

CPB Netherlands Bureau for Economic Policy Analysis

CPB Discussion Paper | 281

Age-Specific Labour Market Effects of Employment Protection

A numerical approach

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July 2014

Abstract

The particular situation of the youngest and oldest individuals on the labour market motivates age-specific labour market analysis. One topical case is employment protection for older workers. The effect of employment protection on the total number of jobs is ambiguous. The positive effect of lower job destruction is counteracted by the negative effect of lower job creation. This ambiguity carries over to the more specific case of age-related employment protection.

Numerical analysis can be illuminating when countervailing effects produce an ambiguity. In this paper, I present a numerical model based on the theoretical set-up of Chéron, Hairault and Langot (2011). Simulations performed with the model highlight age-specific effects of general employment protection measures and effects of measures targeted at particular age-groups on workers outside the target group. Firing taxes and hiring subsidies have age-specific consequences because employment and unemployment rates vary over the lifecycle. Positive effects of employment protection for the target group can be outweighed by negative effects for other workers.

Keywords: age-specific labour markets, matching model of unemployment, lifecycle effects, retirement, older workers, end-game effect

JEL Codes: J64, J63, J21

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1 Introduction

The effect of employment protection measures on the total number of jobs is ambiguous in general. The positive effect of lower job destruction is counteracted by the negative effect of lower job creation, which leaves the net effect ambiguous. This is the result both of analytical studies of theoretical models (e.g., Bertola, 1992) and of empirical studies, whose parameter estimates are often inconclusive (see Addison and Teixeira, 2003, for an overview). This ambiguity carries over to the more specific case of age-related labour market effects. Age-related effects enter the picture via two types of research questions. First, we can ask which age group will be affected the most by a *uniform* employment protection measure. Second, we can ask how an age-specific employment protection measure will affect both the target age group and other groups to which it does not apply. Of particular interest in age-specific labour market analysis are the first years of activity (youth employment) and the years before retirement (old-age employment, pre-retirement). In this paper, I present a numerical model for simulating these age-specific effects based on the theoretical set-up of Chéron, Hairault and Langot (2011; "CHL" in the following). Its working mechanisms are illustrated by simulations of stylised, both general and age-related policy measures.

The appropriate conceptual set-up for analysing employment protection measures is using matching models of the labour market with endogenous job creation and job destruction (see the canonical formulation in Mortensen and Pissarides, 1994). The set-up must take account of transitions into and out of employment because it is precisely these transitions that employment protection policy facilitates or hinders. In the canonical formulation of the matching model, agents are assumed to live infinitely. As a consequence, this formulation cannot capture age-specific effects. In the past few years, however, several authors have presented models that extend the matching approach by using a lifecycle dimension (Bettendorf and Broer, 2005; Saint-Paul, 2009; CHL, 2011, 2012). Workers can then be followed from their start on the labour market until retirement, with the focus particularly on the phase towards the end of labour market activity. Here, an "end-game effect" (Saint-Paul, 2009) can be identified, namely a worsening of the labour market position of older workers that purely results from the shrinking distance to mandatory retirement.

In the analysis of employment protection measures, as always in situations with countervailing effects, the usefulness of a purely analytical approach is limited. The overall effect can only be determined if the partial effects are quantified and netted out against one another. In this respect, there is no difference between the model with the lifecycle dimension and the model with infinitely lived agents. CHL (2011) stretch the analytical approach to the limit. They derive the optimal employment protection policy that corrects for externalities that arise in a matching setting where workers differ by age and search is undirected. In the derivation, they exploit the fact that their model contains only a single labour market distortion, which can be corrected by the proposed policy. However, even under these favourable conditions, the employment effects of the policy measures remain ambiguous. At this point, the numerical approach of the present paper is a step forward in the following respects: (1) It shows that age-independent labour market policies can have agespecific effects. (2) It reveals direct and indirect effects of age-specific policies, e.g. the difference between workers in the target group and those below the eligibility threshold. (3) It illustrates the interaction of flow effects (at the hiring and firing margins) with stocks, which produces differences in the effects of hiring subsidies versus firing taxes.

At the level of the numerical implementation, one issue needs discussing. The theoretical CHL model is based on a continuous distribution of worker productivity. A necessary step in the calculation of the transition probabilities as well as the state values of employment and unemployment is solving the integral over the densities of these distributions. However, an explicit expression for the integral can be derived only for very simple distribution functions. Numerical integration would be time-consuming and practically infeasible with a large number of age classes. Therefore, in this paper I take the approach of discretising the productivity distribution. This creates a follow-up problem. When analysing small policy shocks (which is particularly relevant for marginal analysis), reactions can become non-smooth once the reservation productivity switches between neighbouring bins of the distribution. In Appendix B, I present a refined version of the model, which allows for the endogenous adjustment of bin boundaries. This approach eliminates the problems of non-smooth reactions and turns out to be a reasonable compromise between quantitative precision and computational feasibility.

The body of the present paper is organised as follows. After a quick informal overview of the model in Section 2, its numerical design is laid out in detail in Section 3. The working mechanisms are then illustrated with both age-independent (Section 4) and age-specific (Section 5) employment protection policies. Section 6 discusses a number of possible extensions and Section 7 concludes.

2 An informal overview of the model

In its basic features – except for the discretisation of productivity – the numerical model in this paper follows CHL (2011).¹ This section contains an informal overview of the model; the symbols and equations are introduced in Section 3.

The cornerstone of the model is a matching function à la Mortensen and Pissarides (1994), which combines the stocks of vacancies and unemployed workers to form matches. In contrast to most matching models of the labour market, which assume infinitely lived agents, the workers in the CHL model are characterised by age, which is discrete and finite. When entering the labour market, all workers start unemployed, and at a pre-determined retirement age, they leave it.

Job productivity is stochastic and drawn each period from a given productivity distribution, both for new matches and for existing jobs. If productivity is too low, namely if it is below the endogenous reservation productivity, worker and firm agree to separate. In the case of an existing job, the worker is dismissed; in the case of a new match, the vacancy is not filled. In general, the reservation productivities for new and existing jobs can diverge. This divergence may be caused by different productivity distribution functions or by particular policy measures. Hiring subsidies are only relevant for new jobs, while firing taxes are only relevant for existing jobs.

The matching probability is uniform across jobs and across the age of the unemployed. The underlying assumption is that firms can only post vacancies that are open to everyone (i.e., search cannot be directed to specific age groups). The transition probability from unemployment to employment is not simply identical to the matching probability of an unemployed person, but it rather results from mul-

¹This model is further discussed and extended in CHL (2012).

tiplying the matching probability by the probability of a productivity draw above the reservation productivity for new jobs.

The model is designed for steady-state analysis by accommodating an age dimension, but no time dimension. The assumption is thus that the labour market conditions of workers aged i in the next period are the same as those of workers aged i + 1 in the current period. Therefore, the values of the steady-state period are sufficient to calculate all dynamic value functions. These value functions are recursively formulated for filled jobs (both new and existing), vacancies, employed workers (both in new and in existing jobs) and unemployed workers. Crucial ingredients are the endogenous probabilities of matching, of forming a new job from a match and of separation in an existing job.

Posting a vacancy has a fixed cost per period. The total number of vacancies is then pinned down by the non-profit condition for vacancies. Finally, wages are determined by Nash bargaining, where the match surplus is distributed between worker and firm in constant shares.

3 Formal characterisation of the numerical model

The only difference between the numerical model presented in this section and the analytical model of CHL (2011) is that productivity is discrete. In the following two subsections, I list the variables and equations, while Subsection 3.3 describes the numerical specification. Finally, in 3.4, I introduce the endogenisation of the bin boundaries, which turns out helpful for generating smooth model reactions to policy shocks.

3.1 Indices, variables and parameters of the model

Most variables of the model have two dimensions: age (indexed by i) and productivity (indexed by g).²

²Extended versions of the model with additional dimensions are used in Euwals et al. (2013): (1) a tenure dimension that splits each age-productivity cell of employed or unemployed individuals

$u_{i,g}$	continuing unemployed individuals
$u_{i,g}^0$	new unemployed individuals
$n_{i,g}$	continuing employed individuals
$n_{i,g}^0$	new employed individuals
p	probability of an unemployed worker being matched with a vacancy
q	probability of a vacancy being matched with an unemployed worker
$x_{i,g}$	index: 1 if a job is continued, 0 if it is dissolved
$x_{i,g}^0$	index: 1 if a match results in job creation, 0 otherwise
u_{tot}	total unemployment
v	total vacancies
m	total matches
$J^0_{i,g}$	value of a new job for the firm
$J_{i,g}$	value of an existing job for the firm
$W_{i,g}^0$	value of a new job for the worker
$W_{i,g}$	value of an existing job for the worker
$U_{i,g}$	value of the state of unemployment
$w_{i,g}^0$	wage for new jobs
$w_{i,g}$	wage for existing jobs
τ	lump-sum tax

Table 1: Variables

i	age group index, runs from 0 to T
	"0" is the starting age when everyone is unemployed
	"T" is the retirement age when every one stops working
g,h	productivity index
	productivity is discretised into G bins

Table 2: Indices

into sub-cells corresponding to the number of periods over which the current status has lasted and (2) a sector dimension that allows us to compare different technological or institutional settings in the same model. These dimensions have been eliminated from the following model description in order to improve readability.

t^0	transition probability from productivity h to q when unemployed
$\circ_{i,h,g}$	
C	cost of a vacancy per period
a	productivity parameter of the matching function
b	per-period value of unemployment (home production)
η	share parameter of the matching function
β	discount factor per period
γ	relative bargaining power of the worker
f_i	firing tax
h_i	hiring subsidy

Table 3: Parameters

3.2 Model equations

Flows of employment and unemployment workers

Workers can switch between four states: freshly employed, continuously employed, freshly unemployed and continuously unemployed. The transition probabilities are determined by the distribution of the productivity draws and by the endogenous reservation productivities (captured by the x indices in the discrete setting).

$$u_{i,g} = \sum_{h} (u_{i-1,h} + u_{i-1,h}^{0}) t_{i-1,h,g}^{0} (1 - px_{i,g}^{0})$$

$$u_{i,g}^{0} = \sum_{h} (n_{i-1,h} + n_{i-1,h}^{0}) t_{i-1,h,g} (1 - x_{i,g})$$

$$n_{i,g} = \sum_{h} (n_{i-1,h} + n_{i-1,h}^{0}) t_{i-1,h,g} x_{i,g}$$

$$n_{i,g}^{0} = \sum_{h} (u_{i-1,h} + u_{i-1,h}^{0}) t_{i-1,h,g}^{0} px_{i,g}^{0}$$

These equations are defined for i = 1, ..., T - 1. At the initial age ("0"), everyone is unemployed, e.g., $u_{0,g} = 1/|G|$ with a uniform distribution of unemployment over productivity bins, and $u_{0,g}^0 = n_{0,g}^0 = n_{0,g} = 0$. At the retirement age, everyone is unemployed (retired) again.

In the simplest specification, t is uniform and neither age- nor history-dependent, i.e., $t_{i,h,g} = t_{i,h,g}^0 = 1/|G|$. More elaborated patterns of $t_{i,h,g}$ can be used to model productivity persistence or a random walk.³

³The general form of the transition equations includes the two cases of worker- and match-

Matching technology

Vacancies and unemployed workers are matched according to a Cobb-Douglas function with distribution parameter η . This function provides the matching probabilities p and q for workers and vacancies, respectively.

$$m = au_{tot}^{\eta}v^{1-\eta}$$

$$p = \frac{m}{u_{tot}}$$

$$q = \frac{m}{v}$$

$$u_{tot} = \sum_{i,g} (u_{i,g} + u_{i,g}^{0})$$

Zero-profit condition for vacancies

$$c = \frac{q}{u_{tot}}\beta \sum_{i,h,g} \left(u_{i,h} + u_{i,h}^0 \right) t_{i,h,g}^0 x_{i+1}^0 \left(J_{i+1,g}^0 + h_{i+1} \right)$$

This condition is derived from setting the value equation for vacancies to zero. The expected surplus from vacancies filled in the future must exactly compensate the vacancy costs incurred in the current period.

Value equations for labour market states

The value functions for the different labour market states (filled jobs, employed workers and unemployed workers) take the standard, recursive form. They consist of a surplus in the current period and the discounted values of the possible states in the next period, weighted with the corresponding transition probabilities.

$$J_{i,g}^{0} = e_{g} - w_{i,g}^{0} + \beta \left[\sum_{h} t_{i,g,h} x_{i+1,h} J_{i+1,h} - \left(1 - \sum_{h} t_{i,g,h} x_{i+1,h} \right) f_{i+1} \right]$$

specific productivities. If t and t^0 are uniform, this does not make a difference. However, if productivity is persistent, we must answer the question of whether productivity persistence applies to the unemployed as well.

$$J_{i,g} = e_g - w_{i,g} + \beta \left[\sum_h t_{i,g,h} x_{i+1,h} J_{i+1,h} - \left(1 - \sum_h t_{i,g,h} x_{i+1,h} \right) f_{i+1} \right]$$

$$W_{i,g}^0 = w_{i,g}^0 - \tau + \beta \left[\sum_h t_{i,g,h} x_{i+1,h} W_{i+1,h} + \sum_h t_{i,g,h} (1 - x_{i+1,h}) U_{i+1,h} \right]$$

$$W_{i,g} = w_{i,g} - \tau + \beta \left[\sum_h t_{i,g,h} x_{i+1,h} W_{i+1,h} + \sum_h t_{i,g,h} (1 - x_{i+1,h}) U_{i+1,h} \right]$$

$$U_{i,g} = b - \tau + \beta p \left[\sum t_{i,g,h}^0 x_{i+1,h}^0 W_{i+1,h}^0 + \sum_h t_{i,g,h}^0 (1 - x_{i+1,h}^0) U_{i+1,h} \right]$$

$$+\beta (1 - p) \sum t_{i,g,h}^0 U_{i+1,h}$$

These equations are defined for i = 0, ..., T - 1. In T, all state values are set to zero: $J_{i,g}^0 = J_{i,g} = W_{i,g}^0 = W_{i,g} = U_{i,g} = 0.4$

Wage equations

Wages are determined by Nash bargaining between firm and worker. This bargaining process results in the following sharing rules.

$$W_{i,g}^{0} - U_{i,g} = \gamma \left(J_{i,g}^{0} + h_{i} + W_{i,g}^{0} - U_{i,g} \right)$$
$$W_{i,g} - U_{i,g} = \gamma \left(J_{i,g} + f_{i} + W_{i,g} - U_{i,g} \right)$$

The wages, $w_{i,g}^0$ and $w_{i,g}$ (not explicit in the wage equations, but implicit in the J and W values) are adjusted so that the value of the match for the worker, $W_{i,g}^0 - U_{i,g}$, is a fixed share of the total match value, $J_{i,g}^0 + h_i + W_{i,g}^0 - U_{i,g}$.

Determination of reservation productivity

$$-h_i \stackrel{\geq}{\equiv} J^0_{i,g} \perp 0 \le x^0_{i,g} \le 1$$
$$-f_i \stackrel{\geq}{\equiv} J_{i,g} \perp 0 \le x_{i,g} \le 1$$

⁴These state values need not be zero (e.g., pension income), as long as they are constant across labour market states. The important aspect is that in the last active period, T - 1, the future is irrelevant for hiring and firing decisions.

These equations are the core of the numerical formulation of the model with discrete productivity bins.⁵ If the value of a new job plus the hiring subsidy is larger than zero (the value of a vacancy), the indicator for job creation is at its upper bound, 1. If the value of a new job plus the subsidy is below zero, the indicator is at its lower bound, 0. In the unlikely event that the value of a new job plus the subsidy is exactly zero, an intermediate value of $x_{i,q}^0$ is possible.

Analogous reasoning applies to the case of existing jobs and the firing tax.

Government budget constraint

The government collects revenue from the firing tax (associated with each occurrence of a freshly unemployed worker) and incurs expenses for the hiring subsidy (associated with new jobs).

$$\sum_{i,g} \left(n_{i,g}^0 h_i - u_{i,g}^0 f_i \right) = \tau \sum_{i,g} \left(n_{i,g}^0 + n_{i,g} + u_{i,g}^0 + u_{i,g} \right)$$

The government budget is balanced by an allocatively neutral lump-sum tax, τ , paid by all employed and unemployed workers.

3.3 Numerical specification

The numerical implementation requires all functions to be specified and all parameters to be assigned concrete values.

- The age group length is chosen to be 2.5 years, which gives 18 periods between entry into the labour market at age 20 and mandatory retirement at age 65.
- Productivity is normalised on the [0, 1] interval and discretised in 30 equidistant bins.

 $^{^5\}mathrm{At}$ a technical level, the model is formulated as a mixed complementarity problem in GAMS (Rutherford, 1999).

• Transition between bins is uniform and path-independent, i.e., $t_{i,h,g} = t_{i,h,g}^0 = 1/|G|$.

The values of the exogenous model parameters are listed in Table 4. These parameter values are meant to be illustrative and in an empirically plausible range, without having an empirical foundation in a strict sense. Productivity-related parameters must be seen in relation to the productivity distribution (normal between 0 and 1) and the period length of 2.5 years. A vacancy cost of 0.05 then means 5% of the output, i.e. 3 month of work, of the most productive worker. The value of the perperiod value of unemployment, 0.4, is chosen so that it generates an inner solution for the reservation productivity for all age groups (see Figure 2). Lower values of *b* would have meant that all matches result in jobs for younger workers. The discount factor is equivalent to a 4.3% discount rate per year. Setting the relative bargaining power of the worker, γ , and the share parameter of the matching function, η , both at the salient value of 0.5 avoids intrinsic inefficiencies in the matching process (Hosios, 1990). Finally, the efficiency parameter of the matching function is chosen so that a reasonably hight level of the employment rate (70%, see Figure 1) is reached for prime-aged workers.

С	cost of a vacancy per period	0.05
a	productivity parameter of the matching function	0.4
b	per-period value of unemployment (home production)	0.4
η	share parameter of the matching function	0.5
β	discount factor per period	0.9
γ	relative bargaining power of the worker	0.5

Table 4: Numerical parameter specification

3.4 Endogenous bin size for stochastic productivity

The numerical formulation of the model in Section 3.3 leaves us with one problem. It turns out that dividing the productivity space into bins with fixed boundaries leads to discrete jumps in the model reactions (see Appendix B for an illustration). In principle, this could be fixed by narrowing the bin width. However, this would increase the computational burden enormously by inflating the model to having a large number of unnecessary bins. After all, only the bins close to the reservation productivities are interesting and deserve closer inspection.

A model modification that avoids the problems of jumps at the boundaries of pre-defined productivity bins is the endogenous determination of bin boundaries. In particular, the number of bins is increased by one by dividing one of the even-sized bins of the model in Section 3 into two at precisely the point of the reservation productivity. In this way, the model remains with the advantages of productivity discretisation (i.e., it avoids numerical integration). However, as fixed bin boundaries are necessary in the numerical model solution procedure, this procedure needs to be modified and to take the form of an iterative set-up.

These iterations proceed as follows:

- Solve the model in Section 3 with fixed bin boundaries.
- Calculate the approximate reservation productivities for new and existing jobs by linearly interpolating the value functions $J_{i,g}^0$ and $J_{i,g}$ between the two productivity bins that produce values directly below and directly above the critical value.
- Adjust the bin boundaries so that the endogenous boundary coincides with the calculated reservation productivity. (Bin delineation for new and existing job will not be the same in general.)
- Adjust the transition probabilities $t_{i,g,h}^0$ and $t_{i,g,h}$ according to the new bin sizes.
- Go back to the first step and iterate until the change in reservation productivities is below the convergence criterion.⁶

 $^{^{6}}$ In the specification used for the simulations in Sections 4 and 5, with a convergence criterion of 10^{-5} for the reservation productivities, convergence is normally achieved within four or five iterations.



4 Simulation: age-independent employment policies

Figures 1 and 2 show the effects of a uniform firing tax of 0.1 (10% of the maximum production per period) on employment and the reservation productivities respectively. In the pre-policy situation (solid line in Figure 1), the employment rate is steadily increasing from labour market entry at age 20 until approximately age 30, where it reaches a steady-state value of slightly higher than 70%. After age 55, an "end-game effect" occurs. As workers approach retirement, it becomes less attractive for firms to keep them employed in periods of (perhaps only transitory) low productivity. This means older workers are more likely to be dismissed and, consequently, their employment rate is lower than that of middle-aged workers.

A uniform firing tax (dashed line in Figure 1) has an employment-increasing effect in general. For young workers, this is hardly relevant. However, starting from approximately the age of 30, the effect of fewer dismissals accumulates, leading to a more than 10 percentage point increase in the employment rate of middle-aged workers. For older workers (aged approximately 55 and above), the employment effect is somewhat smaller again.



Figure 2: Reservation productivity with a large firing tax

Figure 2 shows the age-specific reservation productivities and reveals some of the dynamic underpinnings of the employment change. In general, the reservation productivity is flat for young workers and steeply increasing for older workers. This is an alternative illustration of the end-game effect, which makes the labour hoarding of younger workers profitable, but not that of older workers.⁷ In the no-policy case (solid line in Figure 2), the reservation productivities for new jobs (hiring) and existing jobs (firing) are identical. A firing tax drives a wedge between the two. As a direct effect of this tax, the reservation productivity for firing is considerably reduced. However, there is a countervailing effect on hiring. As firms know that, with high probability, hiring will lead to the payment of a firing tax in a later period, they are more reluctant to hire new workers (i.e., the reservation productivity increases). In a purely analytical model, this effect produces ambiguity in the employment reaction. However, in our numerical model specification, the indirect effect does not even come close to outweighing the direct effect on firing. The only group for which employment decreases is young workers, who depend particularly on the probability of finding a job.

⁷Observe that for the oldest workers, immediately before retirement, the reservation productivity is equal to the value of leisure. For younger workers it is substantially lower.



The mirror image of a firing tax is a hiring subsidy. Figures 3 and 4 show the simulation results for a hiring subsidy of 0.1 (i.e., as high as the firing tax considered above). Again, the overall employment effect (Figure 3) is composed of a direct effect of the subsidies on hiring (lower curve in Figure 4) and an indirect, countervailing effect on dismissals (upper curve in Figure 4).⁸ Compared with the case of the firing tax (and judged from the extent of the shifts in the reservation productivities), the countervailing effect is larger in the case of the hiring subsidy.

Interestingly, the employment effect of a hiring subsidy is not uniform across age groups. The employment of the youngest and oldest age groups increases, whereas that of the intermediate age groups decreases. Clearly, for the youngest workers, who start with an employment rate of zero, the positive effect on hiring is crucial. However, from a certain point in the lifecycle, the employment rate becomes so high that the effect on firings (even though small for each individual worker) starts to dominate. Only for the oldest age groups does employment increase again because the firing-increasing effect is almost non-existent for these groups (right-hand side of the curves in Figure 4).

⁸The indirect effect runs via the worker. Firms can pay higher wages to new workers because of the subsidy. Hence, it becomes more attractive for the worker to leave an existing job, even if



Figure 4: Reservation productivity with a hiring subsidy

5 Simulation: age-specific employment policies

In Section 4, we have analysed age-specific effects of a general policy (age-independent hiring subsidy or firing tax). Another important application of the model are agespecific policies. In this section, we look at policies that are targeted at older workers (see OECD (2006) for an overview of policy measures). Because of the end-game effect (see Section 4), older worker are disadvantaged on the labour market, with a higher firing risk and lower re-employment probabilities. The straightforward policy instruments to counter this effect are firing taxes and hiring subsidies exclusively for older workers. In the simulations of this section, the policy for older workers is in effect for workers of 55 or older. This is approximately the point where the endgame effect becomes quantitatively significant in the no-policy scenario (see Figure 1).

Figures 5 and 6 show the case of a firing tax for older workers. For those workers to which the tax applies, the effects on the reservation productivities (Figure 6) are similar to the case of a age-independent tax (Figure 2). A direct effect (lower reservation productivity) results in fewer dismissals, and a countervailing effect reduces

this means a period of unemployment.



Figure 5: Employment effects of a firing tax for older workers

the number of new jobs for older workers. In addition to the effects on the target group, we have anticipation effects for younger workers, in particular for those directly below the eligibility threshold at age 55. For workers aged between 50 and 55, the effects both on hiring and dismissals are negative. Workers in this age group are easier dismissed and more reluctantly hired that without the tax, because firms anticipate that they will soon become eligible for employment protection.

Adding up all these effects and taking accumulation over the life cycle into account gives the picture in Figure 5. Employment decreases just before the eligibility threshold and rises again thereafter. All in all, the employment effects are positive, and older workers benefit significantly in terms of their employment rate.⁹

The situation with a hiring subsidy for older workers (Figures 7 and 8) is, in terms of the reservation productivities (Figure 8), almost a perfect mirror image of the case of the firing tax: positive effect on hiring, negative countervailing effect on dismissals of older workers, and a negative anticipation effect on both hiring and dismissals of the age group directly below the eligibility threshold.

⁹Observe, however, that this does not imply a welfare assessment. Given that the reservation productivity of workers results from their outside option, it is not evident that a situation with higher employment increases their welfare.



Figure 6: Reservation productivity with firing tax for older workers



Figure 7: Employment effects of a hiring subsidy for older workers



Figure 8: Reservation productivity with hiring subsidy for older workers

However, the consequences for employment (Figure 7) are strikingly different from the case of the firing tax. Qualitatively, we have, again, an employment drop for the group under 55, and more positive outcomes for the target group. However, in contrast to the case of the firing tax, the positive effects for the oldest workers are insufficient to compensate the negative anticipation effects. The reason for this quantitative outcome is the difference in the stocks. As far more than one half of the workers are employed (except for the very first and last periods), a policy-induced change in the firing rate has a larger effect on overall employment than a change in the hiring rate. Even though a hiring subsidy for older workers succeeds in increasing their job-finding chances, the stock of older unemployed is too low for this change to compensate the employment loss due to the anticipation effect. Only the very oldest age group (right margin in Figure 7) has slightly higher employment with the subsidy than in the no-policy case.

6 Possible extensions of the model

It needs to be emphasised that the simulations of this paper are not meant as a realistic policy assessment, but rather as illustrations of the working mechanisms of the model. A realistic policy assessment would require to extend the model with a number of mechanisms that are relevant for real-world labour markets. These mechanisms have been discussed in the existing labour-market literature; this discussion is in most cases not age-specific, however.

Period length. In order to approximate the dynamics of real labour markets, the period length of the model must be reduced. In particular, the begin of the working life cannot realistically be captured with a period length of 2.5 years. In simulations designed to reproduce actual labour market flows, normally a period length of a quarter is assumed (e.g. Mortensen and Pissarides, 2003; Michau, 2009). However, this would inflate the model by a factor of 10 and put a serious constraint on computation resources.

Job-to-job transitions. In the model workers can only change jobs via a period of unemployment. Given that the period length is 2.5 years, this is a serious restriction. In principle, it would be possible to extend the model with on-the-job search (Burdett and Mortensen, 1998; Moscarini, 2005). Then active workers are matched with a certain probability with another firm, and if productivity is higher in that firm, they decide to quit their old job.

Directed search. The model follows Chéron et al. (2011) in the assumption that firm can only post general vacancies that are open for everyone, without age restriction (undirected search). Even if job discrimination by age is illegal in many countries, there are certainly ways of indirectly selecting by age (most prominently: work experience), so that the assumption of undirected search seems unrealistic. It is straightforward to modify the model so that vacancies are perfectly specific, only directed to one particular age group (Fujimoto, 2013, focuses on this point). However, finding a plausible intermediate specification, where vacancies target a certain age range of workers rather than only one group, is less straightforward.

Persistence in productivity. In the model it is assumed that the productivity of each worker is updated every period with a new draw from the probability distribution, which is independent of his productivity before. This simplifies the calculations, but it is particularly unrealistic. In a numerical setting, it is simple to include any pattern of persistence, including the one discussed in Chéron et al. (2011), i.e. the productivity remains unchanged with probability λ , and with probability $1 - \lambda$, it is replaced by a new draw.¹⁰

Age-specific productivity. In the model the productivity distribution is independent of age. In the numerical version, it is straightforward to make this distribution age-specific. However, finding an empirical foundation for this variation is a challenge, because it is a disputed question whether productivity falls for older workers, or whether there is only a shift in tasks at which they perform best (see, e.g. Borghans and ter Weel, 2002, Friedberg, 2003, Aubert et al., 2006, Börsch-Supan and Weiss, 2013). A less empirically demanding extension of the model would be to add negative health shocks to the productivity draws of older workers.

Human capital. Experience-related productivity can be treated similarly to the case of age-specific productivity. The model can be extended with a human capital stock that leads to productivity increases in continued jobs. However, this requires to extend the model with a tenure (i.e. human capital) dimension. The empirical calibration of this extension can be based on literature like Kotlikoff and Gokhale, 1992.

Wage rigidity. The Nash bargaining assumption of the model implies that wages are flexible between periods and react instantly to changes in stochastic productivities. This contrasts with a large literature about wage rigidities both in macro and labour market economics (e.g. Corden, 1981, Holden and Wulfsbert, 2009). In particular, the discussion about non-neutrality of severance payments (Lazear, 1990, Garibaldi and Violante, 2005) and about increasing wage profiles (Lazear, 1979, Hashimoto, 1981) are applicable to the case of age-specific labour-market policy. The wage-rigidity issue creates a dilemma for applied modelling, however. Adding ad-hoc wage rigidity is simple, but unsatisfactory, whereas adding a mechanism that creates and explains wage rigidity (e.g. incentive problems as in Lazear, 1979) considerably complicates the model.

Unemployment insurance. The model contains only the most rudimentary of institutions and does not take the consequences of unemployment insurance into account. It is assumed that utility in the state of unemployment is generated by

¹⁰In the setting with variable bin boundaries, one complication arises when implementing "unchanged productivity". Bin boundaries change from one period to the next. As a consequence, one cannot simply define "unchanged productivity" as "remaining in the same productivity bin". A plausible adjustment is asked for.

leisure or some income from informal activities. More realistic is the introduction of an unemployment insurance, financed by contributions of the employed workers. This would generate a fiscal externality (Blanchard and Tirole, 2008), which in turn is a potential target for internalisation when the scheme of hiring subsidies and firing taxes is designed.

A subset of the listed extensions have been implemented in a study prepared for the Dutch Ministry of Social affairs (Euwals et al., 2012). Some of them, namely persistence in productivity and health shocks, can be implemented in the basic model set-up of Section 3. The introduction of human capital and a realistic pattern of labour market institutions (e.g. tenure-dependent unemployment insurance) requires the extension of the model with a tenure dimension.

7 Conclusions

Starting from the analytical set-up of Chéron, Hairault and Langot (2011), this paper presents a numerical labour market model with frictional unemployment and a lifecycle dimension. The model can be used to quantify both the age-specific effects of a uniform employment protection policy (Section 4) and the effects of age-specific labour market measures (Section 5).

In the assessment of these policy measures, the numerical model can highlight effects that are difficult, if not impossible to reveal in a purely analytical treatment.

- It shows the cumulative effects of changes in the reservation productivities over the life cycle.
- It enables us to compare quantitatively the countervailing effects of changes in job creation and job destruction on overall employment.
- It shows that age-independent labour market policies can have age-specific effects. E.g. a uniform hiring subsidy can decrease employment of the middle-aged workers, whereas employment prospects of older workers improve (see Section 4).

- It reveals direct and indirect effects of age-specific policies. E.g. a firing tax for older workers increases employment for the target group, but the effects on the workers below the eligibility threshold are detrimental (see Section 5).
- A particular enlightening role of the model is to illustrate the interaction of flow effects (at the hiring and firing margins) with stocks, which produces a noteworthy difference in the effects of hiring subsidies versus firing taxes for older workers. In terms of employment, older workers benefit from a general firing tax, but not from a general hiring subsidy (see Section 5).

The model, as presented in this paper, is still a considerable step away from a realistic representation of real-world labour markets. In Section 6, I discuss a number of possible extensions: shorter period length, job-to-job transitions, directed search, persistence in productivity, age-specific productivity, human capital and more complex institutions. Each of these extensions is feasible, and some of them have been implemented in the Dutch policy paper of Euwals et al. (2013). However, combining several of the proposed extensions in a single model will quickly surpass the limits of computational feasibility.

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A The model of Chéron *et al.* (2011)

In contrast to the numerical model of this paper, the original model of Chéron et al. (2011) works with a continuous distribution of productivities. Crucial for the working mechanism are reservation productivities R_i (for ongoing jobs) and R_i^0 (for new jobs), which generate transitions according to the productivity distribution function G.

The model has T age classes, indexed by $i \in \{1, ..., T\}$. At age i = 1, workers enter the labour market and are unemployed with certainty $(u_1 = 1)$, age i = T - 1is the last period when workers are active on the labour market, in period T, they retire with certainty.

The probability of an unemployed worker of age i to be employed at age i + 1 is

$$jc_i \equiv p(\theta) \left[1 - G(R_{i+1}^0) \right],$$

where p is the matching rate generated by the matching function for unemployed workers, u, and vacancies, v:

$$p(\theta) = M(u, v)/u$$

The job destruction rate for employed workers is given by¹¹

$$jd_i = G(R_i)$$

The resulting labour market flows generate the following age-profile of unemployment:

$$u_{i+1} = u_i \left\{ 1 - p(\theta) \left[1 - G(R_{i+1}^0) \right] \right\} + G(R_{i+1})(1 - u_i)$$

for¹² $\forall i \in (1, T - 2)$ and the initial condition $u_1 = 1$. Adding up gives the overall level of unemployment:

$$u = \sum_{i=1}^{T-1} u_i$$

 $^{1^{11}}jc_i$ depends on R_{i+1}^0 , but jd_i on R_i . This is slightly anomalous and requires care in the numerical implementation.

¹²In the original paper the index runs from 1 to T - 1. This is incorrect.

Note that the unemployed of period T-1 are part of the unemployment pool and can be matched with a vacancy, even if the do not start working because they are about to reach the retirement age.

The expected value of a vacant position is^{13}

$$V = -c + \beta q(\theta) \sum_{i=1}^{T-2} \frac{u_i}{u} \left\{ \int_{R_{i+1}^0}^1 \left[J_{i+1}^0(x) + H_{i+1} \right] dG(x) + G(R_{i+1}^0) V \right\} + \beta \left[1 - q(\theta) \right] V,$$

which, using the zero-profit condition V = 0, can be written as

$$\frac{c}{q(\theta)} = \beta \sum_{i=1}^{T-2} \frac{u_i}{u} \int_{R_{i+1}^0}^1 \left[J_{i+1}^0(x) + H_{i+1} \right] dG(x)$$

Bellman equations for the values of the different labour market states are formulated in the usual, recursive manner. This gives

$$J_i^0(\epsilon) = \epsilon - w_i^0(\epsilon) + \beta \left[\int_{R_{i+1}}^1 J_{i+1}(x) dG(x) + G(R_{i+1})(V - F_{i+1}) \right]$$

for the value of a filled new job (with outsider wage $w_i^0(\epsilon)$) for $\forall i \in (1, T - 1)$,

$$J_{i}(\epsilon) = \epsilon - w_{i}(\epsilon) + \beta \left[\int_{R_{i+1}}^{1} J_{i+1}(x) dG(x) + G(R_{i+1})(V - F_{i+1}) \right]$$

for the value of a filled continued job (with insider wage $w_i^0(\epsilon)$) and

$$\mathcal{W}_{i}^{0}(\epsilon) = w_{i}^{0} - t_{i} + \beta \left[\int_{R_{i+1}}^{1} \mathcal{W}_{i+1}(x) dG(x) + G(R_{i+1}) \mathcal{U}_{i+1} \right]$$
$$\mathcal{W}_{i}(\epsilon) = w_{i} - t_{i} + \beta \left[\int_{R_{i+1}}^{1} \mathcal{W}_{i+1}(x) dG(x) + G(R_{i+1}) \mathcal{U}_{i+1} \right]$$
$$\mathcal{U}_{i} = b - t_{i} + \beta \left[p(\theta) \int_{R_{i+1}}^{1} \mathcal{W}_{i+1}^{0}(x) dG(x) + p(\theta) G(R_{i+1}^{0}) \mathcal{U}_{i+1} + (1 - p(\theta)) \mathcal{U}_{i+1} \right]$$

¹³The unemployed of age T-1 can be matched with a vacancy, but they add no value, therefore the sum runs only up to T-2.

for the values of insiders, outsiders and unemployed workers, respectively. The final conditions are $\mathcal{W}_T = \mathcal{U}_T$ (exogenous).

Wages are determined by Nash bargaining with bargaining power γ of the worker

$$\mathcal{W}_{i}^{0}(\epsilon) - \mathcal{U}_{i} = \gamma \left[J_{i}^{0}(\epsilon) + H_{i} + \mathcal{W}_{i}^{0}(\epsilon) - \mathcal{U}_{i} \right]$$

$$\mathcal{W}_i(\epsilon) - \mathcal{U}_i = \gamma \left[J_i(\epsilon) + F_i + \mathcal{W}_i(\epsilon) - \mathcal{U}_i \right]$$

B Pitfalls of pre-determined productivity bin boundaries

In a straightforward numerical implementation of the CHL set-up we can discretise the productivity space into bins of equal size and calculate the value functions for each of these bins. An earlier version of this paper was actually based on this straightforward implementation (with 30 equal-distant bins). However, the discretisation produces non-continuous reactions that make the results difficult to interpret. This appendix illustrates the pitfalls and motivates the approach with endogenous bin boundaries used in the main text (Section 3.4).

Figure 9 shows the employment effects of a uniform firing tax in the model with pre-determined bin boundaries. In contrast to the main text, the level of the tax is chosen particularly low (0.01, i.e. 1% of the maximum production per worker and period) in order to highlight the resulting interpretation problems. The no-policy situation (solid line) is rather close to the model with endogenous bin boundaries (Figure 1). The employment rate is first increasing, reaches a steady-state value of slightly higher than 70% after the age of 30 and finally declines because of the end-game effect.

As in the main text, a uniform firing tax (dashed line in Figure 9) has an employment-increasing effect in general. However, the age-specific pattern that emerges is irritating. For age groups between 25 and 50, employment increases considerably, significantly more that what one might have expected as a reaction to the small policy impulse. For workers above age 50, in contrast, the employment effect is hardly noticeable.



This strange age-pattern in the employment effect is an artefact of the productivity discretisation. With discrete productivity (in roughly 3%-points steps when there are 30 bins), any policy will either have a negligible effect¹⁴ (if the reservation productivity does not shift by at least one productivity bin) or a significant effect (if it does). This is highlighted in Figure 10, which shows the pattern of the reservation productivity with and without the firing tax.

The solid line depicts the reservation productivity in the "no policy" case (where the lines for hiring and firing coincide). It is constant until age 55 and then sharply increasing. In the case with the firing tax, the reservation productivity for hiring remains unchanged. This outcome is not unexpected; however, because of general equilibrium feedback, it is not obvious either. The changes take place in the reservation productivity for firing, which in general declines. The changes are in line with the discrete structure of the productivity bins. For the age groups below 50 and above 60, the reservation productivity declines by exactly one bin width (3%-points), for the age groups in-between, it does not change at all.

Patters of this sort are, even if qualitatively enlightening, quantitatively uncon-

 $^{^{14}}$ The effect is not exactly zero, because one variable in the model, vacancies, does adjust continuously.



Figure 10: Reservation productivity with small firing tax

vincing. They produce spurious jumps in the age-specific effects of a uniform policy, depending solely on the coincidental closeness of the reservation productivities to the bin boundaries. Therefore, the model in the main text has been modified to include endogenously adjusting bin boundaries.

Figure 11 shows the age-specific employment effects of a small firing tax (0.01) in this modified model. In both respects that were unsatisfactory in the model with fixed bin boundaries, the situation is improved:

- The employment effects are small and in the order of magnitude of the policy shock analysed.
- The age-pattern of employment effects is smooth, as one would expect with a policy that is not targeted to specific age groups.
- The pattern of reservation productivities (Figure 12) is smooth as as well. In both the no-policy and the firing-tax scenario the end-game effect sets in gradually, not abruptly as in Figure 10.



Figure 11: Employment effects of a small firing tax % f(x)=f(x)



Figure 12: Reservation productivity with small firing tax

Publisher:

CPB Netherlands Bureau for Economic Policy Analysis P.O. Box 80510 | 2508 GM The Hague T (070) 3383 380

July 2014 | ISBN 978-90-5833-651-4