CPB Discussion Paper

No 126

May, 2009

Discounting investments in mitigation and adaptation

A dynamic stochastic general equilibrium approach of climate change

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The responsibility for the contents of this CPB Discussion Paper remains with the author(s). The author greatly acknowledges valuable comments on earlier versions of this paper by Leon Bettendorf, Casper van Ewijk, Christian Gollier, Coen Teulings, Herman Vollebergh, Aart de Zeeuw, Gijsbert Zwart and participants at a CPB seminar. Correspondence via e-mail: aalbers@cpb.nl.

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ISBN: 978-90-5833-405-3

Abstract in English

We use a dynamic stochastic general equilibrium model to determine efficient discount rates for climate (mitigation and adaptation) and non-climate investment in the face of climate change. Our main result is that the non-diversifiable risk in the economy may be related to both shocks in aggregate wealth and shocks in global average temperature. Therefore, both aggregate wealth and global average temperature will carry a risk premium reflecting their contribution to the total amount of non-diversifiable risk. We characterize both climate and non-climate investments by means of a contingent claim and show that climate and non-climate investments will in general be discounted at different rates. We discuss the conditions under which the discount rates of climate investments will be lower than the discount rate of non-climate investments.

Key words: discounting, adaptation, mitigation, climate change, risk premia, dynamic stochastic general equilibrium model

JEL code: G12, H43, Q5, Q54

Abstract in Dutch

Door gebruik te maken van een dynamisch stochastisch algemeen-evenwichtsmodel zijn we in staat om efficiënte discontovoeten te bepalen voor investeringen in adaptatie en mitigatie. Ons belangrijkste resultaat is dat het systematische risico in een economie die geconfronteerd wordt met klimaatverandering, bestaat uit twee componenten. De eerste component reflecteert het systematische risico verbonden aan (financieel) vermogen, de tweede het systematische risico verbonden aan klimaatverandering. Elke component kent zijn eigen risicopremie. Het gevolg hiervan is dat de discontovoet voor investeringen in adaptatie en mitigatie over het algemeen af zal wijken van de discontovoet voor 'normale' investeringen. We bespreken de condities waaronder investeringen in adaptatie en mitigatie met een lagere discontovoet gedisconteerd moeten worden.

Steekwoorden: disconteren, adaptatie, mitigatie, klimaatverandering, risicopremie, dynamisch stochastisch algemeen-evenwichtsmodel

Contents

1 Introduction

Suppose we are asked to value two investments which are equally risky in terms of aggregate wealth. The only difference between these investments is that the first one is a climate investment, i.e. an investment in mitigation or adaptation, whereas the second one is a non-climate investment. Should we then require these investments to earn the *same* rate of return *at each moment* in time? That is, should we apply a *uniform* discount rate to value them?[1] A persuasive argument in favour of a uniform discount rate rests on the notion of the opportunity cost of capital and is at the heart of the logic of the descriptive approach to climate change.[2] Arrow et al. (1996) describe the basic logic behind this approach as follows. Suppose that the rate of return on climate investments lies below the market return of non-climate investments. Total welfare can then be improved by increasing the level of non-climate investments at the expense of climate investments until the difference in their respective rates of return is eliminated.

Although the descriptive approach has been challenged by many inside and outside the economic profession for its use of market-based returns, the use of a uniform discount rate for climate and non-climate investments has, perhaps surprisingly, hardly received any criticism.[3] For example, proponents of the prescriptive approach on climate change, like Cline (1992) and Stern (2007, 2008), have argued on ethical grounds that market interest rates should not be used as a benchmark for the social discount rate (SDR). However, they never challenged the idea that the discount rate should be uniform. This has made the prescriptive approach vulnerable to the counterargument that a low SDR would lead to very high saving rates (see f.e. Nordhaus (2007) and Dasgupta (2008)). Another strand of literature has questioned the tendency of the descriptive approach to extrapolate the level of the current market interest rate (indefinitely) into the future, but not the idea that the discount rate should be uniform. For example, Weitzman (1998) shows that uncertainty about the appropriate rate of return on capital in the far-distant future will lead to a declining discount rate (DDR) over time. Still, all investments should be discounted at a uniform rate because in his model the discount rate for any project depends solely on the time-horizon of the project's benefits.

Although (part of) the motivation for using either the SDR or DDR approach lies within risk associated with climate change,[4] neither of them has formally modelled environmental

[1] Note that discount rates for the time frames under consideration are in fact not observable. This does not, however, interfere with the issue under consideration in this paper: why use the same discount rate for climate and non-climate investments at each point in time?

[2] The term 'descriptive approach' is due to Arrow et al. (1996). The best-known proponent of the descriptive approach is Nordhaus (2008).

[3] Recently, Weitzman (2007) has suggested that that the beta of climate investments might actually be lower than the beta of non-climate investments.

[4] A low value of the SDR has been motivated partly by choosing a low value for the coefficient or relative risk aversion.

uncertainty as *the* central feature of climate change while recognizing that (i) this environmental uncertainty is endogenous, i.e. it is generated by our own activities; and (ii) the environment may have an (uncertain) feedback on economic activity and welfare. In this paper, we develop a stochastic dynamic general equilibrium model that accounts for these drawbacks. In our model climate change is uncertain. It affects, and is affected by, economic activity and welfare. The model encompasses four ways in which climate change may affect welfare. First, climate change may affect welfare indirectly, by changing the marginal value of consumption. Second, climate change may affect welfare directly, by changing the amenity value.[5] Third, climate change may affect both the expected rate of return and the volatility of production. Fourth, climate change may lead to an increase or decrease in volatility of climate change itself. This allows us to study the relationship between the risk premium of an investment on the one hand and the presence of economic and environmental risk on the other hand. Our main results are twofold. First, we show that the discount rate for any investment can be written as a function of (i) the risk free rate; (ii) the risk premium with respect to wealth; and (iii) the risk premium with respect to temperature. Second, and this is our main result, we show that climate and non-climate investments will in general be discounted by a different discount rate. It must be emphasized that our result is fully consistent with any argument on the opportunity cost of capital provided one recognizes that the opportunity cost of capital may be determined by both economic and environmental risk.

This paper is organized in the following way. In Section 2, we present our model and derive expressions for the level of the risk-free interest rate and the equilibrium prices of risk. Section 3 characterizes climate and non-climate investments in terms of contingent claims and shows that in general climate investments are discounted at a different rate than non-climate investments. In section 4, we discuss the results in terms of current knowledge on (the economics of) climate change. Section 5 concludes.

[5] That is, we use a bivariate utility function. Hoel and Sterner (2007) and Gollier (2008) analyze the effect of such a function on the risk free rate under certainty and uncertainty, respectively.

2 The model

We consider a simple general equilibrium model with two aggregate consumption goods. The first is produced consumption, denoted by $C(t)$. The second is an aggregate environmental bad, like temperature, $T(t)$. The index t denotes time. The representative consumer maximizes expected utility of these consumption goods over time:

$$J \equiv E\int_o^{\infty} e^{-\delta t} U[C(t),T(t)]dt \tag{2.1}$$

where $U(.)$ denotes the utility function of the representative consumer and δ is the utility discount rate. Utility is increasing in consumption, $U_C > 0$, decreasing in temperature, $U_T < 0$ and concave in both consumption and temperature $U_{CC}, U_{TT} < 0$. Production of the aggregate consumption good is considered to take place in n different sectors and is described by the following system of stochastic differential equations:[6]

$$dK(t) = I_K \alpha(T,t)dt + I_K G(T,t)dw(t) \tag{2.2}$$

Here, $K(t)$ is a vector of size n representing invested capital in each of the n sectors, I_K is an nxn diagonal matrix whose ith diagonal element is the ith component of $K(t)$ and $w(t)$ is an $n+1$ dimensional Wiener process in R^{n+1}. Finally, $\alpha(T,t)$ is a n dimensional vector and $G(T,t)$ is an $n\times(n+1)$ matrix. $\alpha(T,t)$ denotes the expected rates of return of these production processes, whereas $G(T,t)G(T,t)'$ denotes the covariance matrix of the rates of return. Notice that (2.2) includes the case in which the n production processes are a geometric Brownian motion.

The environmental bad, temperature, evolves according to the following stochastic differential equation:

$$dT(t) = \theta(T(t), \gamma' K(t))dt + s(T(t),t)' dw(t) \tag{2.3}$$

where γ is a vector whose ith element is describing the pollution intensity of the ith sector and $s(T(t),t)' s(T(t),t)$ is the variance of temperature. Here, $s(T(t),t)$ is a vector of dimension $n\times 1$ and γ is of dimension $n\times 1$. The flow of pollution $\gamma' K(t)$ is fully determined by size and distribution of the capital stock over the n sectors. This implies that in each sector the ratio between capital and pollution is fixed. Pollution depends of the expected rate of return in the economy, $E\left(dK(t)\right)/dt = I_K\alpha$. This follows from the observation that the flow of

[6] To facilitate interpretation we present the stochastic differential equation for sector i, where we have divided both sides by $K_i(t)$. This differential equation reads: $dK_i(t)/K_i(t) = \alpha_i(T,t)dt + g_i(T,t)dw(t)$. Here, g_i is the ith row vector of G.

pollution $\gamma' K(t) = \tilde{\gamma}' I_K \alpha$, where $\tilde{\gamma}_i = \gamma_i / \alpha_i$.[7] Pollution leads to an expected drift in temperature, which is co-determined by the current level of temperature, $T(t)$. Although equation (2.3) includes a broad range of stochastic processes for the evolution of temperature over time,[8] we do not defend the position that the impact of carbon dioxide on the stochastic process of temperature can be described anywhere along the lines of equation (2.3). All that is important for our purposes here is that the flow of pollution changes the drift of temperature and that this change is positive, i.e. $\theta_2 > 0$. Moreover, it would be possible - albeit at the cost of greater complexity – to write down a more comprehensive model describing the interaction between the emissions of carbon, the stock of carbon in the atmosphere and oceans, the sea level and temperature.[9]

Equations (2.1)-(2.3) show that temperature may affect utility in three ways. First, it may affect utility directly. Higher temperatures lead to lower utility. Second, it may affect utility indirectly by changing the marginal value of consumption. Third, it may affect utility indirectly by both changing the expected rates of return, the volatility of production in each of the n sectors as well as the volatility in temperature itself.

In the model, abatement is represented by shifting investment in capital from dirty to clean(er) sectors. When abatement is costly we have:

$$0 = \gamma_1 < \gamma_2 \cdots < \gamma_{n-1} < \gamma_n \text{ and } 0 < \alpha_1 < \alpha_2 < \cdots < \alpha_{n-1} < \alpha_n \tag{2.4}$$

That is, sectors with lower pollution intensities, represented by a lower i, have a lower expected rate of return. Since all sectors produce the same aggregate consumption good, one may think of these sectors using different energy inputs. For example, in the case of two sectors, sector 1 may use renewables having zero pollution and a low expected rate of return, , whereas sector 2 may use coal for its production process having a high rate of pollution and a high expected rate of return. The use of a different energy source as input may – besides affecting the expected rate of return – also effect the variability and the covariance of the returns. Notice that under our assumptions it is possible that optimal investment is zero in some sectors. As in CIR (1985) we explicitly allow for this possibility. The covariance between unanticipated changes in $K(t)$ and $T(t)$ is given by $s'G$. For the moment we postpone the discussion of its sign and simply note that this covariance is allowed to be either positive, zero or negative depending on the sector. It may also change sign over time.

[7] Alternatively, pollution could be made dependent on the actual rate of return. We will discuss this case in section 4.

[8] Examples are (Geometric) Brownian motion as well as (Geometric) Mean Reversion (see e.g. Metcalf and Hasset, 1995).

[9] See for example Cox, Ingersoll and Ross (1985) who model the movement of a k-dimensional vector of state variables.

Individuals can lend and borrow at an endogenously determined interest rate r and can invest in each of the n production possibilities and one contingent claim. The value of this claim is governed by the following stochastic differential equation:

$$dF = (F\beta - \zeta)dt + Fh'dw(t) \tag{2.5}$$

and will depend in general on all variables necessary to describe the state of the economy. Here, $F\beta$ is the total mean return on the claim and ζ is the payout received. Notice that it is not necessary to assume relation (2.5). It can be derived within the context of the model (see Appendix A). h is an $n+1$ dimensional vector. The variance of the rate of return on this claim is given by $h'h$.

The representative individual allocates his wealth among the $(n+1)$ investment opportunities in the basis and the riskless opportunity, borrowing or lending.[10] His budget constraint is then described by

$$dW = \left(a'(\alpha - r1)W + b(\beta - r1)W + rW - C\right)dt + W\left(a'G + bh'\right)dw(t) \tag{2.6}$$

where aW and bW denote the amount of wealth invested in each of the production processes and the contingent claim. Expected changes in wealth are determined by the excess return[11] on the production processes and the contingent claim plus the risk free return minus (the flow of) consumption. We want to maximize utility over consumption and the investment strategy. Setting up the Bellman equation we get[12]

$$\delta J(W,T) = \max_{C,a,b}\left\{U(C,T) + \frac{E\{dJ(W,T)\}}{dt}\right\} \tag{2.7}$$

where the maximum is subject to $C \geq 0, a \geq 0$. Using (2.3) and (2.6) and the fact that in equilibrium invested capital must equal the share of wealth allocated to each sector, i.e. $K(t) = aW(t)$, (2.7) can be written as:

$$\begin{aligned} \delta J(W,T) \quad = \quad \max_{C,a,b} \quad & \{U(C,T) + W\mu(W)J_W + \theta J_T + \\ & \tfrac{1}{2}(W^2a'GG'a + 2W^2a'Ghb + W^2bh'hb)J_{WW} + \\ & (Wa'Gs + Wbh's)J_{WT} + \tfrac{1}{2}s'sJ_{TT}\} \end{aligned} \tag{2.8}$$

[10] The basis is defined as the set of production processes and contingent claims such that any other contingent claim can be written as a linear combination of the assets in the basis. See Merton (1977) for a complete description of this concept.

[11] The return in excess of the equilibrium interest rate.

[12] Throughout the paper we assume that the problem is well-defined, i.e. $J_W > 0$, $J_T < 0$, $J_{TT} < 0$, $J_{WW} < 0$ and $J_{WW}J_{TT} - (J_{WT})^2 > 0$.

where $W\mu(W) = E(dW)/dt$ and $\theta = \theta(T(t), \gamma' K(t))$. For the value function subscripts denote derivatives with respect to the states W and T. Differentiating (2.8) with respect to the controls C, a en b gives the first order conditions[13]

$$\begin{aligned}
\psi_C &= U_C - J_W \le 0 \\
C\psi_C &= 0 \\
\psi_a &= (\alpha - r1)WJ_W + \theta_2 \gamma WJ_T + (GG'a + Ghb)W^2 J_{WW} + GsWJ_{WT} \le 0 \\
a'\psi_a &= 0 \\
\psi_b &= (\beta - r)WJ_W + (h'G'a + h'hb)W^2 J_{WW} + h'sWJ_{WT} = 0
\end{aligned} \tag{2.9}$$

where ψ_C, ψ_a, ψ_b are defined implicitly and θ_2 denotes the partial derivative of θ with respect to its second argument, the flow of pollution. We follow CIR (1985) in defining an equilibrium as a set of stochastic processes $(r, \beta; a, C)$ satisfying (2.9) and the market clearing conditions $\sum a_i = 1$ and $b = 0$. We now turn to the determination of the equilibrium values of r and β. The equilibrium interest rate is given by[14]

$$r(W,T,t) = \hat{a}'(\alpha + \theta_2 \gamma \frac{J_T}{J_W}) - \left(\frac{-J_{WW}}{J_W}\right)\left(\frac{\text{var}(W)}{W}\right) - \left(\frac{-J_{WT}}{J_W}\right)\left(\frac{\text{cov}(W,T)}{W}\right). \tag{2.10}$$

while the equilibrium expected excess rate of return on any contingent claim F is given by

$$(\beta - r)F = \varphi_W F_W + \varphi_T F_T \tag{2.11}$$

Here, φ_W and φ_T are the risk premia associated with the state variables wealth and temperature. These are given by

$$\varphi_W = \left(\frac{-J_{WW}}{J_W}\right)\text{var}(W) + \left(\frac{-J_{WT}}{J_W}\right)\text{cov}(W,T) \tag{2.12}$$

$$\varphi_T = \left(\frac{-J_{WT}}{J_W}\right)\text{var}(T) + \left(\frac{-J_{WW}}{J_W}\right)\text{cov}(W,T) \tag{2.13}$$

Here, the risk premium φ_W can be interpreted as the excess expected return on an asset whose value is always equal to W. To see this construct an asset whose value is always equal to W, i.e. $F = W$. For such an asset, we have $F_W = 1$ and $F_T = 0$. Hence, from (2.11) we have $(\beta - r)F = \varphi_W$. An identical interpretation holds for φ_T. Combining equations (2.11) with (2.12) and (2.13) gives

$$(\beta - r) = -\text{cov}(F, J_W)/FJ_W \tag{2.14}$$

[13] We refer to CIR (1985) for the assumptions under which such an equilibrium exists.

[14] Proofs of equations (2.10) - (2.14) are given in Appendix B.

The expected excess return on a contingent claim is equal to the negative of the covariance of its rate of return with the rate of change in the marginal utility of wealth. Individuals demand a higher rate of return on assets that tend to pay of more when marginal utility is lower. Hence, such securities will have a higher risk premium. Our results are similar to the results obtained by CIR (1985) with the exception of (2.10). It is the expected social return on assets, $\hat{a}'(\alpha+\theta_2\gamma J_T/J_W)$, not the expected private return, $\hat{a}'\alpha$, that is relevant for the equilibrium interest rate in the social optimum. The damage caused by an increase in temperature as a result of an additional unit of pollution – measured in terms of the marginal utility of wealth – must be subtracted from the private return on markets in order to get the equilibrium interest rate in the social optimum. This is intuitive as the equilibrium interest rate measures the expected rate of change in the marginal utility of wealth which is equal to the expected social rate of return on wealth plus the covariance of the rate of return on wealth with the rate of change in the marginal utility of wealth.[15] Finally, in case $U(C,T)=U(C)$ it is easy to show that the expression for the risk free rate in (2.10) simplifies to the extended Ramsey equation. Under certainty, it simplifies further to the Ramsey equation.[16] This implies that the discussion on discounting in the literature (see for example Stern (2007), Nordhaus (2007) and Dasgupta (2008) is in fact a discussion on the appropriate level of the risk free rate.

Regarding the intuition behind the risk premia, first consider an asset whose value is perfectly correlated with wealth. Such an asset will command an excess expected return of φ_W. The payoff of this asset will be high (low), when wealth is high (low). Hence, it requires a positive risk premium, which is reflected by the first part of the RHS of (2.12). The sign of the effect of temperate on the risk premium for wealth depends on the signs of J_{WT} and $\mathrm{cov}(W,T)$. If both are positive, the second part of (2.12) will be negative. In that case, the payoff of the asset is high (low) when wealth and temperature are high (low) as the latter are positively correlated. Given that high (low) temperatures lead to a high (low) marginal value of wealth, the payoff of such an asset is high (low) when the marginal value of wealth is high (low). The intuition behind the risk premium for temperature which is displayed in (2.13) is similar.

Before relating our results to the economic literature on global warming, we provide further intuition for the risk premia. To that effect we rewrite equations (2.12) and (2.13) as follows (see Appendix E):

$$\varphi_W=\left(\frac{-U_{CC}}{U_C}\right)\mathrm{cov}(\hat{C},W)+\left(\frac{-U_{CT}}{U_C}\right)\mathrm{cov}(W,T) \tag{2.15}$$

$$\varphi_T=\left(\frac{-U_{CC}}{U_C}\right)\mathrm{cov}(\hat{C},T)+\left(\frac{-U_{CT}}{U_C}\right)\mathrm{var}(T) \tag{2.16}$$

[15] For a proof, see appendix C.

[16] For a proof, see Appendix D.

We are now ready to decompose the risk premia into a risk premium belonging to the level of consumption ('the level effect') and a risk premium belonging to the change in marginal utility ('the utility effect'). Consider first an asset whose value is perfectly correlated with wealth. The risk premium corresponding to the level effect is equal to the product of coefficient of absolute risk aversion times the covariance of optimal consumption and wealth. This is a familiar result in the asset pricing literature, see f.e. Breeden (1979). Notice that although temperature does no longer appear in $\text{cov}(\hat{C}, W)$ it is actually subsumed in it (see Appendix E for details). The risk premium corresponding to the utility effect is equal to the product of the relative change in the marginal utility of consumption times the covariance between wealth and temperature. To give an example of the utility effect, suppose that consumption and temperature are substitutes in the sense of Edgeworth-Pareto, i.e. $U_{CT} < 0$ and that wealth and temperature move together, i.e. $\text{cov}(W, T) > 0$. First, high (low) wealth implies a low (high) marginal value of wealth. Second, we have from the assumption that wealth and temperature are moving together that high (low) wealth will mean high (low) temperature. As consumption and temperature are substitutes high (low) temperature will entail low (high) marginal utility of consumption. Summarizing, we have that high (low) wealth means low (high) marginal utility not only because wealth is high, but also because consumption is valued less. The utility effect reinforces the level effect. This is reversed when consumption and temperature are complements. This is intuitive as when goods are substitutes the value of goods moving together is less compared to the case where they are complements. It is also reversed when wealth and temperature are moving in opposite directions.

Next, we turn to risk premium for temperature. Consider an asset whose value is perfectly correlated with temperature. The risk premium corresponding to the level effect is equal to the product of coefficient of absolute risk aversion times the covariance of optimal consumption and temperature. Again notice that although wealth does no longer appear in $\text{cov}(\hat{C}, T)$ it actually is subsumed in it (see Appendix E for details). The risk premium corresponding to the utility effect is equal to the product of the relative change in the marginal utility of consumption times the variance of temperature. To give an example of the utility effect: if consumption and temperature are substitutes in the sense of Edgeworth-Pareto, i.e. $U_{CT} < 0$, high (low) temperature will mean that the marginal utility of consumption is low (high). From the consumer's point of view, valuing (the marginal unit of) existing consumption less is equivalent to having more consumption (which reduces the value of the marginal unit of consumption). Hence, whenever consumption and temperature are substitutes in consumption it is *as if* they tend to move together. This leads to a positive risk premium as the first part on the RHS of (2.18) shows. This conclusion is reversed when consumption and temperature are complements.

3 Discounting climate investments

The cost-benefit ratio of long-term projects like the prevention of climate change depends crucially on the choice of the discount rate. The Stern Review uses a discount rate of 1.4% per annum and concludes that "the benefits of strong, early action considerably outweigh the costs."[17] In his most recent study, Nordhaus (2008) uses a discount rate of 5.5% for the first half century and 4% for the first century.[18] His conclusion is that "efficient emissions reductions […] involve modest rates of emissions reductions in the near term, followed by sharp reductions in the medium and long terms." That the choice of discount rate is for a large part 'responsible' for the widely diverging conclusions of Stern and Nordhaus has been made plausible by Nordhaus when he recalculates the optimal policy using Stern's discount rate. This Stern Review run of the DICE model produces results similar (in terms of the required carbon tax) to the Stern Review.[19] So, the choice of the discount rate seems to be the crucial variable in judging whether or not early action is the efficient strategy in the face of climate change.

The above argument hinges on the assumption that all investments in the economy – climate and non-climate investments[20] – (should) earn the same rate of return.[21] A lower required return on climate investments compared to non-climate investments would raise the level of climate investments without inducing inefficiencies. This possibility has been brought forward by Weitzman (2007) who has argued that the beta of climate investments might indeed be lower than the beta of non-climate investments. In order to frame this issue in terms of our model, we distinguish between non-climate investments, investments in adaptation and investments in mitigation and describe the benefits of these investments as contingent claims. These are denoted by the superscripts n, a en m respectively. Thus, $F^a(W,T)$ denotes the value of the benefits of an investment in adaptation. The specification of the claim includes a full description all the benefits (payoffs) that may be received from that claim. These benefits may depend on the state variables aggregate wealth and temperature. In a similar way the contingent claims, $F^m(W,T)$ and $F^n(W,T)$, denote the value of the benefits of an investment in mitigation and the value of a non-climate investment respectively.[22] Using (2.11), we have that the difference in the equilibrium expected rate of return between a climate investment, either adaptation or mitigation, and non-climate investments is given by

[17] Stern Review, Executive summary, page vi, http://www.hm-treasury.gov.uk/d/Summary_of_Conclusions.pdf.

[18] "With this pair of assumptions, the real return on capital around 5 ½ percent per year for the first half century of the projections, and this is our estimate of the return on capital." (p. 61) and "The estimated discount rate in the model averages 4 percent per year over the next century" (p. 10)

[19] Similar conclusions have been drawn by Hope (2006a) and Mityakov (2007).

[20] We use the term non-climate investments to denote all investments except climate investments.

[21] Nordhaus is aware of this argument: "the discussion here assumes that climatic investments share the risk properties of other capital investments." (Nordhaus 2008, p. 217, footnote 18).

[22] Here, we have used the fact that the expected rate of return of contingent claims not in the basis are uniquely determined by the equilibrium expected rate of return of those in the basis. CIR (1985, p. 374) show that equation (2.11) holds for all claims, basis and non-basis.

$$\beta^i - \beta^n = \phi_W \left(\frac{F_W^i}{F^i} - \frac{F_W^n}{F^n}\right) + \phi_T \left(\frac{F_T^i}{F^i} - \frac{F_T^n}{F^n}\right) = \phi_T \left(\frac{F_T^i}{F^i} - \frac{F_T^n}{F^n}\right), \quad i = a, m. \tag{3.1}$$

where we have assumed that all investments have equal risk characteristics in terms of wealth.[23] Equation (3.1) shows that the difference in the equilibrium expected rate of return between climate and non-climate investments is given by $\phi_T(F_T^i / F^i - F_T^n / F^n)$, the product of the risk premium with respect to temperature and the difference of the percentage change in the value of the contingent claims with respect to temperature. For non-climate investments and high enough temperatures, this partial derivative will be negative: an increase in temperature will decrease the value of a non-climate investment, i.e. $F_T^n < 0$. Notice that this is simply a restatement of the fact that climate change will damage the economy, that is $J_T < 0$. We recognize that some sectors in some countries might actually benefit from a modest rise in temperature (see f.e. Tol (2008)), but focus on the more long-term and more interesting case in which higher temperatures result in damages. As the benefits of mitigation and adaptation are equal to the avoided damages, we have $F_T^a, F_T^m > 0$. A rise in temperature increases the value of a climate investment (see also Stern, 2007, Ch. 18). Hence, we have that climate investments can be seen as a hedge for non-climate investments with respect to changes in temperature. However, as climate investments may be risky with respect to wealth, they are not necessarily a hedge with respect to changes in wealth.

We are now finally in the position to write down the major contributions of the paper. The following proposition specifies sufficient and necessary conditions for climate investments to have a lower, equal or higher expected equilibrium rate of return than non-climate investments.

Proposition 1 Given equal risk characteristics in terms of wealth, the difference in the equilibrium expected rate of return between climate and non-climate investments given by (3.1) is determined by the sign of the risk premium with respect to temperature, ϕ_T.

The proposition follows immediately from (3.1) and the fact that $F^m, F^a, F^n > 0$ and $F_T^m, F_T^a > 0, F_T^n < 0$. Proposition 1 allows us to make two observations. First, climate and non-climate investments must in general be discounted at a different rate. Only in the special case where $\forall_{W,T} : \mathrm{cov}(W,T) J_{WW} = -\mathrm{var}(T) J_{WT}$, we have that $\phi_T = 0$. Second, in any model without environmental risk,[24] i.e. $\mathrm{var}(T) = 0$, climate and non-climate investment must be discounted at the same rate of return given that they have similar risk characteristics in terms of terms of

[23] Of course, the risk premia of two investments may differ because the investments have different risk characteristics in terms of wealth. As the focus of this paper is on different risk characteristics in terms of temperature, we will assume throughout this section that the risk characteristics in terms of wealth are equal.

[24] To the best of our knowledge almost all the major Integrated Assessment Models fall into this category with the exception of Stern (2006) and Nordhaus (2008, Ch. 7).

wealth. This follows from the fact that $\mathrm{var}(T) = 0$ implies $\mathrm{cov}(W, T) = 0$. Hence, by (2.13) we have $\phi_T = 0$.

4 The sign of the risk premium with respect to temperature

The sign of the risk premium with respect to temperature, ϕ_T, is determined by the sign of the partial derivative of (optimal) consumption with respect to temperature, the cross-derivative of the utility function and the covariance between wealth and temperature. In this section, we discuss possible restrictions on the sign of these determinants and interpret these in terms of literature on (the economics of) climate change.

We start with the case where both the cross-derivative of the utility function, U_{CT}, and the covariance between wealth and temperature, $\mathrm{cov}(W,T)$, are zero. This case is of considerable interest as it includes as a special case the type of climate change model used in the literature (Stern (2007), Nordhaus (2008)).[25] The risk premium with respect to temperature is given by $\phi_T = -U_{CC}/U_C\,\hat{C}_T\,\mathrm{var}(T)$ implying that its sign is fully determined by the sign of the partial derivative of optimal consumption with respect to temperature.[26] What do we know about the change in consumption as a result of a change in temperature when the level of wealth constant? Suppose for the moment that the change in temperature will result in a decrease in future wealth and will leave the variance of the stochastic processes of wealth and temperature unchanged. The decrease of future wealth implies a rise of the marginal value of future wealth. Hence, the consumer would like to increase future wealth at the expense of current wealth by reducing current consumption. Under the conditions specified, both the sign of the partial derivate of consumption with respect to temperature and the risk premium with respect to temperature will be negative. Climate investments are discounted at a rate smaller than non-climate investments. When the increase in temperature affects the (co)variance of the stochastic processes of either wealth or temperature or both, additional effects may come into play. For example, an exogenous increase in uncertainty increases the prudent consumers' willingness to save, which is reflected in higher risk free rate (Leland (1968)). The increased willingness to save decreases current consumption, i.e. the partial derivative of uncertainty on consumption is negative for a prudent consumer. In the context of a stochastic dynamic model of optimal consumption Gollier (2002a, 2002b, 2007) shows that the properties of the term structure of the risk free rate depend on both the properties of the utility function and the properties of the stochastic process of wealth. In the special case where the growth of the economy follows a stationary random process and the representative agent has a constant relative risk aversion, the wealth and uncertainty effect exactly compensate each other.[27]

[25] Both Nordhaus (2008) and Stern (2007) use $U(C,T)=U(C)$ and (implicitly) assume $\mathrm{cov}(W,T)=0$.

[26] There exists a sizeable literature on the Environmental Kuznetz Curve by now. Recent theoretical work includes papers by Andreoni and Levinson (2001) and Johansson and Kriström (2007). Note, however, that his literature is concerned with the sign of the total derivative of consumption with respect to the flow pollution instead of the partial derivative of consumption with respect to temperature (which is a stock variable).

[27] We are not aware of any studies that have empirically tried to estimate the partial effect of temperature on consumption. A number of studies have tried to estimate the partial effect of temperature on GDP though (see f,e. Horowitz (2006) and Dell et. al. (2008)), but we are interested the partial effect on temperature on consumption.

In case consumption and temperature are not independent goods in consumption, $U_{CT} \neq 0$, the risk premium with respect to temperature is given by $\phi_T = -(U_{CC}\hat{C}_T + U_{CT})/U_C \operatorname{var}(T)$. Inspection shows that the effect of the partial derivative is reinforced by cross derivative if their signs are opposite. For example, in case $\hat{C}_T < 0$ and $U_{CT} > 0$ the risk premium with respect to temperature is unambiguously negative. What do we know about the sign of this cross-derivative? Heal (2008) observes that this "is not an issue that has been discussed in the literature, as we almost always work with one-good models." Subsequently, he argues that for 'low temperatures', temperature and consumption are likely to be complements, i.e. $U_{CT} > 0$, whereas for 'high temperatures' they are likely to be substitutes, i.e. $U_{CT} < 0$.[28] The general idea here is that there is a minimum level of ecosystem services needed for survival, i.e. the provision of water, air and basic foodstuffs. For high temperatures, the ecosystem approaches its limits and there is no substitutability between the ecosystem and produced consumption goods. As temperature and the quality of the ecosystem are negatively related, we have that produced consumption and temperature must be substitutes. For low temperatures, we will still have substitutability between consumption and the ecosystem implying that consumption and temperature must be complements. In terms of the risk premium with respect to temperature, this means that as long as consumption and temperature are complements, the risk premium will decrease, thus making abatement and mitigation more attractive. This case is relevant as long as temperature remains 'low'. When the limits of the ecosystem are approached, the risk premium will increase thereby making abatement and mitigation less attractive. This case is relevant when temperature is 'high', i.e. when the effects of climate change are likely to be severe.

If the stochastic processes of wealth and temperature are dependent, the risk premium of temperature is given by $\phi_T = -(U_{CC}\hat{C}_T + U_{CT})/U_C \operatorname{var}(T) - U_{CC}/U_C\, \hat{C}_W \operatorname{cov}(W,T)$. From the first order condition for consumption, we can derive that $\hat{C}_W > 0$. Hence, the sign of the additional term introduced by the dependency of the stochastic processes of wealth and temperature will be equal to the sign of the covariance between wealth and temperature. This covariance captures the instantaneous movement between these state variables. In order to understand the type of effects captured by this covariance rewrite (2.3) and (2.6) as follows:

$$\begin{aligned} W(t+\Delta t) &= W(t) + \left(a'\alpha W(t) - C\right)\Delta t + W(t)a'G\Delta w \\ T(t+\Delta t) &= T(t) + \theta(T(t), \gamma' a W(t))\Delta t + s'\Delta w \end{aligned} \tag{4.1}$$

For small Δt we have that $W(t+\Delta t)$ is equal to $W(t)$ plus the expected change is the period $(t, t+\Delta t)$ plus the unexpected change at time $t+\Delta t$. Notice that during the time period

[28] Heal develops his argument in terms of consumption and environmental quality. As environmental quality and temperature are negatively related, his argument is easily restated in terms of consumption and temperature. Hence, when consumption and environmental quality are substitutes, consumption and temperature are complements are vice versa.

$(t, t+\Delta t)$ both $W(t)$ and $T(t)$ are kept constant. Hence, any effect from temperature on wealth (and vice versa) may not be caused by a change in temperature (wealth). For in that case, it would be captured by their respective drifts.

Under what circumstances may we expect the covariance between wealth and temperature to be different from zero? First of all, emissions at time t may be related to the actual instead the expected rate of return. In that case unexpected changes in wealth will be positively correlated with unexpected changes in temperature as higher (lower) wealth leads to higher (lower) emissions. As a result the covariance between wealth and temperature would be positive. Second, unexpected changes in temperature at time t may lead to (unexpected) changes in wealth in time t. As these changes must be contemporaneous, the effect on wealth may not operate through a change in temperature. This excludes most of the so-called feedback effects mentioned in the literature on climate change as these either increase (positive feedbacks) or decrease (negative feedbacks) temperature. For example, the unexpected release of methane from sinks on land or the deep ocean is a positive feedback. Such an unexpected release would raise global temperatures and hence would decrease future wealth, but would not have a contemporaneous effect on wealth (methane in itself does not damage wealth). A possible exception is an unexpected change in water vapour. Unexpected increases in water vapour may be contemporaneously correlated with more intense precipitation, more severe storms and more intense flooding (Meehl and Stocker, (2007)). This would mean that the covariance between wealth and temperature is negative. We are unaware of any studies that have tried to estimate this instantaneous covariance. Hence, not only the sign but also the magnitude of the covariance between wealth and temperature is undetermined. Finally, notice that a non-zero covariance may be the result of an underlying third factor influencing both wealth and temperature simultaneously. Although we cannot rule out this possibility, we are unaware that such a third factor actually exists.

5 Conclusion

A general insight from the literature on cost-benefit analysis is that investments should be discounted at a rate that reflects their non-diversifiable risk in aggregate wealth. In this paper, we have generalized this insight to include non-diversifiable risk with respect to average global temperature. Using this result, we have shown that climate and non-climate investments will in general be discounted at a different rate. This allows us to substantiate Weitzman´s argument that climate investments may indeed be discounted at a lower rate compared to non-climate investments. Our analysis shows that this is more likely when (i) the partial derivative of optimal consumption with respect to temperature is smaller than zero; (ii) consumption and temperature are complements in consumption; and (iii) the covariance of wealth and temperature is negative. We have discussed when these conditions are likely to be met, but noted that there has been little discussion on these topics in the literature. Weitzman (2007) further suggested that spending more money to combat climate change may be more about insurance than about consumption smoothing. We note that – given a suitably chosen process for average global temperature – our model encompasses the possibility that climate change might be predominantly about small risk and large losses.

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Appendix A

Recognizing that the value of a contingent claim is a function of the state variables W and T , we get, applying Ito's Lemma:

$$dF(W,T) = \left[F_t dt + F_W dW + F_T dT\right] + \tfrac{1}{2}\left[F_{WW}(dW)^2 + 2F_{WT}(dW)(dT) + F_{TT}(dT)^2\right]$$

Using equations (2.3) and (2.6) to substitute out dW and dT we get after some rearrangements:

$$dF = (F\beta - \zeta)dt + Fh'dw(t)$$

where

$$\begin{aligned} F\beta - \zeta \quad = \quad & F_t + F_W(a'\alpha W + b\beta W - C) + F_T\theta + \\ & \tfrac{1}{2}(F_{WW}W^2(\hat{a}'GG'\hat{a} + 2a'Ghb + b^2h'h) + 2F_{WT}W(a'G + bh')s + F_{TT}s's) \end{aligned}$$

and

$$Fh' = F_W Wa'G + F_W bh' + F_T s'$$

Appendix B

For convenience, we repeat equation (2.9) containing the first order conditions:

$$
\begin{aligned}
\psi_C &= U_C - J_W \le 0 \\
C\psi_C &= 0 \\
\psi_a &= (\alpha - r1)WJ_W + \theta_2\gamma WJ_T + (GG'a + Ghb)W^2 J_{WW} + GsWJ_{WT} \le 0 \\
a'\psi_a &= 0 \\
\psi_b &= (\beta - r)WJ_W + (h'G'a + h'hb)W^2 J_{WW} + h'sWJ_{WT} = 0
\end{aligned} \tag{B.1}
$$

We first turn to the proof for the expression of the equilibrium interest rate given in (2.10). From (B.1) we see that the equilibrium solution for a, r and β in terms of the (derivates of the) value function J is partially separable. When $b = 0$ the third and the fourth first order condition determine a and r. Given a and r, the fifth first order condition will determine β. Denote the equilibrium value of the control variables in (B.1) by $(\hat{C}, \hat{a}, \hat{b})$. As in CIR (1985) the optimal value of the equilibrium interest rate can be determined by examining two related planning problems. The first planning problem has the same physical production opportunities and interaction between the economy and the environment, but has no borrowing, lending and contingent claims. The second planning problem is identical to the first planning problem with borrowing and lending allowed (but contingent claims not).

Let $\tilde{a}$ and $\tilde{C}$ denotes the optimal physical investment and consumption strategy for the first planning problem and $\tilde{J}$ the corresponding indirect utility (or value) function. The portfolio allocation of the first planning problem can now be written as a quadratic programming problem:

$$
\begin{aligned}
&\max_a \; a'\phi + a'Da \\
&\text{subject to} \\
&a'1 = 1 \\
&a \ge 0
\end{aligned} \tag{B.2}
$$

where $\phi = \alpha W\tilde{J}_W + \theta_2\gamma W\tilde{J}_T + GsW\tilde{J}_{WT}$ and $D = \frac{1}{2}GG'W^2\tilde{J}_{WW}$. Notice that solving (B.2) gives the third and fourth first order condition of (B.1) when there is no borrowing or lending, i.e. $r = 0$, and there are no contingent claims, i.e. $b = 0$. As $\tilde{a}$ is optimal, then by the Kuhn-Tucker theorem there exists a $\tilde{\lambda}$ such that

$$
\begin{aligned}
\phi - \tilde{\lambda} * 1 + 2D\tilde{a} &\le 0 \\
\tilde{a}'(\phi - \tilde{\lambda} * 1 + 2D\tilde{a}) &= 0
\end{aligned} \tag{B.3}
$$

Here $\tilde{\lambda}$ is the shadow price corresponding to the market clearing condition $a'1 = 1$. Now we turn to the second planning problem with borrowing and lending at $\tilde{r}$ and indirect utility at $\tilde{\tilde{J}}$. Comparing the first and the second planning problem shows that if $\tilde{\tilde{J}} = \tilde{J}$ and $\tilde{r} = \tilde{\lambda}/W\tilde{J}_W$, we have that $(\tilde{r}, \tilde{a}, \tilde{C})$ is the equilibrium for the second planning problem. The equilibrium interest rate in this economy is proportional to the shadow price, $\tilde{\lambda}$, in the first planning problem. Hence, in equilibrium we will have $\hat{C} = \tilde{C}$, $\hat{a} = \tilde{a}$, $r = \tilde{r}$ and $J = \tilde{J} = \tilde{\tilde{J}}$. This gives the following expression for the equilibrium interest rate:

$$
\begin{aligned}
r(W,T,t) &= \frac{\tilde{\lambda}}{WJ_W} = \frac{\hat{a}'(\alpha WJ_W + \theta_2 \gamma WJ_T + GsWJ_{WT} + 2\tfrac{1}{2}GG'\hat{a}W^2 J_{WW})}{WJ_W} \\
&= \hat{a}'(\alpha + \theta_2 \gamma \frac{J_T}{J_W}) - \left(\frac{-J_{WW}}{J_W}\right)\left(\frac{\operatorname{var}(W)}{W}\right) - \left(\frac{-J_{WT}}{J_W}\right)\left(\frac{\operatorname{cov}(W,T)}{W}\right)
\end{aligned}
\tag{B.4}
$$

The last step follows directly by substituting in the expressions for the variance of wealth and the covariance between wealth and temperature evaluated at the optimal values for a and b. These definitions follow directly from (2.3) and (2.6).

Subsequently, we turn to the proof for the expression of the expected rate of return on any contingent claim (2.11) and the associated risk premia (2.12). Rearranging the fifth first order condition of (B.1) and multiplying by F we get:

$$
\begin{aligned}
\beta F &= rF - \frac{J_{WT}}{J_W} Fh's - \frac{WJ_{WW}}{J_W} Fh'G'\hat{a} \\
&= rF - \frac{J_{WT}}{J_W}\left[F_W W\hat{a}'G + F_M s'\right]s - \frac{WJ_{WW}}{J_W}\left[F_W W\hat{a}'G + F_T s'\right]G'\hat{a} \\
&= rF + \varphi_W F_W + \varphi_T F_T
\end{aligned}
\tag{B.5}
$$

Here, we have used the expression for Fh' (evaluated at $b = 0$) that was derived in Appendix A for the second equality and have rearranged terms on F_W and F_T to derive the third equality. The risk premia φ_W and φ_T have been implicitly defined and are equal to

$$
\begin{aligned}
\varphi_W &= -\frac{J_{WT}}{J_W} W\hat{a}'Gs - \frac{WJ_{WW}}{J_W} W\hat{a}'GG'\hat{a} \\
&= \left(-\frac{J_{WW}}{J_W}\right)\operatorname{var}(W) + \left(-\frac{J_{WT}}{J_W}\right)\operatorname{cov}(W,T)
\end{aligned}
\tag{B.6}
$$

$$\begin{aligned}
\varphi_T &= -\frac{J_{WT}}{J_W}s's - \frac{WJ_{WW}}{J_W}s'G'\hat{a} \\
&= \left(-\frac{J_{WT}}{J_W}\right)\mathrm{var}(T) + \left(-\frac{J_{WW}}{J_W}\right)\mathrm{cov}(W,T)
\end{aligned} \tag{B.7}$$

Finally, the proof of equation (2.13) is given by

$$\begin{aligned}
\beta - r &= (\varphi_W F_W + \varphi_T F_T)/F \\
&= -\frac{J_{WW}}{J_W}W^2\hat{a}'GG\hat{a}\frac{F_W}{F} - \frac{J_{WT}}{J_W}W\hat{a}'Gs\frac{F_W}{F} - \frac{J_{WT}}{J_W}s's\frac{F_T}{F} - \frac{J_{WW}}{J_W}W\hat{a}'Gs\frac{F_T}{F} \\
&= -\mathrm{cov}(F, J_W)/FJ_W
\end{aligned}$$

The first equality follows directly from (B.5). The second from substituting in (B.6) and (B.7) as well as the expressions for $\mathrm{var}(W)$, $\mathrm{var}(T)$ and $\mathrm{cov}(W,T)$. Finally, the third equality follows from applying Ito's Lemma to write out $dJ_W(W,T)$ explicitly and using the result of Appendix A to determine $\mathrm{cov}(F, J_W)$. This completes the proof.

Appendix C

We want to show that the equilibrium interest rate is equal to minus the expected rate of change in the marginal utility of wealth which in turn is equal to the expected social rate of return on wealth plus the covariance of the rate of return on wealth with the rate of change in the marginal utility of wealth. First, we develop an expression for the expected rate of change in the marginal utility of wealth by applying Ito´s Lemma to dJ_W. This gives

$$\begin{aligned} dJ_W \;&=\; (-\delta J_W + (\hat{a}'\alpha W - \hat{C})J_{WW} + \theta J_{WT} + \tfrac{1}{2}\hat{a}'GG'\hat{a}W^2 J_{WWW} + \\ &\quad \hat{a}'GWsJ_{WWT} + \tfrac{1}{2}s'sJ_{WTT})dt + (\hat{a}'GWJ_{WW} + J_{WT}s')dw(t) \end{aligned} \quad . \tag{C.1}$$

Hence, the drift term in (C.1) is equal to the expected rate of change in the marginal utility of wealth. Second, we prove that the equilibrium interest rate equals minus the expected rate of change in the marginal utility of wealth. From equation (2.10) we have:

$$\begin{aligned} r(W,T,t) \;&=\; \hat{a}'(\alpha + \theta_2\gamma\frac{J_T}{J_W}) - \left(\frac{-J_{WW}}{J_W}\right)\left(\frac{\operatorname{var}(W)}{W}\right) - \left(\frac{-J_{WT}}{J_W}\right)\left(\frac{\operatorname{cov}(W,T)}{W}\right) \\ &=\; (\tfrac{1}{2}\operatorname{var}(W)J_{WWW} + \operatorname{cov}(W,T)J_{WWT} + (\hat{a}'\alpha W - \hat{C})J_{WW} + \qquad \text{(C.2)} \\ &\quad \theta J_{MT} + \tfrac{1}{2}s'sJ_{WTT} + \delta J_W)/J_W = -\frac{E\{dJ_W\}}{J_w dt} \end{aligned}$$

The second equality follows from differentiating the Bellman equation (2.8) with respect to W recognizing that both C and a are functions of W and using the first order conditions on C and a. The third equality follows from (C.1).

Finally, we show that the equilibrium interest rate is equal to the expected social rate of return plus the covariance of the rate of return on wealth with the rate of change in the marginal utility of wealth.

$$\begin{aligned} r(W,T,t) \;&=\; \hat{a}'(\alpha + \theta_2\gamma\frac{J_T}{J_W}) - \left(\frac{-J_{WW}}{J_W}\right)\left(\frac{\operatorname{var}(W)}{W}\right) - \left(\frac{-J_{WT}}{J_W}\right)\left(\frac{\operatorname{cov}(W,T)}{W}\right) \\ &=\; \hat{a}'(\alpha + \theta_2\gamma\frac{J_T}{J_W}) - \frac{1}{J_W}\left[J_{WW}\,\hat{a}'GW + J_{WT}\,s'\right]\left[\hat{a}'G\right]' \qquad \text{(C.3)} \\ &=\; \hat{a}'(\alpha + \theta_2\gamma\frac{J_T}{J_W}) + \frac{\operatorname{cov}(W,J_W)}{WJ_W} \end{aligned}$$

where the third equality follows from (C.1) and (2.6).

Appendix D

We want to show that the expression in (2.10) simplifies to the extended Ramsey equation if $U(C,T)=U(C)$. Starting from (C.2) we have:

$$\begin{aligned} r(W,T,t) &= \hat{a}'(\alpha+\theta_2\gamma\frac{J_T}{J_W})-\left(\frac{-J_{WW}}{J_W}\right)\left(\frac{\operatorname{var}(W)}{W}\right)-\left(\frac{-J_{WT}}{J_W}\right)\left(\frac{\operatorname{cov}(W,T)}{W}\right) \\ &= (\tfrac{1}{2}\operatorname{var}(W)J_{WWW}+\operatorname{cov}(W,T)J_{WWT}+(\hat{a}'\alpha W-\hat{C})J_{WW}+ \\ &\quad \theta J_{WT}+\tfrac{1}{2}s's J_{WTT}+\delta J_W)/J_W \end{aligned} \tag{D.1}$$

Differentiation of $J_W=U_C$ with respect to W and T gives $J_{WW}=U_{CC}\hat{C}_W$ and $J_{WT}=U_{CC}\hat{C}_T$. Differentiating these expression again with respect to W and T gives $J_{WWW}=U_{CCC}\hat{C}_W^2+U_{CC}\hat{C}_{WW}$, $J_{WWT}=U_{CCC}\hat{C}_W\hat{C}_T+U_{CC}\hat{C}_{WT}$ and $J_{WTT}=U_{CCC}\hat{C}_T^2+U_{CC}\hat{C}_{TT}$. Substituting these expressions into (D.1) and rearranging terms gives:

$$\begin{aligned} r(W,T,t) &= \delta+\left(-\frac{U_{CC}}{U_C}\right)((\hat{a}'\alpha W-\hat{C})\hat{C}_W+\theta\hat{C}_T \\ &\quad +\tfrac{1}{2}\operatorname{var}(W)\hat{C}_{WW}+\operatorname{cov}(W,T)\hat{C}_{WT}+\tfrac{1}{2}\operatorname{var}(T)\hat{C}_{TT})+ \\ &\quad \left(-\frac{U_{CCC}}{U_{CC}}\right)(\tfrac{1}{2}\operatorname{var}(W)\hat{C}_W^2+\operatorname{cov}(W,T)\hat{C}_W\hat{C}_T+\tfrac{1}{2}\operatorname{var}(T)\hat{C}_T^2) \end{aligned} \tag{D.2}$$

Applying Ito's lemma to $d\hat{C}(W,T)$ gives:

$$\begin{aligned} d\hat{C}(W,T) &= \Big(\hat{C}_W(\hat{a}'\alpha W-\hat{C})+\hat{C}_W\theta+\tfrac{1}{2}\hat{C}_{WW}\operatorname{var}(W) \\ &\quad +\tfrac{1}{2}\hat{C}_{WT}\operatorname{cov}(W,T)+\tfrac{1}{2}\hat{C}_{TT}\operatorname{var}(T)\Big)dt \\ &\quad +\Big(\hat{C}_W\hat{a}'GW+\hat{C}_T s'\Big)dw(t) \end{aligned} \tag{D.3}$$

Using this to rewrite (D.3) gives the desired result:

$$r=\delta+\eta(\hat{C})E\{g(\hat{C})\}-\frac{1}{2}P(\hat{C})\eta(\hat{C})\operatorname{var}(g(\hat{C})), \tag{C.4}$$

where $\eta(\bullet)$ and $P(\bullet)$ are respectively the coefficient of relative risk aversion and relative prudence. Equation (C.4) is the extended Ramsey equation. In the absence of uncertainty $\operatorname{var}(g(\hat{C}))=0$ and (C.4) becomes the Ramsey equation.

Appendix E

Using $J_W = U_C$, we obtain by differentiation $J_{WW} = U_{CC}\hat{C}_W$ and $J_{WT} = U_{CC}\hat{C}_T + U_{CT}$.
Here, $\hat{C}_W$ and $\hat{C}_T$ denote the partial derivative of optimal consumption with respect to wealth and temperature respectively. Substituting these expressions into (2.12) and (2.13) we get:

$$\varphi_W = \left(\frac{-U_{CC}\left(\hat{C}_W \ \mathrm{var}(W) + \hat{C}_T \ \mathrm{cov}(W,T)\right)}{U_C} \right) + \left(\frac{-U_{CT} \ \mathrm{cov}(W,T)}{U_C} \right) \tag{E.1}$$

$$\varphi_T = \left(\frac{-U_{CC}\left(\hat{C}_W \ \mathrm{cov}(W,T) + \hat{C}_T \ \mathrm{var}(T)\right)}{U_C} \right) + \left(\frac{-U_{CT} \ \mathrm{var}(T)}{U_C} \right) \tag{E.2}$$

Second, we rewrite (2.15) and (2.16) by applying Ito's Lemma to $dC(W,T)$. From appendix D we have:

$$\begin{aligned} d\hat{C}(W,T) \quad = \quad & \Big(\hat{C}_W (\hat{a}'\alpha W - \hat{C}) + \hat{C}_W \theta + \tfrac{1}{2}\hat{C}_{WW} \ \mathrm{var}(W) \\ & + \tfrac{1}{2}\hat{C}_{WT} \ \mathrm{cov}(W,T) + \tfrac{1}{2}\hat{C}_{TT} \ \mathrm{var}(T)\Big) dt \\ & + \left(\hat{C}_W \hat{a}' G W + \hat{C}_T s'\right) dw(t) \end{aligned} \tag{E.3}$$

Together with (2.3) and (2.6) this gives:

$$\begin{aligned} \mathrm{cov}(\hat{C},W) &= \hat{C}_W \hat{a}' G G \hat{a} W^2 + \hat{C}_T \hat{a}' G s W = \hat{C}_W \ \mathrm{var}(W) + \hat{C}_T \ \mathrm{cov}(W,T) \\ \mathrm{cov}(\hat{C},T) &= C_W \hat{a}' G s W + C_T s' s = C_W \ \mathrm{cov}(W,T) + C_T \ \mathrm{var}(T) \end{aligned} \tag{E.4}$$

Substituting (E.4) into (E.1) and (E.2) we obtain (2.15) and (2.16).