beveridge scheme.

2.1 Intra generational heterogeneity

The model describes the development of individuals in a cohort. The life cycle is split up into twenty age cohorts $j \in (1, \dots 20)$ *i.e.* every model period covers five years in reality. The working cohorts are from $j_w = 5$ up to and including $j_r - 1 = 13$. Income and expenditures of the first 4 cohorts are neglected. The retired cohorts are from j_r up to j_e. The oldest age cohort is $j_e = 20$. Three skill classes s are distinguished: lower, middle and higher skilled. The cohort sizes per skill class, $n_{j,s}$, decline over time $n_{j,s} = \psi_{j,s} n_{j-1,s}$ with conditional survival probabilities smaller than one $\psi_{j,s} < 1$. Next section describes behaviour of an individual over the life cycle. To prevent clumsy notation only the age of an individual is indicated and not time or a specific indicator for the individual. The model makes no difference between men and women. So, the different mortality probabilities by sex play no role in our analysis. The conditional survival probabilities are expected values.

Within a cohort a continuum of different individuals is distinguished. The individuals are characterized by their current state. The state of an individual in cohort j is described with the vector $z_j = (j, s, e_j^p, A_j, e_j)$ in which j denotes the age, s the skill-class, e_j^p earnings points for first pillar pension claims, A_j financial assets holding and e_j the productivity (efficiency). For the age cohort j_w holds $z_{j_w} = (j_w, s, 0, 0, e_{j_w})$.

2.2 The micro economic risks

Whether an individual stays alive at a certain moment in time is uncertain. Assume, the probability of staying alive is uniform and independent over time distributed between zero and one ($u_{j,s} \sim U(0,1)$). The survivors are the individuals for which holds ($u_{j,s} < \psi_{j,s}$). The independence assumption of the survival probabilities over time is adequate for the purpose of our model. Indeed, the model is used to analyze longevity risk. The assumptions guarantee different lifetimes and generates the survival probabilities ($\psi_{j,s}$) consistently.

The productivity of an individual over the life cycle as also uncertain. The productivity (efficiency) of labour over the life cycle (productivity profile) depends on age, a_j and on individual shocks, η_j. It is modelled as in Fehr et al. (2013), *i.e.*

$$\log(e_j) = \beta_0 + \beta_1 a_j + \beta_2 a_j^2/100 + \eta_j - \log 10 \tag{2.1}$$
$$a_j = 5j - 2.5$$

The individual shocks follow an AR(1) process

$$\eta_j = \rho_\eta \eta_{j-1} + \varepsilon_j \tag{2.2}$$

with $\varepsilon_j \sim N(\varsigma_j, \sigma_\varepsilon^2)$ and $\eta_0 = 0$. The means, ς_j, are fixed in such a way that the expected productivity profile becomes equal to the profile without risk.

2.3 Individual behaviour; the assumptions

Individuals derive utility from consumption of goods c_j, and disutility of working $\bar{l} - l_j$, with $\bar{l}$ denoting the maximum available time and l_j leisure, respectively. Work leads to less home production and to labour related consumption expenditures. This labour induced consumption $\underline{c}_j$ does not directly contribute to utility. Net consumption, *i.e.* consumption net of labour induced consumption generates utility. Individuals maximize expected utility over the life cycle. The optimization problem of a j-years old individual with state z_j is

$$V_j(z_j) = \max_{c_j, l_j} \left\{ \left(c_j - \underline{c}_j\right)^{\frac{\gamma-1}{\gamma}} + \frac{\psi_{j+1}}{1+\delta} E_j V_{j+1}^{1-\frac{1}{\gamma}}(z_{j+1} \mid z_j) \right\}^{\frac{\gamma}{\gamma-1}} \tag{2.3}$$

with ψ representing the expected conditional survival probabilities, δ the time preference parameter and γ the inter-temporal elasticity of substitution. Labour induced consumption is modelled as

$$\underline{c}_j = -\alpha^{\frac{1}{\rho}} \frac{\rho}{\rho - 1} \left(l_j^{\frac{\rho-1}{\rho}} - \bar{l}^{\frac{\rho-1}{\rho}} \right) \tag{2.4}$$
$$l_j \leq \bar{l}, \; \rho < 1$$

with ρ marking the intra-temporal elasticity of substitution. The marginal per-period utility of leisure becomes infinite as leisure approaches zero. This guarantees positive values of leisure. The restriction that leisure must be equal to or smaller than the maximum available time has to be explicitly imposed. This utility specification implies the restriction $c_j > \underline{c}_j$ for commodity consumption. Indeed the marginal utility of per-period commodity consumption becomes infinite as the consumption approaches this minimum level. The positive leisure consumption guarantees positive commodity consumption as long as the intra temporal elasticity of substitution is smaller than one ($\rho < 1$). This specification of the per-period utility function was proposed by Greenwood et al. (1988) and is used in the GAMMA model (Draper and Armstrong (2007)).

Individuals maximize their utility given their budget constraint

$$A_{j+1} = (1+r)A_j + W_j + P_j - \tau^p W_j - C_j \qquad (2.5)$$
$$A_j \geq 0$$

with A representing financial wealth, r the return on assets, $W_j = \left(\bar{l} - l_j\right) e_j \omega$ gross wage income and P pension income.[3] Gross wage income is determined by the labour time, $\bar{l} - l_j$, the productivity, e_j, and the wage per productivity unit ω. Note the liquidity constraint $A_j \geq 0$. Deaton (1991) discusses saving and liquidity constraints in the case of uncertainty about labour income. Income consists of net labour income, (in the working years), pension income, P_j (in the retirement period) and capital income.

2.4 The pension system

2.4.1 Pension benefits

Pension income, P_j may consists of an annuity P_j^0, a Beveridge part P_j^1, a Bismarck part P_j^2 or a mixture. These components will be discussed successively.

Annuity system

An annuity is a first best insurance against longevity risk. Wealth of the deceased is distributed over the survivors (Yaari (1965)). Assume such an annuity is available at the statutory pension age j_r

$$P_j^0 = \frac{1-\psi_{j+1}}{\psi_{j+1}} \left[(1+r)A_j - C_j\right] \text{ and } j \geq j_r \qquad (2.6)$$

with $1 - \psi_{j+1}$ the fraction of a cohort that dies and ψ_{j+1} the fraction that survives. This assumption implies that the insurance company distributes total wealth of a cohort over the survivors pro rato an individual's wealth in the previous period. Thus all savings are in fact transferred to the insurance company.

Beveridge system

A means-tested Beveridge part guarantees an income $\kappa_1 E(W)$ at the statutory pension age j_r. A lower income than this guarantee leads to a benefit P_j^1 to bridge the difference. This means

[3] Economic growth is not explicitly included in the model. The simulation results have to be interpreted as scaled by the productivity level. Front loading of consumption has been mimiced by decreasing the rate of return relative to the time preference parameter.

tested Beveridge part of the system reads as

$$P_j^1 = \max\left[\kappa_1 E(W) - \varphi_p P_j^2 - \varphi_a r \max\left[A_j, 0\right], 0\right] \tag{2.7}$$

κ_1 marks the replacement rate for the means tested part and $E(W)$ the unconditional expected wage income of the whole population. The taper rates φ_a and φ_p define the precision of the means test against the stock of liquid assets (or returns on assets) and pensions from the Bismarck part P_j^2, respectively. Without a means test (i.e. $\varphi_p = \varphi_a = 0$) individual pensions from the Beveridge part are uniform for all agents. On the other hand, with a means-test (i.e. $\varphi_p \neq 0$ and (or) $\varphi_a \neq 0$) the amount of the benefits depends on individual characteristics of the retiree.

Bismarck system

The accumulation of earning points in a Bismarck system depends on the work history

$$e_{j+1}^p = e_j^p + \frac{W_j}{E(W)} \tag{2.8}$$

Bismarck pension claims at the retirement age j_r are given by

$$P_j^2 = \frac{e_{j_r}^p}{j_r - j_w} \kappa_2 E(W) \text{ and } j \geq j_r \tag{2.9}$$

in which κ_2 represents the relevant replacement rate. The Bismarck pension is thus different for each agent, depending on the individual earnings history.

Approximation Dutch system

The Dutch pension system has an income independent first pillar scheme and an income dependent second pillar. This pension scheme is approximately a Bismarck system with a means tested ($\varphi_p = 1$ and $\varphi_a = 0$) Beveridge part. This can be seen more easily by writing $P_j^1 + P_j^2 = \kappa_1 E(W) + \left[P_j^2 - \min\left(P_j^2, \kappa_1 E(W)\right)\right]$. The current first pillar is, $\kappa_1 E(W)$, while the current second pillar is approximated by the second term. Note, the financing structure of the first pillar is different and the second pillar has a saving ceiling. Moreover, the build up in the second pillar is slightly bit different. These institutional details of the Dutch system are not not taken into account.

2.4.2 Pension premium

Pension contributions to the funded system are paid during the working ages. As stated before, the objective is to investigate the implications of different schemes for one generation only. For this purpose the closing rule for both pension schemes together is that of a funded scheme, *i.e.* the premium τ^p

$$\tau^p \sum_{j=j_w}^{j_r-1} E(W_j) n_j (1+r)^{-(j-j_w)} = \sum_{j=j_r}^{j_e} E(P_j) n_j (1+r)^{-(j-j_w)} \tag{2.10}$$

makes the net benefits for a generation zero. This closing rule holds both ex ante and ex post because only micro risk is taken into account.

2.5 Unintended bequests

Unintended bequest occur in all systems. The unintended bequests scaled by the cohort size before retirement are

$$B_{j_r-1} = n_{j_r-1}^{-1} \sum_{j=j} (n_j - n_{j+1}) E\left[(1+r)A_j + W_j + P_j - \tau^p W_j - C_j\right] \tag{2.11}$$

The maintained assumption is that these unintended bequests flow to the government who spend it in the public interest. These public expenditures have no significant welfare effects for the individuals by assumption. A system with less unintended bequests give mutatis mutandis a larger welfare.

2.6 Individual behaviour

The assumptions of the previous sections determine consumption and leisure at all ages. Here we present the internal solution, *i.e.* without liquidity constraints and for a person that works. Optimal behaviour implies as leisure demand relation

$$l_j = \alpha p_{lj}^{-\rho} \tag{2.12}$$

with the price of leisure

$$p_{lj} = (1-\tau^p)\, e_j \omega + \frac{\mu_{ej}}{\mu_{Aj}} e_j \frac{\omega}{\overline{\overline{W}}} \tag{2.13}$$

with μ_{ej} the expected optimal value of the marginal utility of earning points and μ_{Aj} the expected optimal value of the marginal utility of wealth.

The simulation results are quite easy to understand given this rather simple labour supply relation. The pension premium τ^p just like the marginal utility of earning points is zero without a pension scheme or in case of an annuity system. These are systems without any distortions: leisure demand is entirely determined by the wages. A Beveridge scheme has a positive pension premium, but the marginal utility of earning points is zero. This results in labour market distortions, because it increases the demand for labour. Lastly, the marginal utility of earning points is positive in the Bismarck system. However, this marginal utility is not constant: it is low at the beginning of the working period and high at the end.

The Euler relation for consumption reads as

$$c_j - \underline{c}_j = \left[\frac{\psi_{j+1}}{1+\delta} E_j \left((1+\widetilde{r}_{j+1}) \left(c_{j+1} - \underline{c}_{j+1}\right)^{-\frac{1}{\gamma}} \right)\right]^{-\gamma} . \tag{2.14}$$

with the return defined as

$$(1+\widetilde{r}_j) = \begin{cases} (1+r)/\psi_j \text{ and } P_j^0 > 0 \\ (1+r) \text{ and } P_j^1 = 0 \\ (1+(1-\varphi_a)r) \text{ and } P_j^1 > 0 \end{cases} \tag{2.15}$$

The consumption development is not uncertain when productivity risk is insured. However, consumption will decline over the lifecycle because individuals want consume their wealth during their life. Indeed, the time preference is corrected for the survival probabilities. An annuity system ($P_j^0 > 0$) gives such a high return that it counter balances this effect: consumption becomes constant. Lastly, the Euler equation reveals that a means test ($P_j^1 > 0$) against wealth distorts consumption: it diminishes savings.

3 Calibration

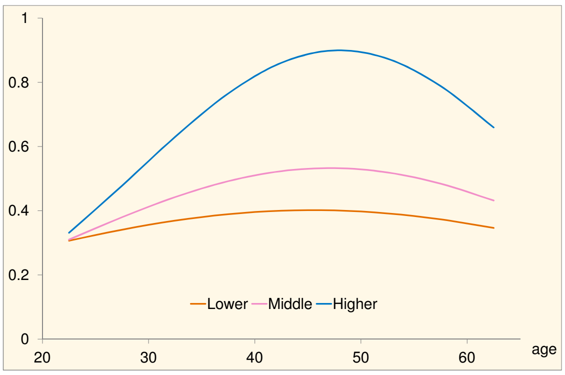
Table 3.1 presents the parameters of the model The coefficients of the productivity profiles are

Table 3.1 Demographic,utility and income parameters

Demography	Preferences	Income	Productivity		
			lower	middle	higher
$j_e = 20$	$\gamma = 0.5$	$\omega = 1$	$\rho_\eta = 0.33$	$\rho_\eta = 0.47$	$\rho_\eta = 0.61$
$j_r = 14$	$\rho = 0.3$	$r_j = 0.15$	$\sigma_\varepsilon^2 = 0.10$	$\sigma_\varepsilon^2 = 0.11$	$\sigma_\varepsilon^2 = 0.15$
$j_w = 5$	$\alpha = 2.7$		$\beta_0 = 1.90$	$\beta_0 = 1.43$	$\beta_0 = 0.61$
	$\delta = 0.15$		$\beta_1 = 0.03$	$\beta_1 = 0.06$	$\beta_1 = 0.11$
	$\mu = 2$		$\beta_2 = -0.04$	$\beta_2 = -0.07$	$\beta_2 = -0.12$

estimated on German household data[4]. Adequate estimates for the Netherlands are currently not available. The implicit assumption is that the productivity profiles of the Netherlands and Germany are not so different. With respect to the preference parameters we set the intertemporal elasticity of substitution γ to 0.5. Estimates of this elasticity typically vary widely in the range between zero and one. Research by Epstein and Zin (1991), which properly distinguishes between the aversion to risk and the aversion to intertemporal substitution, confirms this result. Our value of 0.5 is well within their range of estimated values. The intratemporal elasticity of substitution (which is equal to the wage elasticity of leisure) ρ is set to 0.3. This implies a labour supply elasticity $l_j/\left(\bar{l}-l_j\right)\rho \approx 0.3$ because leisure time is calibrated at about half total time using α. This labour supply elasticity is rather high given the results from a meta-analysis for the

Figure 3.1 Wage income profiles



[4] In order to derive different skill classes, Fehr et al. (2012) have classified individuals between ages 20 and 60 of the years 1984 to 2006 from German Socio-Economic Panel (SOEP) dat into three different educational groups (s=3) according to the International Standard Classification of Education. For low-skilled we have aggregated levels 0–2 (primary and lower secondary education), levels 3 and 4 (higher-secondary and post-secondary education) are merged to middle-skilled and levels 5 and 6 (tertiary education) to high-skilled individuals.

labour supply elasticity (Evers et al. (2005)). The productivity profiles together with the wage elasticity of labour give the wage profiles as presented in Figure 3.1 in the absence of productivity shocks.[5] The coefficient of relative risk aversion $\mu = 2.0$ is relatively small due to the use of the traditional expected utility specification. The leisure preference parameter α is set to 2.7. Finally, we set the time discount rate δ to 0.15 (which implies an annual discount rate of about 3 percent). The rate of return r is set to 0.15 (which implies a yearly return of 0.03). The time preference parameter equals the rate of return, leading to a constant consumption level in case the survival is constant.[6]

Table 3.2 Survival probablilities

Age	lower	middle	higher	average	Age	lower	middle	higher	average
5	1.00	1.00	1.00	1.00	13	0.90	0.93	0.96	0.93
6	1.00	1.00	1.00	1.00	14	0.84	0.88	0.92	0.88
7	1.00	1.00	1.00	1.00	15	0.75	0.80	0.85	0.80
8	0.99	1.00	1.00	1.00	16	0.61	0.68	0.73	0.67
9	0.99	0.99	1.00	0.99	17	0.42	0.49	0.56	0.49
10	0.98	0.99	1.00	0.99	18	0.21	0.28	0.33	0.27
11	0.96	0.98	0.99	0.98	19	0.06	0.11	0.13	0.10
12	0.94	0.96	0.98	0.96	20	0.01	0.03	0.03	0.002

Agents start working at the age of 20 (j_w = 5), are forced to retire at age 65 (j_r = 14) and face a maximum possible life span of 100 years (j_e=20). The used survival probabilities $\prod_{l=0}^{j} \psi_l$ are presented in Table 3.2. The figures are based on data for 2010 from statistics Netherlands (Poelman and Duin (2010)). The differences between skill categories are based on Dutch data (Bonenkamp et al. (2013)).

[5] For the accumulated productivity shocks holds $\sigma_{\eta j}^2 = \rho_\eta^2 \sigma_{\eta j-1}^2 + \sigma_\varepsilon^2$.

[6] See footnote 4.

4 Longevity risk

This section simulates the economy assuming that longevity is the only risk factor. Individuals are thus insured against productivity risk which implies the same income profile over the life cycle for everyone. The individuals determine their labour supply and consumption optimally given different pension schemes. The following policy questions are addressed:

- What are the consequences of no insurance against longevity risk? We show that this leads to relative poverty for those who live longer than expected early in life. This leads to the following question:
- What are the characteristics of full insurance against longevity risk from the mandatory retirement age onwards? The optimal replacement rates are approximated per skill classes. It will be argued that full insurance is difficult to establish. Second best solutions are necessary.
- Does a Beveridge or Bismarck pension scheme mimic full insurance?

Transfers between skill classes do not occur in this section. The ex-ante welfare effects are presented.[7] Moreover, the expected life-cycle paths of consumption, capital and non-capital income are compared. For instance, the Figures 4.1 and 4.3 present the development of expected consumption, capital and non-capital income using stochastic simulations for the middle skilled. Expected consumption, $E_0[C_j \mid l_j > 0]$, is determined conditional on living longer than age j, *i.e.* life expectancy l_j is positive at age j . The two sources of expected income are: labour and pension (annuity) income $E_0[(1-\tau^p)W_j + P_j \mid l_j > 0]$ as well as expected capital income $E_0[(1+r_j)A_j \mid l_j > 0]$. Labour income is generated during the working ages.

The assumptions in this section are:

1. All calibration assumptions made in Section 3, except except the productivity shock assumption.
2. This section assumes full insurance against productivity shocks, *i.e.* all shocks become equal to the expected value: $\varepsilon_j = \varsigma_j$

The different systems explored are:

1. No insurance against longevity risk: $P_j^0 = P_j^1 = P_j^2 = 0$; the replacement rate is endogenous (Section 4.1). This simulation gives answer to the first policy question formulated above.
2. An annuity system: $P_j^0 \neq 0$; $P_j^1 = P_j^2 = 0$ (Section 4.2). This simulation gives endogenous a replacement rate (κ^b) that will be used as bench-mark because the risks are optimally insured.[8]

[7] Welfare changes are measured by percentage changes in the ex-ante expected utility (see equation 2.3) at the age at which individuals enter the labour market (V_{j_w}). Utility is homogeneous in consumption exceeding the labour induced consumption level ($c_j - \underline{c}_j$). The percentage change of utility gives the necessary increase in resources net of labour induced consumption which brings about the same welfare change. By using this ex-ante welfare measure the perspective of individuals is chosen before the veil of ignorance is lifted.

[8] Note there is no annuity for $j < j_r$. Moreover there are still liquidity constraints. This simulation is thus not a first best

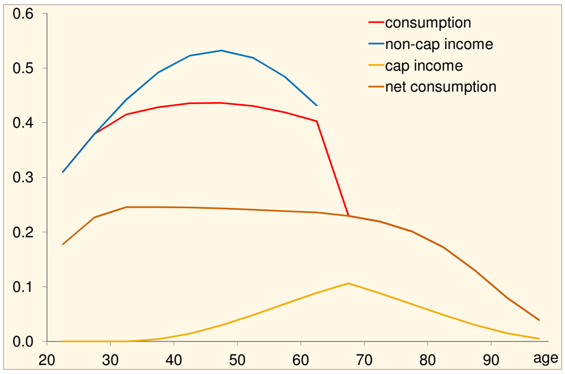
This simulation gives answer to the second policy question.

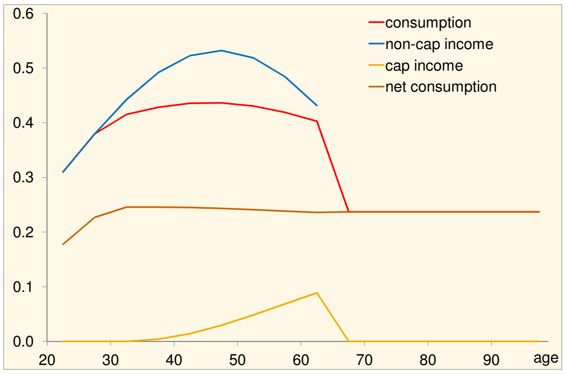
3. A pure Beveridge system: $P_j^1 \neq 0$; $P_j^0 = P_j^2 = 0$; No means testing $\varphi_p = \varphi_a = 0$; Replacement rate $\kappa_1 = \kappa^b$ (Section 4.3).
4. A pure Bismarck system: $P_j^2 \neq 0$; $P_j^0 = P_j^1 = 0$; Replacement rate $\kappa_2 = \kappa^b$ (Section 4.4).
   The last two simulations give answer to the third policy question formulated above.

4.1 No insurance against longevity risk

What are the consequences of the absence of insurance against longevity risk, when individuals are insured against productivity risk? Individuals don't know when they decease. This lack of perfect foresight leads to a consumption plan mainly driven by life expectancy, because utility of consumption is only experienced when alive. Growing older, life expectancy decreases less than age increases. This implies that every year the consumption plan takes this new information into account. After retirement savings are spread over more and more periods than first expected. This leads to a declining consumption profile in the retirement period as the left panel of Figure 4.1 reveals. The developments during the working ages reveal the behavioural assumptions further. Consumption equals the income development in the first ten years due to liquidity constraints. A rather stable development follows after the first 10 years up to the retirement age. A sharp drop in consumption takes place at the retirement age because labour induced consumption disappears.[9] Net consumption, *i.e.* consumption net of labour induced consumption, develops smoothly.

Figure 4.1 Profiles middle skilled without a scheme (left) and in case of an annuity (right)





The left panel of Figure 4.1 reveals further that the expected development of non-capital

solution.

[9] The utility specification implies a positive correlation between commodity consumption and labour supply, which is consistent with excess sensitivity (a positeve correlation between consumption and expected income changes found in the econometric literature (see Flavin (1981)). The utility specification is roughly consistent with the hump shape observed in the data (see Ree and Alessie (2009)).

income is hump shaped. Capital income rises due to capital accumulation in the working ages and declines afterwards. The unintended bequests are the largest without any system of course. They will be presented in the summary Tabel 4.2.

The developments for the different skill classes are qualitatively rather the same. However, the income and consumption levels differ substantially. Average labour income of the lower skilled is 19% lower and for higher skilled 53% higher than the average income of the middle skilled.

In summary, the model describes the phenomenon that elderly save too little for the possibility that they become older than expected. The ratio of average consumption in the retirement period relative to the working period is 0.29, 0.33 and 0.39 for lower, middle and higher skilled labour, respectively. These figures give a relatively favourable picture because actual consumption declines with age in the retirement period. These are relative poverty indicators, which measure how older people fare during retirement in comparison to their living standards in their working live. Relative poverty can hit lower, middle as well as higher skilled. Relative poverty amongst the elderly is a phenomenon of economies without good pension provisions (see Zaidi (2010)). This poverty can be prevented by offering adequate insurance schemes against longevity risk. This will be investigated in the next section.

4.2 Longevity insurance by an annuity market

Assume individuals can fully annuitize their financial wealth ($P_j^0 \neq 0$; $P_j^1 = P_j^2 = 0$) when they retire and are insured against productivity risk. This implies that the wealth of those that decease goes to those who live longer (Yaari (1965)). The effective return on financial wealth increases by this arrangement which makes the annuity optimal for individuals, who buy them voluntary. They consume the annuity amount in the retirement period. After retirement unintended bequests become zero when individuals buy annuities. This is the best approximation of the first best solution for insurance against both risks. It is not exactly the first best solution because there may be still unintended bequests and liquidity constraints before retirement. Moreover, individuals can not optimally choose their retirement age. The replacement rate (κ), defined as the ratio between the annuity and average, expected income in the working years of this annuity, is 0.48, 0.52 and 0.58 for lower, middle and higher skilled labour, respectively. These replacement rates are optimal in the sense that they represent the optimal choice of individuals given full insurance against productivity risk over the whole life cycle, insurance against longevity risk from the retirement age onwards and given possible liquidity constraints. They will be used as a bench-mark in the other simulations. So we will not explore what the optimal replacement rates are in the different systems, but use the here derived values.

Before the retirement age the income and consumption profiles with annuities are exactly the same as the profiles without insurance. Individuals have thus the same wealth at the retirement age in both cases. The unintended bequests are a factor 10 lower which indicates a much better insurance against longevity risk. After retirement the consumption developments are different. Annuitization thus leads to a constant consumption (see the right panel in Figure 4.1) at a much higher level than without annuities after retirement. Net consumption is nearly constant over the whole life cycle except for the first years due to liquidity constraints. The consumption increase leads to a welfare gain for lower, middle and higher skilled retirees of 25%, 22% and 20% at the retirement age, respectively. However, the expected utility increase measured at the start of the working career is only 2%, 1.7%, and 1.5%. The welfare gain at the start of the working ages represents the expected discounted value of the increased consumption possibilities after retirement.

In summary: Annuitization at the retirement age provides the best approximation of the first-best solution for insurance against longevity risk. The annuity system will be used as a bench-mark to evaluate other pension systems.

We investigate how a pension system of a classic Beveridge design performs in comparison with the bench-mark solution in the next section. The discussion is restricted to middle skilled only because the economic mechanisms are the same for lower and higher skilled.

4.3 Beveridge system

Retirees get an equal pension in the Beveridge system (left panel Figure 4.2). A replacement rate $\kappa_1 = 0.52$, taken from the bench-mark case, is obtained with a pension premium of 7.4% for the middle skilled. The labour market distortions diminishes labour supply (-2.8% on average over the working period of a cohort). The expected utility at the start of the working period is 3.8% lower than in the bench-mark case (and is even lower than the no insurance case) and 3.8% higher at the start of the retirement period. Welfare measured at the start of the working period

Figure 4.2 Profiles middle skilled with a Beveridge (left) and Bismarck scheme (right)

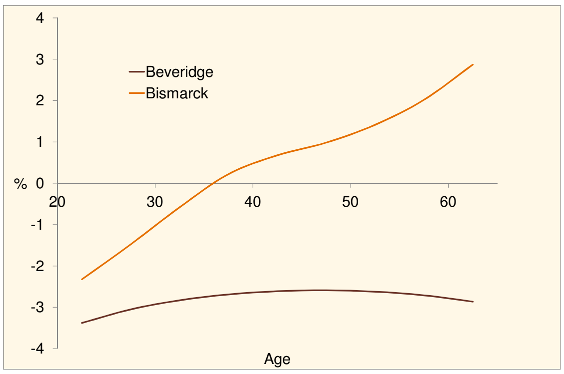
diminishes because individuals become more liquidity constrained due to pension premium payments, which leads to a lower net income. The welfare gain measured at the start of the retirement period is induced by the larger consumption possibilities in the first years of retirement, caused by the shift of consumption towards older ages due to the liquidity constraint. Retirees benefit also from premia paid by persons who died before retirement.

In summary; the Beveridge system mimics the annuity markets as long as longevity is the only risk factor. Consumption over the retirement period is 52% over the retirement period relative to the working period and does not decline. So we may conclude that the Beveridge system prevents poverty amongst elderly. Moreover, the adverse selection problems are less severe. This makes it more easy to establish than the benchmark system. However welfare declines -3.8% relative to the bench-mark at the start of the working period. The system leads to less labour supply due to the missing link between benefits and contributions. This decline in labour supply can be prevented by a Bismarck system as is shown in the next section.

4.4 Bismarck system

The right panel of Figure 4.2 presents the profiles in case of a Bismarck system. The profiles do not look very different. However, this is not really the case as will be shown below (Table 4.1). A replacement rate $\kappa_1 = 0.52$ is obtained again with a pension premium of 7.4% for the middle skilled, which is exactly the same as in the Beveridge system. An important difference is the employment development. The Bismarck system seems not to distort the labour market. Employment increases even a little bit with 0.4% on average relative to the bench-mark. However, this aggregated figure does not reveal the actual difference with the Beveridge system. Figure 4.3 shows, the Bismarck system is distorting, too. The distortions are now caused by the lack of actuarial neutrality. This non-neutrality is caused by a pension built up independent of the age of the worker, combined with a constant premium rule. Indeed, an earning point earned

Figure 4.3 Percentage changes employment relative to the benchmark; middle skilled



at the start of the working period needs less funding than a point earned at the end of the working period due to the difference in investment horizon. This combination leads to less labour supply at the begin of the working period and more labour supply at the end. Moreover, net consumption smoothing is not possible due to liquidity constraints. The consumption net of labour induced consumption increases for older workers and retirees while it diminishes for younger workers relative to the Beveridge system (see Table 4.1).

The expected utility at the start of the working period is 4.4% lower than in the bench-mark case (and is even lower than without insurance or with the Beveridge system) and at the start of the retirement period 5.3% higher than in the benchmark case. The Bismarck system is thus not an obvious improvement for the Beveridge scheme when its design is not actuarially neutral.

Table 4.1 Effects of a Bismarck instead of a Beveridge system

	percentage changes							
age	22.5	32.5	42.5	52.5	62.5	72.5	82.5	92.5
Consumption	1.1	2.3	3.6	4.7	6.0	0.1	3.5	3.7
Non-capital income	1.1	2.3	3.3	4.1	5.9	3.5	3.5	3.5
Net consumption[1)]	– 0.1	– 0.2	0.0	0.3	0.1	0.1	3.5	3.7
Employment	1.1	2.3	3.4	4.1	5.9	0.0	0.0	0.0

[1)] Net consumption is defined as $\left(c_j - \underline{c}_j\right)$

4.5 Summary

This section simulates the economy assuming longevity as the only risk factor. Table 4.2 summarizes these simulations. The bench mark is the annuity system against which the differences are presented. We summarize the results.

- No insurance against longevity risk lead to relative poverty for those who live longer than expected early in life. The replacement rates (κ_i) are low: 0.29, 0.33 and 0.39 for lower, middle and higher skilled labour, respectively. These figures give a relatively favourable picture because actual consumption declines with age in the retirement period.
- Annuitization at the retirement age prevents poverty. However, few individuals voluntarily annuitize their savings in reality. An important reason is a mismatch between the optimal consumption path and the annuity income (Davidoff et al. (2005)) for instance due to large unexpected health care costs. Full annuitizing is thus not optimal when unexpected large expenditures can occur in the future, which can not be paid with the annuity and liquidity constraints make borrowing impossible. Moreover, individuals have private information about their health at the age 65. When they have a short life expectancy they prefer bequeathing above annuitizing their wealth (if they have bequest motives in their utility function).[10] An annuity pension system is thus not a realistic option.
- This explains why we explore whether a Beveridge or Bismarck pension scheme is able to

[10] Heijdra and Reijnders (2012) investigate the consequences of asymmetric information thoroughly.

mimic full insurance. Aggregated (over time) employment (a in the table) is different between both systems. The Bismarck system gives less labour market distortions on average, but distorts labour supply over the life cycle due to non acturarial neutral premiums. The consumption possibilities become larger, but can not be smoothed over the life cycle due to liquidity constraints. This leads to the non expected conclusion that the Bismarck system is not an obvious improvement for the Beveridge scheme.

- The unintended bequests (B_{jr-1}) are very large without a scheme. An annuity system gives a factor ten lower unintended bequests. They are still large relative to a Beveridge or Bismarck system. Those systems lead to mandatory savings and a decline of private savings.

The goal of the first pillar is to prevent old age poverty. Both the Beveridge and Bismarck systems manage to deal with this problem in case there is no uncertainty about productivity. It is just not clear which one performs better. Hence, we do the same exercise in an uncertain productivity setup, too. Labour income uncertainty will be included in the following analysis in addition to longevity risk to obtain more insight into the differences.

Table 4.2 Summary simulation results with only longevity risk[1)]

	κ_i	B_{jr-1}	τ	V_{jw}	V_{jr}	a
Lower			(Δ)	(%)	(%)	(%)
Annuity	0.48	0.17	0	0	0	0
No insurance	0.29	0.75	0	-2.0	-19.5	0
Middle						
Annuity	0.52	0.16	0	0	0	0
No insurance	0.33	1.56	0	-1.6	-18.2	0
Beveridge	0.52	0.04	7.4	-3.8	3.6	-2.8
Bismarck	0.52	0.03	7.4	-4.4	5.3	0.4
Higher						
Annuity	0.58	0.20	0	0	0	0
No insurance	0.39	2.55	0	-1.5	-16.5	0

[1)] A % points to a percentage change and a Δ to a difference relative to the bench-mark case.

5 Productivity and longevity risk

This section extends the previous analysis by including productivity risk apart from longevity risk. The setup is the same as in the previous section. This section addresses the following policy questions:

- What are the consequences of no insurance against both productivity and longevity risk? We show that this leads to relative poverty for those who live longer than expected early in life and for those who have low earning capacity.
- What are the characteristics of full insurance against longevity risk, *i.e.* annuitization of financial wealth, from the mandatory retirement age onwards? It will be shown that annuitization does not give a solution for those who have a too low earning capacity.
- Does a Beveridge or Bismarck pension scheme give a solution to both productivity and longevity risk?
- Can the current Dutch pension system be improved by extending the Bismarck part (second pillar) or by extending means testing?

Transfers between skill classes do not occur in this section. The ex-ante welfare effects are presented. Moreover, the expected life-cycle paths of consumption, capital and non-capital income are compared.

The assumptions in this section are:

1. All calibration assumptions made in Section 3 are maintained without any exception.

The different systems investigated in this section are:

1. No insurance against longevity risk: $P_j^0 = P_j^1 = P_j^2 = 0$ (Section 5.1); the replacement rate is endogenous. This simulation gives answer to the first policy question formulated above.
2. Longevity insurance by an annuity market: $P_j^0 \neq 0$; $P_j^1 = P_j^2 = 0$ (Section 5.2). This simulation gives endogenous replacement rate that are not optimal due to precautionary saving effects.[11] This simulation gives answer to the second policy question.
3. Pure Beveridge system: $P_j^1 \neq 0$; $P_j^0 = P_j^2 = 0$ (Section 5.3); No means testing $\varphi_p = \varphi_a = 0$; Replacement rate $\kappa_1 = \kappa^b$, with κ^b the bench-mark replacement rate determined in Section 4.2 .
4. Bismarck system: $P_j^2 \neq 0$; $P_j^0 = P_j^1 = 0$ (Section 5.4); Replacement rate $\kappa_2 = \kappa^b$, with κ^b the bench-mark replacement rate determined in Section 4.2.
   Simulation 3 and 4 give answer to the third policy question.
5. Current Dutch pension system: $P_j^0 \neq 0$; $P_j^1 \neq 0$; $P_j^2 \neq 0$ (Section 5.5); $\varphi_p = 1$; $\varphi_a = 0$; $\kappa_2 = \kappa^b$; $\kappa_1 = 0.25$. This simulation together with the simulation of the pure Bismarck system (simulation

[11] Note there is no annuity for $j < j_r$. Moreover there are still liquidity constraints. This simulation is thus not a first best solution.

4) gives answer to the question whether the current Dutch pension system can be improved by extending the Bismarck part.

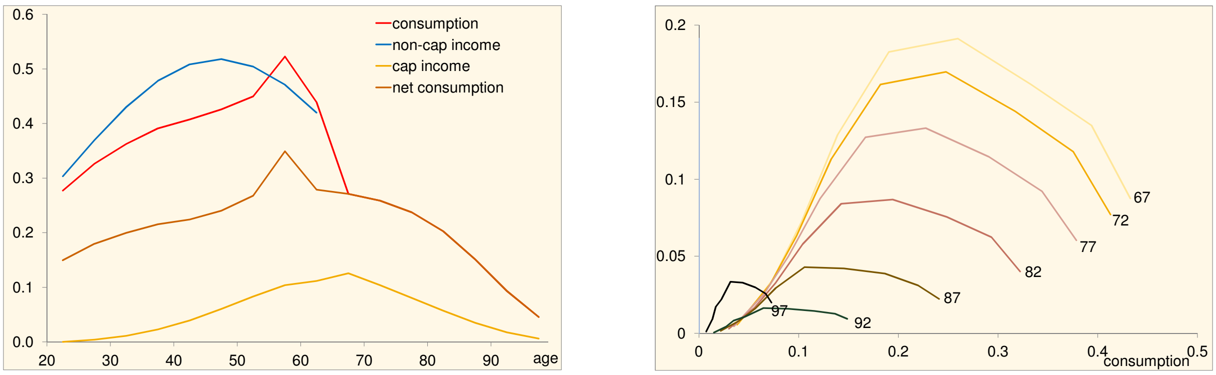
6. Extended means testing: $P_j^0 \neq 0$; $P_j^1 \neq 0$; $P_j^2 \neq 0$ (Section 5.5); $\varphi_p = 1$; $\varphi_a = 1$; $\kappa_2 = \kappa^b$; $\kappa_1 = 0.25$. This simulation together with simulation (5) gives answer to the question whether the current Dutch pension system can be improved by extending means testing with wealth.

The discussion is restricted to middle skilled in Section 5.1, 5.2, 5.3 and 5.4. Section 5.5 discusses results for lower and higher educated, too. Section 5.6 summarizes the results.

5.1 No insurance against longevity- and productivity risk

Productivity risk leads to a decline in labour supply. Indeed the literature points to a wage elasticity of labour supply smaller than one. Such a wage elasticity implies a smaller impact on labour supply from positive than from negative productivity shock so that the shocks do not cancel out. Uncertain labour income has thus a negative wage (price) effect on labour supply. But is this not cancelled out by a positive income effect? It is true that future income uncertainty may induce precautionary labour supply, but this intertemporal considerations are not taken into account in our model. Our approach is supported by evidence that finds zero or small intertemporal effects (Lumsdaine and Mitchell (1999)). On balance the non-capital income is lower (Figure 5.1) than with insurance against productivity risk (Figure 4.1)

Figure 5.1 Profiles middle skilled in case of no scheme (left) and consumption distributions (right)



Without insurance, households can only save by themselves for the retirement period[12]. Productivity risk leads to precautionary savings. Precautionary savings make the liquidity constraints less severe and leads to a growing consumption profile during the working ages[13].

[12] The non-capital income profile is not exactly the same in Figure 5.1 as in Figure 4.1 through the endogenous labour supply reactions without insurance against labour income risk.

[13] We use Monte Carlo integration methods to determine the expected developments. This may explain the sharp increase around age 60, while a more smoothed development is expected. Judd et al. (2011) advocates exact integration using the Gauss-Hermite quadrature to get more accuracy. This is a possible future improvement of the used method.

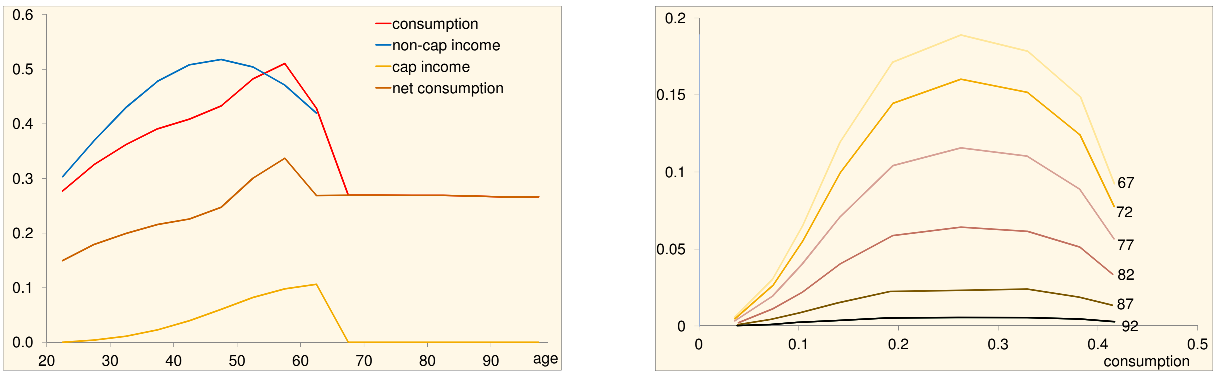
Consumption drops again at the retirement age to a much lower level, because labour induced consumption becomes zero. Consumption stays declining in the retirement period due to consumption planning based on expected longevity. The limited consumption possibilities, when individuals become older than expected, signal relative poverty.However, poverty does not occur due to longevity only, but also due to low earning capacity. This is illustrated in the right panel of Figure 5.1 which presents the consumption distributions for different ages multiplied with the probability of staying alive $P[a < C_j < b]\,P[l_j > 0]$. The whole surface below the line of a certain age group gives the probability of being alive at that age $P[l_j > 0]$ while the surface between the consumption levels a and b represents the probability of a consumption level in between a and b multiplied with the probability of being alive. The older the age cohort, the further to the left the distribution is located.

The Figure reveals that poverty does not only hit the oldest age cohorts but also the younger with low earning capacity. Indeed, some individuals have very low consumption possibilities at every age cohort as the consumption distributions reveal. As will be shown in the next section, poverty can be prevented only partly by the introduction of annuities.

5.2 An annuity market

Assume individuals annuitize their financial wealth when they retire and consume the constant annuity amount during the retirement period. The replacement rate (κ) of this annuity for middle skilled individuals is on average 0.60. This replacement rate is higher than in the bench-mark case with insurance against both longevity and productivity risk. Income uncertainty leads to precautionary savings and thus to more financial wealth, which brings about a larger replacement rate after annuitizing.Expected utility at the retirement age increases through the

Figure 5.2 Profiles middle skilled in case of an annuity (left) and consumption distribution (right)

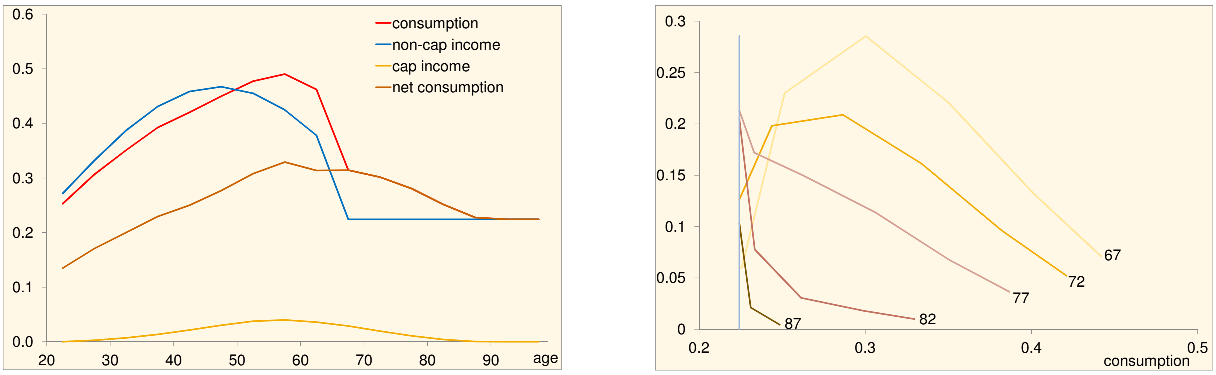


introduction of annuities with 18.9% relative to no insurance against both productivity and longevity risk. This is again a huge welfare gain. The utility of the 20 years old increases only with 0.6% relative to no insurance against both productivity and longevity risk[14], *i.e.* the economy described in section 5.1. Households save to smooth consumption but also for precautionary reasons compared to the bench-mark solution as discussed in Section 4.2. This leads to the relatively low welfare increase for the younger generations and the large gain for the older. The unintended bequests are larger through the precautionary saving effect, which gives an additional explanation for the small welfare gain of the younger.

The positive effect remains as was discussed in Section 4.2: longevity risk is insured. However, the annuity system does not prevent all poverty as households with low earning capacity now run into problems due to productivity risk, which wasn't there before! Moreover, a private annuity market will not be established due to for instance asymmetric information problems as stated before.

In summary, a private or mandatory annuity market seems not very realistic and does not prevent poverty within all age cohorts. Moreover, precautionary saving effects lead to a larger replacement rate than the bench-mark value, which approximates the first best solution. The Beveridge system which guarantees income at retirement is investigated in the next section. The bench-mark replacement rate will be used and not the here obtained larger value because the latter one is due to inefficient precautionary saving effects.

Figure 5.3 Profiles middle skilled in case of a Beveridge scheme (left) and consumption distribution (right)



5.3 Beveridge system

Retirees get a uniform pension in a Beveridge system. Income is thus optimally insured in the retirement period. The bench-mark value of the replacement rate[15] for middle skilled $\kappa_1 = 0.52$ is obtained with a pension premium for middle skilled of 7.4%. This pension premium leads to a

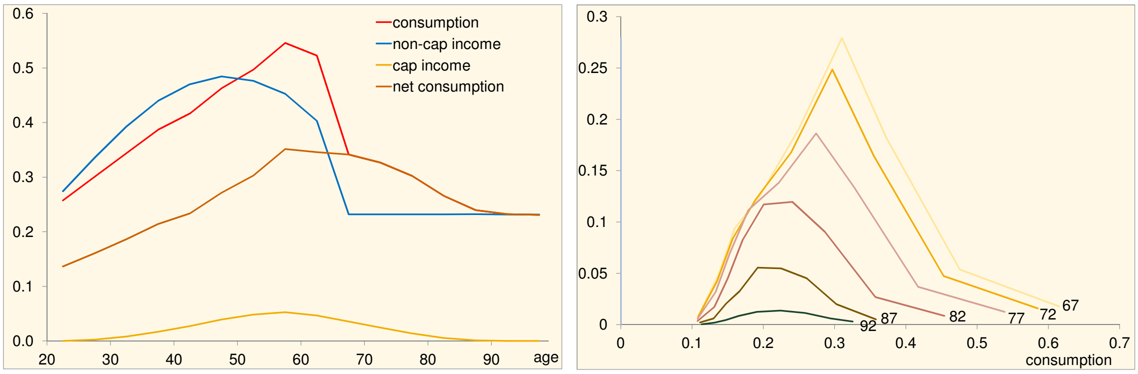
[14] This value is the difference of V_{jw} between the row annuity and the row no insurance in the right column of Table 5.3.

[15] We don't use use the annuity-replacement rate of 60% from Section 5.2 because this higher value is caused by non optimal, precautionary savings.

decline in middle skilled labour supply of 2.8% relative to no insurance against both productivity and longevity risk, *i.e.* the economy described in section 5.1.Nevertheless, the 20 years old experience a welfare gain of 0.6% relative to no insurance against both productivity and longevity risk[16], which coincidentally equals the welfare effect of annuitization. The income insurance in the Beveridge system leads to an expected utility increase of 45% relative to no insurance against both productivity and longevity risk at the start of the retirement period. This increase is much larger than the utility increase with an annuity system. Many individuals have larger consumption possibilities due to the income guarantee of the Beveridge system, as the consumption distribution in the right panel of Figure 5.3 reveals. A major disadvantage of the Beveridge system is the negative effect on labour supply caused by the missing link between contributions and benefits. This adverse effect on labour supply does not occur in a Bismarck system, which we investigate in the next section.

5.4 Bismarck system

Figure 5.4 Profiles middle skilled in case of a Bismarck scheme (left) and consumption distribution (right)



The bench-mark value of the replacement rate for middle skilled $\kappa_2 = 0.52$ is obtained with a pension premium 7.5%. Pensions depend on the labour market history in a Bismarck pension scheme. This link between individual earnings and the subsequent pension claims leads to a small labour supply increase (0.3%) in comparison to no insurance at all. However, expected utility at the start of the working period decreases now with 0.5% relative to no insurance against both productivity and longevity risk[17], *i.e.* the economy described in section 5.1. The main reason is the labour market distortion caused by the lack of actuarial neutrality of the Bismarck system. This unfairness leads to a shift in labour supply from younger to older ages. The liquidity constraint becomes more tight. Income uncertainty remains both in the working ages as

[16] This value is the difference of V_{jw} between the row Beveridge and the row no insurance in the right column of Table 5.3.

[17] This value is the difference of V_{jw} between the row Bismarck and the row no insurance in the right column of Table 5.3.

well as in the retirement period. Uncertainty leads to precautionary savings and thus to more unintended bequests. The negative effects from precautionary savings and from the liquidity constraint dominate the positive effects of more labour supply. Retirees take advantage of these precautionary savings and experience a welfare gain of 42.5% relative to no insurance against both productivity and longevity risk. The main reason for the welfare increase in the retirement period are the larger consumption possibilities. So the Bismarck system stimulates employment but does not diminish the labour market distortions.The Bismarck system does not lead to an overall increase of welfare. Moreover, it does not prevent poverty within age cohorts. To prevent poverty within age groups social assistance is necessary. For this reason, a Bismarck system extended by a means-tested Beveridge part is investigated in the next section.

5.5 Bismarck system with means-tested Beveridge part

A Bismarck system extended with a means-tested Beveridge part guarantees an income $\kappa_1 E(W)$. A lower income than this guarantee leads to a benefit P_j^1 to bridge the difference. This means tested Beveridge part of the system reads as

$$P_j^1 = \max\left[\kappa_1 E(W) - \varphi_p P_j^2 - \varphi_a r A_j, 0\right] \text{ and } A_j \geq 0 \tag{5.1}$$

κ_1 marks the replacement rate for the means tested part. The taper rates φ_a and φ_p define the precision of the means test against the stock of liquid assets (or returns on assets) and pensions from the Bismarck part P_j^2, respectively. Without a means test (i.e. $\varphi_p = \varphi_a = 0$) individual pensions from the Beveridge part are uniform for all agents. On the other hand, with a means-test (i.e. $\varphi_p \neq 0$ and (or) $\varphi_a \neq 0$) the amount of the benefits depends on individual characteristics of the retiree.

The current Dutch two pillar system can be approximated by setting $\varphi_p = 1$ and $\varphi_a = 0$.[18] The replacement rate for the Beveridge part is set at $\kappa_1 = 0.25$.[19] The bench-mark value of the replacement rate for middle skilled $\kappa_2 = 0.52$ is obtained with a pension premium of 7.5%. The means test implies a little bit higher pension premium relative to a pure Bismarck system. Moreover, the marginal utility of earning points decreases by the means test. This leads to less labour supply (see Table 5.1). The budget constraint becomes more tight leading to less consumption and a decrease of welfare at the start of the working period. At the end of the life

[18] This can be seen more easily by writing $P_j^1 + P_j^2 = \kappa_1 E(W) + \left[P_j^2 - \min\left(P_j^2, \kappa_1 E(W)\right)\right]$. The current first pillar is, $\kappa_1 E(W)$, while the current second pillar is approximated by the second term. Note, the financing structure of the first pillar is different and the second pillar has a saving ceiling. Moreover, the build up in the second pillar is a little bit different. These institutional details of the Dutch system are not not taken into account.

[19] A full AOW is about equal to a social assistance benefit. A full (100%) gross AOW (household with men and women) is equal to the gross minimum wage, while the minimum wage is about half average wages. The model distinguishes individuals and not households. This leads to the following parameterization: $\kappa_1 = 0.5 \times 0.5 = 0.25$. This is a rather low value given that hardly anybody has an income below 0.1 in our simulations. So the effects can not be large of means testing.

cycle consumption increases a little bit due to the income guarantee. Welfare increases slightly relative to a pure Bismarck system for retirees.

Table 5.1 Effects of a means test ($\varphi_p = 1$ and $\varphi_a = 0.$) relative to a pure Bismarck system; middle skilled

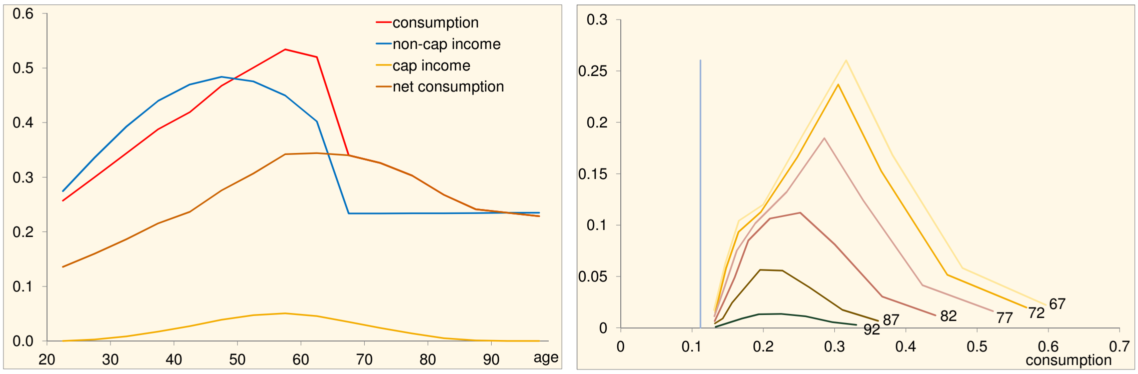
	percentage changes							
Age	22.5	32.5	42.5	52.5	62.5	72.5	82.5	92.5
Consumption	– 0.2	0.0	0.6	0.7	– 0.5	– 0.1	0.8	1.1
Non-capital income	0.0	0.0	– 0.1	– 0.2	– 0.2	0.7	0.9	1.4
Capital income	0.0	2.8	0.7	– 2.1	– 1.7	– 1.6	– 0.8	0.0
Net consumption	– 0.4	0.1	1.2	1.2	– 0.5	– 0.1	0.8	1.1
Employment	0.0	– 0.1	– 0.1	– 0.1	– 0.2	0.0	0.0	0.0

The results for the extended means test ($\varphi_p = \varphi_a = 1$) against second pillar income and wealth are ambiguous. The reason is that the means test leads to different effects for low and high incomes within a skill group. Low income earners will decrease their savings to increase their first pillar pensions. So they will increase their consumption early in life. High income earners will increase their savings to get the same pension as before the means test. Moreover, the effects of the means test against the returns on assets are small because the introduction of mandatory savings for pensions leads to a decline of free savings (see Figure 5.4). The results do not point to a welfare increase of the introduction of a means test against wealth income.

Table 5.2 Effects of an extended means test ($\varphi_p = \varphi_a = 1$) relative to the means test ($\varphi_p = 1$ and $\varphi_a = 0.$)

	percentage changes							
Age	22.5	32.5	42.5	52.5	62.5	72.5	82.5	92.5
Lower								
Consumption	0.1	– 0.1	– 0.2	0.3	0.3	– 0.3	– 0.6	0.1
Non-capital income	0.0	0.0	0.0	0.1	– 0.1	– 0.2	– 0.1	0.0
Capital income			0.7	1.5	0.8	– 2.6	– 8.0	
Net consumption	0.1	– 0.2	– 0.3	0.5	0.6	– 0.3	– 0.6	0.1
Employment	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Middle								
Consumption	0.6	0.1	1.1	– 2.2	– 0.2	– 1.9	– 2.3	0.0
Non-capital income	0.1	0.2	0.2	– 0.4	0.0	– 0.1	0.1	0.1
Capital income	0.0	– 3.3	– 2.6	– 0.9	– 5.1	– 9.0	– 15.9	0.0
Net consumption	1.0	0.0	1.6	– 2.6	– 0.4	– 1.9	– 2.3	0.0
Employment	0.0	0.1	0.1	– 0.2	0.0	0.0	0.0	0.0
Higher								
Consumption	– 0.1	– 0.9	0.5	– 0.8	1.8	1.9	0.9	– 0.2
Non-capital income	– 0.1	– 0.1	0.2	0.0	0.2	– 0.1	– 0.1	– 0.2
Capital income	0.0	0.0	1.0	– 2.0	6.1	5.7	3.5	0.0
Net consumption	– 0.1	– 1.4	0.4	– 1.1	2.3	1.9	0.9	– 0.2
Employment	0.0	– 0.1	0.1	– 0.1	0.2	0.0	0.0	0.0

Figure 5.5 Profiles middle skilled in case of a means tested Bismarck scheme ($\varphi_p = \varphi_a = 1$) (left) and consumption distribution (right)



5.6 Summary

Table 5.3 summarizes the analyses relative to the bench-mark system in which insurance takes place against both risks (the annuity simulation in Section 4.2). This section extends the previous analysis by including productivity risk apart from longevity risk.

- No insurance against productivity risk has a large impact on welfare at the start of the working period (V_{jw}). Moreover productivity risk leads to less labour supply (a) due to subsitutition effects in the absence of income effects. The unintended bequests are the biggest without insurance and for all other systems approximately the same.
- Relative poverty does not only hit the oldest age cohorts but also the younger with low earning capacity without insurance. Indeed, every age cohort has individuals with very low consumption possibilities with only self insurance.
- Relative poverty can be only partly prevented by the introduction of annuities. Moreover, precautionary saving effects lead to a larger replacement rate than the bench-mark value, which approximates the first best solution.
- The pure Beveridge scheme gives a larger utility than a pure Bismarck pension scheme. This leads again to the conclusion that the Bismarck system is not an obvious improvement for the Beveridge scheme.
- The current Dutch (first and second pillar) pension system is approximated by a Bismarck system with a Beveridge part means-tested against second pillar pensions. This pension system gives a clear welfare gain for retirees relative to no scheme. The welfare effect evaluated at the start of the working period is less clear cut. Low income workers experience a welfare gain, but middle and high income workers enjoy no welfare gain of the current system at the start of the working period.
- The results do not point to a clear welfare gain of an extension of the Bismarck part of the system for retirees. An extension of the Beveridge part of the system seems even more

welfare improving for retirees. A lack of actuarial neutrality of the investigated Bismarck system give unexpected poor welfare effects.

- Lastly, the simulations do not point to a clear welfare gain of a means test against wealth income.

Table 5.3 Summary simulation results[1)]

	Longevity and productivity risk					
	κ_i	B_{jr-1}	τ	V_{jw}	V_{jr}	a
Lower			(Δ)	(%)	(%)	(%)
No insurance	0.36	2.09	0	−14.5	−15.6	−8.1
Means tested ($\varphi_p = 1$, $\varphi_a = 0$)	0.48	0.33	6.20	−12.5	9.2	−2.4
Means tested ($\varphi_p = \varphi_a = 1$)	0.48	0.33	6.18	−11.8	9.2	−2.3
Middle						
Annuity	0.60	0.40	0	−19.1	−4.6	−3.3
No insurance	2.60	0.70	0	−19.7	−19.7	−3.3
Beveridge	0.44	0.37	7.40	−19.1	16.7	−6.1
Bismarck	0.52	0.37	7.46	−20.2	14.4	−3.0
Means tested ($\varphi_p = 1$, $\varphi_a = 0$)	0.52	0.37	7.53	−20.7	14.5	−3.2
Means tested ($\varphi_p = \varphi_a = 1$)	0.52	0.38	7.50	−20.7	11.8	−3.1
Higher						
No insurance	3.90	0.65	0	−27.8	−27.3	−4.1
Means tested ($\varphi_p = 1$, $\varphi_a = 0$)	0.58	0.34	9.28	−32.0	4.7	−3.8
Means tested ($\varphi_p = \varphi_a = 1$)	0.58	0.35	9.27	−32.0	7.0	−3.8

[1)] A % points to a percentage change and a Δ to a difference relative to the bench-mark case.

6 Conclusions

This paper has investigated alternative approaches to the current Dutch first pillar pension scheme (AOW). To do so, it has presented a unique stochastic partial equilibrium model to study these alternatives for the Dutch situation. The analyses have explored the welfare, labour-market, saving and unintended-bequest effects of a shift from a Beveridge towards a Bismarck system of the first pillar pension system in the Netherlands. This change is investigated using a model with micro-economic labour productivity risk and longevity risk and without intergenerational redistribution. Full insurance against labour productivity risk and full annuitization of wealth at the exogenous retirement age yields benchmark replacement rates of 0.48, 0.52 and 0.58 for lower-, middle-, and higher-skilled workers. This benchmark solution is difficult to establish due to for instance (not modelled) adverse selection or moral hazard problems. This benchmark solution is used to evaluate other pension schemes.

A full Beveridge and Bismarck system is investigated for the middle skilled. The replacement rate, fixed at the benchmark value, leads to the same premium in both systems. Expected utility at the start of the working period decreases more in the pure Bismarck system than in a pure Beveridge system. The Beveridge system leads to labour-market distortions as does the investigated Bismarck system. A lack of actuarial neutrality of the investigated Bismarck system together with liquidity constraints yields the unexpectedly poor welfare effects.

The full Bismarck system could lead to relative poverty amongst the elderly with low earning capacity during their working life. As a consequence, the Bismarck system has to be extended by a means-tested Beveridge first pillar. The effects of a Bismarck system with a means-tested Beveridge first pillar are limited without redistribution between skill classes, i.e., when the replacement rate is defined in a percentage of expected income per skill class. It still yields lower welfare compared to a Beveridge system with more income certainty in the retirement period. The two pillar system with a Beveridge pension, which is means tested against second pillar income, is an approximation of the current Dutch system (apart from the financing structure and the saving ceiling in the second pillar). This leads to the first main conclusion of this paper: A shift of the first pillar pensions from a Beveridge towards a Bismarck system in which the pension rights depend on labour-market history is not necessarily welfare improving from an ex ante insurance perspective, i.e. before the veil of ignorance is lifted. The second main conclusion is that a means test of the first pillar against second pillar income and against wealth income is not welfare improving. Self insurance seems insufficient because it leads to relative poverty among the elderly.

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Appendix A Solution method parameterized expectations

To solve the constrained optimization problem in the main text, the first-order and Kuhn-Tucker conditions are derived. The Fischer-Burmeister function is used to implement these Kuhn-Tucker conditions. The first-order conditions contain several conditional expectations, *i.e.* expectations conditional on the state of the individual. This state is determined by their skill level, financial wealth, earning points, which determine the pension rights, and the efficiency. We use parametrized expectations to model these conditional expectations (see Heer and Maussner (2005), Judd et al. (2009)[20]). In particular, Hermite polynomials in the state variables are used to model expectations. The presented results are obtained with third order polynomials. The parameters of these polynomials are determined using Kalman filters in an interactive way. To increase the efficiency of the simulations we use antithetic draws (see Hendry (1995)).

This method is known as parameterized expectations or approximate dynamic programming. It is used in an overlapping generations framework with macro risk in Draper and Westerhout (2011). An overlapping generations model with both idiosyncratic and aggregate economic risk is Harenberg and Ludwig (2013).

A.1 Derivation of first order conditions

Use as utility measure

$$U_j \equiv \frac{\gamma}{\gamma-1} V_j^{\frac{\gamma-1}{\gamma}} \tag{A.1}$$

and define the optimization problem recursively

$$U_j(z_j) = \max_{c_j,l_j} \left\{ \left(\frac{\gamma}{\gamma-1}\right) u_j(c_j,l_j)^{1-\frac{1}{\gamma}} + \frac{\psi_{j+1}}{1+\delta} E_j U_j(z_{j+1} \mid z_j) - \frac{\psi_{j+1}}{1+\delta} \mu_{Aj} \lambda_{lj} \left[l_j - \bar{l}\right] + \right. \tag{A.2}$$

$$\left. + \frac{\psi_{j+1}}{1+\delta} \mu_{Aj} \lambda_{Aj} \left[(1+r)A_j + (1-\tau^p) W_j + P_j - c_j p_{cj}\right] \right\}$$

with $z_j = (j, e_j^p, A_j, e_j)$, λ_{lj} the Lagrange parameter with respect to the leisure restriction $\bar{l} > l_j$ and λ_{Aj} the Lagrange parameter with respect to the liquidity constraint $A_{j+1} \geq 0$. Note, both Lagrange parameters[21] are multiplied with the constant μ_{Aj} which is the expected optimal value of the marginal utility of wealth $\mu_{Aj} \equiv E_j \left(\frac{\partial U_{j+1}}{\partial A_{j+1}}\right)$. The derivation makes use of the expected optimal value of the marginal utility of earning points which is defined as $\mu_{ej} \equiv E_j \left(\frac{\partial U_{j+1}}{\partial e_{j+1}^p}\right)$. The wealth equation reads as

$$A_{j+1} = (1+r)A_j + (1-\tau^p) W_j + P_j - c_j p_{cj} \text{ and } P_j = P_j^1 + P_j^2 \ . \tag{A.3}$$

[20] Se also , Judd et al. (2011) and Judd et al. (2012). This method seems promising for solving overlapping generations models (Hasanhodzic and Kotlikoff (2013)).

[21] Scaling of the Lagrange parameter after the derivation of the first order conditions is a more clumsy procedure.

Wages are defined as

$$W_j = \left(\bar{l} - l_j\right) e_j \omega \ . \tag{A.4}$$

First pillar pensions, means tested against second pillar pension income and wealth income read as

$$P_j^1 = \max\left[\kappa_1 E(W) - \left(\varphi_p P_j^2 + \varphi_a r \max\left[A_j, 0\right]\right), 0\right] \ . \tag{A.5}$$

Second pillar pensions

$$P_j^2 = \left[\frac{e_{j_r}^p}{j_r - j_w - 1}\right] \kappa_2 E(W) \tag{A.6}$$

depend on the earning points

$$e_{j+1}^p = e_j^p + \frac{W_j}{E(W)} \ . \tag{A.7}$$

The first order conditions are:

1. for leisure:

$$\frac{\partial U_j(z_j)}{\partial l_j} = u_j^{\frac{-1}{\gamma}} u_{lj} - \frac{\psi_{j+1}}{1+\delta} \widetilde{p}_{lj} \mu_{Aj} = 0 \tag{A.8}$$

with u_{lj} the marginal utility of leisure and

$$\widetilde{p}_{lj} = (1 - \tau^p) e_j \omega (1 + \lambda_{Aj}) + \frac{\mu_{ej}}{\mu_{Aj}} e_j \frac{\omega}{\overline{\overline{W}}} + \lambda_{lj} \tag{A.9}$$

the price of leisure. Note for pensioners $e_j = 0$ holds.

2. for consumption:

$$\frac{\partial U_j(z_j)}{\partial c_j} = u_j^{\frac{-1}{\gamma}} - \frac{\psi_{j+1}}{1+\delta} \widetilde{p}_{cj} \mu_{Aj} = 0 \tag{A.10}$$

with $\widetilde{p}_{cj} = p_{cj}(1 + \lambda_{Aj})$.

The Benveniste Scheinkman conditions are:

1. for financial wealth:

$$\frac{\partial U_j(z_j)}{\partial A_j} = \frac{\psi_{j+1}}{1+\delta} \mu_{Aj} (1 + \widetilde{r}_j) \tag{A.11}$$

with

$$(1 + \widetilde{r}_j) = \begin{cases} (1+r)(1+\lambda_{Aj}) \text{ and } P_j^1 = 0 \\ \left(1 + \lambda_{Aj}\right)(1 + (1 - \varphi_a) r) \text{ and } P_j^1 > 0 \end{cases} \tag{A.12}$$

2. and for the earning points

$$\frac{\partial U_j(z_j)}{\partial e_j^p} = \frac{\psi_{j+1}}{1+\delta} \mu_{ej} \ . \tag{A.13}$$

The Kuhn Tucker conditions are

$$0 = \lambda_{lj}\left[\bar{l} - l_j\right] \; , \tag{A.14}$$
$$0 = \lambda_{Aj}\left[(1+r)A_j + (1-\tau^p)W_j + P_j - c_j p_{cj}\right] \; .$$

Substitute the definition of μ_{ej} into equation A.13

$$\frac{\partial U_j(z_j)}{\partial e_j^p} = \frac{\psi_{j+1}}{1+\delta} E_j \left(\frac{\partial U_{j+1}}{\partial e_{j+1}^p} \right) \tag{A.15}$$

and the definition of μ_{Aj} into equation (A.11)

$$\frac{\partial U_j(z_j)}{\partial A_j} = \frac{\psi_{j+1}}{1+\delta}(1+\widetilde{r}_j) E_j \left(\frac{\partial U_{j+1}}{\partial A_{j+1}} \right) \; . \tag{A.16}$$

Define

$$\begin{aligned} \widetilde{X}_j &= c_j \widetilde{p}_{cj} + l_j \widetilde{p}_{lj} \\ &= c_j \widetilde{p}_{cj} + \alpha \left(\frac{\widetilde{p}_{lj}}{\widetilde{p}_{cj}} \right)^{1-\rho} \widetilde{p}_{cj} \\ &= \left(c_j - \underline{c}_j + \left[\underline{c}_j + \alpha \left(\frac{\widetilde{p}_{lj}}{\widetilde{p}_{cj}} \right)^{1-\rho} \right] \right) \widetilde{p}_{cj} \end{aligned} \tag{A.17}$$

and

$$\begin{aligned} \widetilde{x}_j &= u_j = c_j - \underline{c}_j \\ \widetilde{x}_j^l &= \left[\underline{c}_j + \alpha \left(\frac{\widetilde{p}_{lj}}{\widetilde{p}_{cj}} \right)^{1-\rho} \right] \\ \widetilde{p}_{xj} &= \widetilde{p}_{cj} \\ \widetilde{X}_j &= \left[c_j \widetilde{p}_{cj} + l_j \widetilde{p}_{lj} \right] = \left(\widetilde{x}_j + \widetilde{x}_j^l \right) \widetilde{p}_{xj} \; . \end{aligned} \tag{A.18}$$

The first order condition for consumption (A.10) can be written as

$$\widetilde{x}_j^{\frac{-1}{\gamma}} - \frac{\psi_{j+1}}{1+\delta} \widetilde{p}_{xj} \mu_{Aj} = 0 \; . \tag{A.19}$$

Equation (A.11) and (A.19) imply

$$\frac{\partial U_j(z_j)}{\partial A_j} = (1+\widetilde{r}_j) \widetilde{x}_j^{\frac{-1}{\gamma}} \frac{1}{\widetilde{p}_{xj}} \; . \tag{A.20}$$

Substitute equation (A.20) in (A.16)

$$\widetilde{x}_j^{-\frac{1}{\gamma}} = \frac{\psi_{j+1}}{1+\delta} E_j \left((1+\widetilde{r}_{j+1}) \widetilde{x}_{j+1}^{\frac{-1}{\gamma}} \frac{\widetilde{p}_{xj}}{\widetilde{p}_{xj+1}} \right) \; . \tag{A.21}$$

Division of (A.8) and (A.10) leads

$$l_j = \alpha \left(\frac{\widetilde{p}_{lj}}{\widetilde{p}_{xj}} \right)^{-\rho} \; , \tag{A.22}$$

after substitution of the marginal utility of leisure to.

The model for workers consists of the labour supply relation (A.22), the Euler equation for consumption (A.21) which determines together with the labour induced consumption relation (2.4) consumption according to (A.18). The budget relations are equation (A.3) up to and including (A.7). The price equations for consumption, leisure and total consumption are defined in (A.10), (A.9) and (A.18) respectively. The model is completed with the Kuhn Tucker conditions (A.14), the Euler equation for earning points (A.15) and the expected marginal utility of wealth (A.19). This system can be solved in closed form for age $j \geq j_r - 1$ because these generation does not have labour income uncertainty. The other cohorts have to form expectations conditional on the state they are.

The presented derivations assume $P_j^0 = 0$. With an annuity system changes the return of wealth. The most important consequence is that the survival rate drops out the Euler equation (A.21). This implies constant consumption after retirement due to the calibration assumption of an equal rate of return and time preference parameter.

A.2 Closed form for age $j \geq j_r - 1$

Add the wealth equation (A.3) and the Kuhn Tucker conditions (A.14) and use the already derived price of leisure (A.9) to obtain

$$A_{j+1} = (1+\widetilde{r}_j)A_j + \left[\widetilde{p}_{lj} - \frac{\mu_{ej}}{\mu_{Aj}} e_j \frac{\omega}{\overline{W}}\right] \left[\overline{l} - l_j\right] - c_j \widetilde{p}_{cj} + \widetilde{R}_j \tag{A.23}$$

with $\widetilde{r}_j$ defined in (A.12) and $\widetilde{R}_j = (1+\lambda_{Aj})R_j = (1+\lambda_{Aj})\left(R_j^1 + R_j^2\right)$ with

$$R_j^1 = \begin{cases} 0 \text{ and } P_j^1 = 0 \\ \kappa_1 E(W) - \varphi_p P_j^2 \text{ and } P_j^1 > 0 \end{cases} \tag{A.24}$$

$$R_j^2 = P_j^2 \tag{A.25}$$

Solve the wealth equation forward

$$A_j = \sum_{i=j}^{j_e} \left(c_i \widetilde{p}_{ci} - \left[\widetilde{p}_{li} - \frac{\mu_{ei}}{\mu_{Ai}} e_i \frac{\omega}{\overline{W}}\right] \left[\overline{l} - l_i\right] - \widetilde{R}_i \right) d_o(i) \tag{A.26}$$

with

$$d_o(i) \equiv \prod_{l=j}^{i} (1+\widetilde{r}_l)^{-1} \tag{A.27}$$

Define pension wealth

$$W_j^p \equiv \sum_{i=j}^{j_e} \left[\widetilde{R}_i - \frac{\mu_{ei}}{\mu_{Ai}} e_i \frac{\omega}{\overline{W}} \left[\overline{l} - l_i\right]\right] d_o(i) \tag{A.28}$$

and define net human wealth

$$W_j^h \equiv \sum_{i=j}^{j_e} \left[\bar{l}\widetilde{p}_{li} - \widetilde{x}_i^l \widetilde{p}_{xi}\right] d_o(i) \tag{A.29}$$

Substitution into the budget equation leads to

$$W_j \equiv A_j + W_j^p + W_j^h = \sum_{i=j}^{j_e} \widetilde{x}_i \widetilde{p}_{xi} d_o(i) \tag{A.30}$$

using the definitions A.18. Euler equation (A.21) implies

$$\widetilde{x}_i \widetilde{p}_{xi} = \widetilde{x}_j \widetilde{p}_{xj} \left(1+\widetilde{r}_j\right)^{-\gamma} \left[\frac{\widetilde{p}_{xj}}{\widetilde{p}_{xi}}\right]^{\gamma-1} \left(\frac{d_s(i)}{d_o(i)}\right)^{\gamma} \tag{A.31}$$

with

$$d_s(i) \equiv \left(\frac{\psi_j}{1+\delta}\right)^{-1} \prod_{l=j}^{i} \left[\frac{\psi_l}{1+\delta}\right] \tag{A.32}$$

Substitution into the wealth equation results in

$$W_j = \sum_{i=j}^{j_e} \widetilde{x}_i \widetilde{p}_{xi} d_o(i) = \widetilde{x}_j \widetilde{p}_{xj}^{\gamma} \left(1+\widetilde{r}_j\right)^{-\gamma} p_{wj}^{1-\gamma} \tag{A.33}$$

with

$$p_{wj} \equiv \left(\sum_{i=j}^{j_e} \widetilde{p}_{xi}^{1-\gamma} d_o(i)^{1-\gamma} d_s(i)^{\gamma}\right)^{\frac{1}{1-\gamma}} \tag{A.34}$$

Leading to the consumption at age j, net of labour induced consumption and the consumption plan for the other years

$$\widetilde{x}_i = \widetilde{x}_j \left(1+\widetilde{r}_j\right)^{-\gamma} \left[\frac{\widetilde{p}_{xj} d_s(i)}{\widetilde{p}_{xi} d_o(i)}\right]^{\gamma} = \frac{W_j}{p_{wj}} \left(\frac{\widetilde{p}_{xi} d_o(i)}{p_{wj} d_s(i)}\right)^{-\gamma} \tag{A.35}$$

Substitution of the total consumption relations in the utility equation gives

$$V_j(z_j) = \frac{W_j}{p_{wj}} \tag{A.36}$$

The marginal utility of earning points can be written in closed form for age $j_r - 1$

$$\mu_{e\,j_r-1} = \mu_{A\,j_r-1} \left(\sum_{i=j_r}^{j_e} \left[\frac{\partial \widetilde{R}_i^1}{\partial e_i^p} + \frac{\partial \widetilde{R}_i^2}{\partial e_i^p}\right] d_o(i)\right) \tag{A.37}$$

$$\frac{\partial R_i^1}{\partial e_i^p} = \frac{\partial P_i^1}{\partial e_i^p} = \begin{cases} 0 \text{ and } P_j^1 = 0 \\ -\varphi_p \frac{\partial P_i^2}{\partial e_i^p} \text{ and } P_j^1 > 0 \end{cases} \tag{A.38}$$

$$\frac{\partial R_i^2}{\partial e_i^p} = \frac{\partial P_i^2}{\partial e_i^p} = \left[\frac{1}{j_r - j_w - 1}\right] \kappa_2 E(W)$$

$$\frac{\mu_{e\,j_r-1}}{\mu_{A\,j_r-1}} = \sum_{i=j_r-1}^{j_e} \left[\frac{\partial \widetilde{R}_i^1}{\partial e_i^p} + \frac{\partial \widetilde{R}_i^2}{\partial e_i^p}\right] d_o(i) \tag{A.39}$$

The model for retirees and workers one year before retirement consists of the labour supply relation (A.22), the net consumption equation (A.35) which determines together with the labour-induced consumption relation (2.4) consumption. The wealth relations are (A.30, A.23, A.28 and A.29). The price equations for consumption, leisure, total consumption and wealth are defined in (A.10), (A.9), (A.18) and (A.34) respectively. The model is completed with the Kuhn Tucker conditions (A.14) and the equation for the marginal value of earning points (A.37).

A.3 Implementation of the Kuhn Tucker conditions

The Fischer-Burmeister function will be used to implement the Kuhn Tucker conditions (see Munson et al. (1999) for an extension of this approach). More precisely the equations (A.14) will be approximated with

$$\begin{aligned} \lambda_{lj} &= \lambda_{lj} + \beta_l \phi(\lambda_{lj}, \bar{l} - l_j) \\ \lambda_{Aj} &= \lambda_{Aj} + \beta_A \phi\left(\lambda_{Aj}, (1+r_j)A_j + W_j - \tau^p W_j - c_j p_{cj}\right) \\ \phi(a,b) &= \sqrt{a^2+b^2} - a - b \end{aligned} \tag{A.40}$$

A.4 Implementation of minimum conditions

Write the first pillar pensions as

$$P_j^1 = x_a - \min\left[x_a, x_b\right] \tag{A.41}$$

with $x_a = \kappa_1 E(W)$ and $x_b = \left(\varphi_p P_j^2 + \varphi_a r_j \max\left[A_j, 0\right]\right)$. The first pillar pension benefit is approximated with

$$P_j^1 = x_a - x = x_a - \left[(x_a)^{-\rho} + (x_b)^{-\rho}\right]^{-\frac{1}{\rho}} \tag{A.42}$$

and the derivatives with

$$\frac{\partial P_i^1}{\partial x_b} = -\frac{\partial x}{\partial x_b} = -\left(\frac{x}{x_a}\right)^{\rho+1} \tag{A.43}$$

$$\frac{\partial P_i^1}{\partial e_i^p} = -\frac{\partial x}{\partial x_b}\frac{\partial x_b}{\partial e_i^p} = -\frac{\partial x}{\partial x_b}\varphi_p \frac{\partial P_i^2}{\partial e_i^p} \tag{A.44}$$

$$\frac{\partial P_i^1}{\partial e_i^p} + \frac{\partial P_i^2}{\partial e_i^p} = \left(1 - \frac{\partial x}{\partial x_b}\varphi_p\right)\frac{\partial P_i^2}{\partial e_i^p} \tag{A.45}$$

A.5 Conditional expectations

This section follows Judd et al. (2012). The variables

$$\mu_{Aj} = E_j \frac{\partial U_{j+1}}{\partial A_{j+1}},\ \mu_{ej} = E_j \frac{\partial U_{j+1}}{\partial e_{j+1}^p} \text{ and } E_j U_j$$

are the conditional expectations given the information $z_{ij} \in (e_j^p, A_j, e_j)$ at age j, *i.e.* the state at age j. The description of the method is restricted to $u_j \equiv \frac{\partial U_{j+1}}{\partial A_{j+1}}$because the same method is used in the other cases. The conditional expectation $\mu_{Aj} = E_j u_j$ is a function of the state variable z_j.We seek an approximating function. For instance $u_j = \sum_i \alpha_{ij} z_{ij} + \alpha_{0j} + \xi_j$, with ξ_j an error term leading to the conditional expectation $\mu_{Aj} = E_j u_j = \sum_i \alpha_{ij} z_{ij} + \alpha_{0j}$. The data will be scaled to enhance numerical stability. Define

$$y_j = \frac{u_j - \mu_{uj}}{\sigma_{uj}} \text{ and}$$
$$x_{ij} = \frac{z_{ij} - \mu_{zij}}{\sigma_{zij}}$$

with μ_{uj} the average and σ_{uj} standard deviation of u_j and μ_{zij} the average and σ_{zij} the standard deviation of z_{ij}. A constant term is not necessary due to the transformation. Assume both y and x depend on the productivity shocks ε. Given an initial guess for the parameters, the Monte Carlo approach is used to generate pairs of y_j and x_{ij} using the model. Then we regress in our example

$$y_j = \sum_i \alpha_{ij}^+ x_{ij} + \xi_j \tag{A.46}$$

to obtain new values for the coefficients. The original parameters are obtained from this regression as

$$\alpha_{ij} = \frac{\sigma_{uj}}{\sigma_{zij}} \alpha_{ij}^+ \tag{A.47}$$
$$\alpha_{0j} = \mu_{uj} - \sum_i \alpha_{ij} \mu_{zij} \tag{A.48}$$

with $\overline{u}_j$ and $\overline{z}_{ij}$ the sample mean. In the same way the conditional expectations of the marginal utility of earning points can be obtained. Judd et al. (2012) advocates to use higher order Hermite polynomials instead of the first order polynomial used in this section for presentation purposes. The next section gives details on the construction of a complete set of Hermite polynomials

A.6 Hermite polynomial representation

A Hermite polynomial $h_i(m)$ of order m in variable x_i (the age index j is not included to maintain simple notation) is defined as

$$h_i(0) = 1 \tag{A.49}$$
$$h_i(1) = x_i$$
$$h_i(m) = x_i h_i(m-1) - (m-1) h_i(m-2) \text{ and } m > 1$$

We construct a complete set of polynomials of degree p in n variables using the ordinary polynomials

$$z = \left\{ \prod_{i=0}^{n} h_i(l_i) \mid \sum_{i=1}^{n} l_i = j,\ j = 0, \ldots, p \right\} \tag{A.50}$$

For instance the complete set of degree zero ($p = 0$) in two variables ($n = 2$) is

$$z = \{h_1(0)h_2(0)\} \tag{A.51}$$

of degree one ($p = 1$) in two variables ($n = 2$) is

$$z = \{[h_1(0)h_2(0)], [h_1(1)h_2(0)], [h_1(0)h_2(1)]\} \tag{A.52}$$

of degree two ($p = 2$) in two variables ($n = 2$) is

$$z = \{[h_1(0)h_2(0)], [h_1(1)h_2(0)], [h_1(0)h_2(1)], [h_1(1)h_2(1)], [h_1(2)h_2(0)], [h_1(0)h_2(2)]\} \tag{A.53}$$

The complete set of degree three ($p = 3$) in two variables ($n = 2$) extends the complete set of order two with

$$\{[h_1(2)h_2(1)], [h_1(1), h_2(2)], [h_1(3)h_2(0)], [h_1(0)h_2(3)]\} \tag{A.54}$$

The polynomial approximations of orders four and five have 15 and 21 coefficients, respectively. The constant term will not be included because all variables will be normalized. Note, in case state variable x_i is not relevant (for instance earning points in case of a Beveridge system or without first pillar pensions) the reduced complete set of polynomials is obtained by setting the corresponding hermite polynomials $h_i = 0$.

A.7 Numerical recipes

The simulations are restricted to the ninety percent confidence interval for the productivity shocks ε_j. 'Extreme' events are therefore excluded. The parameterized expectation functions can be better determined with a restricted interval.

The simulations of Section 4 and 5 are based on a complete set of Hermite polynomials of order three. To improve on the explanatory power transformations of the state variables are used. More specifically a net-consumption possibility variable $(c_r - \underline{c})^{-1}$ is used with c_r maximum possible consumption given the liquidity constraint $A \geq 0$. This variable gives a much better explanation for the marginal utility of wealth. This specification is used for all systems except the Bismarck system. Simultaneity in the expectations formation leads to problems in the simulation exercises. This brings about a smaller explanatory degree of the parameterized expectations functions for the Bismarck system.

Another possibility to improve on the accuracy of the method is the use of another integration method. Monte Carlo integration methods are used to determine the expected developments. Judd et al. (2011) advocates exact integration using the Gauss-Hermite quadrature to get more accuracy.

Euler equation A.16 leads to numerical problems during the iteration process. When a low marginal utility of financial wealth (or equally high consumption) is expected for next year, the marginal utility in this year will be low, too. This implies a high consumption level, which may lead to liquidity constraints by which the consumption in next year will be much lower than expected in first instance. The expectation function will be changed in the next iteration, but convergence appears to be difficult. Convergence improved a lot by relaxation: the parameters α_{ij} are update slowly between between the iteration l: $\alpha_{ij}^{l} = (1-\beta)\,\alpha_{ij}^{l-1} + \beta\alpha_{ij}$. The used relaxation factor was $\beta = 0.1$.

A.8 Kalman filter

Rational expectations imply that the agents understand the working of the economy, i.e. they have a very good model to predict the future, given the state of the economy. To get a good approximation a Kalman filter approach is used. This section follows Harvey (1986) page 106 up to 110. Equation (A.46) can be summarized by

$$y_t = z_t'\alpha_t + \xi_t \tag{A.55}$$

with $\xi_t \sim WS(0,\sigma^2)$ in which WS indicates the variable has a mean and a variance; WS stands for 'wide sense'. Suppose constant parameters

$$\alpha_t = \alpha_{t-1} \tag{A.56}$$

We use a_t for the the minimum mean square linear estimator (MMSLE) of α_t at time t. The covariance matrix of a_{t-1} is $\sigma^2 P_{t-1}$. The covariance matrix of the estimation error is

$$a_{t-1} - \alpha_t \sim WS(0,\sigma^2 P_{t-1}) \tag{A.57}$$

The error made in prediction y_t at time $t-1$ is

$$\upsilon_t = z_t'(\alpha_t - a_{t-1}) + \xi_t \tag{A.58}$$

with variance

$$Var(\upsilon_t) = \sigma^2\left(z_t' P_{t-1} z_t + 1\right) \equiv \sigma^2 f_t \tag{A.59}$$

Updating rule for the covariance matrix

$$P_t = P_{t-1} - P_{t-1} z_t z_t' P_{t-1}/f_t \tag{A.60}$$

The updating rule for the state vector is

$$a_t = a_{t-1} + P_{t-1} z_t \upsilon_t / f_t \tag{A.61}$$

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