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Abstract

We study how the possibility of renegotiation affects optimal bail-out policies for countries under asymmetric information on a country's cost of reforms. To that end, we solve the Bellman equation describing the optimal bail-out mechanism in a multiple-period principal-agent model with adverse selection and renegotiation. In each period, the agent can incrementally raise its level of reforms. The principal values these reforms and negotiates with the agent over reforms and payments. We show that the first-best efficient outcome is reached after two periods when spill-over benefits are quadratic. The principal can use market discipline to improve the outcome. Market discipline can lower the rents the principal has to pay and speed up the renegotiation process.

Keywords: renegotiation, bail-out, financial crisis

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1. Introduction

The sovereign debt crisis that has swept the euro area following the worldwide financial crisis has featured a bail-out of countries such as Greece, Portugal, and Ireland. In essence, in such

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a bail-out countries receive money, for example in the form of loans against favourable interest rates, in exchange for policy reforms. A common feature of bail-outs is that these are often renegotiated after some time.

Greece serves as an example of how bail-out may be followed by renewed bail-outs and renegotiation of the contract terms. In May 2010 Greece signed a 110 billion euro bail-out package of which 30 billion was financed by the International Monetary Fund and 80 billion by bilateral loans with members of the Eurozone. The bail-out was conditional on the implementation of a fiscal adjustment package of 11 percentage points of GDP over three years, and structural reforms that meant to restore competitiveness and growth.

In July 2011 in the slipstream of a second rescue package, financed by the European Financial Stability Fund (EFSF) and the private sector, a partial renegotiation of the first package took place, in the form of an extension of the maturity of existing bilateral EU loans. In November 2012 the terms of the adjustment program were more thoroughly renegotiated. The fiscal adjustment path was lengthened by two years to 2016. Privatization targets were lowered. Eurozone countries agreed to reduce the interest rate on the Greek Loan Facility, to lengthen maturities on all of their lending, and to transfer profits earned by the ECB on Greek debt back to Greece. In addition, Greece's European partners agreed to provide roughly 26 billion euro in additional financing for the period 2012 – 2016.

At first sight, such renegotiation might seem to indicate that initial contracts were not well structured. In this paper we want to study bail-out contracts from a principal-agent perspective. We show that when the agent (the country that is bailed out) is better informed about the costs of reforms than the principal (i.e., those who provide the bail-out), and when the principal cannot commit never to renegotiate, a contract that features some renegotiation may be the optimal result from the point of view of the principal.

We study renegotiation in a dynamic procurement model. An agent, which could be either a bank or a country, can raise its economic health by making investments with private costs. These costs arise for example because reforms are socially costly in the case of countries, or because raising new equity is costly to the current owners, when considering bail-outs of banks.

The agent can have either high or low cost. The principal, which could be another country, a group of other countries, or a regulator, benefits from such an investment, for example because there are spillovers from the agent's health level. Spillovers between countries can arise for example due to linkages between the banking sectors of different countries or contagion effects through financial markets. In a full-information setting, the principal will induce either type of agent to choose an optimal health level, which will depend on the costs. When there is asymmetric information on costs, this full information benchmark is not incentive compatible, as the low-cost agent will benefit from pretending to be high-cost. As is standard, the optimal strategy for the principal in a one-period static model will involve paying rents to low-cost agents, in order to induce them to reveal their lower costs. To reduce these information rents, the principal will simultaneously distort the investments by high-cost types downward.

In a multi-period game, however, it is well-known that the latter one-period outcome is not renegotiation-proof. A principal observing agents' contract choices will find it ex-post profitable to contract with the high-cost agent to increase its health to the efficient level. A low-cost agent, however, will anticipate such renegotiation and the additional rents it brings in the subsequent period.

We study the optimal renegotiation-proof contract for the principal in an infinite period model, where the agent can invest to increase its health level in each period. We do so by formulating the principal's problem in terms of a Bellman equation. We demonstrate that in general, when the posterior probability of the agent being low-cost is sufficiently low, all remaining low types separate and produce optimal quantities. In the special case when the spill-over benefits are quadratic, we show that such separation occurs immediately. For more general spill-over benefits, there can be initial periods in which some low-cost types pool with high-cost types, until the posterior probability has dropped sufficiently for complete separation to take place.

The anticipation of a bail-out in adverse conditions may invite moral hazard in good times. Indeed, the Bank for International Settlements (BIS) recently argued that very low interest rates may lead countries to postpone structural reforms (BIS, 2013), while the president of the

Bundesbank Jens Weidman argued in his speech "The euro - political project and prosperity promise" that implicit guarantees for sovereigns caused shareholders, investors, governments and voters to worry less about risk. The fear is that unlimited purchases of government debt by the ECB could eliminate capital market discipline on borrowing countries. Some therefore argue that market discipline should be an important element in disciplining countries to conduct prudent policies and implement reform.

When a crisis arises, however, it is less clear that such a policy is optimal. Why should countries face the threat of a debt crisis if this doesn't affect past behaviour? We study one potential alternative role for market discipline that is related to adverse selection and the possibility to renegotiate. We model market discipline as an additional (punishment) cost to the agent contingent on his producing low levels of health, at some cost to the principal as well. The prospect of such market discipline relaxes the incentive compatibility constraint and can reduce rents left to the low-cost agent. In non-quadratic models where separation does not take place immediately, market discipline also speeds up revelation of efficient types.

Summarizing, we have three results. First, renegotiation can be the second-best outcome in the presence of information asymmetry if support providing countries are unable to commit ex ante to a given bail-out policy. Second, we find that also in a setting with continuous cumulative investments, renegotiation need not continue for long periods. In our model optimal outcomes are reached very fast in the case of quadratic concave spill-over benefits. Third, we find that market discipline may help even if there are no ex ante benefits in terms of reducing moral hazard, since it reduces expected rents and can reduce the time until efficiency is reached. However, whether market discipline is optimal or not depends on the cost it imposes on the principal.

Our paper fits in a strand of literature on long-term contracts with renegotiation, as well as on the Coase conjecture. In methodology our paper is related to Laffont and Tirole (1990), who analyse a twice repeated procurement situation with adverse selection over the agent's costs of production. While, in their context, the optimal contract for the principal would be to commit to repeating twice the one-period (static) optimal contract, this contract is not renegotiation-proof. After period one, when the agent's type has been revealed, principal and agent can jointly improve on the static outcome, undoing the distortions that were introduced in the optimal first-period contract. Since both parties to the contract can be made better off, they can agree to renegotiate the original contract. The anticipation of such renegotiation however alters the first-period game. Laffont and Tirole (1990) show that the optimal renegotiation-proof contract involves mixing behaviour of low-cost agents: with positive probability, low-cost types pool with high-cost types in period one, so that their true type is not yet fully revealed after period one.

Also in our framework we have a procurement situation, where the principal "buys reforms" from an agent whose cost of reforms is unknown to the buyer. Our analysis departs from Laffont and Tirole (1990) in two dimensions. First, we consider a model with an unlimited number of periods available to both parties, and second, we consider a gradual build-up of the stock of 'goods' (the reforms) that are procured, rather than a repetition of identical procurements without memory of past quantities bought.

With respect to the infinite repetition of buyer-seller interactions, our paper relates to Hart and Tirole (1988), who, building on literature on the Coase conjecture, (see e.g. Fudenberg, Levine and Tirole, 1985; Gul, Sonnenschein and Wilson, 1986), consider a monopolist repeatedly setting price in order to ultimately sell a unit good to a buyer whose willingness to pay is either high or low. Here high value buyers, with some probability, pool with low value buyers and price keeps decreasing over time until the good is sold. Our model generalizes this by assuming a divisible good, that can be bought and sold in small increments.

Closest to our work is Maestri (2013). He analyses a similar situation to ours where a monopolist repeatedly sells goods to consumers having different tastes for quality, and the monopolist can adapt quality over time in a continuous manner. In the renegotiation-proof equilibrium, the monopolist offers two qualities, one which is efficient for the high valuation type, and one which is below that for the low valuation type. High types mix between the two, while low types always choose the lower quality product. Maestri (2013) shows that over time, low type quality increases towards its efficient level.

In contrast with Maestri (2013), we solve the Bellman equation of a game that is similar to his explicitly. In particular, we show that the game unravels immediately when the principal's valuations are quadratic. In addition, in the context of bail-outs that we study, we address the question how market discipline affects that equilibrium.

We introduce the model and derive the Bellman equation in section 2. In section 3, we analyse that model in general and solve it analytically for the case in which the spill-over benefits for the principal are quadratic. We also investigate how that solution is affected by the introduction of market discipline. Section 4 concludes.

2. The model

We study an infinite-period contracting model, where in each period the agent can permanently raise its economic health Q by making costly investments, e.g. by raising equity (banks) or by executing socially costly policy reforms. The principal – which could be the regulator in the case of banks, or a partner country in an international context – benefits from such investments, in the form of spill-over effects. Without contracting among the two, total health Q would be too low from their joint perspective.

The agent's net marginal costs of adding reforms Q to increase its health level beyond the privately optimal level are assumed constant, and can either be high (c_H) or low (c_L) with probabilities $\phi_H, \phi_L = 1 - \phi_H$. The agent's cost is not observable for the principal. The principal can observe the increases in Q (the amount of reforms) in each period.

The principal values the agent's reforms Q as $V(Q)$ in each period. He benefits from an increase in the agent's health level resulting from reforms, which means $V'(Q) > 0$. Marginal benefits from an increased health level decrease with health, implying concave per-period benefits, $V''(Q) < 0$.

The principal can offer a menu of compensations t_n^i and corresponding increases in the agent's health q_n^i to agent $i = L, H$ for each period $n \in \{0, 1, ..., \infty\}$. The per-period discount factor is denoted by δ. Starting from no additional reforms, $Q = 0$, we want to find the optimal renegotiation-proof menu.

Let us first consider the full information benchmark where the principal observes the agent's type. The optimal contract would be to bring the agent to the joint-surplus-maximizing level of Q immediately in period $n = 0$. Since the present value of total benefits for the principal equals $\frac{V(Q)}{1-\delta}$, the optimum is to set levels of investment $Q^*_{H,L}$ satisfying

$$V'(Q^*_{H,L}) = (1-\delta)c_{H,L}$$

and to make a time zero transfer to the agent equal to $t_0^{H,L} = c_{H,L}Q^*_{H,L}$ to remunerate its incurred costs.

When there is asymmetric information on costs, the full information benchmark is not incentive compatible, as the low-cost agent will benefit from pretending to be high-cost. As is standard, the optimal strategy for the principal in a one-period model will involve paying rents to low-cost agents, in order to induce them to reveal their lower costs. In order to reduce these information rents, the principal will simultaneously distort the investments by high-cost types downward, $Q_H < Q^*_H$,

In a multi-period game, however, the resulting outcome is not renegotiation-proof; in the next period, a principal observing the inefficient Q_H will find it ex-post profitable to contract with the agent to increase its Q to the efficient level Q^*_H. Anticipating such renegotiation, the low type will need to be offered higher remuneration for revealing immediately (Laffont and Tirole, 1990).

We therefore look for the optimal renegotiation-proof contract of the dynamic contracting model. As in Laffont and Tirole (1990), we will investigate solutions involving partial pooling of the efficient low types with the high types: in each period n a number $\psi_n \geq 0$ of the low-type agents reveal their type, so that at the start of period n, a cumulative fraction $\Sigma_{i\leq(n-1)}\psi_i \leq \phi_L$ will have separated to efficiency. These agents will from that moment onward keep their socially optimal amount of health Q^*_L.

Equivalently, at any time n, a total of $\Psi_n = \phi_L - \Sigma_{i \leq (n-1)} \psi_i$ low types have not yet revealed themselves. Initially, when no separation has yet occurred, $\Psi = \phi_L$. If ultimately all low types have revealed themselves, $\Psi = 0$. We denote by $Q_n \leq Q_H^*$ the quantity attained up to that point in time by the pool of high types and those low types that have not yet revealed their type so far.

At each point in time n, the principal will want to optimize his continuation welfare given pooling branch quantity and number of pooling low types at that point, $W(Q_n, \Psi_n)$. Doing so means optimizing over the (positive) change q_n in Q_n during that period (bringing total quantity in the pooling branch to $Q_{n+1} = Q_n + q_n$) and the number of pooling low types that separate to efficiency in that period, ψ_n (leading to a drop in remaining pooling types, $\Psi_{n+1} = \Psi_n - \psi_n$). The flows of utility in each period n are depicted in the graph below.

$$\begin{tikzcd}
\cdots & Q_L^*, \phi_L - \Psi_n \arrow[r] & Q_L^*, \phi_L - \Psi_{n+1} \arrow[r] & \cdots \\
\cdots & Q_n, \Psi_n + \phi_H \arrow[r] \arrow[ur, "\psi_n"] & Q_{n+1}, \Psi_{n+1} + \phi_H \arrow[r] \arrow[ur, "\psi_{n+1}"] & \cdots
\end{tikzcd}$$

The top left-hand corner denotes the $\phi_L - \Psi_n$ low types that are already at their optimum level Q_L^* at the beginning of period n. These will remain there and produce a flow of utility for the principal equal to $(\phi_L - \Psi_n)V^*$ in this period.

Second, on the diagonal arrow, we have the ψ_n low-types revealing in this period. They move from the pooling to the separating branch, producing $q_n^L = Q_L^* - Q_n$ to attain the efficient low-type quantity, and receiving a remuneration for that equal to t_n^L.

Finally, the lower arrow represents those $\Psi_n - \psi_n$ low types that remain in the pooling branch, as well as the ϕ_H high types. These produce an additional q_n^H in this period, bringing their aggregate health to $Q_{n+1} = Q_n + q_n^H$, and receive t_n^H in return.

At the end of this period, the principal's continuation utility will be equal to W evaluated at the new values of the state variables, $Q_{n+1} = Q_n + q_n$, $\Psi_{n+1} = \Psi_n - \psi_n$. The principal will optimize his net utility over changes q_n, ψ_n, as well as the required payments $t_n^{H,L}$. This is captured in the Bellman equation for W:

$$
\begin{aligned}
W(Q,\Psi) = \max_{q,\psi,t^H,t^L} & (\phi_L - \Psi)V^* + \psi\left(V^* - t^L\right) + \\
& (\Psi - \psi + \phi_H)\left(V(Q+q) - t^H\right) + \delta W(Q+q, \Psi - \psi),
\end{aligned}
$$

subject to the agent's participation and incentive constraints that we turn to presently.

The principal has to design the contract such that ex ante, both types will want to accept. As is usual, only the inefficient type's participation constraint is binding. Secondly, in each period, revealing low types have to be compensated for giving up the information rents associated with remaining in the pooling branch. We can then write the binding constraints in equilibrium as

$$
\begin{aligned}
PC_H: &\quad t_n^H = c_H q_n^H \\
IC_L: &\quad t_n^L - c_L(Q_L^* - Q) = \sum_{k=0}^{\infty} \delta^k (t_{n+k}^H - c_L q_{n+k}^H)
\end{aligned}
$$

The first equality states that high types get exactly their costs remunerated in each period.[1] The second states that in each period, the low type's rents from separating equal his anticipated rents from pooling. If this is satisfied in each period, low types will be indifferent about when they separate, a necessary condition for a mixed strategy to be optimal.

From $Q_L^* > Q_H^* \geq \sum_{n=0}^{n=\infty} q_n^H$, it is obvious that then the high types will never want to mimic the efficient low types.

We can now rewrite the Bellman equation by inserting the expressions for t_n^H and t_n^L. It is convenient to write the latter in terms of the continuation utility $\Phi(Q,\Psi)$ of the low types, i.e.

[1] In principle, one could allow for higher payments in some periods to be compensated by lower payments in others to get the same net present value. Requiring a binding constraint in each period separately is the obvious choice, but is without loss of generality.

the rents that they need to earn to be indifferent between separating and pooling:

$$t^L - c_L q^L \equiv \Phi(Q, \Psi) = \Delta c q^* + \delta \Phi(Q + q^*, \Psi - \psi^*). \tag{1}$$

Here the quantities q^* and ψ^* are those that are chosen by the principal to optimize his welfare, given Q and Ψ. The principal chooses these quantities by solving the Bellman equation

$$\begin{aligned} W(Q, \Psi) = &\max_{q,\psi} (\phi_L - \Psi) V^* \\ &+ \psi \left(V^* - c_L(Q_L^* - Q) - \Delta c q - \delta \Phi(Q + q, \Psi - \psi) \right) \\ &+ (\Psi - \psi + \phi_H) \left(V(Q + q) - c_H q \right) \\ &+ \delta W(Q + q, \Psi - \psi) \end{aligned} \tag{2}$$

We are looking for a solution to the joint equations (1,2) that in addition satisfies two boundary conditions. When all low types have revealed, i.e., $\Psi = 0$, the principal will offer a contract to the remaining high types, $(t, q) = (c_H(Q_H^* - Q), Q_H^* - Q)$, eliminating any remaining distortions.

And secondly, when the agents in the lower pooling branch produce the efficient amount Q_H^*, the principal will offer the remaining Ψ low types no rents and a contract $(t, q) = (c_L(Q_L^* - Q_H^*), Q_L^* - Q_H^*)$, bringing them to efficiency. Together this results in the following boundary conditions.

$$\begin{aligned} W(Q, 0) &= \phi_L \frac{V^*}{1-\delta} + \phi_H \frac{V_H^*}{1-\delta} - \phi_H c_H (Q_H^* - Q), \\ W(Q_H^*, \Psi) &= \phi_L \frac{V^*}{1-\delta} + \phi_H \frac{V_H^*}{1-\delta} - \Psi c_L (Q_L^* - Q_H^*), \\ \Phi(Q_H^*, \Psi) &= 0 \\ \Phi(Q, 0) &= \Delta c (Q_H^* - Q). \end{aligned}$$

3. Analytical Solution

We now explore the analytical solution to the set of Bellman equations (1,2). We then specialize to quadratic spill-over benefits $V(Q)$, and see that efficiency is reached fast.

To analyse the coupled Bellman equations for future rents Φ and value W, we first note a useful simplifying characteristic of the associated first-order conditions.

Lemma 1 *In an interior equilibrium Q_{i+1} and Ψ_{i+1} do not depend on Q_i.*

The lemma follows because the first-order conditions depend only on $Q_i + q_i = Q_{i+1}$.

This means that the continuation from period i onward of an equilibrium path will only depend on the share of efficient agents Ψ_i that are still pooling in that period, and not on the quantity Q_i that has been reached at that point; hence

$$\Psi_{i+1} = \Psi_i - \psi_i^* = \Psi(\Psi_i),$$
$$Q_{i+1} = Q_i + q_i^* = Q(\Psi_i).$$

which defines the optimal strategies $\Psi(\Psi)$ and $Q(\Psi)$.

With the help of this observation, we now proceed with the analysis of the solution to the Bellman equation. Our strategy is as follows. We will work with the working hypothesis that, if Ψ is close enough to zero (i.e. nearly all efficient types have revealed themselves), the policy q, ψ that optimizes $W(Q, \Psi)$ will be a corner solution $\psi = \Psi$, so that from that value of Ψ, in the next step all remaining pooling efficient types will separate. We will show that if some putative solution $W^{(0)}$ and $\Phi^{(0)}$ satisfy this working hypothesis, their mapping under one iteration of the Bellman equation do so, too. Hence, also the solution to the Bellman equation that is the fixed point under this mapping satisfies the working hypothesis (see e.g. Lucas and Stokey, 1989, p. 81,82). In the process, we will establish what the maximum value of Ψ is for the complete revelation solution to hold.

So, as a working hypothesis, suppose

$$\Psi_{i+1}(\Psi_i) = 0 \text{ for all } i \geq 0 \text{ and } \Psi_i \leq \bar{\Psi}, \tag{WH}$$

where $\bar{\Psi}$ is to be determined, and where we used lemma 1 to argue that Ψ_{i+1} is in fact only a function of Ψ_i.

The implication of this working hypothesis will be that, in the domain $\Psi < \bar{\Psi}$, we have complete unravelling in the next period. This means that we can find the explicit expression for functions $\Phi^{(0)}(Q, \Psi)$ and $W^{(0)}(Q, \Psi)$ that satisfy (WH) in this domain. By lemma 1, we know that optimization over q results in reaching $Q(\Psi)$, so that the optimal capacity increment q at that point equals $q = Q(\Psi) - Q$. Secondly, by the assumption we are in the regime where the optimal $\psi = \Psi$, i.e. full revelation. Finally, we have the boundary conditions

$$\begin{aligned} \Phi^{(0)}(Q(\Psi), 0) &= \Delta c(Q_H^* - Q(\Psi)) \\ W^{(0)}(Q(\Psi), 0) &= \phi_L \frac{V^*}{1-\delta} + \phi_H \frac{V_H^*}{1-\delta} - \phi_H c_H (Q_H^* - Q(\Psi)) \end{aligned}$$

These conditions simply mean that after the boundary $\psi = \Psi$ has been reached, in the final period the high-types will be offered a contract that requires them to produce their efficient output Q_H^* (and forever after). Of course, the low types will have to be paid a rent in order to prevent them from switching to this contract in the previous period.

We can then compute the value of $\Phi^{(0)}(Q, \Psi)$ and $W^{(0)}(Q, \Psi)$ by just substituting all these expressions. We summarize the result as follows.

Lemma 2 *For any trial solutions $W^{(0)}, \Phi^{(0)}$ to the Bellman equation satisfying the working hypothesis (WH), the explicit expressions for the values $W^{(0)}$ and $\Phi^{(0)}$ in the domain $\Psi < \bar{\Psi}$*

are

$$
\begin{aligned}
W^{(0)}(Q,\Psi) = {}& \phi_L \frac{V^*}{1-\delta} - \Psi c_L(Q^* - Q_H^*) - \Psi c_L(Q_H^* - Q(\Psi))(1-\delta) \\
& + \phi_H \left(V(Q(\Psi)) + \frac{\delta V_H^*}{1-\delta} \right) \\
& - (\Psi + \phi_H) c_H \left[(Q(\Psi) - Q) + \delta \left(Q_H^* - Q(\Psi) \right) \right]
\end{aligned} \tag{3}
$$

and

$$
\Phi^{(0)}(Q,\Psi) = \Delta c(Q(\Psi) - Q) + \delta \Delta c(Q_H^* - Q(\Psi)). \tag{4}
$$

If furthermore $Q < Q(\Psi)$, then $Q(\Psi)$ is defined by

$$
\phi_H V'(Q(\Psi)) = \phi_H c_H (1-\delta) + \Psi \Delta c (1-\delta). \tag{5}
$$

With this characterization of $W^{(0)}$ and $\Phi^{(0)}$ in the domain $\Psi < \bar{\Psi}$ in place, we can now verify whether iterating the Bellman mapping on the functions $W^{(0)}, \Phi^{(0)}$ produces updated functions $W^{(1)}, \Phi^{(1)}$ that again satisfy the working hypothesis (WH), and hence are also of the form as described in lemma 2 in the domain $\Psi \in [0, \bar{\Psi}]$. In doing so we will find a condition on the value of the bound $\bar{\Psi}$.

We therefore consider $W^{(1)}$ and $\Phi^{(1)}$ defined by applying the Bellman mapping to $W^{(0)}$ and $\Phi^{(0)}$, substituting the expressions for $W^{(0)}(Q+q, \Psi-\psi)$ and $\Phi^{(0)}(Q+q, \Psi-\psi)$ from lemma 2.

$$
\begin{aligned}
W^{(1)}(Q,\Psi) = \max_{q,\psi} & (\phi_L - \Psi) V^* \\
& + \psi \left[V^* - c_L(Q^* - Q - q) - \delta \Phi^{(0)}(Q+q, \Psi-\psi) \right] \\
& + (\Psi - \psi + \phi_H) V(Q+q) - (\Psi + \phi_H) c_H q \\
& + \delta W^{(0)}(Q+q, \Psi-\psi)
\end{aligned}
$$

The first-order condition for ψ now reads

$$V^* - c_L(Q^* - Q - q) - \delta\Phi^{(0)}(Q+q, \Psi-\psi) + \delta\psi\Phi_2^{(0)}(Q+q, \Psi-\psi)$$
$$- V(Q+q) - \delta W_2^{(0)}(Q+q, \Psi-\psi) \geq 0$$

where $\Phi_2^{(0)}$ and $W_2^{(0)}$ denote the derivatives of the $\Phi^{(0)}$ and $W^{(0)}$ to their second arguments. To check whether the optimal policy ψ for any $\Psi \leq \bar{\Psi}$ is the corner solution $\psi = \Psi$, we need to verify whether this inequality is verified at the corner, $\psi = \Psi$. If this is the case, then also $W^{(1)}$ and $\Phi^{(1)}$ satisfy the working hypothesis, and by extension, the actual solution W, Φ of the Bellman equations will do so, too. By substituting the expressions for $\Phi^{(0)}, W^{(0)}$ and their derivatives, we find this indeed to be the case if $\bar{\Psi}$ satisfies the following condition.

Proposition 1 *The solution to the Bellman equations involves full revelation, $\psi = \Psi$, as long as $\Psi \leq \bar{\Psi}$, with $\bar{\Psi}$ satisfying*

$$V^* - V(Q(\bar{\Psi})) - c_L(Q^* - Q(\bar{\Psi}))(1-\delta) + \delta\bar{\Psi}\Delta c(1-\delta) \left.\frac{dQ(\bar{\Psi})}{d\bar{\Psi}}\right|_{\Psi=0} = 0 \tag{6}$$

with (from equation 5)

$$\frac{dQ}{d\Psi} = \frac{\Delta c(1-\delta)}{\phi_H V''(Q)} < 0 \tag{7}$$

In principle, there might be no value for Ψ satisfying the condition for a corner solution. In that case, we would have $\bar{\Psi} = 0$ and effectively, the working hypothesis that we started from would prove incorrect. However, we will next consider more specifically the case of quadratic value $V(Q)$, and show that in this case such a region does exist. Perhaps somewhat surprisingly, we find that the corner solution holds for all Ψ.

Proposition 2 *If $V(Q)$ is quadratic, in the optimum we have immediate full revelation, $\psi_1 = \phi_L$. In the second period, the high types are brought to the efficient level Q_H^*.*

proof We can now evaluate this condition for quadratic V, to find up to which value of Ψ a corner solution is reached immediately. In that case, from equation (7), $dQ/d\Psi$ is constant (and negative), hence $Q(\Psi)$ itself is linear in Ψ.

$$Q(\Psi) = Q_H^* + \frac{\Delta c(1-\delta)}{\phi_H V''}\Psi \leq Q_H^*,$$

with V'' the negative constant second derivative of the quadratic function $V(Q)$. Furthermore,

$$V^* - V(Q(\Psi)) - c_L(Q^* - Q(\Psi))(1-\delta) = -\frac{1}{2}V''(Q^* - Q(\Psi))^2$$

and

$$V'(Q_H^*) - V'(Q^*) = \Delta c(1-\delta) = -V''(Q^* - Q_H^*),$$

so combining that,

$$Q^* - Q(\Psi) = Q^* - Q_H^* - \frac{\Delta c(1-\delta)}{\phi_H V''}\Psi = \frac{\Delta c(1-\delta)}{-\phi_H V''}(\phi_H + \Psi)$$

We can next substitute these expression for quadratic V in the general first-order condition to find for which values of Ψ that holds:

$$\begin{aligned}
& V^* - V(Q(\Psi)) - c_L(Q^* - Q(\Psi))(1-\delta) + \delta\Psi\Delta c(1-\delta) \left.\frac{dQ(\Psi)}{d\Psi}\right|_{\Psi=0} \geq 0 \\
\text{so } & -\frac{1}{2}V''(Q^* - Q(\Psi))^2 + \delta\Psi\Delta c(1-\delta)\frac{\Delta c(1-\delta)}{\phi_H V''} \geq 0 \\
\text{so } & (\phi_H + \Psi)^2 - 2\delta\Psi\phi_H \geq 0
\end{aligned}$$

which reduces to

$$\phi_H^2 + \Psi^2 + 2\phi_H\Psi(1-\delta) \geq 0,$$

and this is true for all $\Psi \geq 0$. So with quadratic V, we have a corner solution for all Ψ. *Q.E.D.*

If the benefit function is not quadratic, the region in which full revelation occurs right away can be smaller. In that case, Q will be gradually increased each time period, until Ψ is sufficiently close to zero to end the game in the next step.

From proposition 2, with quadratic V, we have immediate full revelation of the efficient types. In period 2, then, the inefficient types are brought to their efficient quantities Q_H^*. The value function is therefore as defined in lemma 2 for all Ψ, and for all $Q < Q(\Psi)$. In particular, since initially $\Psi = \phi_L$, $Q = 0$, we can rewrite equation (3) from lemma 2 as

Corollary 1 *With quadratic V, the principal's time zero welfare is*

$$W(0,\phi_L) = \underbrace{\phi_L \frac{V^*}{1-\delta} + \phi_H \left(V(Q_H) + \frac{\delta V_H^*}{1-\delta} \right)}_{benefits} - \underbrace{\left(\phi_L c_L Q_L^* + \phi_H c_H \left[Q_H + \delta\left(Q_H^* - Q_H\right)\right]\right)}_{costs}$$
$$- \underbrace{\phi_L \Delta c \left[Q_H + \delta(Q_H^* - Q_H)\right]}_{rents} \tag{8}$$

with Q_H the period 1 investment by the inefficient types,

$$\phi_H V'(Q_H) = \phi_H c_H(1-\delta) + \phi_L \Delta c(1-\delta). \tag{9}$$

We can compare this renegotiation-proof equilibrium with the outcome that the principal can obtain if he can commit to terminate the game after one period. In that second-best case, where the game would end after one period, as usual the high-cost type's contract is inefficient in order to keep in check the rents received by the low-cost type. By standard arguments, the principal chooses quantities Q_L and Q_H to optimize his expected welfare under commitment,

$$W^c = \max_{Q_L,Q_H} \phi_L \left(\frac{V(Q_L)}{1-\delta} - c_L Q_L - \Delta c Q_H \right) + \phi_H \left(\frac{V(Q_H)}{1-\delta} - c_H Q_H \right)$$

where the final term in the first brackets represents the information rents for the low-cost types. We find that indeed $Q_L = Q_L^*$, while Q_H is given by equation (9), i.e. the optimal high-cost type quantity in the second-best case (with commitment) equals the period one quantity for these types in the non-commitment case with renegotiation. It is therefore straightforward to compare the principal's welfare in the case with renegotiation, and the second-best welfare with commitment.

Corollary 2 *With quadratic V, failure to commit not to renegotiate leads to a loss of surplus* *for the principal equal to*

$$W(0,\phi_L) - W^c = \underbrace{\phi_H \frac{\delta}{1-\delta}\left[V_H^* - V(Q_H)\right]}_{benefits} - \underbrace{\delta\phi_H c_H(Q_H^* - Q_H)}_{costs} - \underbrace{\delta\phi_L \Delta c(Q_H^* - Q_H)}_{rents}$$
$$= \tfrac{1}{2}\delta\phi_H \frac{1-\delta}{V''}\left(\frac{\phi_L}{\phi_H}\right)^2 (\Delta c)^2 < 0$$

Clearly, the principal benefits from the added Q in the renegotiation; however, the extra rents paid to the low types (in addition to the costs to realize the added Q) outweigh these benefits, and the final principal's welfare is lower than the second-best.

Even if the principal cannot credibly promise never to renegotiate an ex-post inefficient outcome, perhaps there are other means of reducing the rents paid to low-cost types. If the principal can somehow make the option of producing the low quantity (Q_H) less profitable, it will be less attractive for low types to pose as a high-cost type, and the required rents could be lowered.

We suggest that introducing 'market-discipline' can achieve that task and make the principal's position stronger. Suppose that the principal can make sure that the market penalizes bad performance, i.e., low observations of $Q < Q_L^*$.[2] Suppose the agent's costs of such market discipline equals K_A. High-cost types cannot avoid those costs K_A, and their participation constraint is therefore unchanged.[3] For low-cost types, on the other hand, the incentive compatibility constraint is relaxed,

$$IC_L' : \quad t_n^L - c_L(Q_L^* - Q) = \sum_{k=0}^{\infty} \delta^k (t_{n+k}^H - c_L q_{n+k}^H) - K_A.$$

As a result, in the equilibrium, the payments to low-cost types t^L will be reduced.

The downside is that market discipline, when exercised, may be costly also for the principal,

[2]As an example, in the context of EU country bail-outs, one might imagine that while the EU countries cannot credibly promise never to bail out a distressed member state, the central bank, ECB, can credibly restrict its interventions to keep the distressed country's borrowing rate in check.

[3]We implicitly assume that K_A is not so high that high-cost agents want to copy the low cost agent's strategy to avoid being exposed to discipline.

for instance as a result of spill-over effects. Suppose those costs for the principal equal K_P. Since all high types will unavoidably fall prey to market discipline, the expected costs for the principal of introducing market discipline will equal $\phi_H K_P$.

From this discussion, we see that introducing market discipline has two effects on the expected surplus for the principal.

Proposition 3 *Suppose that introducing market discipline leads to additional costs for the agent, K_A, for low realizations of Q, (i.e. $Q < Q_L^*$), as well as spill-over costs K_P for the principal in those cases. Then, in the quadratic case, such market discipline changes the principal's surplus by*

$$\Delta W = \phi_L K_A - \phi_H K_P.$$

It will therefore be attractive to introduce market discipline for the principal if the benefits of lower transfers to low types, of an expected amount $\phi_L K_A$, exceed the costs for the principal when the agent is genuinely of high-cost, $\phi_H K_P$.

In the quadratic case, the outcome of the game is not altered: with and without market discipline, all low types reveal at once, while high types reach their lower efficient level after two periods. It is not hard to see that in the general case explored in proposition 1, market discipline can have an additional effect. If, in the Bellman equation (2), transfers t_L are reduced by K_A, then the left-hand side of the first-order condition for ψ (in proposition 1) increases by that amount. As a result, the domain $[0, \bar{\Psi}]$ from which full revelation is reached will expand, and the end result with full revelation will be reached sooner. In other words, market discipline can speed up the renegotiation process.

4. Conclusions

Greece serves as an example of how bail-out may be followed by renewed bail-outs and renegotiation of the contract terms. We study renegotiation in a dynamic procurement model where an agent can raise its health by making investments with private costs.

We introduce a Bellman equation describing the optimal bail-out mechanism in a multiple-period principal-agent model with adverse selection on a country's cost of reforms and renegotiation. We solve this Bellman equation analytically when the functional form of the principal's value for the agent's health level is quadratic, even though the Bellman equation itself is not quadratic, which makes the problem non-trivial. We show that the first-best efficient outcome is reached after two periods. For more general spill-over benefits, the renegotiation process may take additional steps until efficiency is reached.

We also study how the principal can use market discipline to improve outcome. We find that market discipline can lower the rents the principal has to pay and, in non-quadratic situations, can speed up the renegotiation process. It will be attractive for the principal to introduce market discipline if the benefits of lower transfers to low types exceed the costs for the principal when the agent is genuinely of high-cost.

An interesting extension would be to endogenise the choice of market discipline by the principal to study the question what the optimal level of market discipline is. An important question here is the mechanism through which the principal can influence market discipline.

We leave a more detailed analysis of the non-quadratic case for further research. In that case, not all low types will reveal at once and the renegotiation process can carry on for several periods. Finding an analytical solution may be difficult, however, in which case one may have to resort to numerical methods.

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