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Safe Dike Heights at Minimal Costs

An Integer Programming Approach

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Abstract

Optimal dike heights are of crucial importance to the Netherlands as almost 60% of its surface is under threat of flooding from sea, lakes, or rivers. This area is protected by more than 3,500 kilometres of dunes and dikes. These dunes and dikes require substantial yearly investments of more than 1 billion euro. In this paper we propose an integer programming model for a cost-benefit analysis to determine optimal dike heights. We improve upon the model proposed by Brekelmans et al. (2012), which is in turn an improvement of the model by Van Dantzig (1956). The model by Van Dantzig (1956) was introduced after a devastating flood in the Netherlands in 1953. Our model provides an alternative approach with almost complete flexibility towards input-parameters for flood probabilities, damage costs and investment costs for dike heightening. In contrast to Brekelmans et al. (2012), who present a dedicated approach with no optimality guarantee, we present an easy-to-implement algorithm that provides an optimal solution to the problem. We briefly discuss robust optimization approaches to deal with uncertainty, e.g. climate change. The method has been implemented and tested for the most recent data on flood probabilities, damage and investment costs, which are presently being used by the government to determine how the safety standards in the Dutch Water Act should be changed.

Subject classifications: flood prevention; climate change adaptation; cost-benefit analysis; MIP; IP; robust optimization.

JEL classification: C61; D61; Q54.

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Samenvatting

Het bepalen van de optimale dijkhoogte is van cruciaal belang voor Nederland, omdat ongeveer 60% van het oppervlak wordt bedreigd door overstromingen vanuit de zee of de rivieren. Dit gebied wordt beschermd door meer dan 3500 kilometer duinen en dijken. Deze duinen en dijken vragen elk jaar voor meer dan 1 miljard euro aan investeringen en beheer en onderhoud.

Dit paper werkt een model uit voor een maatschappelijke kosten-batenanalyse om de optimale dijkhoogtes en -sterktes te bepalen. We stellen een integer programmeringsmodel voor. Dit model is een uitbreiding van het model zoals voorgesteld door Brekelmans et al. (2012). Recent is hun model gebruikt voor het bepalen van nieuwe, economisch optimale waterveiligheidsnormen voor vrijwel alle dijkringdelen in Nederland in het Deltares-rapport Maatschappelijke kosten-batenanalyse Waterveiligheid 21e eeuw (MKBA WV21; Kind, 2011).

Het belangrijkste voordeel van het model in deze paper boven het model van Brekelmans et al. (2012) is de vrijwel volledige flexibiliteit met betrekking tot de invoergegevens. Deze invoergegevens betreffen de overstromingskansen, de schadekosten in geval van een overstroming en de kosten voor dijkverhoging of -versterking. Het model van Brekelmans et al. (2012) vereist dat deze invoergegevens worden gevat in enkele specifieke functionele verbanden. Door deze flexibiliteit van ons model kan bijvoorbeeld het verschil tussen de overschrijdingskans (= de kans dat het water over de dijk heen komt waardoor de dijk faalt) en de totale overstromingskans (= de kans op falen van de dijk door alle mogelijke hydraulische oorzaken) herkenbaar worden gemodelleerd. Het model van Brekelmans et al. (2012) beschouwt alleen overschrijdingskansen.

Daarnaast presenteren wij een eenvoudig te implementeren oplossingsalgoritme voor het bepalen van de bewezen optimale oplossing voor het probleem. De oplossingsprocedure van Brekelmans et al. (2012) betreft een specifiek voor hun model ontwikkelde aanpak waarmee een oplossing wordt gevonden waarvan niet bekend is in welke mate de gevonden oplossing afwijkt van de echte optimale oplossing.

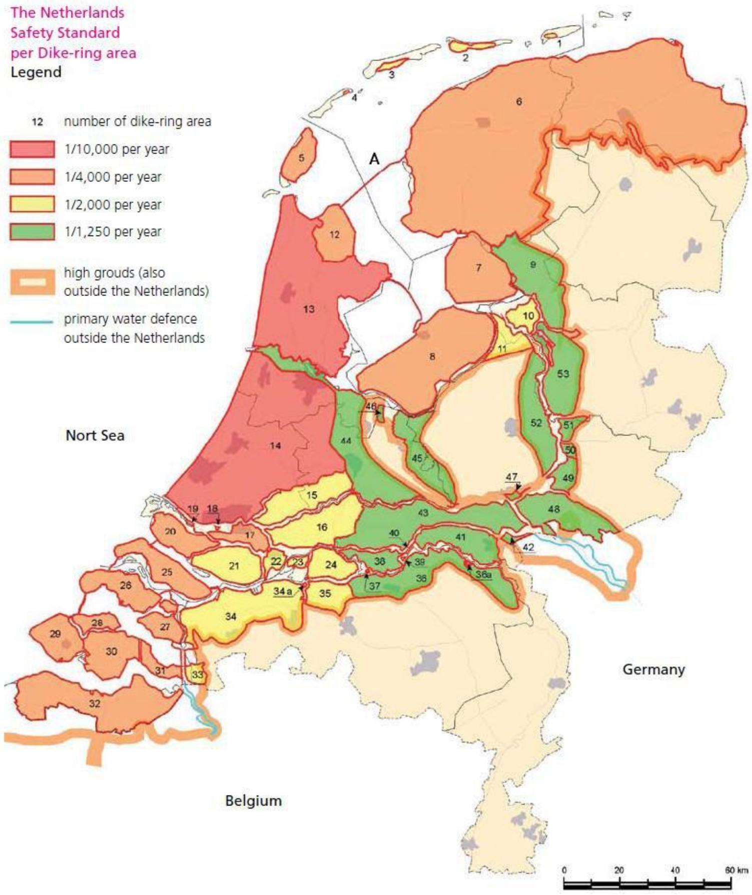
Het ontwikkelde model en het oplossingsalgoritme zijn geïmplementeerd en getest door een vergelijking van de uitkomsten met Brekelmans et al. (2012). Met behulp van ons oplossingsalgoritme kon binnen een minuut de bewezen optimale oplossing voor alle probleeminstanties worden gevonden.

1. Introduction

The 1953 flood in the South-western part of the Netherlands is after almost 60 years still in the Dutch collective memory. The flood occurred in the night and resulted into the death of 1,835 people. Almost 200,000 hectares of land were flooded, 67 dike breaches arose and immense economic damage resulted (10% of Dutch GDP). The government rapidly appointed the so-called Delta Committee in order to design measures for preventing similar disasters in the future. The Delta Committee asked Van Dantzig (1956) to develop a mathematical approach to formulate and solve the economic cost-benefit decision model concerning the optimal dike height problem.

The work of the Delta Committee, including the work by Van Dantzig (1956), finally resulted in statutory minimal safety standards. The current safety standards against flooding are defined on the basis of a dike ring area, see Figure 1.

Figure 1: Dike ring areas and safety standards in the Netherlands.



Note: 'A' indicates the location of the Afsluitdijk ('barrier dam'), which protects the Northern part of the Netherlands

A dike ring is an uninterrupted ring of water defences. In total, there are 53 dike ring areas, each having a certain minimal safety standard (i.e. maximum flood probability). The tightest (i.e. lowest) flood probability is 1/10,000 per year for the most populated part of the Netherlands. This number is derived from Van Dantzig (1956).

At present, protection against flooding is an important issue worldwide (Adikari and Yoshitania, 2009; Syvitski et al., 2009). Several devastating flows occur each year, e.g. in Bangladesh, Pakistan, and Germany. Other well-known examples are the serious flooding in and around New Orleans in 2005, which killed about 1,500 people and created enormous damage, and the 2012 flood in New York City.

Renewed interest in determining optimal dike heights in the Netherlands arose - again - after a critical situation in 1995. The rising water levels of the major rivers Rhine and Meuse forced 200,000 people to evacuate. Fortunately, no serious flooding occurred. This event triggered the Dutch government to ask CPB Netherlands Bureau for Economic Policy Analysis to develop an economic cost-benefit analysis to determine optimal safety levels for dike rings along the river Rhine. The results of this analysis are presented in Eijgenraam (2005, 2006) and Eijgenraam et al. (2010). The government initiated an investment project of several billion euros, Room for the River, to bring the dikes adjacent to the major Dutch rivers up to standards (Ministry of Infrastructure and the Environment, 2009).

Gradually, awareness grew in the Netherlands that the current safety levels against flooding are about 60 years old and are in need of a thorough reconsideration. Since then, both the population size and the economic value of the protected land have increased significantly. Moreover, the knowledge about the causes of flooding has increased, as well as the portfolio of civil engineering and other measures to prevent flooding or reduce its consequences. And last but not least, the sea level and the discharge levels of the rivers during winter have risen during this period. Therefore, the Dutch Central Government initiated a safety programme as part of an overall new Delta Programme (Delta Programme Commissioner, 2012), with the aim of developing and setting down new water safety standards and implementing the EU Flood Risks Directive (EU, 2007). Each year, a new Delta Programme must be written due to the Delta Act and be presented to both the Senate and the House of Representatives. In 2017, the new standards will be set down in the Water Act.

Several research projects are initiated to prepare these new standards. A new economic cost-benefit analysis (CBA) was carried out by the hydraulic research and consultancy company Deltares (Kind, 2011). This CBA is based upon a mathematical model developed by Brekelmans et al. (2012). This model is an extension of the previous models by Van Dantzig (1956) and Eijgenraam (2006). The model closely follows the mathematical modelling of flood probability and investment costs by Van Dantzig (1956) and Eijgenraam (2006).

The Dutch State Secretary on Water Infrastructure (Atsma, 2011) decided to initiate additional policy actions towards the new minimal safety standards in 2017. The immediate cause was the publication of the cost-benefit analysis by Deltares (Kind, 2011). In a letter to the Dutch House of Representatives, the State Secretary considers this analysis to be ‘only a rough approximation of the actual situation’, since ‘the underlying starting points and assumption strongly determine the final results’. Some crucial assumptions of the underlying model by Brekelmans et al. (2012) are rather restrictive. Hence, the State Secretary asked for a more made-to-measure approach, which is able to include more local-details for safety measures, especially low-cost solutions, to increase safety. Given the ongoing economic recession together with substantial government deficits, this is of course a natural reaction. In the coming years, this decision process of central and local governments (municipalities, counties and district waterboards) should result in a final proposal to the Dutch Cabinet for new safety standards for flood risks.

Recently, the Minister of Infrastructure and the Environment (Schultz van Haegen-Maas Geesteranus, 2013) approved the result of the cost-benefit analysis to increase safety in the specific regions (Eijgenraam et al., 2013; see also the video presentation by the Minster at the Franz Edelman Award 2013). However, in the same letter to the Parliament, she also announced to adjust the previously derived optimal probabilities because of the huge differences in expected damage within a dike ring area.

We present a new modelling approach to determine the optimal timing and extent of dike heightening or strengthening. We will use the terms heightening and strengthening interchangeably throughout this paper. Our new modelling approach entails three major advantages in comparison with Brekelmans et al. (2012).

First, the model provides substantial flexibility with respect to the input data. Hence, the previously mentioned made-to-measure characteristics of possible flood protection measures can be included in the model. Therefore, the model is well equipped to be used in coming research projects. Several examples exist in which this flexibility is shown to be crucial, e.g.:

- For the major rivers (for example the Rine), a maximum exists to the discharge that can enter the Netherlands. Hence, the overflow probability of dikes will become zero in cases where these dikes are above a certain height. This effect on the flood probability cannot be modelled with the exponential distribution used by Brekelmans et al. (2012) (Kind, 2011; CPB, 2011).
- Damage that occurs when a dike (ring) fails differs up to a factor 20 to 100, depending on the exact location of the breach (VNK2, CPB, 2011). This clearly contrasts the assumption of identical damage for all locations in Brekelmans et al. (2012).
- For some dikes (e.g. the Afsluitdijk, Grevers and Zwaneveld, 2011), it is possible to renovate certain constructions, like vessel locks and drainage sluices, up to a certain safety level at relatively low costs. These very specific cost characteristics cannot be modelled properly by the specified cost functions of Brekelmans et al. (2012).
- After a 'standard' strengthening of a dike (by increasing its height and width), additional safety can be obtained by means of tailor-made, low cost measures (like 'strengthened' grass for a more robust dike covering), which yield a safety equivalent of 50 centimetres heightening of a dike.
- A time-varying discount rate may be appropriate and is already prescribed in France and the UK (Hepburn, 2007; UK Dft, 2011). Brekelmans et al. (2012) only allow for a time-invariant discount rate.

Second, we present an easy to implement solution procedure, which solves the dike heightening problem to optimality. Ease of implementation is not only very important for the use of our results in Dutch practice, but also to dissimilate our results to less wealthy countries.

Third, our problem definition and solution procedure improves upon the heuristic and case-specific approach by Brekelmans et al. (2012). No optimality guarantee is given for this approach. Our solution procedure solves all considered problem instances to proven optimality.

2. Cost-benefit model as an integer programming model

In this section, we present an integer programming (IP-model) formulation for the presently available methods to determine optimal dike heights. These methods base the optimal timing and heightening of a dike ring on cost-benefit analysis. In this type of analysis we attempt to minimize the total (discounted) social costs, consisting of the investment costs for heightening the dikes and the remaining expected loss by flooding.

The basic dilemma is the trade-off between paying up the investment costs of heightening a dike ring or accepting a (higher) probability of dike failure with all associated costs of flooding. The costs of flooding include damage costs, cost of evacuation, rescue costs and immaterial costs (e.g. victims, sufferings). These costs of flooding for a specific dike ring are based upon extensive simulations by engineers of Deltares, using the information system HIS-SSM (Kind, 2011).

We present our IP-model, called model C, for the case of non-homogeneous dike rings. This problem was first proposed by Brekelmans et al. (2012) and considers the case in which a dike ring consists of several segments. All segments can be heightened independently of each other. The model by Brekelmans et al. (2012) is in actual practical use in the Netherlands (Kind, 2011) and is solved using a procedure as described in Brekelmans et al. (2012).

The aim of our paper is to provide a model formulation with maximum flexibility with respect to the input-parameters that represent investment costs and expected damage costs. Hence, we set up our model C with generally applicable exogenous input data.

We provide two additional IP-models in Appendix A and B. Appendix A discusses the problem as considered by Eijgenraam (2006) and Eijgenraam et al. (2010). It considers the so-called homogeneous case, which means that a dike ring can be considered as one homogeneous dike. The model by Eijgenraam (2006) is based upon specific functional formulations for investment costs and expected damage costs as initially introduced by Van Dantzig (1956). The model and solution procedure by Eijgenraam (2006) is in actual practical use in the Netherlands and is solved by closed form formulas. The IP formulation of this problem, called model A, is derived straightforward from our general IP-formulation by setting the number of dike segments in a dike ring equal to one. Due to the ease of using model A and solving it by its LP-relaxation, we expect that our model A is well equipped to be used in practice.

Appendix B provides an alternative formulation of the non-homogenous case, as described by Brekelmans et al. (2012). This model B is more intuitive and easier to understand, but less general than our model C for the non-homogenous case.

2.1 IP-model for homogeneous dike rings

2.1.1 Notation

The (binary) decision variables represent the decision to heighten (or more generally spoken: strengthen) a dike ring in a particular year from height/safety level h_1 to safety level h_2. The set H represents all possible safety levels/heights of this dike ring. For example, one may consider heightening a dike ring with steps of 50 cm.

From a practical point of view, steps smaller than 20 cm need not to be considered. Many sizable uncertainties exist about the actual strength of a dike ring: the unknown settlement of the dike itself and its underground after heightening of a dike ring and uncertainties with respect to the calculation methods to determine the flood probability given a certain dike height/strength. Dikes are heightened with the use of clay. Due to a-priori unknown thickness of the clay, it is almost impossible to assess a priori the new height of a dike after heightening with more precision than 20 cm.

Besides the probability of water overtopping or overflowing the crest of a dike ring, a safety level h_2 can also refer to the probability of failure due to mechanisms like piping and the quality of some structures. Hence, these safety levels provide the flexibility to fit the model towards practical requirements.

The set T represents all considered time periods. A time period $t \in T$ may represent one year, but can also represent a certain time period, say 5 or 10 years. In practice, the exact timing of a dike heightening cannot be planned with great precision due to legislation, consulting local authorities, communication and negotiation with land owners and inhabitants and planning uncertainties due to civil engineering works. For example, the construction of a higher dike takes at least five years. Deviations of more than 10 years from the planned completion time occur in practice (Kind, 2011). The minimal time period between consecutive strengthenings of a dike ring involves 10 years due to legal and civil engineering restrictions. Year '0' is used to represent the starting conditions of a dike ring. The present height is also denoted by '0'. Without loss of generality, we can denote and order the different safety level/heights by $H = \{0,1,2,\ldots,|H|-1\}$ and the time period/years by $T = \{0,1,2,\ldots,|T|-1\}$.

A dike ring, which protects a certain area of land against water floods, is said to be non-homogeneous if it consists of, say $|L|$ $(|L|>1)$, different segments or links. All segments can be heightened independently of each other. Moreover, each segment has its own properties with respect to investment costs and flood probabilities. To indicate the dependence of a model parameter on a particular dike segment, an index or superscript l $(l=1,..,|L|)$ will be added to this parameter or decision variable. The set of all segments will be denoted by L.

This problem is extensively described in Brekelmans et al. (2012). Characteristics of their problem definition are:

i. The expected damage in case of flooding does not depend on the location or segment in which a breach arises first.
ii. A dike ring fails first at its weakest point or, to state it differently, a dike ring starts to fail at a segment with the highest flood probability. Hence, the flood probability of a dike ring as a whole can be calculated by taking the maximum of the individual flood probabilities over all segments. Since damage in case of flooding is presumed to be identical for all segments, the overall expected damage of a dike ring can be calculated by taking the maximum of the expected damage per segment. The latter formulation is used in the model formulation of Brekelmans et al. (2012).

Next, we present a more general model for the non-homogeneous case, which is able to include, for example, segment-specific damage costs. Hence, we relax upon the above mentioned characteristic (i).

Brekelmans et al. (2012) also present the possibility that the expected damage in case of flooding depends on the (absolute) height of a certain dike height segment. This dike segment will be denoted by l^F. In case of a flood, the water runs downhill within the dike ring area into a certain direction, i.e. segment l^F. Hence, the dike height of this segment determines for some time (i.e. at least until the moment the water overtops this dike segment) water depths within the area. We did not encounter this situation in practice. Therefore, we will ignore this at first. Later on, we will explain how this aspect can be included in our modelling approach.

2.1.2 General IP-model formulation: model C

The decision variables are:

$CY(t,l,h_1,h_2) =$ 1, if segment or link *l* of the dike ring is updated in time period *t* from height/safety level h_1 up to height/safety level h_2. If $h_1 = h_2$ then this dike ring segment is not strengthened in period *t* and remains at its previous height. This decision variable is used for bookkeeping investment (and maintenance) costs.
0, otherwise.

$DY(t,l,h_2) =$ 1, if segment *l* with height /strength h_2 represents the 'weakest link' in period *t*, i.e. the segment with the highest flood probability such that a dike ring starts to fail at this segment. This decision variable is used for bookkeeping flood probabilities and related expected damage costs.
0, otherwise.

The input-parameters are:

$cost(t,l,h_1,h_2) =$ costs for investment and maintenance, if segment *l* of the dike ring is strengthened in time period *t* from h_1 to h_2. If $h_1 = h_2$, the dike ring segment is not strengthened in period *t* and these costs only represent maintenance costs.

$prob(t,l,h_2) =$ flood probability, if the resulting height of segment *l* in period *t* equals h_2.

$damage(t,l,h_2) =$ damage (i.e. 'damage in case the dike ring starts to fail at segment *l*'), if the resulting height of segment *l* in period *t* equals h_2.

All input parameters are calculated in net present value of a certain year (i.e. 2015, which is the starting year for our calculations) and represent price levels in a certain year. In our calculations, we also presume that dike heightening takes place immediately at the start of the time period $t \in T$. The final time period $|T|-1$ includes the expected damage from this time period until infinity.

Model C reads as follows:

$$
\begin{aligned}
Min \; & \sum_{t\in T}\sum_{l\in L}\sum_{h_1\in H^l}\sum_{h_2\ge h_1\in H^l} cost(t,l,h_1,h_2)\cdot CY(t,l,h_1,h_2) \\
& +\sum_{t\in T}\sum_{l\in L}\sum_{h_2\in H^l} prob(t,l,h_2)\cdot damage(t,l,h_2)\cdot DY(t,l,h_2)
\end{aligned} \tag{C.1}
$$

subject to

$$CY('0',l,'0','0')=1;\, CY('0',l,h_1,h_2)=0 \quad \forall l\in L; h_1,h_2\in H^l; h_2\ge h_1 \wedge h_2 > '0' \tag{C.2}$$

$$\sum_{h_1\le h_2\in H^l} CY(t-1,l,h_1,h_2)=\sum_{h_3\ge h_2\in H^l} CY(t,l,h_2,h_3) \quad \forall t\in T/\{'0'\}, l\in L, h_2\in H^l \tag{C.3}$$

$$
\begin{aligned}
& \sum_{h_1\in H^l}\sum_{\substack{h_2\ge h_1\in H^l:\\ prob(t,l,h_2)>prob(t,l^*,h_2^*)}} CY(t,l,h_1,h_2)+\sum_{l_h\in L}\sum_{\substack{h_2\in H^{l_h}:\\ prob(t,l_h,h_2)\le prob(t,l^*,h_2^*)}} DY(t,l_h,h_2)\le 1 \\
& \qquad \forall t\in T/\{'0'\}, l\in L, l^*\in L, h_2^*\in H^{l^*}
\end{aligned} \tag{C.4}
$$

$$\sum_{l\in L}\sum_{h_2\in H^l} DY(t,l,h_2)=1 \quad \forall t\in T/\{'0'\} \tag{C.5}$$

$$CY(t,l,h_1,h_2)\in\{0,1\} \quad \forall t\in T, l\in L, h_1\in H^l, h_2\ge h_1\in H^l \tag{C.6}$$

$$DY(t,l,h_2)\in\{0,1\} \quad \forall t\in T, l\in L, h_2\in H^l \tag{C.7}$$

The objective function (C.1) minimizes the total cost for investments (first term) and expected damage (second term). Constraints (C.2) define the starting condition for each segment *l* of the dike ring (i.e. its present height/strength). Constraints (C.3) ensure that the final height of each segment *l* of the dike ring in a period *t-1* equals the starting height of the segment in the consecutive period *t*. Constraints (C.4) determine the value of $DY(t,l,h_2)$ in each period. If the overall safety level associated with segment l^* and height h_2^* is selected, then no segment is allowed to have a strength (l,h_2) below this safety level. Note that this is equivalent to a higher failing probability: $prob\;\;t,l,h_2\;\;>prob(t,l^*,h_2^*)$. Hence, all segments need to be at least as safe as level (l^*,h_2^*) prescribes. Constraints (C.5) state that in each time period one and only one safety level must be selected. Constraints (C.6) and (C.7) declare the decision variables as binary.

As previously mentioned, Brekelmans et al. (2012) present the possibility that the expected damage in case of a flood depends on the (absolute) height of a certain dike segment. Model C is not capable to model this aspect in detail, but model C proved to be sufficient to duplicate the results of Brekelmans et al. (2012). However, model C can easily be extended to incorporate this aspect. This extended model, called model D, is presented in Appendix C.

An additional constraint that must be added to model C (and models A, B and D likewise) comes from the already mentioned fact, that a dike ring segment cannot be updated twice within a certain time period. We denote this minimum *update period* of segment *l* by *up(l)*. Hence, we can add the following constraints:

$$\sum_{h=t+1,\dots,t+up(l)} \sum_{h_1 \in H^l} \sum_{h_2 > h_1 \in H^l} CY(h,l,h_1,h_2) \le 1 \qquad \forall l \in L, t = 0,\dots,|T| - up(l) \tag{C.8}$$

Constraints C.8 are described given one-year time periods. If multi-year periods are used, the constraints need to be changed in a straightforward manner. Another real-world example of a crucial side-constraint is the obligation that a segment must be updated before a certain year. This is due to the fact, that certain segments wore out and need to be thoroughly reconstructed at a certain point in time.

Model C requires input for several parameters. Many of these parameters are uncertain. In Appendix D, we present a robust optimization approach to deal with uncertainty, e.g. climate change.

3. Implementation, solution procedure and numerical results

We have implemented models A, B, C and D in GAMS and used CPLEX to solve the models to optimality by using branch-and-cut. Due to the total unimodularity of model A, optimal integer solutions to that IP-model can be easily found by solving its LP-relaxation. The addition of side -constraints similar to C.8 doesn’t influence this solution approach. Instances of model A require a few seconds to obtain the proven optimal solution.

Tests with model B and C clearly show superior solution times for model C. Since model C is also more general in its formulation, we only present computational results for model C. We briefly discuss the computational results of model D in Appendix C.

We explicitly aim to show, that our modelling and solution approach requires minimal programming efforts. Easy and inexpensive implementation is of crucial importance for practical use and dissemination of our results. This holds in the Netherlands and probably even more in less wealthy countries that suffer from flooding. Therefore, we deliberately use standard and easily available software (GAMS and CPLEX) and avoid complex programming tricks.

3.1 Discretization schemes

As briefly mentioned earlier, the problem size of all models depends critically on the chosen number of time periods and possible heightenings. Several aspects play a role in the choice of the discretization scheme for time periods and heigthenings. First, the level of refinement determines the problem size and consequently the solution time. Secondly, using a more refined discretization scheme provides the possibility to determine the optimal timing and extent of dike heightening more precisely. Finally, practical considerations, as earlier mentioned, indicate, that very fine discretization schemes are only of theoretical importance. In practice, both the timing and the extent of heightening cannot be determined very precisely. Experts from Rijkswaterstaat (Dutch government agency responsible for - among others - dike ring construction and maintenance) indicate, that discretization schemes finer than five-year time periods and heightening steps of 20 centimetre are of no value to them.

In addition, one should realize that only model results with respect to the near future, say 20 to 30 years from now, are being used in practice. The Dutch government only makes decisions on actions that need to be initiated during this time period. How to proceed after this period, is the responsibility of future generations (and governments). Postponing the decision on the required dike heigthenings in 30 years from now, is also economically sensible. More information will then be available on climate change, flood risks, new technical solutions to prevent flooding or reduce its consequences, and the occurring economic damage in case of flooding. This principle is also laid down in the recently enacted Water Act (in Dutch: ‘Waterwet’). This law enforces that ‘the effectiveness and consequences of water safety norms must be reconsidered every twelve years’.

Having said this, it is this still important to take time periods in the distant future into consideration. Possible dike heightenings in the distant future may influence dike heightening decisions in the first few decades. Therefore we consider a time frame up to the year 2314, which is in line with Brekelmans et al. (2012) and Kind (2011). Of course, discretization schemes of dike heigthenings in the (distant) future can be selected more roughly.

We use the following discretization scheme:

- Heightenings in steps of 10 cm up to 1 m, in steps of 20 cm for heights between 1 m and 2 m, and steps of 30 cm for heights between 2 m and 4 m.
- For the years from 2015 up to 2100 five-year periods are being used, while after 2100 ten-year periods are being used.

After solving the model with several other discretization schemes, we concluded that the optimal solution hardly depends on the used discretization scheme.

3.2 Preprocessing: reducing the problem size

By using preprocessing techniques based upon the structure of our problem, the problem size can be reduced substantially. This is useful, because it limits computing times of the branch-and-cut procedure.

We present three preprocessing techniques. Each technique aims at reducing the number of decision variables. We use the fact that in all cases we have encountered, damage in case of flooding only depends on a specific year. Hence, we can replace the parameter $damage(t,l,h_2)$ by the parameter $damage(t)$. The presented preprocessing techniques can be adjusted to allow for segment and height specific $damage(t,l,h_2)$. The techniques can also be adjusted for use in model D.

Techniques 1: identical heightening one period later always better?

This technique relies on the idea that given a feasible solution with a certain decision variable the objective function value can always be improved by replacing the decision variable with another decision variable.

Consider any feasible solution in which a segment *l* is heightened in period $t < |T| - 1$ from height h_1 up to $h_2 > h_1$. Given this solution, another feasible solution can be constructed in which this heightening takes place one period later, i.e. in $t+1$. The minimal improvement of the objective function value if segment *l* is not further heightened in period $t+1$, equals:

$$\begin{aligned} MxChngT1(t,l,h_1,h_2) = & -cost(t+1,l,h_1,h_2) + cost(t,l,h_1,h_2) \\ & +[prob(t,l,h_2) - prob(t,l,h_1)] \cdot damage(t) \\ & -cost(t,l,h_1,h_1) \end{aligned}$$

The first line represents the improvement in investments costs due to the incurred postponing of the dike heightening. The second line represents the *maximum* change (i.e. worsening) in expected damage. The last line represents the extra maintenance costs due to maintaining segment l at height h_1 in period t.

The change in expected damage is the maximum change since we implicitly assume that segment l determines the overall flood probability of the dike ring area in period t. If another segment determines the overall flood probability, than the change in expected damage equals zero. Note that the following holds due to the fact that h_2 is safer than h_1:

$$[prob(t,l,h_2) - prob(t,l,h_1)] \cdot damage(t) < 0 \quad \forall t \in T, l \in L, h_2 > h_1 \in H^l$$

It may also occur that segment l is also heightened in period $t+1$ in the considered feasible solution. Suppose that this segment obtains height h_3 in period $t+1$, then the improvement will be higher, namely:

$$\begin{aligned} MxChng2T1(t,l,h_1,h_2) = & -cost(t+1,l,h_1,h_3) + cost(t,l,h_1,h_2) + cost(t+1,l,h_2,h_3) \\ & +[prob(t,l,h_2) - prob(t,l,h_1)] \cdot damage(t) \\ & -cost(t,l,h_1,h_1) > MxChngT1(t,l,h_1,h_2) \end{aligned}$$

The latter inequality holds since:

$$\begin{aligned} & -cost(t+1,l,h_1,h_3) + cost(t,l,h_1,h_2) + cost(t+1,l,h_2,h_3) \\ & \quad > -cost(t+1,l,h_1,h_2) + cost(t,l,h_1,h_2) \quad \forall t < |T|-1, l \in L, h_3 > h_2 > h_1 \in H^l \end{aligned} \tag{1}$$

Inequality (1) results from the high fixed costs of heightening a dike ring segment.

Hence, if $\exists t < |T|-1, l \in L, h_2 > h_1 \in H^l : MxChngT1(t,l,h_1,h_2) \geq 0$, then variable $CY(t,l,h_1,h_2)$ is dominated by variable $CY(t+1,l,h_1,h_2)$ and the former can be set equal to zero (or removed from the problem formulation).

<u>Technique 2: heightening not optimal</u>

This technique relies on the idea, that heightening a segment in one of the latest time periods will probably be not efficient. We derive which heightenings $CY(t^*,l,h_1,h_2)$ can be removed from the problem instance since they can always be replaced by other heightening in any feasible solution and this will results in an improvement of the objective function value. Hence, these variables will not be included in the optimal solution.

We compare a certain heightening of a segment at time period *t** with not heightening the segment at all from period *t** onwards. We start with the final time period $t^* = |T| - 1$. After investigating all heightenings in this period, we apply a similar procedure for the previous period.

First, we introduce parameter *NoHeight(t,l,h).* This parameter denotes whether or not it may be optimal to heighten segment *l* in one of the later periods if this segment has height *h* in period *t*. Setting this parameter to ‘one’ means that no later heightenings have to be considered, zero otherwise.

Set t*:= |T|-1; Set $NoHeight\ \ t*+1, l, h\ \ := 1\ \ \forall l \in L, h \in H^l$

While t*>0 Do

BEGIN

For $\forall l \in L, h_1 \in H^l$

  Set $NoHeight\ \ t*, l, h_1\ \ := 1$

  For all $h_2 > h_1$ do

    If $NoHeight\ \ t*+1, l, h_2\ \ := 1$ and

$$\begin{aligned} &[cost(t^*, l, h_1, h_2) + \sum_{t>t^*} cost(t, l, h_2, h_2) - \sum_{t\geq t^*} cost(t, l, h_1, h_1)]\ + \\ &\quad [\sum_{t\geq t^*} prob(t^*, l, h_2) - \sum_{t\geq t^*} prob(t^*, l, h_1)] \cdot damage(t^*) > 0 \end{aligned} \qquad (2)$$

    Then Set $CY(t^*, l, h_1, h_2) := 0$

    Else Set $NoHeight\ \ t*, l, h_1\ \ := 0$

Set t*:= t^*-1;

END

Note that the first term in (2) represents difference in costs. The second term represents the (maximum!) difference in expected damage. Maximum due to the assumption that segment *l* determines the overall flood probability of the dike ring area. If inequality (1) holds then keeping segment l from period t^* onwards at height h_1, improves all feasible solutions in comparison with heightening this segment to height h_2 in period t^*. Hence, we can set $CY(t^*, l, h_1, h_2)$ to zero (or remove it from the problem instance). If $NoHeight\ \ t^*, l, h_1$ remains equal to one after investigating all $h_2 > h_1$, we know that heightening segment l from height h_1 will not be optimal in period t^*.

Technique 3: Heightening in two steps better than in one step?

This third technique relies on the idea that certain large heightenings at once are so expensive that it is always better to heighten the segment in two consecutive steps. The procedure is as follows. We use mxh_l to denote the height of segment l with the lowest flood probability (i.e. the biggest height).

Do $\forall t_1 \in T; l \in L; h_1, h_3 \in H^l; h_3 > h_1$

Set $stop := 1$

Do $\forall h_2 \in H^l : h_3 > h_2 > h_1$

Set $t_2 := t_1 + 1$ or if a minimal update period is defined, Set $t_2 := t_1 + up(l)$

While $(stop = 1) \wedge (t_2 \leq |T| - 1)$ do

If

$$\begin{aligned}&[cost(t_1, l, h_1, h_3) + \sum_{t_2 \geq t > t_1} cost(t, l, h_3, h_3) \\ &- cost(t_1, l, h_1, h_2) - \sum_{t_2 > t > t_1} cost(t, l, h_2, h_2) - cost(t_2, l, h_2, h_3)] \ + \\ &[\sum_{t_2 > t \geq t_1} \{prob(t, l, mxh_l) - prob(t, l, h_2)\} \cdot damage(t)] > 0\end{aligned} \tag{3}$$

Then Set $CY(t^*, l, h_1, h_3) := 0$; Set $stop := 0$

Set $t_2 := t_2 + 1$;

If condition (3) is met, then we can improve any feasible problem instance in which segment l is heightened in time period t_1 from h_1 up to h_3. This improved solution implies heightening segment l in time period t_1 from h_1 up to h_2 and heightening segment l in time period t_2 from h_2 up to h_3. The first term of (2) calculates the cost reduction if this segment is heightened in two consecutive steps. Note that this may result in a negative number, i.e. a cost increase. Due to higher flood probability, the second term of (2) represents the maximum difference in expected damage. Maximum due to the assumption that segment l determines the overall flood probability of the dike ring area. We use minimum flood probability (due to mxh_l) to be sure, that we do not underestimate the increase in expected damage. The reason is because segment l may be heightened from height h_3 again in the given feasible solution between time period t_1 and t_2.

3.3 Results

The results of our optimization approach are compared with the results of the MINLP-approach by Brekelmans et al. (2012). Ruud Brekelmans (Tilburg University) kindly provided us all data to allow for a direct comparison. All cases presented in Brekelmans et al. (2012) use only time-dependent damage cost. Hence, the parameter *damage*(t,l,h_2) of model C can be reduced to *damage*(t).

Our computing times are measured on a Windows Server 2003 based computer with Intel Xeon E5-2670 processors and refer to the CPLEX-based branch-and-cut procedure only. We specify that CPLEX should first branch on variable *DY*. Apart from this, we use default CPLEX settings. Computing times of Brekelmans et al. (2012) were obtained for a PC with an Intel Core 2 CPU processor.

Table 1 shows, besides the objective function values from equation C.1, also the True Objective values. The difference between these two objective values results from using time periods of 5-10 years. The solution of model C is derived under the assumption, that the segment with the highest total expected damage over all years within one period determines the expected damage for the dike ring in that period. However, the weakest segment in a dike ring might change between two consecutive years within a period. The True Objective recalculates the objective value of the optimal solution with the expected damage determined by the weakest segment per year. Therefore, the True Objective will always be larger than the objective function value of Model C. The procedure of Brekelmans et al. (2012) also uses similar discretization schemes of time periods. Hence, their results show similar differences between the objective function value of their MINLP-model and the True Objective.

Table 1 Comparison of MINLP-approach of Brekelmans et al. (2012) and model C

Dike ring	Number of segments	MINLP approach of Brekelmans et al. (2012) MINLP Objective (M)	True Objective (M)	Solution time (min)	model C Model C Objective (M)	True Objective (M)	Solution time (min)	Difference in True Objectives (%)
10	4	107.51	107.51	0.52	108.26	108.26	0.06	0.69%
13	4	10.38	10.38	0.07	10.33	10.33	0.03	-0.48%
14	2	94.04	94.04	0.54	94.57	94.57	0.03	0.56%
16	8	1044.45	1046.08	6.24	1064.65	1064.80	0.36	1.79%
17	6	377.05	377.05	3.33	380.65	380.66	0.10	0.96%
21	10	217.4	217.71	2.23	221.62	221.62	0.89	1.80%
22	5	373.98	374.08	7.62	378.67	378.68	0.16	1.23%
36	6	395.65	395.65	60.19	395.34	395.34	0.10	-0.08%
38	3	136.26	136.29	59.33	136.75	136.76	0.05	0.34%
43	8	486.72	488.1	1.65	492.66	492.66	0.48	0.93%
47	2	16.57	16.57	8.54	16.63	16.63	0.03	0.36%
48	3	42.92	42.92	2.77	43.37	43.37	0.04	1.05%
average				12.75			0.19	

As Table 1 shows, model C could be solved to optimality within one minute for all 12 dike rings. The average solution time of the branch-and-cut procedure is 0.19 minute. The procedure of Brekelmans et al. (2012) needed on average a solution time of 12.75 minutes. Furthermore, the difference in values for the True Objective function between the MINLP-model of Brekelmans et al. (2012) and model C is small, i.e. less than 2%. A detailed investigation of the optimal solutions of both models shows that they are almost identical in all instances. Due to the fact that de solution procedure of Brekelmans et al. (2012) allows for very fine heightenings (e.g. 7.292 cm), their True Objective is in most cases slightly better than the corresponding True Objective of model C. In a few cases (dike ring 13 and 36), model C provided a slightly better solution than Brekelmans et al. (2012). This may be a consequence of the fact that we solve model C to proven optimality, while Brekelmans et al (2012) apply a heuristic algorithm. It may also be a consequence of the fact that both methods apply different discretization schemes. Clearly, we can refine our discretization scheme in a 'second round' to find mathematically better solutions. However, this has no relevance in practice whatsoever. From the results we conclude, that both methods find (almost) identical solutions in all tested situations.

The table below provides information on the effect of using preprocessing techniques for the solution time of IP model C. The reduction in solution time for the branch-and-cut procedure is substantial, i.e. roughly 40%. However, in terms of absolute solution time, preprocessing techniques reduce the solution times of the branch-and-cut procedure in most cases by a few seconds. The table below indicates furthermore, that the use of preprocessing technique 1 only, already reduces the problem size substantially. Although the use of preprocessing techniques appears not to be very relevant for the solution time of the investigated problem instances, they could be relevant for more detailed or complex problem instances.

Table 2 Effect of the preprocessing techniques on the solution of branch-and-cut algorithm of model C

Dike ring	Number of segments	Number of variables before pre-processing	Reduction in the number of variables by the pre-processing rules				Solution time (min)	
			technique 1	technique 2	technique 3	total	with pre-processing	without pre-processing
10	4	42988	44%	0%	11%	56%	0.06	0.12
13	4	42988	64%	0%	3%	67%	0.03	0.11
14	2	21494	35%	0%	10%	45%	0.03	0.04
16	8	85976	33%	0%	18%	51%	0.36	0.55
17	6	64482	49%	0%	14%	63%	0.10	0.27
21	10	107470	34%	0%	16%	50%	0.89	1.34
22	5	53735	36%	0%	15%	51%	0.16	0.50
36	6	64482	25%	0%	15%	40%	0.10	0.12
38	3	32241	29%	0%	12%	41%	0.05	0.08
43	8	85976	53%	1%	15%	68%	0.48	0.70
47	2	21494	63%	1%	10%	74%	0.03	0.03
48	3	32241	45%	0%	18%	64%	0.04	0.07

3.4 Effect of considering failure mechanisms other than overflow

The main advantage of model C over the MINLP-approach by Brekelmans et al. (2012) is its flexibility with respect to the functional form for flood probabilities, damage and investment costs. In principle, every type of functional form could be specified. This adaptability to the actual situation is not only important from a theoretical perspective, but is crucial for a successful real-world application (Zwaneveld en Verweij, 2014). Important examples of required flexibility with respect to the actual situation were provided in the introduction. For policymakers to accept the model results, it is crucial that the actual situation is well represented in the model. Otherwise, the results will not be accepted as a basis for policy decisions, even if they are mathematically correct.

In this paragraph, we present an example in which this flexibility is crucial for the acceptance of the results. The MINLP-model of Brekelmans et al. (2012, see p.1343) only allows for height-based failure mechanisms, like overflow, and not for strength-based failure mechanisms, like piping or lack of structural quality. As Brekelmans et al. (2012) state on page 1343, this modelling approach presumes that actual problems with piping and the quality of some of the structures are solved before further improvements in the safety level are considered.

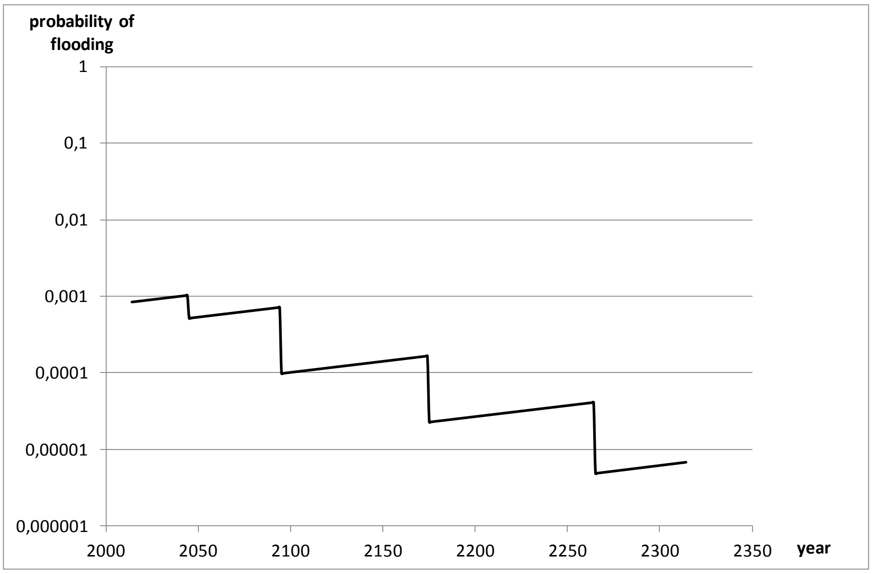
In practice however, there is a need to asses the optimal timing of these anti-piping measures or renovations of constructions and whether or not a simultaneous heightening should be included. In their MKBA WV21 study (Kind, 2011, p. 28, second reference situation), Deltares states that currently the overall flood probability of about half of all dike rings is to a large extent (>50%) determined by other failure mechanism than overflow. Hence, the ability to correctly model these other failure mechanisms is of utmost importance to obtain optimal dike strengthening. Due to the flexibility of our modelling approach, these mechanisms can be easily included.

To illustrate this flexibility, we adjust the input parameters for dike ring 10 to correctly represent the actual flood probability. The failure mechanisms piping and slope instability contribute substantially to the flood probability of dike ring 10. According to the latest insights (Kind, 2011), the total flood probability of this dike ring area is twice as high as the overflow probability. Hence, we have to double the previously used initial flood probability.

Possible measures to solve the problems related to piping and slope instability are the construction of a sheet pile wall or a partial broadening of the dike. The construction costs of these anti-piping measures are in general low in comparison with the (fixed) cost of a dike heightening. For dike ring 10, the cheapest solution to solve the piping and slope instability problems costs 78 million euro. The minimum costs of heightening (including anti-piping measures) dike ring 10 amount to 122 million euro. Given that on average a dike is heightened by about 60 cm, the ‘regular’ costs of a dike heightening are 273 million euro for dike ring 10. After the implementation of these anti-piping measures the flood probability is reduced by 50%.

This ‘jump’ in the flood probability and the specific costs of these anti-piping measures can be easily modelled in model C. The construction of anti-piping measures only is represented in the model by introducing safety level ‘$h=1$’. Higher safety levels involve both the actual heightening of the dike and the construction of anti-piping measures. This represents the fact that in the case of dike heightening, the additional costs of anti-piping measures are relatively low.

Figure 3: Flood probabilities for dike ring 10 according to the optimal investment strategy, with the option of constructing anti piping measures only.



The figure above depicts a typical pattern for economic optimal dike heightening. Due to the existence of fixed investment costs, the dikes are periodically heightened/strengthened. After heightening, the probability of flooding will gradually increase as a result of higher water levels and the subsidence of the dike. In the long run, the economic optimal level of flooding probabilities will decline, because the economic value of goods and people behind the dikes will increase with continued economic growth.

The figure shows the relevance of allowing the construction of anti-piping measures only. It turns out to be optimal to take these anti-piping measures in the year 2045, without any further heightening at that time. This can be seen in the figure since the flood probability is halved in 2045. The dike ring should be heightened in 2095. Clearly, it is non-optimal to take these anti-piping measures directly in 2015.

4. Concluding remarks

This paper considers the dike height optimization problem: what is the economical optimal dike investment strategy to protect against floods? This has been a very important problem in the Netherlands for decades and recent flooding in other deltas shows that it is becoming an important issue all over the world.

We propose an integer programming model for a cost-benefit analysis to determine optimal dike heights and strengths. We improve upon the model proposed by Brekelmans et al. (2012). Our approach, as discussed in this paper, has three important advantages:

1. Virtually complete flexibility towards input-parameters and functional specifications for flood probabilities, damage costs and investments costs for dike heightening. This flexibility facilitates the inclusion of more location specific safety measures and is crucial for the acceptance of the model results by policy makers.
2. Proven optimal solutions are found for all problem instances.
3. The model is easy to implement with the use of standard software. Ease of implementation is not only important for the use of our results in Dutch practice, but also for the dissimilation of our results to less wealthy countries.

The only possible drawback of our formulation of the dike safety problem, is the required a priori discretization of the time periods and the amount of dike strengthening. However, comparing our results with the study of Brekelmans et al. (2012), based on a more continuous approach, shows a difference in values for the objective function smaller than 2%. This is small given the amount of uncertainty in the input data of the model. Moreover, a discretization finer than 10 cm or 5 year periods is not considered to be relevant in practice.

The solution times for our IP model are all below one minute. This is quite fast, especially in comparison with the heuristic approach by Brekelmans et al. (2012). We were able to assess that the procedure of Brekelmans et al (2012) yields near-optimal solutions in the tested problem instances. In a few cases, we were able to find better solutions than Brekelmans et al. (2012). Use of preprocessing techniques improves the solution time of the branch-and-cut procedure somewhat. Because the preprocessing techniques reduce the number of decision variables by roughly 50 %, their use might become relevant in more elaborate problem instances.

In the years to come, more and more detailed local information on flood risk and different prevention measures will become available. Given the recent economic recession together with huge government deficits, policy makers will increasingly ask for tailor-made low costs measures. The models presented in this paper, including the robust optimization approach, are well suited to meet this demand.

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Appendix A: Model A - Homogeneous dike rings

This appendix discusses the problem as considered by Eijgenraam (2006) and Eijgenraam et al. (2010). Model A considers the so-called homogeneous case, which means that a dike ring can be considered as one homogeneous dike.

The binary decision variables are:

$Y(t,h_1,h_2)=$ 1, if the dike ring is updated in year *t* from height/safety level h_1 up to height h_2. If $h_1 = h_2$, then this dike ring is not strengthened in period *t* and remains at its previous height.
0, otherwise.

The input-parameters are:

cost(t,h_1,h_2) = costs for investment and maintenance if the dike ring is strengthened in year *t* from h_1 to h_2. If $h_1 = h_2$, the dike ring is not strengthened in year *t* and these costs only represent maintenance costs.

damage(t,h_2) = expected damage (i.e. 'probability' times 'damage in case of occurrence'), if the resulting height of the dike ring in period *t* equals h_2. Due to economic growth, sea level rising, subsidence and/or increasing river discharges, expected damage increases in time *t*. In some particular instances, expected damage also depends on resulting dike height h_2, Dike height determines water level within a dike ring after a flood and this water level determines damage costs.

The homogeneous dike ring problem reads as follows:

$$Min \sum_{t\in T}\sum_{h_1\in H}\sum_{h_2\geq h_1\in H}[cost(t,h_1,h_2)+damage(t,h_2)]\cdot Y(t,h_1,h_2) \qquad (A.1)$$

subject to

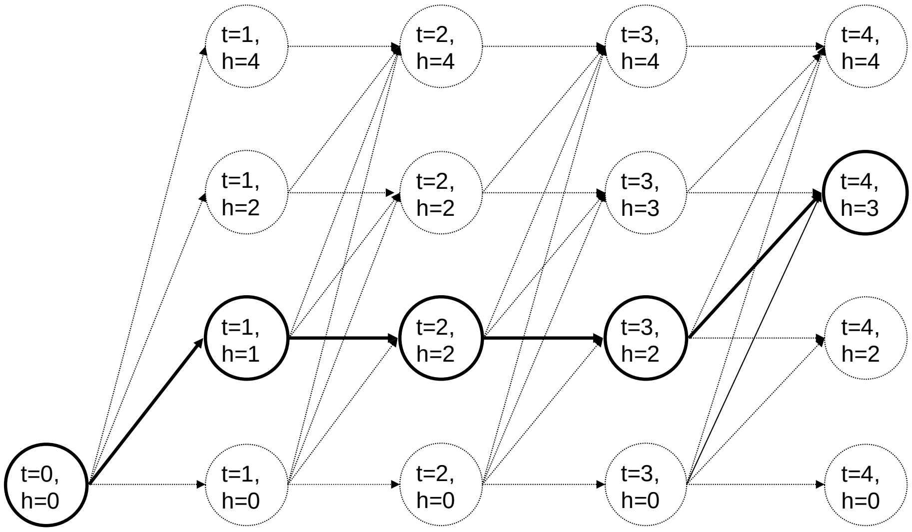
$$Y('0','0','0')=1;\quad Y('0',h_1,h_2)=0 \quad \forall h_1,h_2(\text{with } h_2\geq h_1 \wedge h_2>'0')\in H \qquad (A.2)$$

$$\sum_{h_1\leq h_2\in H} Y(t-1,h_1,h_2)=\sum_{h_3\geq h_2\in H} Y(t,h_2,h_3) \qquad \forall t\in T/\{'0'\},h_2\in H \qquad (A.3)$$

$$Y(t,h_1,h_2)\in\{0,1\} \qquad \forall t\in T,h_1\in H,h_2\geq h_1\in H \qquad (A.4)$$

The objective function (A.1) minimizes the total cost of investments and expected damage. Constraint (A.2) defines the starting condition for the dike ring (i.e. its present height/strength). Constraint (A.3) ensures that the final height/strength of the dike ring in period *t-1* equals the starting height of the dike ring in the consecutive period *t*. The latter constraints (A.4) declare the decision variable $Y(t, h_1, h_2)$ as binary.

Figure A: Homogeneous dike ring problem in network representation for five time periods and four height/safety levels.



A careful investigation of model A shows, that this model exactly satisfies the most fundamental of all network flow problems (Ahuja et al., 1993), namely the minimum cost flow model. Figure A gives a graphical representation of an instance of this model. The decision variables represent arcs in a directed network. The nodes *{t, h}* in this network are represented by a specific choice of time period *t* and dike height *h*. Hence, constraints (A.3) can be seen as *flow* or *mass balance constraints*. The *inbound flow* in node *{t, h_2}* is represented by the first or left-hand side term. The *outbound flow* in this node is represented by the second or right-hand side term and must equal inbound flow.

Since the restriction matrix *A* of the corresponding LP-problem (min *cx*, subject to *Ax=b*) is totally unimodular (Ahuja et al., 1993) for any minimal cost flow model, we can easily obtain an optimal - integer - solution for model A by solving its LP-relaxation.

Hence, the homogeneous dike ring problem can be formulated as a minimum cost flow model and solved by using linear programming. From a practical point of view, this result is very relevant. Model A can be solved with minimal programming effort using commonly available and user-friendly software, such as GAMS with the LP/IP-solver CPLEX. Moreover, no restrictions on investment and damage costs apply to this network flow approach.

In recent and present Dutch research, model A is solved using closed form formulas, as presented in Eijgenraam (2006). This approach requires rather strict functional representations of investment and damage costs. The model was also solved by Chahim et al. (2012) using an Impulse Control approach. The solution procedure presented by Chahim et al (2012) requires similar strict functional representations and it uses a mathematical approach, which is unfamiliar to many and requires substantial programming efforts. Finally, the Dynamic Programming approach in Eijgenraam et al. (2010) can also be used to solve model A to optimality. Probably due to the fact that this approach requires a dedicated programmed algorithm, it is not used in practice. Moreover, a Dynamic Programming approach is less flexible in including additional side-constraints in comparison with the use of model A and an IP-solver (or even a LP-solver) such as CPLEX.

Appendix B: Model B: non-homogeneous dike rings, the simplest approach

This appendix gives an alternative formulation of the non-homogenous case, as described by Brekelmans et al. (2012). Model B is more intuitive, but less general than model C from the main text. This model is a straightforward expansion of model A.

The decision variables are:

$Y(t,l,h_1,h_2) =$ 1, if segment or link *l* of the dike ring is updated in period *t* from height/safety level h_1 up to height/safety level h_2. If $h_1 = h_2$ then this dike ring is not strengthened in period *t* and remains at its previous height/safety level.

0, otherwise.

$DY(t) =$ Expected overall damage costs of the dike ring in period *t*.

The input-parameters are:

$cost(t,l,h_1,h_2) =$ costs for investment and maintenance if segment *l* of the dike ring is strengthened in period *t* from h_1 to h_2. If $h_1 = h_2$, the dike ring segment is not strengthened in period *t* and these costs only represent maintenance costs.

$damage(t,h_2) =$ expected damage (i.e. 'probability' times 'damage in case of occurrence'), if the resulting height of the dike ring segment in period *t* equals h_2.

The non-homogeneous dike ring problem reads as follows:

$$Min \sum_{t\in T}\sum_{l\in L}\sum_{h_1\in H^l}\sum_{h_2\geq h_1\in H^l} cost(t,l,h_1,h_2)\cdot Y(t,l,h_1,h_2)+\sum_{t\in T} DY(t) \qquad \text{(B.1)}$$

subject to

$$Y('0',l,'0','0')=1; Y('0',l,h_1,h_2)=0 \quad \forall l\in L, h_1,h_2 (\text{with } h_2\geq h_1 \wedge h_2>'0')\in H^l \qquad \text{(B.2)}$$

$$\sum_{h_1\leq h_2\in H^l} Y(t-1,l,h_1,h_2)=\sum_{h_3\geq h_2\in H^l} Y(t,l,h_2,h_3) \quad \forall t\in T/\{'0'\}, l\in L, h_2\in H^l \qquad \text{(B.3)}$$

$$\sum_{h_1\in H^l}\sum_{h_2\geq h_1\in H^l} damage(t,h_2)\cdot Y(t,l,h_1,h_2)\leq DY(t) \quad \forall t\in T/\{'0'\}, l\in L \qquad \text{(B.4)}$$

$$Y(t,l,h_1,h_2)\in\{0,1\} \quad \forall t\in T, l\in L, h_1\in H^l, h_2\geq h_1\in H^l \qquad \text{(B.5)}$$

$$DY(t)\geq 0 \quad \forall t\in T \qquad \text{(B.6)}$$

The objective function (B.1) minimizes total costs for investments (first term) and expected damage (second term). Constraint (B.2) defines the starting condition for each segment *l* of the dike ring (i.e. its present height/strength). Constraint (B.3) ensures that the final height of each segment *l* of the dike ring in a period *t-1* equals the starting height of the segment in the consecutive period *t*. Constraints (B.4) gives the value of *DY(t)* in every period *t* by determining the maximum of the expected damage cost over all segments. Constraints (B.5) declare the decision variable $Y(t,l,h_1,h_2)$ as binary. Constraints (B.6) define the decision variables *DY(t)* to be continuous and non-negative.

Appendix C: Model C extended

As mentioned in paragraph 2.2.2, damage costs may depend on the height of a certain dike ring segment. This segment is denoted by l^F. Here, we extend model C to include this aspect.

To introduce the dependence between damage and the height of segment l^F, we replace decision variable $DY(t,l,h_2)$of model C by $DY(t,l,h_2,h^F)$. Height h^F refers to the height of dike segment l^F. This decision variable is defined as:

$DY(t,l,h_2,h^F) =$ 1, if segment l with height /strength h_2 represents the 'weakest link' in period t, i.e. the segment with highest flood probability such that a dike ring starts to fail at this segment. Segment l^F has height h^F. This decision variable is used for bookkeeping flood probability and related expected damage costs.
0, otherwise.

The damage parameter becomes:

$damage(t,h^F) =$ damage (i.e. 'damage occurrence if the dike ring starts to fail at segment l'), if the resulting height of segment l^F in period t equals h^F.

The required adjustments to equation C.1 until C.7 due to these new definitions, are straightforward. Apart from these adjustments, the following additional equations must be included in the model:

$$\sum_{h_1 \le h^F \in H^l} CY(t,l^F,h_1,h^F) = \sum_{l_h \in L} \sum_{h_2 \in H^{l_h}} DY(t,l_h,h_2,h^F) \quad \forall t \in T/\{'0'\}, h^F \in H^{l^F}$$

These constraints connect the 'damage' decision variables $DY(t,l,h_2,h^F)$ with the actual height of segment l^F, such that only damage decision variables may be selected that correctly represent the height of this segment. Model C including all required adjustments is presented below. We refer to this model as model D.

The decision variables are:

$CY(t,l,h_1,h_2) =$ 1, if segment or link l of the dike ring is updated in time period t from height/safety level h_1 up to height h_2. If h_1 = h_2 then this dike ring segment is not strengthened in period t and remains at its previous height. This decision variable is used for bookkeeping investment (and maintenance) costs.

0, otherwise.

$DY(t,l,h_2,h^F)=$ 1, if segment l with height /strength h_2 represents the ‘weakest link’ in period t, i.e. the segment with highest flood probability such that a dike ring starts to fail at this segment. Segment l^F has height h^F. This decision variable is used for bookkeeping flood probability and related expected damage costs.

0, otherwise.

The input-parameters are:

cost(t,l,h₁,h₂) — $cost(t,l,h_1,h_2)=$ costs for investment and maintenance if segment l of the dike ring is strengthened in time period t from h_1 to h_2. If $h_1 = h_2$, the dike ring segment is not strengthened in period t and these costs only represent maintenance costs.

$prob(t,l,h_2)=$ flood probability if the resulting height of segment l in period t equals h_2.

$damage(t,h^F)=$ damage (i.e. ‘damage in case occurrence if the dike ring start to fail at segment l’), if the resulting height of segment l^F in period t equals h^F.

Model D reads as follows. Model equations refer to similarly numbered equations of model C in paragraph 2.2.2.

$$\begin{aligned} Min \ &\sum_{t\in T}\sum_{l\in L}\sum_{h_1\in H^l}\sum_{h_2\geq h_1\in H^l} cost(t,l,h_1,h_2)\cdot CY(t,l,h_1,h_2) \\ &+\sum_{t\in T}\sum_{l\in L}\sum_{h_2\in H^l}\sum_{h^F\in H^{l^F}} prob(t,l,h_2)\cdot damage(t,h^F)\cdot DY(t,l,h_2,h^F) \end{aligned} \tag{D.1}$$

subject to

$$CY('0',l,'0','0')=1; CY('0',l,h_1,h_2)=0 \quad \forall l\in L; h_1,h_2\in H;^l\ h_2\geq h_1\wedge h_2>'0' \tag{D.2}$$

$$\sum_{h_1\leq h_2\in H^l} CY(t-1,l,h_1,h_2)=\sum_{h_3\geq h_2\in H^l} CY(t,l,h_2,h_3) \quad \forall t\in T/\{'0'\}, l\in L, h_2\in H^l \tag{D.3}$$

$$\begin{aligned} &\sum_{h_1\in H^l}\ \sum_{\substack{h_2\geq h_1\in H^l:\\ prob(t,l,h_2)>prob(t,l^*,h_2^*)}} CY(t,l,h_1,h_2)+\sum_{l_h\in L}\ \sum_{\substack{h_2\in H^{l_h}:\\ prob(t,l_h,h_2)\leq\\ prob(t,l^*,h_2^*)}}\ \sum_{h^F\in H^{l^F}} DY(t,l_h,h_2,h^F)\leq 1 \\ &\qquad \forall t\in T/\{'0'\}, l\in L, l^*\in L, h_2^*\in H^{l^*} \end{aligned} \tag{D.4}$$

$$\sum_{l \in L} \sum_{h_2 \in H^l} \sum_{h^F \in H^{l^F}} DY(t,l,h_2,h^F) = 1 \qquad \forall t \in T / \{'0'\} \tag{D.5}$$

$$CY(t,l,h_1,h_2) \in \{0,1\} \quad \forall t \in T, l \in L, h_1 \in H^l, h_2 \geq h_1 \in H^l \tag{D.6}$$

$$DY(t,l,h_2,h^F) \in \{0,1\} \quad \forall t \in T, l \in L, h_2 \in H^l, h^F \in H^{l^F} \tag{D.7}$$

$$\sum_{h_1 \leq h^F \in H^l} CY(t,l^F,h_1,h^F) = \sum_{l_h \in L} \sum_{h_2 \in H^{l_h}} DY(t,l_h,h_2,h^F) \quad \forall t \in T / \{'0'\}, h^F \in H^{l^F} \tag{D.8}$$

The objective function (D.1) minimizes the total costs for investments (first term) and expected damage (second term). Constraints (D.2) define the starting condition for each segment *l* of the dike ring (i.e. its present height). Constraints (D.3) ensure that the final height of each segment *l* of the dike ring in a period *t-1* equals the starting height of the dike ring in the consecutive period *t*. Constraints (D.4) determine the value of $DY(t,l,h_2,h^F)$ in each period. If the overall safety level associated with segment l^* and height h_2^* is selected, then each segment is not allowed to have a strength (l,h_2) below this safety level. Note that this is equivalent with higher failing probability: $prob\ \ t,l,h_2\ \ > prob(t,l^*,h_2^*)$. Hence, all segments need to be at least as safe as level (l^*,h_2^*) prescribes. Constraints (D.5) state that in each time period one and only one safety level must be selected. Constraints (D.6) and (D.7) declare the decision variables as binary. Constraints (D.8) connects the 'damage' decision variables $DY(t,l,h_2,h^F)$ with the actual height of segment l^F, such that only a damage decision variable may be selected which correctly represent the height of this segment.

We implemented model D in GAMS and solved the model with CPLEX. No preprocessing techniques and default GAMS and CPLEX settings were used. We adjusted the original problem instances of dike ring 10, 21 and 48 to include height based damage costs. These problem instances could be solved to proven optimality within a few minutes.

Appendix D: Robust optimization

Model A, B, C and D require values for several parameters. Many of these parameters are uncertain. We present robust optimization approaches to take these uncertainties into account. First, we discuss and copy the robust optimization approach by Brekelmans et al. (2012). Next, we propose an alternative robustness approach. Finally, we discuss how uncertainty is dealt with in the Dutch policy practice. We refer to Brekelmans et al. (2012) and Van der Pol et al. (2013) for a discussion of the literature on robust optimization in the context of dike height optimization.

Two types of uncertainties can be identified. First and most important, there are data uncertainties with respect to climate change and economic growth. Problem instances of the presented models in the main text are specified given a specific scenario for climate change and a social-economic scenario. The climate change scenario determines the increase of flood probabilities in time. In addition, subsidence of the subsoil of dikes increases flood probabilities. The social-economic scenario describes the economic and population growth and therefore the development in time of total damage costs in case of flooding. Since both climate change (including subsidence) and economic growth are uncertain, we are interested in finding a solution that performs well for a broad range of realistic climate and socio-economic scenarios, i.e. we are searching for a ‘robust’ solution. In the Netherlands, four climate scenarios (KNMI, 2006) and four socio-economic scenarios (CPB et al., 2006) are available. Both are broadly accepted as a starting point for analysis and they are regularly updated.

Secondly, data uncertainties also exist given a specific choice of scenarios for climate change and economic growth. For example, the cost of heightening a dike ring one meter is not exactly known in advance. However, good estimates on the expected value are provided by civil engineering consultants, in most cases together with a 95% confidence interval. Expected values for flood probabilities are available given a climate change scenario. However, no confidence intervals for flood probabilities given a climate change scenario are available.

We consider a robustness approach with a finite set of scenarios, for example all combinations of the previously mentioned climate and socio-economic scenarios. A scenario represents an instance of expected values for all model parameters including climate and socio-economic parameter settings. Let $S = \{1, ..., |S|\}$ denote the finite set of scenarios. An index or superscript *s* is added to the model parameters and decision variables to indicate the reference to scenario $s \in S$.

The fundamental problem of uncertainty is, that a decision on an investment plan for dike heightening has to be made before the uncertain parameter values become known. Once the actual scenario is known, a measure of the quality of the decision can be obtained by the difference between the total investment and (expected) damage cost of the chosen investment plan and the total costs of the investment plan that would have been optimal for the actual scenario. This difference is called the regret.

Formally, the regret for model C of a given investment plan CY for scenario *s* is defined by

$$\text{regret}(CY, s) = z(CY, s) - z(CY^s, s) \tag{1}$$

, where $z(CY, s)$ is the objective function value or total costs of investment plan CY evaluated for scenario *s*. Investment plan CY^s is the optimal investment plan for scenario *s*. Therefore, $z(CY^s, s)$ represents the best obtainable objective function value for scenario *s*. Hence, $regret(CY, s) \geq 0$.

Several robustness or regret criteria can be used. A logical regret measure is to find an investment plan that minimizes the average regret over all scenarios. Another choice would be to minimize the maximum regret ('min-max regret') over all scenarios. Brekelmans et al. (2012) present results for both regret criteria. The results indicate that both provide quite similar results.

We only select the average regret criterion, since this approach is best linked to the Dutch practice and guidelines for conducting cost-benefit analysis (CPB and NEI, 2000). From a practical point of view, this average regret approach has the advantage of being easy to implement. A disadvantage of the min-max regret criterion is, that this approach aims to protect against a worst-case instance. Hence, results will be highly dependent on this single instance: a very undesirable characteristic of this min-max regret criterion.

Mathematically, average regret is equivalent to

$$\min_{CY}\left\{\frac{1}{|S|}\sum_{s\in S}(z(CY,s)-z(CY^{s},s)):CY \text{ is a feasible investment plan}\right\}$$

Note that it is easy to show, that this minimal average regret solution of Model C can be obtained by solving model C with the parameters in the objective functions replaced by the (arithmetic) average parameters over all scenarios. If probabilities can be attributed to each scenario, the average regret approach can easily be adapted to a *weighted* average regret. In that case, parameter values in the objective function should represent the *weighted* average over all possible scenarios, instead of the arithmetic average. Hence, in order to solve the (weighted) average regret problem, no dedicated model formulation or solution procedure needs to be designed. Assuming symmetric probability distributions will in most cases produce trivial results.

Up to this point, we closely followed the robust optimization approach by Brekelmans et al. (2012). A major drawback for the robustness approach of Brekelmans et al. (2012) to be used in practice, is its implicit assumption that a single decision upon an investment plan for all considered time periods $t \in T$, has to be made *now*. This can be seen from the fact that a single optimal investment strategy is calculated for the next 300 years. In practice, there is no need to do so. The Dutch government is only interested in a (detailed) investment plan for, say, the next two or three decades. This is among others due to the fact, that it takes 10 to 20 years to heighten a dike ring. In addition, the investment plan for the Dutch Government as laid down in the yearly published Multi-Year Plan for Infrastructure, Spatial Planning and Transport (Ministry of Infrastructure and the Environment. 2012), involves at present the period up to the year 2028, i.e. the next 17 years. Moreover, water safety decisions are to be reconsidered every twelve years as stated in the Dutch Water Act.

As a new robustness approach, we propose to look for one specific investment plan for the first 30 years. This investment plan should minimize (weighted) average regret over all scenarios. For later years, we allow for scenario-specific solutions to take into account that more information on the actual climate and socio-economic developments will be available in 30 years time. Of course, scenario-specific investment plans from year 30 onwards must be consistent with the single investment plan in the first 30 years. The objective function value of each scenario for the period after the year 30 has to be weighted by its probability (or just arithmetically). To model this robustness approach for model C, we need to introduce scenario-specific decision variables *CY* and *DY*. The resulting IP-formulation will be very similar to the original model C. The implicit assumption of this robustness approach is that after 30 years of time more (to be precise: complete) information will be available on which scenario will become reality. The assumption of a single moment of perfect learning after 30 years may be relaxed by introducing gradual learning. The latter will complicate the model and introduce the need to assess and model gradual learning.

An obstacle for using robustness approaches in the Dutch practice is the fact that no well established weights are available for the social-economic and climate scenarios. Let alone that there is a common opinion on how to assess gradual learning or on whether the assumption of perfect learning at a single moment in time is plausible. Although several proposed weightings can be found in the scientific literature (e.g. Meinshausen et al., 2009), responsible Dutch authorities state that weighting of scenarios is not feasible. Therefore, no robust optimization approaches as presented by Brekelmans et al. (2012), van der Pol et al. (2013) and in this appendix are used in the Dutch practice. Due to lack of policy relevance, no numerical results are derived for our robustness approach.

Within Dutch cost-benefit analysis practice, uncertainties are assessed by performing ‘sensitivity analyses’. These analyses involve at first the calculation of an optimal investment plan for dike heightening given so-called base-case model parameter settings (‘base-case scenario’). Secondly, the sensitivity of this optimal investment plan is investigated in relation to adjusted input parameters. In most cases, these sensitivity analyses involve investigating other climate and socio-economic scenarios and discount rates. Based upon the base-case scenario together with the results of the sensitivity analyses, a specific optimal investment plan is proposed.

With respect to Dutch water safety policy, the model results are translated into a policy recommendation with respect to dike ring specific safety standards against flooding. For example, it is stated that a dike ring area should have a maximum flood probability (i.e. the statutory safety norm) of 1:2,000 per year in the year 2050. Each dike ring area is assessed every 6 years to check whether it still meets the safety norm. If not, further actions are initiated to determine which actions are appropriate to increase the safety such that the dike ring will meet its statutory safety norm. The most appropriate action needs to be determined by means of a dedicated cost-benefit analysis. Zwaneveld and Verweij (2013) show that this procedure is quite robust with respect to uncertain developments. If the best possible safety norm is selected for each dike ring, a correct signal on when to strengthen the dike ring is provided in 95% of investigated instances. These instances varied with respect to climate scenario, socio-economic scenario and discount rate.

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