CPB Discussion Paper

No 111

September, 2008

Gravity with Gravitas: Comment

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ISBN 978-90-5833-373-5

Abstract in English

In GRAVITY WITH GRAVITAS: A SOLUTION TO THE BORDER PUZZLE, Anderson and Van Wincoop (2003) estimate what trade between US states and Canadian provinces would have been if the border between Canada and the United States had not existed. They showed that computing the border effect requires solving a non-linear system of multilateral price indexes. This note shows that the non-linear system can be solved analytically, such that a numerical approximation is no longer needed. The exact solution yields a reduced-form log-linear gravity equation that can be estimated using standard econometric techniques. After estimation, the calculation of treatment effects like the border effect is straightforward. Using the same data and assumptions, I find that the border effect for Canada is half as large as reported by Anderson and Van Wincoop.

Key words: Gravity equation, Multilateral resistance

JEL code: F10, F15

Abstract in Dutch

In GRAVITY WITH GRAVITAS: A SOLUTION TO THE BORDER PUZZLE, Anderson en Van Wincoop (2003) schatten wat de handel tussen de staten van de VS en Canadese provincies zou zijn geweest als de grens tussen Canada en de Verenigde Staten niet had bestaan. Ze laten zien dat om het grenseffect te kunnen berekenen, een niet-lineair systeem van multilaterale prijsindices moet worden opgelost. Deze notitie laat zien dat het niet-lineaire systeem een analytische oplossing heeft, zodat een nummerieke benadering niet meer noodzakelijk is. De exacte oplossing levert een log-lineaire herleide vorm zwaartekrachtvergelijking op, welke kan worden geschat met standaard econometrische technieken. Na het schatten, kunnen behandelingseffecten zoals het grenseffect eenvoudig worden berekend. Gebruikmakend van dezelfde gegevens en veronderstellingen, blijkt het grenseffect voor Canada de helft van wat door Anderson en Van Wincoop wordt gerapporteerd.

Steekwoorden: zwaartekrachtvergelijking, multilaterale weerstand

Contents

Summary[1]

In GRAVITY WITH GRAVITAS: A SOLUTION TO THE BORDER PUZZLE, Anderson and Van Wincoop (2003) estimate what trade between US states and Canadian provinces would have been if the border between Canada and the United States had not existed. They used a custom non-linear program to obtain an approximate solution of the border effect. This note shows that A-vW's non-linear problem can be rewritten in log-linear form, such that it can be solved analytically. The solution can subsequently be used to substitute for the multilateral resistances in the gravity equation. The resulting transformed gravity equation is log-linear and can be estimated using standard econometric techniques. Counterfactual trade flows can be obtained straightforwardly once parameters have been estimated.

Repeating A-vW's two-country analysis using the exact solution yields border effects for Canadian provinces that are smaller than reported previously. If the border between Canada and the United States had not existed, the ratio of intra-national trade to international trade would drop by a factor five for Canada and a factor four for the United States. These numbers differ from those of A-vW, who report a factor 10.5 for Canada and 2.6 for the United States.

The analytical solution of the system brings along four improvements over A-vW's non-linear procedure. First, it leads to more precise results as no numerical approximations have to be made. The results presented below indicate that both methods can perform quite differently. Second, the system can be solved independently from estimation. This not only makes it possible to compare treatment effects for different parameter estimates, but also to obtain dynamic treatment effects obtained from panel data estimation. Third, estimation of the treatment effect no longer requires an estimate of the elasticity of substitution. A fourth, rather practical, improvement is that a single software package can be used for all problems within A-vW's theoretical framework, making gravity-based policy evaluation easier to do.

[1] I would like to thank the following people for comments and suggestions: Leon Bettendorf, Harry Garretsen, Henri de Groot, Albert van der Horst, Arjan Lejour, Gert-Jan Linders, and Bas ter Weel. All errors are mine.

1 Introduction

In GRAVITY WITH GRAVITAS James Anderson and Eric van Wincoop (2003) demonstrate how trade flows would change if the border between Canada and the United States had not existed. They are able to infer counterfactual trade flows through a theoretical foundation of the gravity equation. The theoretical framework yields a modified gravity equation supplemented with a non-linear system of multilateral price indexes. Anderson and Van Wincoop (A-vW) did not solve this system analytically, but used a custom non-linear program (NLP) to obtain an approximate solution of the border effect.

This note shows that A-vW's non-linear system can be rewritten in log-linear form, such that it can be solved analytically. The solution can subsequently be used to substitute for the multilateral resistances in the gravity equation. The resulting transformed gravity equation is log-linear and can be estimated using standard econometric techniques. Counterfactual trade flows can be obtained straightforwardly once parameters have been estimated.

John McCallum's (1995) finding that trade between Canadian provinces is a factor 22 larger than trade between Canadian provinces and US states, ran against the intuition of many economists that national borders—especially this one—did no longer form an important barier to trade. Obstfeld and Rogoff (2001) identified this border puzzle as one of the six major puzzles in international macroeconomics. A-vW solved part of the puzzle: they concluded that if the border between Canada and the United States had not existed, the ratio of intra-Canadian trade to US-Canada trade would drop by a factor 10.5. Repeating A-vW's two-country analysis using the exact solution yields a factor five, which is half the border effect reported by A-vW. For the United States, I find a somewhat larger border effect of around four where A-vW's approach yields a factor 2.6 (see Table 4.2 for details).[2]

The analytical solution of the system brings along four improvements over A-vW's NLP procedure. First, it leads to more precise results as no numerical approximations have to be made. The results presented below indicate that both methods can perform quite differently. Second, the system can be solved independently from estimation. This not only makes it possible to compare treatment effects for different parameter estimates, but also to obtain dynamic treatment effects obtained from panel data estimation. Third, estimation of the treatment effect no longer requires an estimate of the elasticity of substitution. A fourth, rather practical, improvement is that a single software package[3] can be used for all problems within A-vW's theoretical framework, making gravity-based policy evaluation easier to do.[4]

[2] Balistreri and Hillberry (2007) argue that part of puzzle is caused by the data used for intra-U.S. trade.

[3] The STATA package AVWTRANSFORM is available upon request from the author.

[4] Although the large number of papers citing A-vW would suggest otherwise—Google Scholar reports 984 citations—the A-vW method has, to my knowledge, only been applied to the border between Canada and the United States and not to other borders, Free Trade Areas, or other determinants of trade cost.

Recently, Baier and Bergstrand (2007) adopted the strategy of linearly approximating the equations of A-vW's system. The linear approximation of the system can be solved analytically and used to obtain a reduced form gravity equation. The reduced form gravity equation is log-linear and can be estimated with OLS. For this reason Baier and Bergstrand named their method BONUS VETUS OLS, abbreviated BVO.

The main difference between their approach and the one proposed below is that theirs relies on an approximate solution and therefore leads to less precise estimates of counterfactual trade flows. Paradoxically, linear approximation still turns out to be advantageous when it comes to estimating the parameters of the gravity equation. In contrast to the reduced form gravity equation based on the exact solution, the BVO gravity equation does not suffer from endogeneity bias.[5]

The exact solution of the non-linear system of resistance terms is introduced in Section 2. Estimation results for A-vW's two-country data are presented in Section 3 and border effects are compared in Section 4. Section 5 concludes.

[5] Novy (2008) and Jacks et al. (2008) show that trade cost can also be estimated by substituting out the multilateral resistance terms. Their method does not provide a way to calculate counterfactual trade flows.

2 A log-linear solution

The gravity equation resulting from A-vW's theoretical framework (reproduced below) differs from the classical gravity equation in that it includes a price index for the exporting region and a price index for the importing region.[6]

$$x_{ij} = \frac{y_i y_j}{y_w}\left(\frac{t_{ij}}{P_i P_j}\right)^{1-\sigma} \tag{2.1}$$

Throughout this paper, the notation of A-vW is preserved, such that x_{ij} is the value of trade exported by region i to region j, y is total expenditure, t_{ij} is the (symmetric) trade cost between i and j, P is the price index, σ is the elasticity of substitution, and y_w is the total expenditure of all n regions, $\sum_{i=1}^{n} y_i$.

The price indexes—or multilateral resistance terms—are unobserved, but as shown in A-vW's equation (12) form a non-linear system with σ, θ_i, and t_{ij} as exogenous variables:

$$P_j^{1-\sigma} = \sum_{i=1}^{n} P_i^{\sigma-1}\theta_i t_{ij}^{1-\sigma} \ . \tag{2.2}$$

Here, θ_i is region i's expenditure relative to the total (y_i/y_w). If each price index would depend on only one other index, then the equation would be log-linear and solving it would be straightforward. The theorem below shows that the right-hand-side of (2.2) can be rewritten as the product of geometric means, yielding a log-linear system of equations.

Theorem 1. *Provided that* (2.1) *holds, the system formed by* (2.2) *is equivalent to the system formed by*

$$\ln P_j = -\sum_{i=1}^{n} w_{ij}\ln P_i + \frac{1}{1-\sigma}\left(\ln N_j + \ln\Theta_j\right) + \ln T_j \tag{2.3}$$

where $N_j \equiv \prod_{i=1}^{n} w_{ij}^{-w_{ij}}$, $\Theta_j \equiv \prod_{i=1}^{n}\theta_i^{w_{ij}}$, $T_j \equiv \prod_{i=1}^{n} t_i^{w_{ij}}$ *and* $w_{ij} = x_{ij}/\sum_{h=1}^{n} x_{hj}$.[7]

Proof. The proof proceeds in two steps. First, the proof of Lemma 2 in the appendix shows that (2.2) can be written as (2.3) with $w_{ij} \equiv \frac{P_i^{\sigma-1}\theta_i t_{ij}^{1-\sigma}}{\sum_h P_h^{\sigma-1}\theta_h t_{hj}^{1-\sigma}}$. Second, the proof of Lemma 3 shows that $w_{ij} = x_{ij}/\sum_h x_{hj}$ if (2.1) holds. ■

The system formed by (2.3) can be written in matrix notation as

$$\tilde{\mathbf{P}} = -\mathbf{W}'\tilde{\mathbf{P}} + \frac{1}{1-\sigma}\left(\tilde{\mathbf{N}} + \tilde{\mathbf{\Theta}}\right) + \tilde{\mathbf{T}} \ , \tag{2.4}$$

with a tilde indicating a vector of logarithmic values and $\mathbf{W}$ being a matrix of all w_{ij}.

[6] The price indexes are also known as multilateral resistance terms.

[7] Θ and T are the weighted geometric means of expenditure and trade cost, respectively. N is the anti-log of Shannon's entropy and can interpreted as an index of product variety (Straathof, 2007).

The system can be solved for $\tilde{\mathbf{P}}$, provided that the matrix $\left(\mathbf{I}+\mathbf{W}'\right)$ can be inverted:

$$\tilde{\mathbf{P}} = \left(\mathbf{I}+\mathbf{W}'\right)^{-1}\left(\frac{1}{1-\sigma}\left(\tilde{\mathbf{N}}+\tilde{\mathbf{\Theta}}\right)+\tilde{\mathbf{T}}\right) \tag{2.5}$$

With the system solved in terms of $\ln P_i$, an expression for $\ln\left(P_i^{1-\sigma}\right)$ is readily available.

$$\ln\left(P_i^{1-\sigma}\right) = \bar{\mathbf{w}}_i\left(\tilde{\mathbf{N}}+\tilde{\mathbf{\Theta}}+(1-\sigma)\tilde{\mathbf{T}}\right) \tag{2.6}$$

Here, $\bar{\mathbf{w}}_i$ is the i-th row vector of the inverted matrix. Solutions for $\ln\left(P_i^{1-\sigma}\right)$ and $\ln\left(P_j^{1-\sigma}\right)$ can be inserted into (2.1), such that a reduced-form equation now summarizes the system formed by (2.2) and (2.1):

$$\ln x_{ij} = \ln\left(\frac{y_i y_j}{y_w}\right) + (1-\sigma)\ln t_{ij} - \left(\bar{\mathbf{w}}_i+\bar{\mathbf{w}}_j\right)\left(\tilde{\mathbf{N}}+\tilde{\mathbf{\Theta}}+(1-\sigma)\tilde{\mathbf{T}}\right) \ . \tag{2.7}$$

This reduced form gravity equation can be estimated using linear regression under the assumption that t is a log-linear function of observed variables.[8] The first two terms on the right-hand-side are traditional components of the gravity equation, while the last term is new and contains the multilateral resistance effects. The vectors $\bar{\mathbf{w}}_i$ and $\bar{\mathbf{w}}_j$ reflect that multilateral resistances can be different across regions. The next section presents an application of (2.7) in which t depends on the border between Canada and the United States.

First-order treatment effects can be obtained from the reduced form by setting the t_{ij}'s to their counterfactual values. The resulting trade flows can be used to update the weights in a second iteration. This procedure can be repeated until trade flows have converged to their counterfactual values.

[8] Henderson and Millimet (2008) test whether the assumption of log-linearity is restrictive. Using nonparametric methods, they arrive at the conclusion that this assumption does not appear to be a reason for concern.

3 Estimation

In Gravity with Gravitas trade cost between two regions are a function of the distance between the two regions d_{ij} and whether the two regions are in a different country β_{ij}.[9] The trade cost function used by A-vW is $t_{ij} = b^{\beta_{ij}} d_{ij}^{\rho}$, where the coefficient b measures the border effect on trade and ρ indicates the effect of distance on trade.

Substitution for t_{ij} in (2.3) yields a system of price indexes for this specification of trade cost.

$$\ln P_j = -\sum_{i=1}^{n} w_{ij} \ln P_i + \frac{1}{1-\sigma} (\ln N_j + \ln \Theta_j) + \ln b \tilde{\mathbf{B}} + \rho \tilde{\mathbf{D}} \tag{3.1}$$

The elements of the vectors $\tilde{\mathbf{B}}$ and $\tilde{\mathbf{D}}$ are logarithms of the geometric means $D_j \equiv \prod_i d_{ij}^{w_{ij}}$ and $B_j \equiv \prod_i \exp[w_{ij}\beta_{ij}]$, respectively.

After solving this system for P_i and P_j , we obtain a gravity equation that can be estimated with conventional econometric techniques. Given the specification of trade cost defined above, the empirical equivalent of (2.7) becomes a linear function of two transformed variables:

$$z_{ij} = k + a_1 \ln d_{ij}^* + a_2 \beta_{ij}^* + \varepsilon_{ij} \ . \tag{3.2}$$

Here, $z_{ij} \equiv \ln\left(\frac{x_{ij}}{y_i y_j}\right) + (\bar{\mathbf{w}}_i + \bar{\mathbf{w}}_j)\left(\tilde{\mathbf{N}} + \tilde{\mathbf{\Theta}}\right)$ is the regressor used by A-vW plus the variety and GDP indexes, $\ln d_{ij}^* \equiv \ln d_{ij} - (\bar{\mathbf{w}}_i + \bar{\mathbf{w}}_j)\tilde{\mathbf{D}}$, is the log of distance minus multilateral resistance and $\beta_{ij}^* \equiv \beta_{ij} - (\bar{\mathbf{w}}_i + \bar{\mathbf{w}}_j)\tilde{\mathbf{B}}$ is the border dummy minus multilateral resistance. Parameters k, a_1, and a_2 are the same as in A-vW.[10]

Table 3.1 compares five sets of parameter estimates. Column (1) repeats the estimates A-vW obtained with a non-linear program (NLP). The second set of estimates is obtained by regressing $\ln\left(\frac{x_{ij}}{y_i y_j}\right)$ on $\ln(d_{ij})$, β_{ij}, and dummies for each province and state. A-vW suggested that including dummies in a gravity equation yields unbiased estimates of a_1 and a_2.[11]

Columns (3) to (5) present alternative results based on reduced form gravity equations. Column (3) refers to estimating equation (3.2) using ordinary least squares. The coefficient on distance of -1.15 lies in between the A-vW estimate of -0.79 and the unbiased estimate of -1.25. The coefficient on the border dummy (-1.49) lies below that of both the A-vW model (-1.65) and the unbiased model (-1.55).

The presence of the term $\bar{\mathbf{w}}_i + \bar{\mathbf{w}}_j$ on both sides of (3.2) is likely to lead to endogeneity bias. This could be the reason why the parameter estimates based on the exact solution of the price index system deviate from the unbiased estimates. The bias is not as large as for A-vW's NLP estimation approach.

[9] A-vW used a dummy $\delta_{ij} = 1 - \beta_{ij}$ indicating whether two regions are in the *same* country.

[10] $k = y_w$, $a_1 = (1-\sigma)\rho$, and $a_2 = (1-\sigma)\ln b$.

[11] The parameters are identical to those reported by A-vW (p. 188) and Feenstra (2004, Table 5.2, column (5)).

Table 3.1 Estimation results gravity equation

	NLP	Unbiased	Reduced form		
	(1)	(2)	(3)	(4)	(5)
Distance: $(1-\sigma)\rho$	-0.79	-1.25	-1.15	-1.26	-1.23
	(0.03)	(0.04)	(0.04)	(0.05)	(0.04)
Border: $(1-\sigma)\ln b$	-1.65	-1.55	-1.49	-1.53	-1.53
	(0.08)	(0.07)	(0.07)	(0.08)	(0.09)
Constant		5.53	-3.06	-3.51	-2.92
		(0.40)	(0.06)	(0.03)	(0.07)
Transformation	none	none	exact	BVO	exact
Region dummies	no	yes	no	no	no
Instruments	none	none	none	none	BVO
R^2-adj.		0.65	0.53	0.51	0.53

Robust standard errors are shown in parenthesis.

Column (4) shows the results for Baier and Bergstrand's BVO approach.[12] As they have reported themselves, the parameter estimates are very close to the unbiased estimates obtained with region dummies. Being insensitive to endogeneity bias, the BVO gravity equation outperforms estimates based on the exact solution of the price index system.

In order to avoid bias through endogeneity, model (5) uses the BVO-transformed variables for log distance and the border dummy as instruments. These variables correlate with $\ln d_{ij}^{*}$ and b_{ij}^{*}, but do not rely on trade data for their construction. The instrumented regression yields coefficients closer to those of the unbiased specification.

For the reduced form models, the severity of the endogeneity bias can be evaluated by means of a Hausman test. Under the null hypothesis, the alternative model is unbiased and more efficient than model (2). The null hypothesis is tested against the hypothesis that the alternative model is biased. As the variables of the alternative models are transformed versions of the variables in the unbiased model, a standard Hausman test cannot be used. Instead, both models are incorporated in a seemingly unrelated regression model. Afterward, the equality of coefficients can be tested with a standard Wald test. Table 3.2 displays χ^2-statistics for Wald tests comparing models (3) to (5) with the unbiased model (2). Tests are performed for the coefficients separately as well as jointly.

The hypothesis of unbiased estimates is rejected for the exactly transformed model without instruments (3) as the overall p-value is just 0.01. The BVO specification performs best with an overall p-value of 0.86. Using BVO-transformed variables as instruments for the exactly

[12] Baier and Bergstrand (2007) propose two reduced form gravity equations, one which uses weights based on GDP and one which uses the number of countries ("n-weights"). The results presented in column (4) of Table 3.1 are based on the latter weights, which are more robust to endogeneity bias.

Table 3.2 Hausman test results for reduced form estimates (χ^2)

	(3)	(4)	(5)
Transformation	exact	BVO	exact
Instruments	none	none	BVO
Distance: $(1-\sigma)\rho$	6.51	0.03	0.28
	(0.01)	(0.85)	(0.60)
Border: $(1-\sigma)\ln b$	0.96	0.29	0.09
	(0.33)	(0.59)	(0.76)
Both coefficients	8.66	0.31	0.37
	(0.01)	(0.86)	(0.83)

The probability of the null hypothesis (no bias) is shown in brackets.

transformed model leads to a slightly smaller p -value. Given an adjusted R^2 of 0.65 there is no indication that the unbiased specification is less efficient than the other specifications.[13] Therefore, the estimates of regression (2) are to be preferred above the others.

[13] In applications involving panel data the number of dummies included in regression (3) could become large as the country dummies would then have to be interacted with time dummies. As the degrees of freedom used by the reduced form regressions do not depend on the dimensions of the data set, important differences in efficiency can arise.

4 The border effect

Once parameters have been estimated, the effect of the border can be calculated by comparing actual trade flows x_{ij} with counterfactual trade flows x_{ij}^T. The latter can be derived from (3.2) with the border dummy set to zero for all trade flows ($\beta_{ij} = 0\,\forall\, i,\, j$). Ignoring second-order effects due to changes in weights, the ratio between actual and the borderless trade between i and j is approximately given by

$$\frac{x_{ij}}{x_{ij}^T} = \exp\left[a_2\left[\beta_{ij} - (\bar{\mathbf{w}}_i + \bar{\mathbf{w}}_j)\,\tilde{\mathbf{B}}\right]\right] \; . \tag{4.1}$$

This equation is equivalent to equation (23) in A-vW. Following A-vW, the total effect x_{ij}/x_{ij}^T can be decomposed into an effect due to bilateral resistance x_{ij}/x_{ij}^B and multilateral resistance x_{ij}/x_{ij}^M:

$$\frac{x_{ij}}{x_{ij}^B} = \exp\left[a_2\beta_{ij}\right] \tag{4.2}$$

$$\frac{x_{ij}}{x_{ij}^M} = \exp\left[-a_2\left(\bar{\mathbf{w}}_i + \bar{\mathbf{w}}_j\right)\tilde{\mathbf{B}}\right] \tag{4.3}$$

Table 4.1 compares the decompositions reported by A-vW with those obtained using the alternative approach based on the analytical solution of the price index system. The top panel is based on the A-vW's NLP parameter estimates, the bottom panel is based on A-vW's unbiased parameter estimates. The bilateral ratios of actual trade to borderless trade have been averaged for intra-U.S. trade, intra-Canada trade and U.S.-Canada trade.

Starting with the top panel, the results for intra-U.S. trade are identical for both methods, but the they lead to different results for intra-Canada and cross-border trade. Where A-vW suggest that actual intra-Canada trade is four times larger than it would be without the border, the alternative method predicts that actual trade is only 15 percent larger. For cross-border trade, A-vW report that actual trade flows are just 41 percent of potential trade in the absence of a border. The alternative method yields a comparable 50 percent.

A more detailed look at the decomposition reveals that the especially the estimated effects of multilateral resistance differ between the two approaches. Apparently, the numerical approximation of A-vW overestimates the multilateral resistance.

Turning to the bottom panel, the last three columns of the top panel are replicated using unbiased parameter estimates. The border now has a larger impact on cross-border trade than reported by A-vW, but intra-Canada trade is not affected as dramatically. The last three columns of the bottom panel display the ratio of aggregate trade flows in stead of the average ratio of trade flows.[14] Compared to the averaged ratio, the ratio of aggregates suggests a somewhat smaller effect of multilateral resistance for intra-U.S. trade and slightly higher effects for cross-border and intra-Canada trade.

[14] The ratio of aggregate trade flows is defined as $\sum x_{ij}/\sum x_{ij}^Z$ $Z \in T, B, M$; the average ratio is defined as $\frac{1}{n}\sum\left(x_{ij}/x_{ij}^Z\right)$.

Table 4.1 Impact of border barriers on bilateral trade

NLP parameter estimates (average ratios)

	NLP ($\sigma = 5$)			Exact solution		
	US-US	CA-CA	US-CA	US-US	CA-CA	US-CA
Total effect (x_{ij}/x_{ij}^T)	1.05	4.31	0.41	1.05	1.15	0.50
	(0.01)	(0.34)	(0.02)	(0.06)	(0.05)	(0.02)
Bilateral (x_{ij}/x_{ij}^B)	1.00	1.00	0.19	1.00	1.00	0.45
	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)
Multilateral (x_{ij}/x_{ij}^M)	1.05	4.31	2.13	1.05	1.15	1.10
	(0.01)	(0.34)	(0.09)	(0.06)	(0.05)	(0.05)

Unbiased parameter estimates (exact solution only)

	Average ratio			Ratio of aggregates[a]		
	US-US	CA-CA	US-CA	US-US	CA-CA	US-CA
Total effect (x_{ij}/x_{ij}^T)	1.10	1.33	0.26	1.06	1.41	0.28
	(0.12)	(0.11)	(0.03)	(0.12)	(0.11)	(0.03)
Bilateral (x_{ij}/x_{ij}^B)	1.00	1.00	0.21	1.00	1.00	0.21
	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.01)
Multilateral (x_{ij}/x_{ij}^M)	1.10	1.33	1.21	1.06	1.41	1.30
	(0.12)	(0.11)	(0.12)	(0.12)	(0.11)	(0.12)

Standard deviations are shown in parenthesis and reflect both variation in bilateral trade flows and parameter uncertainty.
[a] Standard deviations reported for the ratio of aggregates are approximated by the standard deviations for the average ratio and can be regarded as upper bounds.

Another way of looking at the impact of the border is to compare the actual ratio between cross-border and intra-national trade with the counterfactual ratio. The border effect for Canada is then defined as

$$Border_{Can} = \frac{x_{US,CA}/x_{CA,CA}}{x_{US,CA}^T/x_{CA,CA}^T}. \tag{4.4}$$

Table 4.2 displays the border effect for the same cases as those of Table 4.1. The border effects reported by A-vW are 2.6 for the United States and 10.5 for Canada. The exact solution of the system yields more modest—but still sizable–effects of 2.1 and 2.3 for the United States and Canada, respectively. For unbiased parameter estimates, the border effect is notably larger: around 4 for the United States and around 5 for Canada.

Table 4.2 Impact border on intra-national trade relative to international trade

	NLP parameter estimates		Unbiased parameter estimates	
	NLP ($\sigma = 5$) average	Exact average	Exact average	Exact aggregate
United States	2.56	2.10	4.30	3.82
Canada	10.51	2.31	5.17	5.09

5 Concluding remarks

Anderson and Van Wincoop have proposed a valuable framework that can be used for evaluating all kinds of trade-related policies. Despite being widely cited, however, no attempts have been made to apply their approach in contexts other than the U.S.-Canadian border. They have proposed a theoretical framework for the gravity equation, which resulted in a modified gravity equation accompanied by a system of non-linear price indexes. One of the factors behind the lack of revealed popularity of their approach is that the computation of counterfactual trade flows requires a custom program for solving the non-linear system of price indexes.

The analytical solution to the system of price indexes put forward in this note enables four improvements over A-vW's NLP approach. First, it is more precise as it avoids numerical or linear approximations. The case of the U.S.-Canadian border shows that the gain in precision can be of an important magnitude. Second, it enables the comparison of treatment effects for different parameter estimates—including estimates obtained with panel data techniques. Third, an estimate of the elasticity of substitution is no longer needed. Fourth, it substantially reduces the effort required to compute counterfactual trade flows. These four technical improvements make it easier to apply A-vW's framework for counterfactual analysis to trade barriers beyond the U.S.-Canadian border.

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Appendix A Lemmas 2 and 3

Lemma 2. *Equations 2.2 and 2.3 are equivalent where*

$$w_{ij} \equiv \frac{P_i^{\sigma-1}\theta_i t_{ij}^{1-\sigma}}{\sum_{h=1}^n P_h^{\sigma-1}\theta_h t_{hj}^{1-\sigma}} \ . \tag{A.1}$$

Proof. Take logs in equation (2.2) and multiply by $\sum_{h=1}^n w_{hj} = 1$,

$$\ln P_j^{1-\sigma} = \ln\left(\sum_{i=1}^n P_i^{\sigma-1}\theta_i t_{ij}^{1-\sigma}\right)\sum_{h=1}^n w_{hj} \ . \tag{A.2}$$

Within brackets, multiply by $P_h^{\sigma-1}\theta_h t_{hj}^{1-\sigma}/P_h^{\sigma-1}\theta_h t_{hj}^{1-\sigma}$ to get

$$\ln P_j^{1-\sigma} = \sum_{h=1}^n w_{hj}\ln\left(\frac{P_h^{\sigma-1}\theta_h t_{hj}^{1-\sigma}}{\frac{P_h^{\sigma-1}\theta_h t_{hj}^{1-\sigma}}{\sum_i P_i^{\sigma-1}\theta_i t_{ij}^{1-\sigma}}}\right) \ . \tag{A.3}$$

Apply the definition of w and rearrange.

$$\ln P_j^{1-\sigma} = \sum_{h=1}^n w_{hj}\ln\left(\frac{P_h^{\sigma-1}\theta_h t_{hj}^{1-\sigma}}{w_{hj}}\right) \tag{A.4}$$

$$\ln P_j^{1-\sigma} = \sum_{h=1}^n w_{hj}\ln\frac{1}{w_{hj}} - \sum_{h=1}^n w_{hj}\ln\left(P_h^{1-\sigma}\right) \tag{A.5}$$

$$+\sum_{h=1}^n w_{hj}\ln\theta_h + (1-\sigma)\sum_{h=1}^n w_{hj}\ln t_{hj} \tag{A.6}$$

Apply the definitions of N, Θ and T in order to complete the proof. ■

Lemma 3. *If equation 2.1 holds, then* $w_{ij} = x_{ij}/\sum_{h=1}^n x_{hj}$.

Proof. Take the share of the value of imports from i in the total imports of region j and substitute for x using equation 2.1.

$$\frac{x_{ij}}{\sum_{h=1}^n x_{hj}} = \frac{\frac{y_i y_j}{y_w}\left(\frac{t_{ij}}{P_i P_j}\right)^{1-\sigma}}{\sum_{h=1}^n \frac{y_h y_j}{y_w}\left(\frac{t_{hj}}{P_h P_j}\right)^{1-\sigma}} \tag{A.7}$$

Let the P_j and y_j cancel out and use $\theta_i \equiv y_i/y_w$ to get

$$\frac{x_{ij}}{\sum_{h=1}^n x_{hj}} = \frac{P_i^{\sigma-1}\theta_i t_{ij}^{1-\sigma}}{\sum_{h=1}^n P_h^{\sigma-1}\theta_h t_{hj}^{1-\sigma}} = w_{ij} \ . \tag{A.8}$$

■