CPB Memorandum

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Description Gamma Model; version pension study

The paper presents a technical description of the GAMMA model as used for the pension study. Moreover, the paper presents the theoretical background of the behavioural relations and details on the calibration method.

Contents

1 The Gamma model

The description of the GAMMA model presented in this paper is related to the version used for the pension study (Westerhout et al. (2004)). The main difference with the version of Draper and Westerhout (2002) and Draper et al. (2003) concerns labour supply. Labour supply is now endogenous contrary to the earlier versions. Future research will focus on welfare analysis, endogenizing human capital, infrastructural capital and its influence on labour productivity

1.1 Introduction

GAMMA is an intertemporal applied general equilibrium model of the Dutch economy. GAMMA extends the generational accounting framework (Ewijk et al. (2000)) in order to account for relevant economic behavioural effects. The GAMMA model strikes a balance between a tractable and consistent CGE model and a faithful representation of the Dutch economy. As always, we consider the Netherlands to be small relative to the outside world. In particular, Dutch supply and demand for capital are unable to affect the interest rate, which is determined on world capital markets. Also, locally produced goods are perfectly substitutable with those produced abroad. The model is deterministic: lifetime uncertainty is recognised, but perfect capital markets allow households to insure against this type of risk. Abstracting from other types of risks, the model features an exogenous and constant equity premium. The return to bonds is equal to the world interest rate while equity returns are higher.

The model features rational expectations and perfect foresight (i.e. economic agents in GAMMA let their current decisions depend on the expectations of future variables, and these expectations coincide with realisations).

Demography

The demographic model of GAMMA describes the development of the population, as has been based on the model of Statistics Netherlands (CBS (2002)). The demographic model is made such that it can reproduce the projection of the CBS, with one concession: it assumes a maximum age of 99. The demographic model distinguishes men and women, with different mortality probabilities. Men and women are then seperated into age cohorts. Immigration, emigration, birth and mortality determine the change of the population-cohort-size. Immigrants and emigrants are treated in the economic model as representative agents for convenience. This means for instance that immigrants have the same financial wealth as other inhabitants.

The public sector

The government and social security (including first pillar pensions) are consolidated into the public sector. Public sector revenues consist of revenues from income taxation, corporate taxation and indirect taxation, as well as social security contributions. Income taxes consist of labour income taxes, transfer income taxes, taxes on public pensions, taxes on private pensions, taxes on imputed income from wealth and other income taxes. Corporate taxes are levied on firm's profits; indirect taxes are levied on consumption and investment. The government has also income from the sale of natural gas and from public assets. All government expenditure grows with wages. The demographic impact on government expenditure is accounted for by extrapolating given age profiles for each type of expenditure.

Households

The life cycle model provides the basic theoretical framework for modelling household behaviour.

According to the life cycle theory, households rationally choose levels of current and future consumption. Labour supply is age-dependent and endogenous. Labour supply features a zero income effect, so that there is no intertemporal substitution of labour. Households fully take into account the implicit tax in pension contributions. Every household is represented by a finitely-lived adult. Lifetime uncertainty is assumed to be diversified by letting each household receive an annuity from a life insurance company in return for bequeathing it its remaining assets upon its decease (Yaari (1965)). The model abstracts from bequests.

Firms

Production takes place with labour and capital according to a CES production technology. The Dutch economy is considered to be small relative to the outside world. Factor prices are determined on world markets. Goods produced at home are perfectly substitutable with those produced abroad. The firm can sell its product at the given market price. Capital deteriorates at a constant rate. The productivity of labour is assumed to depend on both age and calendar time. In particular, different age cohorts have different productivity levels. Apart from their productivity, labour supplied by households of different ages is homogeneous.

Firms maximize the discounted value of their future dividend flows. The dividend payments equal revenue minus the wage bill, corporate taxes and investments. The tax base for corporate taxes consists of revenues minus the wage bill and fiscal depreciation allowance. Fiscal depreciation is based on the historical cost price of investment, and is geometric.

Pension funds (second pillar)

Three different pension systems are distinguished: a final wage system, an average wage system and a defined contirbution (DC) system. The systems do not exist simultaneously but can be chosen with a set of dummy variables.

Private pension funds adjust their contribution rates and value of pension rights in order to achieve equilibrium at their actuarial balance. Equilibrium at the actuarial balance holds if the discounted value of future liabilities equals the sum of the discounted value of future premiums and current wealth. Liabilities consist of current and future benefits for present-day pensioners and of future claims of present-day workers. Old-age benefits are indexed to prices and partly to productivity, reflecting the financial situation for the average Dutch pension fund.

1.2 Notation conventions

Age is indexed to j. We distinguish three states over the life cycle: childhood ends at the age of $j_w - 1$; the working ages are j_w up to $j_r - 1$; and the retired period starts at the age j_r and ends at the age j_e. In the base run the values are set at $j_w = 20$; $j_r = 65$ and $j_e = 99$. Assume individuals of the same age are homogeneous. Stocks are measured at the end of a period. Stocks are indicated with a superscript s. A stock variable, $x^s(j)$, related to an individual of age j multiplied by the population size of the same age $b^s_{ah}(j)$ results in the cohort variable $b^s_{ah}(j)x^s(j)$. Assume, the population size with age j at the end of period t is relevant for the flows into period $t+1$. A flow variable, $x(j)$, related to an individual of age j multiplied by the population size of the previous year $b^s_{ah}(j-1,t-1)$, results in the cohort variable $b^s_{ah}(j-1,t-1)x(j)$. One population unit refers to thousand persons. Stocks and flows per population unit are also measured in thousands. This results in total values per cohort in millions. Macro variables are obtained by aggregating over the cohorts $10^{-3}\sum_j b^s_{ah}(j)x^s(j)$ and $10^{-3}\sum_j b^s_{ah}(j-1,t-1)x(j)$. So macro values are in billions. For convenience we represent only the relations for individuals. To make notation more clear cut, time indices are left out if all variables are related to the same period. A list of symbols is given at the end of the paper. Values are indicated with a capital letter, volumes with a lower case letter. Prices start with p, quotes with q, taxes and premiums with τ. Discounted values and summations of discounted values are expressed in a difference equation format because this is the format in which the model has been programmed.

1.3 Consumers

1.3.1 Budget

Assets change through savings, revaluation and intra-generational transfers

$$S^s_{ah}(j,t) = S^s_{ah}(j-1,t-1) + S_{heh}(j,t) + Y_{ah}(j,t) - X_{ah}(j,t) + G_{en}(j,t),\ j \in \{j_w,..,j_e\} \tag{1.1}$$

At the beginning of their working life (at age 19) people have no assets

$$S^s_{ih}(j) = 0,\ j \in \{0,..,j_w-1\},\ i = a,b,s \tag{1.2}$$

Individuals sell bonds if their financial wealth becomes negative. Individuals own a portfolio of bonds and shares if their financial wealth is positive. This leads to

$$S^s_{bh}(j) = \begin{cases} S^s_{ah}(j) \text{ if } S^s_{ah}(j) < 0,\ j \in \{j_w,..,j_e\} \\ qsbhS^s_{ah}(j) \text{ if } S^s_{ah}(j) > 0,\ j \in \{j_w,..,j_e\} \end{cases} \tag{1.3}$$

$$S_{bh}(j) = S^s_{bh}(j) - S^s_{bh}(j-1,t-1),\ j \in \{j_w,..,j_e\} \tag{1.4}$$

for bonds and to

$$S^s_{sh}(j) = \begin{cases} 0 \text{ if } S^s_{ah}(j) < 0,\ j \in \{j_w,..,j_e\} \\ (1-q_{sbh})S^s_{ah}(j) \text{ if } S^s_{ah}(j) > 0,\ j \in \{j_w,..,j_e\} \end{cases} \tag{1.5}$$

$$S_{sh}(j) = S^s_{sh}(j) - S^s_{sh}(j-1,t-1),\ j \in \{j_w,..,j_e\} \tag{1.6}$$

for shares. In case the households sell bonds, they have to partly pay the high interest rate

$$S^s_{bhn}(j) = q_{sbh}S^s_{ah}(j)\ ,\ j \in \{j_w,..,j_e\} \tag{1.7}$$

$$S^s_{bhr}(j) = \begin{cases} (1-q_{sbh})S^s_{ah}(j) \text{if } S^s_{ah}(j) < 0,\ j \in \{j_w,..,j_e\} \\ 0 \text{ if } S^s_{ah}(j) > 0 \end{cases} \tag{1.8}$$

Savings equal total income minus expenditure

$$S_{vh}(j) = Y_{ah}(j) - X_{ah}(j),\ j \in \{j_w,..,j_e\} \tag{1.9}$$

Intra-generational transfers

$$\begin{aligned} G_{en}(j) &= q_{bdh}(j)S^s_{ah}(j) \\ &= \frac{q_{bdh}(j)}{1-q_{bdh}(j)}\left(S^s_{ah}(j-1,t-1) + S_{heh}(j) + Y_{ah}(j) - X_{ah}(j)\right),\ j \in \{j_w,..,j_e-1\} \end{aligned} \tag{1.10}$$

Revaluation of assets

$$S_{heh}(j) = S^s_{sh}(j-1,t-1)\left(\frac{1+r_s(t)}{1+\frac{D_{iv}(t)}{S^s_{se}(t)}} - 1\right),\ j \in \{j_w,..,j_e\} \tag{1.11}$$

Inflation correction

$$s_{ac}(j) = s^s_{ah}(j-1,t-1)\left(\frac{p(t-1)}{p(t)} - 1\right),\ j \in \{j_w,..,j_e\} \tag{1.12}$$

Changed wealth through immigration and emigration

$$S^h_{ab}(j,t) = q_{beh}(j,t)\left[S^s_{ah}(j-1,t-1) + S_{heh}(j,t) + S_{vh}(j,t)\right] \tag{1.13}$$

$$S^h_{am}(j,t) = q_{bih}(j,t)\left[S^s_{ah}(j-1,t-1) + S_{heh}(j,t) + S_{vh}(j,t)\right] \tag{1.14}$$

1.3.2 Income

Individual's income includes transfers, old age benefits (second pillar), labour income and capital income.

$$Y_{ah}(j) = Y_{th}(j) + P^b_u(j) + Y_{wa}(j) + Y_{zh}(j),\ j \in \{0,..,j_e\} \tag{1.15}$$

Total transfer income includes social and children's assistance, public old-age benefits (first pillar), disability and unemployment benefits

$$Y_{th}(j) = B_{ijs}(j) + A_{kws}(j) + A_{owo}(j) + W_{aoz}(j) + W_{klh}(j),\ j \in \{0,..,j_e\} \tag{1.16}$$

Individuals labour income

$$Y_{wa}(j,t) = \frac{l_{de}(j,t)}{b^s_{ah}(j-1,t-1)}p_{ywe}(j,t) + \frac{l_{dp}(j,t)}{b^s_{ah}(j-1,t-1)}p_{ywp}(j,t),\ j \in \{j_w,..,j_e\} \tag{1.17}$$

Total capital income equals interest income plus dividend income

$$Y_{zh}(j) = Z_{bh}(j) + Z_{sh}(j),\ j \in \{j_w,..,j_e\} \tag{1.18}$$

Interest income

$$Z_{bh}(j,t) = r_b(t)S^s_{bhn}(j-1,t-1) + (r_b(t) + r_{isk})S^s_{bhr}(j-1,t-1),\ j \in \{j_w,..,j_e\} \tag{1.19}$$

Dividend income

$$Z_{sh}(j,t) = D_{iv}(t)\frac{S^s_{sh}(j-1,t-1)}{S^s_{se}(t)\left(\frac{1+\frac{D_{iv}(t)}{S^s_{se}(t)}}{1+r_s(t)}\right)},\ j \in \{j_w,..,j_e\} \tag{1.20}$$

1.3.3 Taxes

The total tax bill of households consists of consumption taxes and income taxes

$$T_{ah}(j) = T_{co}(j) + L_{ito}(j),\ j \in \{j_w,..,j_e\} \tag{1.21}$$

Consumption tax

$$T_{co}(j) = \tau_{co} C_{nh}(j),\ j \in \{0,..,j_e\} \tag{1.22}$$

Income taxes

$$L_{ito}(j) = L_{iak}(j) + L_{iuk}(j) + L_{i65}(j) + L_{iap}(j) + L_{iki}(j) + L_{iov}(j),\ j \in \{j_w,..,j_e\} \tag{1.23}$$

Taxes (including public pensions) on labour income

$$L_{iak}(j) = (\tau_{iak} + \tau_{pp}) \left[Y_{wa}(j) - P_p^b(j) \right],\ j \in \{j_w,..,j_r - 1\} \tag{1.24}$$

Tax (including public pensions) on transfer income

$$L_{iuk}(j) = (\tau_{liuk} + \tau_{pp}) \left[B_{ijs}(j) + W_{aoz}(j) + W_{klh}(j) \right],\ j \in \{j_w,..,j_r - 1\} \tag{1.25}$$

Tax on pensions

$$L_{i65}(j) = (\tau_{li65}) \left[A_{owo}(j) + P_u^b(j) \right],\ j \in \{j_r,..,j_e\} \tag{1.26}$$

Tax on annuities

$$L_{iap}(j,t) = L_{iap}(j,t-1) \frac{p_{ywee}(t)}{p_{ywee}(t-1)} \frac{p_{ro}(t)}{p_{ro}(t-1)} \frac{p_{gef}(t)}{p_{gef}(t-1)} L_{iap}^{au}(t),\ j \in \{0,..,j_e\} \tag{1.27}$$

Other income taxes

$$L_{iov}(j,t) = L_{iov}(j,t-1) \frac{p_{ywee}(t)}{p_{ywee}(t-1)} \frac{p_{ro}(t)}{p_{ro}(t-1)} \frac{e_{vut}(j,t)}{e_{vut}(j,t-1)} L_{iov}^{au}(t),\ j \in \{0,..,j_e\} \tag{1.28}$$

Public pension contribution (AOW)

$$T_{pp}(j) = \tau_{pp} \left[B_{ijs}(j) + W_{aoz}(j) + W_{klh}(j) + Y_{wa}(j) - P_p^b(j) \right],\ j \in \{j_w,..,j_r - 1\} \tag{1.29}$$

Tax on financial wealth

$$L_{iki}(j) = \tau_{liki} S_{ah}^s(j-1,t-1),\ j \in \{j_w,..,j_e\} \tag{1.30}$$

1.3.4 Expenditures

Total individual expenditure includes consumption and taxes

$$X_{ah}(j) = C_{nh}(j) + T_{ah}(j) + P_p^b(j),\ j \in \{0,..,j_e\} \tag{1.31}$$

Gross individual expenditures on consumption equal net outlays on consumption plus indirect tax payments

$$C_{onp}(j) = C_{nh}(j) + T_{co}(j),\ j \in \{j_w,..,j_e\} \tag{1.32}$$

Nominal net outlays on consumption equal the real net outlays times the price index

$$C_{nh}(j) = pc_{nh}(j), \; j \in \{j_w, .., j_e\} \tag{1.33}$$

At the end of the life cycle people consume their total wealth

$$c_{nh}(j_e,t) = \frac{S^s_{ah}(j_e-1,t-1) + S_{heh}(j_e,t) + Y_{ah}(j_e,t) - T_{ah}(j_e,t)}{p(t)} \tag{1.34}$$

Consumption

$$c_{nh}(j,t) = \left(\frac{1+\beta}{1+r_{hr}(j+1,t+1)} \frac{1+\tau_{co}(t+1)}{1+\tau_{co}}\right)^{\gamma} c_{nh}(j+1,t+1), \; j \in \{j_w, .., j_e-1\} \tag{1.35}$$

and for the last simulation year

$$c_{nh}(j,T) = \left(\frac{1+\beta}{1+r_{hr}(j+1,T)}\right)^{\gamma} c_{nh}(j+1,T) \frac{p_{ro}(T)}{p_{ro}(T-1)}, \; j \in \{j_w, .., j_e-1\} \tag{1.36}$$

with T being the last simulation year.

Discount factor

$$d_{ish}(t) = d_{ish}(t-1) \frac{1}{1+r_{hr}(t)} \tag{1.37}$$

1.3.5 Human wealth

Human wealth has been written as a difference equation

$$h(j,t) = y(j,t) + \frac{1-q_{bdh}(j,t)}{1+r_{hr}(t+1)} h(j+1,t+1), \; j \in \{j_w, .., j_e-1\} \tag{1.38}$$

$$h(j_e,t) = y(j_e,t) \tag{1.39}$$

$$y(j) = y_{th}(j) + p^b_u(j) + y_{wa}(j) - \left(l_{iak}(j) + l_{iuk}(j) + l_{i65}(j) + l_{iap}(j) + l_{iov}(j) + p^b_p(j)\right) \tag{1.40}$$

1.4 Firms

1.4.1 The value of the firm

The value of a private firm equals the value of the capital stock with a correction for taxes and depreciation allowances minus the value of firms owned by the government.

$$S^s_{se} = \left(1 - \frac{\tau_{vpb} v}{v + r_s}\right) pk^s_{ae} - \frac{(1-\tau_{vpb}) r_b + b_0}{r_s + b_0} S_{be} + \tau_{vpb} \left(\frac{v}{r_s + v} i_{ge}(t) p(t) + \frac{1-v}{r_s+v} A_f(t)\right) - K^s_h(t) \tag{1.41}$$

$$K_h^s(t) = \left[G_{asb}(t+1) + V_{mid}(t+1) + (1-\tau_{vpb})V_{mip}(t+1) + P_e\right](1+r_s)^{-1}$$
$$+ K_h^s(t+1)(1+r_s)^{-1} \tag{1.42}$$

Minus the hidden reserves of firms

$$R_{ee}^s = S_{se}^s + S_{be}^s + K_h^s - K_{ae}^s \tag{1.43}$$

$$S_{se}(t) = S_{se}^s(t) - S_{se}^s(t)\frac{1+\frac{D_{iv}(t)}{S_{se}^s(t)}}{1+r_s(t)} \tag{1.44}$$

Total liabilities of firms

$$S_e = S_{be} + S_{se} \tag{1.45}$$

1.4.2 The budget

Profits

$$P_{rf}(t) = Y_{ge}(t) - Y_{we}(t) - r_b(t)S_{be}^s(t-1) - V_{mip}(t) \tag{1.46}$$

Dividend payments

$$D_{iv} = P_{rf} - I_{ge} - V_{pdb} - G_{asb} - V_{mid} - P_e + \Delta S_{be}^s \tag{1.47}$$

Other income payments

$$Y_{ze} = D_{iv} + G_{as} + V_{mid} + V_{mip} + P_e \tag{1.48}$$

The government levies a proportional tax on net profits

$$V_{pdb} = \tau_{vpb}\left[P_{rf} - A_f\right] \tag{1.49}$$
$$V_{pdb}(j) = \frac{v_{pdb}^{ci}(j)}{v_{pdb}^{ci}}V_{pdb} \tag{1.50}$$

with A_f being the fiscal depreciation allowance. Fiscal depreciation is based on the historical cost price of investment .

$$A_f(t) = \nu A^s(t-1)$$
$$A^s(t) = (1-\nu)A^s(t-1) + p(t)i_{ge}(t) \tag{1.51}$$

1.4.3 Production, capital and labour demand.

Production

$$y_{ge} = \left(\kappa k^s_{ae}(t-1)^{\frac{\sigma-1}{\sigma}} + \alpha l_{ee}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{1.52}$$

$$Y_{ge} = p y_{ge} \tag{1.53}$$

$$Y_{ne} = Y_{ge} - I_{de} \tag{1.54}$$

$$y_{ne} = \frac{Y_{ne}}{p} \tag{1.55}$$

Capital

$$k^s_{ae}(t) = l_{ee}(t+1)\left(\frac{\kappa}{\alpha}\frac{p_{ywee}(t+1)10^{-3}}{p_{k_{ae}}(t+1)}\right)^{\sigma} \tag{1.56}$$

$$K^s_{ae} = p k^s_{ae} \tag{1.57}$$

Marginal product of capital

$$p_{k^s_{ae}} = \frac{1}{1-\tau_{vpb}}\left(\frac{p(t-1)}{p}(1+r_s) - (1-\delta_s)\right)\times \tag{1.58}$$

$$\left[1 - \frac{\tau_{vpb}\nu}{\nu + r_s} - b_1 + \frac{(1-\tau_{vpb})r_b + b_0}{r_s + b_0} b_1\right] p \tag{1.59}$$

Gross investments

$$I_{ge}(t) = K^s_{ae}(t) - (1-\delta_s)K^s_{ae}(t-1)\frac{p(t)}{p(t-1)} \tag{1.60}$$

Capital deteriorates at a fixed rate

$$I_{de}(t) = \delta_s K^s_{ae}(t-1)\frac{p(t)}{p(t-1)} \tag{1.61}$$

Net investment equals gross investment minus depreciation

$$I_{ne} = I_{ge} - I_{de} \tag{1.62}$$

Employment and wages

$$p_{ywee} = 10^3 \alpha^{-\frac{\sigma}{1-\sigma}} p \left[1 - \kappa^{\sigma}\left(\frac{p_{k_{ae}}}{p}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{1.63}$$

$$p_{ywe}(j) = p_{ywee} l_{sh}^{ci}(j) p_{ro},\ j \in \{j_w, .., j_e\} \tag{1.64}$$

$$F_{l_{ee}} = \left(\frac{y_{ge}}{l_{ee}}\right)^{\frac{1}{\sigma}} \alpha \text{ with } l_{ee} = \sum_j l_{de}(j) l_{sh}^{ci}(j) p_{ro},\ j \in \{j_w, .., j_e\} \tag{1.65}$$

$$l_{de}(j) = \frac{l_{de}}{l_{da}} l_{da}(j) \text{ with } l_{de} = l_{da} - l_{dp},\ j \in \{j_w, .., j_e\} \tag{1.66}$$

$$Y_{we} = \sum_j p_{ywe}(j) l_{de}(j) 10^{-3},\ j \in \{j_w, .., j_e\} \tag{1.67}$$

$$p_{ywe} = \frac{\sum_j p_{ywe}(j) l_{de}(j)}{l_{de}} \tag{1.68}$$

1.5 Government

1.5.1 Public debt and assets

Government debt, deficit and wealth

$$O_{vhs}^s(t) = O_{vhs}^s(t-1) + F_{ntk}(t) \tag{1.69}$$

$$F_{ntk} = V_{rtk} + F_{acr} + F_{ine} \tag{1.70}$$

$$O_{vvm}^s = K_{gto}^s + F_{acr}^s + F_{acb}^s + K_{gga}^s + P_{cht}^s + D_{nba}^s - O_{vhs}^s \tag{1.71}$$

Government investments in firms

$$K_{h2}^s = K_{gga}^s + P_{cht}^s + D_{nba}^s \tag{1.72}$$

Net bond position of the government

$$S_{bg}^s = O_{vhs}^s - F_{acr}^s \tag{1.73}$$

EMU deficit

$$V_{rtk} = U_{ito} - I_{kto} \tag{1.74}$$

$$V_{rt1} = V_{rtk} - D_{tot} \tag{1.75}$$

Net change in bonds issued by the government

$$S_{bg} = V_{rtk} + F_{acb} \tag{1.76}$$

Central bank assets

$$D_{nba}(t) = D_{nba}^s(t) - D_{nba}^s(t-1) \tag{1.77}$$

$$D_{nba}^s(t) = D_{nba}^s(t-1) \frac{Y_{ne}(t)}{Y_{ne}(t-1)} \tag{1.78}$$

Lease of parcels of land

$$P^s_{cht}(t) = P^s_{cht}(t-1)\frac{Y_{ne}(t)}{Y_{ne}(t-1)} \tag{1.79}$$

Capital

$$K^s_{gto} = K^s_{ggb} + K^s_{gif} + K^s_{gsh} \tag{1.80}$$

$$K^s_{gj}(t) = K_{gj}(t-1)\frac{p(t)}{p(t-1)} + I_{nj}(t);\ j = gb, if, sh \tag{1.81}$$

Financial assets

$$F_{acr}(t) = F^s_{acr}(t) - F^s_{acr}(t-1) \tag{1.82}$$

$$F^s_{acr}(t) = F^s_{acr}(t-1)\frac{Y_{ne}(t)}{Y_{ne}(t-1)} \tag{1.83}$$

$$F_{ine}(t) = F^s_{acb}(t) - F^s_{acb}(t-1)\frac{p(t)}{p(t-1)} \tag{1.84}$$

$$F_{acb}(t) = F^s_{acb}(t) - F^s_{acb}(t-1) \tag{1.85}$$

$$F^s_{acb}(t) = F^s_{acb}(t-1)\frac{Y_{ne}(t)}{Y_{ne}(t-1)} \tag{1.86}$$

1.5.2 Public expenditures

Employment and wages

$$l_{dp}(t) = l_{dp}(t-1)\frac{b_{lh}(t)}{b_{lh}(t-1)} \tag{1.87}$$

$$Y_{wp} = Y_{wa} - Y_{we} \tag{1.88}$$

Wage rate of the government

$$p_{ywp}(j,t) = p_{ywp}(j-1,t-1)\frac{p_{ywe}(j,t)}{p_{ywe}(j-1,t-1)} \tag{1.89}$$

Depreciation

$$D_{tot} = D_{gb} + D_{if} + D_{sh} \tag{1.90}$$

$$D_i(j,t) = \frac{1}{b^s_{ah}(t-1)}\delta K^s_{kj}(t-1)\frac{p(t)}{p(t-1)},\ i = gb, if \tag{1.91}$$

$$D_{sh}(j,t) = \frac{O^{ci}_{ndw}(j)}{O^{ci}_{ndw}}\delta K^s_{ksh}(t-1)\frac{p(t)}{p(t-1)} \tag{1.92}$$

Investment

$$I_{bto} = I_{bgb} + I_{bif} + I_{bsh} \quad (1.93)$$

$$I_{bif}(t) = I_{bif}(t-1)\frac{Y_{ne}(t)}{Y_{ne}(t-1)} \quad (1.94)$$

$$I_{bi}(j,t) = I_{bi}(j,t-1)\frac{p_{ywee}(t)}{p_{ywee}(t-1)}\frac{p_{ro}(t)}{p_{ro}(t-1)},\ j \in \{0,..,j_e\},\ i = gb,sh \quad (1.95)$$

$$I_{nto} = I_{ngb} + I_{nif} + I_{nsh} \quad (1.96)$$

$$I_{nj} = I_{bj} - D_j,\ j = gb,if,sh \quad (1.97)$$

Public consumption includes expenditures on defense, education, health care and public administration

$$C_{ap} = D_{fns} + O_{ndw} + Z_{org} + O_{pbs} + D_{tot} \quad (1.98)$$

Material consumption of the government equals government total consumption minus the government wage bill

$$C_{np} = C_{ap} - Y_{wo} \quad (1.99)$$

Clearing of land for building

$$N_{grv}(j,t) = N_{grv}(j,t-1)\frac{p_{ywee}(t)}{p_{ywee}(t-1)}\frac{p_{ro}(t)}{p_{ro}(t-1)},\ j \in \{0,..,j_e\} \quad (1.100)$$

Public outlays on health care per person grows with productivity, inflation and an endogenous age-specific index

$$Z_{org}(j,t) = Z_{org}(j,t-1)\frac{p_{ywee}(t)}{p_{ywee}(t-1)}\frac{p_{ro}(t)}{p_{ro}(t-1)}\frac{z^{au}_{org}(j,t)}{z^{au}_{org}(j,t-1)}Z^{au}_{org}(t),\ j \in \{0,..,j_e\} \quad (1.101)$$

The endogenous age-specific index depends on the health care costs related to death and on age-specific health costs

$$z^{au}_{org}(j) = q_{bdh}(j)s^{ci}_{tek} + (1-q_{bdh}(j))z^{ci}_{org}(j) + w^{ci}_{tz}(j) \quad (1.102)$$

Public outlays on education per person grows with productivity and inflation, and an exogenous index

$$O_{ndw}(j,t) = O_{ndw}(j,t-1)\frac{p_{ywee}(t)}{p_{ywee}(t-1)}\frac{p_{ro}(t)}{p_{ro}(t-1)}O^{au}_{ndw}(t),\ j \in \{0,..,j_e\} \quad (1.103)$$

Age-specific social and children's assistance, and public old-age benefits grow with labour productivity, price inflation and an exogenous index

$$B_{ijs}(j,t) = B_{ijs}(j,t-1)\frac{p_{ywee}(t)}{p_{ywee}(t-1)}\frac{p_{ro}(t)}{p_{ro}(t-1)}B^{au}_{ijs}(t),\ \ j \in \{0,..,j_e\} \tag{1.104}$$

$$A_{kws}(j,t) = A_{kws}(j,t-1)\frac{p_{ywee}(t)}{p_{ywee}(t-1)}\frac{p_{ro}(t)}{p_{ro}(t-1)}A^{au}_{kws}(t),\ \ j \in \{0,..,j_e\} \tag{1.105}$$

$$A_{owo}(j,t) = A_{owo}(j,t-1)\frac{p_{ywee}(t)}{p_{ywee}(t-1)}\frac{p_{ro}(t)}{p_{ro}(t-1)}A^{au}_{owo}(t),\ \ j \in \{0,..,j_e\} \tag{1.106}$$

Age-specific disability and unemployment benefits grow with labour productivity, price inflation, an exogenous index, and labour participation rate

$$W_{aoz}(j,t) = W_{aoz}(j,t-1)\frac{p_{ywee}(t)}{p_{ywee}(t-1)}\frac{p_{ro}(t)}{p_{ro}(t-1)}W^{au}_{aoz}(t)\frac{p_{gef}(t)}{p_{gef}(t-1)},\ \ j \in \{0,..,j_e\} \tag{1.107}$$

$$W_{klh}(j,t) = W_{klh}(j,t-1)\frac{p_{ywee}(t)}{p_{ywee}(t-1)}\frac{p_{ro}(t)}{p_{ro}(t-1)}W^{au}_{klh}(t)\frac{p_{gef}(t)}{p_{gef}(t-1)},\ \ j \in \{0,..,j_e\} \tag{1.108}$$

Government expenditures on public administration, on subsidies to firms, on defense and on transfers to foreigners grow with production enterprises and with an exogenous index

$$S_{ubs}(j,t) = \frac{S_{ubs}(t-1)}{b^s_{ah}(t-1)}\frac{Y_{ne}(t)}{Y_{ne}(t-1)}S^{au}_{ubs}(t) \tag{1.109}$$

$$O_{pbs}(j,t) = \frac{O_{pbs}(t-1)}{b^s_{ah}(t-1)}\frac{Y_{ne}(t)}{Y_{ne}(t-1)}O^{au}_{pbs}(t) \tag{1.110}$$

$$O_{vbu}(j,t) = \frac{O_{vbu}(t-1)}{b^s_{ah}(t-1)}\frac{Y_{ne}(t)}{Y_{ne}(t-1)}O^{au}_{vbu}(t) \tag{1.111}$$

$$D_{fns}(j,t) = \frac{D_{fns}(t-1)}{b^s_{ah}(t-1)}\frac{Y_{ne}(t)}{Y_{ne}(t-1)}D^{au}_{fns}(t) \tag{1.112}$$

Public interest payments

$$R_{utg}(t) = r_b O^s_{vhs}(t-1) \tag{1.113}$$

1.5.3 Public revenues

$$V_{mir}(t) = r_b(t)F^s_{acr}(t-1) \tag{1.114}$$

$$V_{mib}(t) = r_b(t)F^s_{acb}(t-1) \tag{1.115}$$

$$V_{mid}(t) = r_b(t)D^s_{nba}(t-1) \tag{1.116}$$

$$V_{mip}(t) = r_b(t)P^s_{cht}(t-1) \tag{1.117}$$

Total income from public assets

$$V_{mit} = V_{mir} + V_{mid} + V_{mip} + V_{mib} \tag{1.118}$$

Non-tax income of the government

$$Y_{zg} = V_{mit} + G_{asb} \tag{1.119}$$

Seigniorage grows with production enterprises

$$S_{gnr}(j,t) = \frac{I_{rto}(j,t)}{I_{rto}(t)} S_{gnr}(t-1) \frac{Y_{ne}(t)}{Y_{ne}(t-1)} \tag{1.120}$$

Indirect taxes

$$I_{rto} = T_{co} + I_{rin} \tag{1.121}$$

Indirect taxes on investment goods

$$I_{rin}(t) = I_{rin}(t-1) \frac{I_{ntc}(t)}{I_{ntc}(t-1)} \tag{1.122}$$

$$I_{rin}(j) = \frac{v^{ci}_{pdb}(j)}{v^{ci}_{pdb}} I_{rin} \tag{1.123}$$

1.5.4 National accounts data

Indirect taxes minus subsidies according to the national accounts

$$I_{nsu} = I_{rto} - S_{ubs} \tag{1.124}$$

Net public expenditures include public consumption, gross public investment, transfers to foreigners, subsidies, social security, and social insurance transfers

$$X_{np} = C_{ap} + I_{bto} + O_{vbu} + S_{ubs} + B_{ijs} + A_{kws} + A_{owo} + W_{aoz} + W_{klh} \tag{1.125}$$

Aggregate public expenditures equal net public expenditures plus interest payments.

$$U_{ito} = X_{np} + R_{utg} - D_{tot} \tag{1.126}$$

The aggregate tax burden by definition includes revenues from land sales and seigniorage

$$T_{ax} = N_{grv} + S_{gnr} + I_{rto} + L_{ito} + V_{pdb} \tag{1.127}$$

Aggregate public revenue equals gas revenues plus the tax burden (minus seigniorage) and income from public assets, and minus public expenditures on the acquisition of central bank assets and grounds.

$$I_{kto} = G_{asb} + (T_{ax} - S_{gnr}) + V_{mit} \tag{1.128}$$

1.5.5 Net profits from the government

Benefits

$$
\begin{aligned}
B_{ato}(j) = {} & D_{fns}(j) + O_{pbs}(j) + D_{geb}(j) + R_{geb}(j) + D_{inf}(j) + R_{inf}(j) \\
& + Z_{org}(j) + O_{ndw}(j) + D_{sch}(j) + R_{sch}(j) + S_{ubs}(j) + W_{klh}(j) \\
& + W_{aoz}(j) + B_{ijs}(j) + A_{owo}(j) + A_{kws}(j) + O_{vbu}(j) \qquad (1.129)
\end{aligned}
$$

Benefits from government buildings

$$R_{geb}(j,t) = \frac{1}{b^s_{ah}(t-1)} r_{br}(t) K_{ggb}(t) \qquad (1.130)$$

$$R_{inf}(j,t) = \frac{1}{b^s_{ah}(t-1)} r_{br}(t) K_{gif}(t) \qquad (1.131)$$

$$R_{sch}(j,t) = \frac{1}{b^s_{ah}(t-1)} r_{br}(t) K_{gsh}(t) \qquad (1.132)$$

Costs

$$L_{sto}(j) = L_{ito}(j) + V_{pdb}(j) + I_{rto}(j) + N_{grv}(j) + S_{gnr}(j) \qquad (1.133)$$

Net profits

$$N_{prf}(j) = B_{ato}(j) - L_{sto}(j) \qquad (1.134)$$

Discounted net benefits from the government

$$p_{g2}(j,t) = \frac{1}{1+r_{br}} N_{prf}(j,t) + \frac{1-q_{bdh}(j,t)}{1+r_{br}} p_{g2}(j+1,t+1),\ j \in \{j_w,..,j_e-1\} \qquad (1.135)$$

$$p_{g2}(j_e,t) = \frac{1}{1+r_{br}} N_{prf}(j_e,t) \qquad (1.136)$$

$$p_{g2}(j,t) = p_{g12}(j_w, t+j_w-j)\ ,\ j < j_w \qquad (1.137)$$

with

$$p_{g12}(j,t) = p_{g2}(j,t) d_{isc}(t),\ j \in \{j_w,..,j_e-1\} \qquad (1.138)$$

discount factor

$$d_{isc}(t) = d_{isc}(t-1) \frac{1}{1+r_{br}(t)} \qquad (1.139)$$

1.6 Private pension funds

We do not exactly follow the pension study, but a later version (GAMMA7e).

Pension premium rate

$$\tau^b_{plb} = \delta^f_1 \tau^f_{plb} + \delta^a_1 \tau^a_{plb} + \delta^d_1 \tau^d_{plb} \tag{1.140}$$

Basis premium

$$\tau^b_{plb1} = \delta^f_1 \tau^f_{plb1} + \delta^a_1 \tau^a_{plb1} + \delta^d_1 \tau^d_{plb1} \tag{1.141}$$

Catching up premium

$$\tau^b_{plb2} = \delta^f_1 \tau^f_{plb2} + \delta^a_1 \tau^a_{plb2} + \delta^d_1 \tau^d_{plb2} \tag{1.142}$$

The premium receipts

$$P^b_p(j,t) = \delta^f_1 P^f_p(j,t) + \delta^a_1 P^a_p(j,t) + \delta^d_1 P^d_p(j,t) \tag{1.143}$$

Premium base

$$G^b_{pp}(j,t) = G(j,t)\frac{l_{da}(j,t)}{b^s_{ah}(j-1,t-1)} \tag{1.144}$$

$$G(j,t) = S_{lo}(j,t) - F_{ran}(t) \tag{1.145}$$

Gross wages

$$S_{lo}(j,t) = S_{lo}(j,t-1)\frac{p_{ywee}(t)p_{ro}(t)}{p_{ywee}(t-1)p_{ro}(t-1)} \tag{1.146}$$

Franchise

$$F_{ran}(t) = F_{ran}(t-1)\left(\frac{p_{ywee}(t)p_{ro}(t)}{p_{ywee}(t-1)p_{ro}(t-1)}\right)^{\delta^{in}_{aow}} \tag{1.147}$$

Pension benefits

$$P^b_u(j,t) = \delta^f_1 P^f_u(j,t) + \delta^a_1 P^a_u(j,t) + \delta^d_1 P^d_u(j,t) \tag{1.148}$$

Financial wealth at the end of the year

$$V^{bs}_m(t) = V^{fs}_m(t) + V^{as}_m(t) + V^{ds}_m(t) \tag{1.149}$$

Financial wealth at the start of the year

$$V^{bs}_{mb}(t) = V^{fs}_{mb}(t) + V^{as}_{mb}(t) + V^{ds}_{mb}(t) \tag{1.150}$$

Change financial wealth

$$V_m^b(t) = V_m^f(t) + V_m^a(t) + V_m^d(t) \tag{1.151}$$

Bonds

$$S_{bp}^s(t) = q_{sbp} V_m^b(t) \tag{1.152}$$

$$S_{bp}(t) = S_{bp}^s(t) - S_{bp}^s(t-1) \tag{1.153}$$

Shares

$$S_{sp}^s(t) = (1 - q_{sbp}) V_m^b(t) \tag{1.154}$$

$$S_{sp}(t) = S_{sp}^s(t) - S_{sp}^s(t-1) \tag{1.155}$$

Savings

$$S_{vp}(t) = Y_{zp}(t) + P_p^b(t) - P_u^b(t) \tag{1.156}$$

Revaluation of assets

$$S_{hep} = S_{sp}^s(t-1) \left(\frac{1 + r_s(t)}{1 + \frac{D_{iv}(t)}{S_{se}^s(t)}} - 1 \right) \tag{1.157}$$

Pension capital export and import through migration

$$S_{am}^p(j,t) = \delta_1^f S_{am}^f(j,t) + \delta_1^a S_{am}^a(j,t) + \delta_1^d S_{am}^d(j,t) \tag{1.158}$$

$$S_{ab}^p(j,t) = \delta_1^f S_{ab}^f(j,t) + \delta_1^a S_{ab}^a(j,t) + \delta_1^d S_{ab}^d(j,t) \tag{1.159}$$

Total capital income equals interest income plus dividend income

$$Y_{zp} = Z_{bp} + Z_{sp} + P_e \tag{1.160}$$

Interest income

$$Z_{bp}(t) = r_b(t) S_{bp}^s(t-1) \tag{1.161}$$

Dividend income

$$Z_{sp}(t) = D_{iv}(t) \frac{S_{sp}^s(t-1)}{S_{se}^s(t) \frac{1 + \frac{D_{iv}(t)}{S_{se}^s(t)}}{1 + r_s(t)}} \tag{1.162}$$

Acquired rights of pensioners and workers

$$P_r^b = \delta_1^f P_r^f + \delta_1^a P_r^a + \delta_1^d P_r^d \tag{1.163}$$

Pensioner's discount factor for their pension rights

$$c_{wp}(j,t) = \begin{cases} \frac{p(t)}{p(t-1)} \frac{p_{ro}(t)}{p_{ro}(t-1)} \frac{\xi(j,t)}{1+r_p(t+1)} \left(1+c_{wp}(j+1,t+1)\right), \; j \in \{j_r,..,j_e-1\} \\ \text{in the DC system} \\ \frac{\xi(j,t)}{1+r_b(t+1)} \left(1+c_{wp}(j+1,t+1)\right), \; j \in \{j_r,..,j_e-1\} \\ \text{in the DB sytems} \end{cases} \tag{1.164}$$

$$c_{wp}(j_e,t) = 0 \tag{1.165}$$

Change cohort

$$\xi(j,t) = (1-q_{bdh}(j,t))(1-q_{bih}(j,t)+q_{beh}(j,t)) \tag{1.166}$$

Passed working years

$$d_j(j_w-1,t) = 1.1 \tag{1.167}$$

$$d_j(j,t) = d_j(j-1,t-1) + \frac{l_{da}(j,t)}{b^s_{ah}(j-1,t-1)} \tag{1.168}$$

Future working years

$$d_{jt}(j_e,t) = 0 \tag{1.169}$$

$$d_{jt}(j,t) = d_{jt}(j+1,t+1) + \frac{l_{da}(j+1,t+1)}{b^s_{ah}(j,t)} \tag{1.170}$$

Discount factor pension rights

$$d_{rw}(j,t) = \frac{\xi(j,t)}{1+r_b(t+1)} d_{rw}(j+1,t+1) \text{ and } j = j_w,.., \; j_r-2 \tag{1.171}$$

$$d_{rw}(j_r-1,t) = \frac{\xi(j_r-1,t)}{1+r_b(t+1)} \tag{1.172}$$

Worker's discount factor for their premium base

$$C^h_{lt}(j,t) = \begin{cases} \frac{G^b_{pp}(j+1,t+1)}{G^b_{pp}(j,t)} \frac{\xi(j,t)}{1+r_p(t+1)} \left(1+C^h_{lt}(j+1,t+1)\right), \; j \in \{j_w,..,j_r-2\} \\ \text{DC system} \\ \frac{\xi(j,t)}{1+r_b(t+1)} \left(1+C^h_{lt}(j+1,t+1)\right), \; j \in \{j_w,..,j_r-2\} \\ \text{DB system} \end{cases} \tag{1.173}$$

$$C^h_{lt}(j_r-1,t) = 0 \tag{1.174}$$

The discounted value of the premium base of a worker aged j

$$C_{lt}(j,t) = [1+r_p(t)]^{-1} G^b_{pp}(j,t) \left(1+C^h_{lt}(j,t)\right), \; j \in \{j_w,..,j_r-1\} \tag{1.175}$$

Coverage rate

$$q_{ka}(t) = \frac{V^b_{mb}(t)}{P^b_r(t)} \tag{1.176}$$

1.6.1 Final wage system

Pension premium rate

$$\tau^{f}_{plb} = \tau^{f}_{plb1} + \tau^{f}_{plb2} \tag{1.177}$$

Basis premium

$$\tau^{f}_{plb1} = \frac{P^{f}_{t}}{C_{lt}} \tag{1.178}$$

Catching up premium

$$\tau^{f}_{plb2} = \begin{cases} \min\left\{0.5 - \tau^{f}_{plb1}, \tau^{f}_{plbh}\right\} \\ \\ \tau^{f}_{pbh} \quad \text{if} \quad \tau^{f}_{\tau lbh} < 0 \end{cases} \tag{1.179}$$

with

$$\tau^{f}_{plbh} = \frac{P^{b}_{r}}{G^{b}_{pp}(t-1)}\left[\lambda q^{s}_{ka}(t) + (1-\lambda) q_{ka}(t-1) - q^{fau}_{ka}(t)\right] \tag{1.180}$$

$$q^{fau}_{ka}(t) = \frac{1}{P^{b}_{r}}\Big[(1+r_{pe}(t-1))V^{b}_{mb}(t-1) + \tag{1.181}$$

$$+\tau^{b}_{plb1}G^{b}_{pp}(t-1) - P^{b}_{u}(t-1) - S_{abp}(t-1) + S_{amp}(t-1)\Big]$$

The premium receipts

$$P^{f}_{p}(j,t) = \tau^{f}_{plb}G^{b}_{pp}(j,t),\ j \in \{j_w, .., j_r - 1\} \tag{1.182}$$

Pension benefit

$$P^{f}_{u}(j,t) = \begin{cases} P^{f}_{u}(j-1,t-1)\left(\frac{p(t)}{p(t-1)}\right)^{p^{ci}}\left(\frac{p_{ro}(t)}{p_{ro}(t-1)}\right)^{p^{ci}_{ro}}\left(\frac{p_{ywee}(t)p(t-1)}{p_{ywee}(t-1)p(t)}\right)^{p^{ci}_{l}} \\ ,\ j \in \{j_r + 1, .., j_e\} \\ o^{ci}_{pb} d_j(j_r - 1, t-1) G(j_r - 1, t-1)\left(\frac{p(t)}{p(t-1)}\right)^{p^{ci}} \times \\ \times\left(\frac{p_{ro}(t)}{p_{ro}(t-1)}\right)^{p^{ci}_{ro}}\left(\frac{p_{ywee}(t)p(t-1)}{p_{ywee}(t-1)p(t)}\right)^{p^{ci}_{l}},\ j = j_r \end{cases} \tag{1.183}$$

Acquired rights of pensioners and workers

$$P^{f}_{r} = P^{f}_{r1} + P^{f}_{r2} \tag{1.184}$$

The present value of total pension benefits of a pensioner aged j

$$P^{f}_{r1}(j,t) = [1 + r_p(t)]^{-1} P^{f}_{us}(j,t)(1 + c_{wp}(j,t)),\ j \in \{j_r, .., j_e\} \tag{1.185}$$

Acquired rights of pensioners and workers without current year

$$P_{rh}^{f} = P_{r1h}^{f} + P_{r2}^{f} \tag{1.186}$$

The present value of total pension benefits of a pensioner aged j, excluding current year

$$P_{r1h}^{f}(j,t) = [1 + r_p(t)]^{-1} P_u^{f}(j,t) c_{wp}(j,t),\ j \in \{j_r, .., j_e\} \tag{1.187}$$

The discounted value of future pension benefits of a present-day worker aged j

$$P_w^{f}(j,t) = P_{r1}^{f}(j_r,t) d_{rw}(j,t) \tag{1.188}$$

of which already built up

$$P_{r2}^{f}(j,t) = \frac{d_j(i,t)}{d_j(i,t) + d_{jt}(i,t)} \frac{G(j,t)}{G(j_r - 1,t)} P_w^{f}(j,t) \tag{1.189}$$

and to be built up

$$P_t^{f}(j,t) = \frac{d_{jt}(i,t)}{d_j(i,t) + d_{jt}(i,t)} \frac{G(j,t)}{G(j_r - 1,t)} P_w^{f}(j,t) \tag{1.190}$$

Wealth at the start of the year

$$V_{mb}^{fs}(t) = V_m^{fs}(t-1) + V_{mau}^{fs} \tag{1.191}$$

Wealth at the end of the year

$$V_m^{fs}(t) = V_{mb}^{fs}(t) + V_m^{f}(t) \tag{1.192}$$

Change in wealth

$$V_m^{f}(t) = \delta_1^{f} \left(S_{vp}(t) + S_{hep}(t)\right) + S_{am}^{f}(t) - S_{ab}^{f}(t) \tag{1.193}$$

Change in wealth through immigration and emigration

$$S_{am}^{f}(j,t) = q_{bih}(j,t) \left[P_{r1}^{f}(j,t) + P_{r2}^{f}(j,t)\right] \tag{1.194}$$

$$S_{ab}^{f}(j,t) = q_{beh}(j,t) \left[P_{r1}^{f}(j,t) + P_{r2}^{f}(j,t)\right] \tag{1.195}$$

1.6.2 Average wage system

Pension premium rate

$$\tau_{plb}^{a} = \tau_{plb1}^{a} + \tau_{plb2}^{a} \tag{1.196}$$

Basis premium

$$\tau^a_{plb1} = \frac{P^a_t}{C_{lt}} \tag{1.197}$$

Catching up premium

$$\tau^a_{plb2} = \begin{cases} \min\left\{0.5 - \tau^a_{plb1}, \tau^a_{plbh}\right\} \\ \\ \tau^a_{pbh} \quad \text{if} \quad \tau^a_{\tau lbh} < 0 \end{cases} \tag{1.198}$$

with

$$\tau^a_{plbh} = \frac{P^b_r}{G^b_{pp}(t-1)}\left[\lambda q^s_{ka}(t) + (1-\lambda)q_{ka}(t-1) - q^{aau}_{ka}(t)\right] \tag{1.199}$$

$$q^{aau}_{ka}(t) = \frac{1}{P^b_r}\Big[(1+r_{pe}(t-1))V^b_{mb}(t-1) + \tag{1.200}$$

$$+\tau^b_{plb1}G^b_{pp}(t-1) - P^b_u(t-1) - S_{abp}(t-1) + S_{amp}(t-1)\Big]$$

The premium receipts

$$P^a_p(j,t) = \tau^a_{plb}G^b_{pp}(j,t), \; j \in \{j_w,..,j_r-1\} \tag{1.201}$$

Pension benefit

$$P^b_u(j,t) = \begin{cases} G_{em}(j-1,t-1)\left(\frac{p(t)}{p(t-1)}\right)^{p^{ci}}\left(\frac{p_{ro}(t)}{p_{ro}(t-1)}\right)^{p^{ci}_{ro}}\left(\frac{p_{ywee}(t)p(t-1)}{p_{ywee}(t-1)p(t)}\right)^{p^{ci}_l}, \\ j \in \{j_r,..,j_e\} \;\; \text{DB average wage} \end{cases} \tag{1.202}$$

Rights in the average wage system

$$G^b_{em}(j,t) = \begin{cases} o^{ci}_{pb}d_j(j-1,t-1)G(j-1,t-1) \;\; j \in \{j_w..j_r-1\} \\ \text{before introduction average wage} \\ P^f_u(j,t) j \in \{66,..,j_e\} \text{ before introduction average wage} \\ G^b_{em}(j-1,t-1)\left(\frac{p(t)}{p(t-1)}\right)^{p^{ci}}\left(\frac{p_{ro}(t)}{p_{ro}(t-1)}\right)^{p^{ci}_{ro}}\left(\frac{p_{ywee}(t)p(t-1)}{p_{ywee}(t-1)p(t)}\right)^{p^{ci}_l} + \\ o_{pmi}\frac{l_{da}(j,t)}{b_{ah}(j-1,t-1)}G(j,t) \end{cases} \tag{1.203}$$

Acquired rights of pensioners and workers

$$P^a_r(j,t) = P^a_{r1}(j,t) + P^a_{r2}(j,t) \tag{1.204}$$

The present value of total pension benefits of a pensioner aged j

$$P^a_{r1}(j,t) = [1+r_p(t)]^{-1}P^a_u(j,t)\,(1+c_{wp}(j,t)), \; j \in \{j_r,..,j_e\} \tag{1.205}$$

Acquired rights of pensioners and workers excluding the current year

$$P^a_{rh}(j,t) = P^a_{r1h}(j,t) + P^a_{r2}(j,t) \tag{1.206}$$

The present value of total pension benefits of a pensioner aged j, excluding the current year

$$P^a_{r1h}(j,t) = [1 + r_p(t)]^{-1} P^a_u(j,t) c_{wp}(j,t),\ j \in \{j_r, .., j_e\} \tag{1.207}$$

The discounted value of future pension benefits of a present-day worker aged j

$$P^a_w(j,t) = P^a_{r1}(j_r,t) d_{rw}(j,t)$$

of which already built up

$$P^a_{r2}(j,t) = \frac{d_j(i,t)}{d_j(i,t) + d_{jt}(i,t)} \frac{G(j,t)}{G(j_r - 1,t)} P^a_w(j,t) \tag{1.208}$$

and to be built up

$$P^a_t(j,t) = \frac{d_{jt}(i,t)}{d_j(i,t) + d_{jt}(i,t)} \frac{G(j,t)}{G(j_r - 1,t)} P^a_w(j,t) \tag{1.209}$$

Wealth at the start of the year

$$V^{as}_{mb}(t) = V^{as}_m(t-1) + V^{as}_{mau} \tag{1.210}$$

Wealth at the end of the year

$$V^{as}_m(t) = V^{as}_{mb}(t) + V^a_m(t) \tag{1.211}$$

Change in wealth

$$V^a_m(t) = \delta^a_1 \left(S_{vp}(t) + S_{hep}(t) \right) + S^a_{am}(t) - S^a_{ab}(t) \tag{1.212}$$

Change in wealth through immigration and emigration

$$S^a_{am}(j,t) = q_{bih}(j,t) \left[P^a_{r1}(j,t) + P^a_{r2}(j,t) \right] \tag{1.213}$$

$$S^a_{ab}(j,t) = q_{beh}(j,t) \left[P^a_{r1}(j,t) + P^a_{r2}(j,t) \right] \tag{1.214}$$

1.6.3 Defined Contribution

Discounted value pension target

$$P^d_w(j,t) = B_{str} d^d_r(j,t) \tag{1.215}$$

Pension target

$$B_{str}(t) = (1+c_{wp}(j_r,t))G(j_r-1,t-1)\ o_{pdc}\ d_j(j_r,t) \tag{1.216}$$

Premium rate

$$\tau^d_{plb}(t) = \frac{P_p(t)}{G_{pp}(t)} \tag{1.217}$$

$$\tau^d_{plb}(j,t) = \tau^d_{plb1}(j,t) + \tau^d_{plb2}(j,t)$$

$$\tau^d_{plb2}(j,t) = 0$$

$$\tau^d_{plb1}(t,j) = \begin{cases} \frac{P^d_w(j,t)-V^d_{mb}(j,t)}{C_{lt}(j,t)},\ j \in \{j_w+1..j_r-1\} \ \text{ in year of transition} \\ \tau^d_{plb}(t-1,j-1),\ j \in \{j_w+1..j_r-1\} \\ \frac{P^d_w(j,t)}{C_{lt}(j,t)},\ j = j_w \end{cases} \tag{1.218}$$

The premium receipts

$$P^d_p(j,t) = \tau^d_{plb}(j,t)G^b_{pp}(j,t),\ j \in \{j_w,..,j_r-1\} \tag{1.219}$$

Pension benefits

$$P^b_u(j,t) = V^s_{mb}(j,t)\frac{1+r_p}{1+c_{wp}(j,t)},\ j \in \{j_r,..,j_e\} \ \text{ DC} \tag{1.220}$$

Discount factor

$$d^d_r(j,t) = \frac{p(t)p_{ro}(t)}{p(t-1)p_{ro}(t-1)}\frac{\xi(j,t)}{1+r_p(t+1)}d^d_r(j+1,t+1),\ j \in \{j_w,..,j_r-2\} \tag{1.221}$$

$$d^d_r(j_r-1,t) = \frac{p(t)p_{ro}(t)}{p(t-1)p_{ro}(t-1)}\frac{\xi(j_r-1,t)}{1+r_p(t+1)} \tag{1.222}$$

Wealth at the start of the year

$$V^{ds}_{mb}(j,t) = V^{ds}_m(j-1,t-1) + V^{ds}_{mau}(j,t) \tag{1.223}$$

Wealth at the end of the year

$$V^{ds}_m(j,t) = \begin{cases} \left[V^d_{mb}(j,t)(1+r_{pe}) - P^d_u(j,t)\right]\frac{b_{ahb}(j,t)}{b_{ah}(j,t)},\ j \in \{j_r..j_e\} \\ \text{after the year of transition} \\ \left[V^{ds}_{mb}(j,t)(1+r_{pe}) + \tau^d_{plb}(t,j)G^b_{pp}(j,t)\right]\frac{b_{ahb}(j,t)}{b_{ah}(j,t)},\ j \in \{j_w..j_r-1\} \\ \text{after the year of transition} \end{cases} \tag{1.224}$$

Change in wealth through immigration and emigration

$$S^d_{am}(j,t) = q_{bih}(j,t)\left[V^{ds}_{mb}(j,t)(1+r_{pe}) - P^d_u(j,t) + \tau^d_{plb}(t,j)G^b_{pp}(j,t)\right] \tag{1.225}$$

$$S^d_{ab}(j,t) = q_{beh}(j,t)\left[V^{ds}_{mb}(j,t)(1+r_{pe}) - P^d_u(j,t) + \tau^d_{plb}(t,j)G^b_{pp}(j,t)\right] \tag{1.226}$$

1.6.4 Wealth transfer between pension systems

$$V_{mau}^{fs}(t) = \delta_2^f \delta_2^a \left[V_m^f(t-1) - V_m^a(t-1)\right] + \delta_2^f \delta_2^d \left[V_m^f(t-1) - V_m^d(t-1)\right] \tag{1.227}$$

$$V_{mau}^{as}(t) = \delta_2^a \delta_2^f \left[V_m^a(t-1) - V_m^f(t-1)\right] + \delta_2^a \delta_2^d \left[V_m^a(t-1) - V_m^d(t-1)\right] \tag{1.228}$$

$$V_{mau}^{d}(j,t) = \begin{cases} \delta_2^d \delta_2^f \left[V_m^d(t-1) - V_m^f(t-1)\frac{P_{r1h}^f(j-1,t-1)}{P_{rg}^f}\right] + \\ +\delta_2^d \delta_2^a \left[V_m^d(t-1) - \frac{P_{r1h}^a(j-1,t-1)}{P_{rg}^a} V_m^a(t-1)\right] \text{ and } j \geq j_r \\ \delta_2^d \delta_2^f \left[V_m^d(t-1) - V_m^f(t-1)\frac{P_{r2}^f(j-1,t-1)}{P_{rg}^f}\right] + \\ +\delta_2^d \delta_2^a \left[V_m^d(t-1) - \frac{P_{r2}^a(j-1,t-1)}{P_{rg}^a} V_m^a(t-1)\right] \text{ and } j < j_r \end{cases} \tag{1.229}$$

$$P_{rg}^f = \sum_{j=0}^{j_e-1} b_{ah}(j-1,t-1)\left[P_{r1h}^f(j-1,t-1) + P_{r2}^f(j-1,t-1)\right] \tag{1.230}$$

$$P_{rg}^a = \sum_{j=0}^{j_e-1} b_{ah}(j-1,t-1)\left[P_{r1h}^a(j-1,t-1) + P_{r2}^a(j-1,t-1)\right] \tag{1.231}$$

1.6.5 Net benefits of pension funds

Define net benefits of pension funds of the current workers as

$$\begin{aligned} p_{p2}(j,t) =& \frac{1}{1+r_{pr}}\left[p_u^b(j,t) + s_{abp}(j,t) - p_p^b(j,t) - s_{amp}(j,t)\right] + \\ &+ \frac{1-q_{bdh}(j,t)}{1+r_{pr}} p_{p2}(j+1,t+1),\ j \in \{j_w,..,j_e-1\} \end{aligned} \tag{1.232}$$

$$p_{p2}(j_e,t) = \frac{1}{1+r_{pr}}\left[p_u^b(j_e,t) + s_{abp}(j_e,t) - p_p^b(j_e,t) - s_{amp}(j_e,t)\right] \tag{1.233}$$

$$p_{p2}(j,t) = p_{p12}(j_w, t+j_w-j)\ ,\ j < j_w \tag{1.234}$$

with

$$p_{p12}(j,t) = p_{p2}(j,t) d_{isp}(t),\ j \in \{j_w,..,j_e-1\} \tag{1.235}$$

discount factor

$$d_{isp}(t) = d_{isp}(t-1)\frac{1}{1+r_{pr}(t)} \tag{1.236}$$

1.6.6 Output variables pension system

Average premiums

$$\tau^b_{plg} = \frac{\tau^b_{ppp} G^b_{pp}}{Y_{wa}} \tag{1.237}$$

$$\tau^b_{plg1} = \frac{\tau^b_{plb1} G^b_{pp}}{Y_{wa}} \tag{1.238}$$

$$\tau^b_{plg2} = \frac{\tau^b_{plb2} G^b_{pp}}{Y_{wa}} \tag{1.239}$$

target replacement rate

$$q^b_{asp} = \begin{cases} (w_{er1} - 5) o^{ci}_{pbb} \frac{G(j_r - 1,t)}{S_{lo}(j_r - 1,t)} & \text{final wage} \\ \\ (w_{er1} - 5) o^{ci}_{pmi} \frac{1}{S_{lo}(j_r - 1,t)} \frac{\sum_{j=j_w}^{j_r - 1} G(j,t)}{40} & \text{average wage} \end{cases} \tag{1.240}$$

Actual pension rate at the age of 65

$$q_{pn}(j_r,t) = \begin{cases} o^{ci}_{pbb} d_j(j_r,t) & \text{final wage} \\ \\ \frac{P^b_u(j_r t)}{G(j_r - 1,t-1)} & \text{defined benefit} \\ \\ \frac{G^b_{em}(j_r - 1,t-1)}{G(j_r - 1,t-1)} & \text{average wage} \end{cases} \tag{1.241}$$

Actual average pension rate

$$q_{pn} = \frac{\sum_{j=j_r}^{j_e} b^s_{ah}(j-1,t-1) P^b_u(j,t)}{\sum_{j=j_r}^{j_e} b^s_{ah}(j-1,t-1) p_{ywe}(j_r - 1,t-1)} \tag{1.242}$$

Effective start of the pension period

$$p^{ci}_{lfe}(t) = 50 + p^{ci}_{ex}(50,t) \tag{1.243}$$

$$p^{ci}_{ex}(j,t) = \begin{cases} 0 \\ \\ \left(p^{ci}_{ex}(j+1,t) + 1\right) \frac{b_{lh}(j+1,t)}{b^s_{ah}(j+1,t)} \frac{b^s_{ah}(j,t)}{b_{lh}(j,t)} \end{cases} \tag{1.244}$$

Acquired rights

$$q_{pr} = \frac{P^b_r}{Y_{wa}/(1+\tau_{wp})} \tag{1.245}$$

Rights to be built up

$$q_{pt} = \frac{P^b_t}{Y_{wa}/(1+\tau_{wp})} \tag{1.246}$$

Premium paid by present day workers

$$q_{pp2} = \frac{P_p}{Y_{wa}/(1+\tau_{wp})} \tag{1.247}$$

Pensions

$$q_{pu2} = \frac{P_u^b}{Y_{wa}/(1+\tau_{wp})} \tag{1.248}$$

Wealth

$$q_{vm} = \frac{V_m^b}{Y_{wa}/(1+\tau_{wp})} \tag{1.249}$$

Income from wealth

$$q_{in} = \frac{r_{pr} V_m^b (t-1)}{Y_{wa}/(1+\tau_{wp})} \tag{1.250}$$

Surplus

$$q_{sal} = q_{in} + q_{pp2} - q_{pu2} \tag{1.251}$$

1.7 The labour market

Labour supply and leisure are not consistently modelled with consumption. So, we present it here separately from household behaviour.

Labour supply trend

$$b_l(g,j,t) = b_l(g,j,t-1) \frac{p_{gef}(j,t)}{p_{gef}(j,t-1)} \frac{b_a^s(g,j-1,t-1)}{b_a^s(g,j-1,t-2)},\ j \in \{j_w,..,j_e\},\ g = f,m \tag{1.252}$$

$$b_{lh}(j) = \sum_g b_l(g,j),\ j \in \{j_w,..,j_e\},\ g = f,m \tag{1.253}$$

Aggregate labour supply trend

$$b_{lh} = \sum_j b_{lh}(j),\ j \in \{j_w,..,j_e\} \tag{1.254}$$

Employment increases with aggregate labour supply in the base run

$$l_{dab}(j,t) = l_{dab}(j,t-1) \frac{b_{lh}(j,t)}{b_{lh}(j,t-1)},\ j \in \{j_w,..,j_r-1\} \tag{1.255}$$

moreover, employment changes with the price of labour in other simulations than the base run

$$l_{da}(j,t) = l_{dab}(j,t) \left(\frac{p_v}{p_{vb}}\right)^{0.3} \tag{1.256}$$

Price of leisure

$$p_v(j,t) = (1-(\tau_{iak}(t)+\tau_{pp}(t)))\,\frac{0.9p_{ywe}(j,t)+0.1p_{ywp}(j,t)-\tau^b_{ppp}G(j,t)}{(1+\tau_{co}(t))p(t)} + \\ + \frac{p_{v2}(j,t)}{(1+\tau_{co}(t))p(t)} \tag{1.257}$$

Discounted value pension rights

$$p_{v2}(j,t) \equiv \begin{cases} (1+r_{hr}(t))\,(1-\tau_{li65}(t))o^{ci}_{pb}G(j,t)\widetilde{c}_{fw}(j,t) \text{ final wage system} \\ \\ (1+r_{hr}(t))\,(1-\tau_{li65}(t))o_{pmi}G(j,t)\widetilde{c}_{aw}(j,t) \text{ average wage system} \\ \\ (1+r_{hr}(t))\,(1-\tau_{li65}(t))\frac{p_{pr}(j,t)G(j,t)}{1+c_{wp}(65,t)}\widetilde{c}_{dc}(j,t) \text{ DC system} \end{cases} \tag{1.258}$$

$$\widetilde{c}_{fw}(j,t) = d_{fw}(j,t)\left[\delta(j,t)+\widetilde{c}_{fw}(j+1,t+1)\right] \tag{1.259}$$

and $\delta(j,t)=1$ for $j \geq j_r$ else $\delta(j,t)=0$

$\widetilde{c}_{fw}(n_T,t)=0$

$$d_{fw}(j,\tau) = \begin{cases} \frac{G(j+1,\tau+1)}{G(j,\tau)}\frac{1-\tau_{li65}(\tau+1)}{1-\tau_{li65}(\tau)}\frac{1-q_{bdh}(j,\tau)}{1+r_h(\tau+1)} \text{ and } j \leq j_r-2 \\ \\ p_{in}(\tau+1)\frac{1-\tau_{li65}(\tau+1)}{1-\tau_{li65}(\tau)}\frac{1-q_{bdh}(j,\tau)}{1+r_h(\tau+1)} \text{ and } j \geq j_r-1 \end{cases} \tag{1.260}$$

$$\widetilde{c}_{aw}(j,t) = d_{aw}(j,t)\left[1+\widetilde{c}_{aw}(j+1,t+1)\right] \tag{1.261}$$

and $\delta(j,t)=1$ for $j \geq j_r-1$ else $\delta(j,t)=0$

$\widetilde{c}_{aw}(n_T,t)=0$

$$\widetilde{c}_{aw}(j,t) = d_{aw}(j,t)\left[1+\widetilde{c}_{aw}(j+1,t+1)\right] \tag{1.262}$$

and $\delta(j,t)=1$ for $j \geq j_r-1$ else $\delta(j,t)=0$

$\widetilde{c}_{aw}(n_T,t)=0$

$$d_{aw}(j,t) = p_{in}(t+1)\frac{1-\tau_{li65}(t+1)}{1-\tau_{li65}(t)}\frac{1-q_{bdh}(j,t)}{1+r_h(t+1)} \tag{1.263}$$

$$\widetilde{c}_{dc}(j,t) = d_{dc}(j,t)\left[\delta(j,t)+\widetilde{c}_{dc}(j+1,t+1)\right] \tag{1.264}$$

and $\delta(j,t)=1$ for $j \geq j_r-1$ else $\delta(j,t)=0$

$\widetilde{c}_{dc}(n_T,t)=0$

$$d_{dc}(j,t) = \begin{cases} \frac{1+c_{wp}(j_r,t)}{1+c_{wp}(j_r,t+1)}(1+r_p(t+1))\frac{1-\tau_{li65}(t\tau+1)}{1-\tau_{li65}(t)}\frac{1-q_{bdh}(j,t)}{1+r_h(t+1)} \text{ and j} \leq j_r-1 \\ \\ p_{in}(t+1)\frac{1-\tau_{li65}(t+1)}{1-\tau_{li65}(t)}\frac{1-q_{bdh}(j,t)}{1+r_h(t+1)} \text{ and } j \geq j_r \end{cases} \tag{1.265}$$

1.8 Capital market

The nominal bond rate

$$r_b(t) = (1 + r_{br}(t))\frac{p(t)}{p(t-1)} - 1 \tag{1.266}$$

Rate of return on shares

$$r_s = r_b + r_{isk} \tag{1.267}$$

The after-tax real rate of return to households

$$r_{hr} = \frac{p(t-1)}{p(t)}\left[1 + r_b(t) + (1 - q_{sbh})r_{isk} - \tau_{liki}(t)\right] - 1 \tag{1.268}$$

Average rate of return on assets of pension funds

$$r_p(t) = r_b + (1 - q_{sbp})r_{isk} \tag{1.269}$$

Real rate of return of pension funds

$$r_{pr}(t) = (r_p(t) + 1)\frac{p(t-1)}{p(t)} - 1 \tag{1.270}$$

1.9 National aggregates and the balance of payments

$$D_{eto} = I_{de} + D_{tot} \tag{1.271}$$

$$I_{ntc} = I_{ge} + I_{bto} - N_{grv} \tag{1.272}$$

$$I_{na} = I_{ne} + I_{nto} - N_{grv} \tag{1.273}$$

Net product

$$Y_{na} = Y_{ne} + Y_{wp} \tag{1.274}$$

Gross domestic product

$$B_{bpr} = D_{eto} + I_{rto} - S_{ubs} + Y_{na} \tag{1.275}$$

$$B_{bpf} = Y_{na} + D_{eto} \tag{1.276}$$

Total consumption (private and government)

$$C_{ont} = C_{onp} + C_{ap} \tag{1.277}$$

National expenditures

$$N_{bes} = C_{ont} + I_{ntc} \tag{1.278}$$

Gross national income

$$B_{ni} = N_{bes} + B_{blr} \tag{1.279}$$

Current account

$$B = Y_{gp} + Y_{ge} - C_{onp} - I_{ntc} - C_{ap} + I_{rto} - S_{ubs} \tag{1.280}$$

$$Z_{br} = R_{utg} - Y_{zh} - Y_{zg} + D_{tot} - Y_{zp} + Y_{ze} \tag{1.281}$$

$$S_{vf} = -B + Z_{br} + O_{vbu} \tag{1.282}$$

$$B_{blr} = -S_{vf} \tag{1.283}$$

change of net foreign assets

$$S_{af} = S_{vf} + S_{ab} - S_{am} + S_{hef} \tag{1.284}$$

Net foreign assets and portfolio

$$S^s_{af} = S^s_a - (O_{vvm} + S^s_{ah} + V^{bs}_m) \tag{1.285}$$

$$S^s_{sf} = S^s_{se} - S^s_{sh} - S^s_{sp} - F^s_{acb} \tag{1.286}$$

$$S^s_{bf} = S^s_{af} - S^s_{sf}$$

New bonds issued by firms

$$S_{bf} = S_{af} - S_{sf} \tag{1.287}$$

Revaluation of foreign assets

$$S_{hef} = S_{se} - S_{heh} - S_{hep} \tag{1.288}$$

Total assets in the Netherlands

$$S^s_a = S^s_{se} + S^s_{be} + K^s_{gto} + K^s_{gga} + P^s_{cht} + D^s_{nba} \tag{1.289}$$

Total capital stock

$$K^s_a = K^s_{gto} + K^s_{ae} \tag{1.290}$$

Valuation difference (government versus firms) of government's claim on firms

$$R^s_{eg} = K^s_{h2} - K^s_h \tag{1.291}$$

Asset changes by emigration and immigration

$$S_{ab}(t) = \sum_j b_{eh}(j,t)\,(S^s_{ah}(j-1,t-1) + S_{heh}(j,t) + S_{vh}(j,t))\,,\; j \in \{j_w, .., j_e\} \tag{1.292}$$

$$S_{am}(t) = \sum_j b_{ih}(j,t)\,(S^s_{ah}(j-1,t-1) + S_{heh}(j,t) + S_{vh}(j,t))\,,\; j \in \{j_w, .., j_e\} \tag{1.293}$$

Portfolio change households

$$S_{sh} = S^s_{sh} - S^s_{sh}(-1) \tag{1.294}$$

$$S_{bh} = S^s_{bh} - S^s_{bh}(-1) \tag{1.295}$$

1.10 Demography

population levels

$$b^s_a(g,j,t) = b^s_a(g,j-1,t-1) + b_i(g,j,t) - b_d(g,j,t) - b_e(g,j,t),\; j \in \{1,..,j_e\} \tag{1.296}$$

$$b^s_a(g,0,t) = b_b(g,t) + b_i(g,0,t) - b_d(g,0,t) - b_e(g,0,t) \tag{1.297}$$

Births by gender

$$b_b(g) = q_{bb}(g) b_{bh},\; g \in \{f,m\} \tag{1.298}$$

number of births;

$$b_{bh} = \sum_{j=16}^{50} b_{ff}(j) \tag{1.299}$$

fertility levels of fertile women;

$$b_{ff}(j,t) = q_{bff}(j,t)\frac{b^s_a(f,j-1,t-1) + b^s_a(f,j,t)}{2},\; j \in \{16,...,50\} \tag{1.300}$$

numbers of deaths;

$$b_d(g,j,t) = q_{bd}(g,j,t) b^s_a(g,j-1,t-1),\; j \in \{1,..,j_e\} \tag{1.301}$$

$$b_d(g,0,t) = q_{bd}(g,0,t) b_b(g,t),\; g \in \{f,m\} \tag{1.302}$$

numbers of emigrants;

$$b_e(g,j) = q_{be}(g,j) b^s_a(g,j,t-1),\; j \in \{1,..,j_e\} \tag{1.303}$$

$$b_e(g,0) = q_{be}(g,0) b_b(g),\; g \in \{f,m\} \tag{1.304}$$

numbers of immigrants are exogenous until 2015, and assumed constant thereafter.

$$b_i(g,j,t) = b_i(g,j,t-1), t > 2015$$
$$j \in \{0,1,....,j_e\},\ g \in \{f,m\} \tag{1.305}$$

birth rate per gender group is calculated conditional on the number of $0-$year olds per gender group (at the end of the year) until 2100, and taken to be constant thereafter

$$q_{bb}(g,t) = \frac{b_a^s(g,0,t) - b_i(g,0,t) + b_d(g,0,t) + b_e(g,0,t)}{b_{bh}}, \text{for} t \leq 2100 \tag{1.306}$$
$$= q_{bb}(g,t-1), \text{for} t > 2100,\ g \in \{f,m\} \tag{1.307}$$

aggregate birth rate (a control variable, that must equal one in order to reproduce CBS statistics.

$$q_{bbh} = \sum_g q_{bb}(g),\ g \in \{f,m\} \tag{1.308}$$

death rates are exogenous until 2100, and assumed constant thereafter;

$$q_{bd}(g,j,t) = q_{bd}(g,j,t-1), j \in \{0,....,j_e\},\ g \in \{f,m\}, t > 2100 \tag{1.309}$$

fertility rates of fertile women are exogenous until 2050, and assumed constant thereafter;

$$q_{bff}(j,t) = q_{bff}(j,t-1),\ j \in \{16,...,.50\}, t > 2050 \tag{1.310}$$

number of emigrants is exogenous until 2050, and emigration rates are assumed constant thereafter;

$$q_{be}(g,j,t) = q_{be}(g,j,t-1),\ j \in \{0,....,j_e\},\ g \in \{f,m\}, t > 2050 \tag{1.311}$$

immigration rates.

$$q_{bi}(g,j) = \frac{b_i(g,j)}{b_a^s(g,j-1,t-1)},\ j \in \{1,..,j_e\} \tag{1.312}$$
$$q_{bi}(g,0) = \frac{b_i(g,0)}{b_b(g)},\ g \in \{f,m\} \tag{1.313}$$

death rates;

$$q_{bdh}(j,t) = \frac{b_{dh}(j,t)}{b_{ah}^s(j-1,t-1) + b_{ih}(j,t) - b_{eh}(j,t)},\ j \in \{1,..,j_e\} \tag{1.314}$$
$$q_{bdh}(0,t) = \frac{b_{dh}(j,t)}{b_{bh}(t) + b_{ih}(j,t) - b_{eh}(j,t)} \tag{1.315}$$

emigration rates;

$$q_{beh}(j,t) = \frac{b_{eh}(j,t)}{b^s_{ah}(j-1,t-1)},\; j \in \{1,..,j_e\} \tag{1.316}$$

$$q_{beh}(0,t) = \frac{b_{eh}(j,t)}{b_{bh}(t)} \tag{1.317}$$

immigration rates.

$$q_{bih}(j,t) = \frac{b_{ih}(j,t)}{b^s_{ah}(j-1,t-1)},,\; j \in \{1,..,j_e\} \tag{1.318}$$

$$q_{bih}(0,t) = \frac{b_{ih}(j,t)}{b_{bh}(t)} \tag{1.319}$$

population levels

$$b^s_a(g) = \sum_{j=0}^{j_e} b^s_a(g,j),\; g \in \{f,m\} \tag{1.320}$$

$$b^s_{ah}(b) = \sum_g b^s_a(g,b) \tag{1.321}$$

$$b^s_{ah}(j) = \sum_g b^s_a(g,j),\; j \in \{0,....,j_e\} \tag{1.322}$$

$$b^s_{ah} = \sum_g b^s_a(g) \tag{1.323}$$

number of deaths;

$$b_d(g) = \sum_{j=0}^{j_e} b_d(g,j),\; g \in \{f,m\} \tag{1.324}$$

$$b_{dh}(j) = \sum_g b_d(g,j),\; j \in \{0,....,j_e\} \tag{1.325}$$

$$b_{dh} = \sum_g b_d(g) \tag{1.326}$$

number of emigrants;

$$b_e(g) = \sum_{j=0}^{j_e} b_e(g,j),\; g \in \{f,m\} \tag{1.327}$$

$$b_{eh}(j) = \sum_g b_e(g,j),\; j \in \{0,....,j_e\} \tag{1.328}$$

$$b_{eh} = \sum_g b_e(g) \tag{1.329}$$

number of immigrants.

$$b_i(g) = \sum_{j=0}^{j_e} b_i(g,j),\ g \in \{f,m\} \tag{1.330}$$

$$b_{ih}(j) = \sum_g b_i(g,j),\ j \in \{0,....,j_e\} \tag{1.331}$$

$$b_{ih} = \sum_g b_i(g) \tag{1.332}$$

2 Derivations

2.1 Consumer behavior

2.1.1 Intergenerational transfers

Total financial wealth at the end of period t

$$S^s_{ah}(j) = S^s_{ah}(j-1,t-1) + S_{heh}(j) + Y_{ah}(j) - X_{ah}(j) + G_{en}(j) \tag{2.1}$$

Number of people in a cohort at the end of a period

$$b^s_{ah}(j) = \sum_g b^s_a(g,j) \tag{2.2}$$

$$= \sum_g \left(b^s_a(g,j-1,t-1) + b_i(g,j) - b_d(g,j) - b_e(g,j)\right) \tag{2.3}$$

$$= b^s_{ah}(j-1,t-1) + b_{ih}(j) - b_{dh}(j) - b_{eh}(j) \tag{2.4}$$

$$= b^s_{ah}(j-1,t-1)\left(1 + q_{bih}(j) - q_{beh}(j)\right)\left(1 - q_{bdh}(j)\right) \tag{2.5}$$

The generational transfers follow from the condition that at the cohort level the following holds

$$b^s_{ah}(j)S^s_{ah}(j) = b^s_{ah}(j-1,t-1)\left(1 - q_{beh}(j) + q_{bih}(j)\right) \times \tag{2.6}$$
$$\times \left(S^s_{ah}(j-1,t-1) + S_{heh}(j) + Y_{ah}(j) - X_{ah}(j)\right)$$

At the micro level hold

$$S^s_{ah}(j) = S^s_{ah}(j-1,t-1) + S_{heh}(j) + Y_{ah}(j) - X_{ah}(j) + G_{en}(j) \tag{2.7}$$

or

$$b^s_{ah}(j-1,t-1)(1 - q_{beh}(j) + q_{bih}(j))\, S^s_{ah}(j) =$$

$$b^s_{ah}(j-1,t-1)\left(1 - q_{beh}(j) + q_{bih}(j)\right) \times \tag{2.8}$$

$$\left(S^s_{ah}(j-1,t-1) + S_{heh}(j) + Y_{ah}(j) - X_{ah}(j) + G_{en}(j)\right)$$

From the equations 2.6 and 2.8 we derive the intergenerational transfers

$$\left(b^s_{ah}(j,t) - \left(1 - q_{beh}(j) + q_{bih}(j)\right) b^s_{ah}(j-1,t-1)\right) S^s_{ah}(j)$$
$$= -\left(1 - q_{beh}(j) + q_{bih}(j)\right) b^s_{ah}(j-1,t-1) G_{en}(j) \tag{2.9}$$

$$G_{en}(j) = \left(1 - \frac{b^s_{ah}(j,t)}{\left(1 - q_{beh}(j) + q_{bih}(j)\right) b^s_{ah}(j-1,t-1)}\right) S^s_{ah}(j)$$
$$= q_{bdh}(j) S^s_{ah}(j) \tag{2.10}$$

$$S^s_{ah}(j_e) = S^s_{ah}(j_e-1,t-1) + S_{heh}(j_e) + G_{en}(j_e) + Y_{ah}(j_e) - X_{ah}(j_e) = 0 \tag{2.11}$$

$$G_{en}(j_e) = q_{bdh}(j_e)S^s_{ah}(j_e) = 0 \tag{2.12}$$

$$X_{ah}(j_e) = C_{nh}(j_e) + T_{ah}(j_e) \tag{2.13}$$

$$C_{nh}(j_e) = S^s_{ah}(84,t-1) + S_{heh}(j_e) + Y_{ah}(j_e) - T_{ah}(j_e) \tag{2.14}$$

2.1.2 The budget restriction

$$S^s_{ah}(j) = S^s_{ah}(j-1,t-1) + S_{heh}(j) + Y_{ah}(j) - X_{ah}(j) + G_{en}(j) \tag{2.15}$$

$$\begin{aligned} X_{ah}(j) =& C_{nh}(j) + T_{ah}(j) + P^b_p(j) \qquad (2.16)\\ =& C_{nh}(j)+T_{co}(j) + L_{ito}(j) + P^b_p(j)\\ =& C_{nh}(j)+T_{co}(j) + L_{iak}(j) + L_{iuk}(j) + L_{i65}(j) + L_{iap}(j) + L_{iki}(j) + L_{iov}(j) + P^b_p(j)\\ =& C_{onp}(j)+L_{iak}(j) + L_{iuk}(j) + L_{i65}(j) + L_{iap}(j) + \tau_{liki}S^s_{ah}(j-1,t-1) + L_{iov}(j) + P^b_p(j) \end{aligned}$$

$$\begin{aligned} Y_{ah}(j)+S_{heh}(j) =& Y_{th}(j) + P^b_u(j) + Y_{wa}(j) + Y_{zh}(j) + S_{heh}(j)\\ =& Y_{th}(j) + P^b_u(j) + Y_{wa}(j) + r_b S^s_{bh}(j-1,t-1) + \frac{D_{iv}}{S^s_{se}(t-1)} S^s_{sh}(j-1,t-1) \qquad (2.17)\\ &+ \left(\frac{S^s_{se}}{S^s_{se}(t-1)} - 1\right) S^s_{sh}(j-1,t-1)\\ =& Y_{th}(j) + P^b_u(j) + Y_{wa}(j)\\ &+ \left[r_b \frac{S^s_{bh}(j-1,t-1)}{S^s_{ah}(j-1,t-1)} + \left(\frac{D_{iv}}{S^s_{se}(t-1)} + \left(\frac{S^s_{se}}{S^s_{se}(t-1)} - 1\right)\right) \frac{S^s_{sh}(j-1,t-1)}{S^s_{ah}(j-1,t-1)}\right] S^s_{ah}(j-1,t-1)\\ =& Y_{th}(j) + P^b_u(j) + Y_{wa}(j) + (r_b + (1-q_{sbh})r_{isk})S^s_{ah}(j-1,t-1) \text{ if}\\ r_s =& \left(\frac{D_{iv}}{S^s_{se}(t-1)} + \left(\frac{S^s_{se}}{S^s_{se}(t-1)} - 1\right)\right) = r_b + r_{isk} \qquad (2.18) \end{aligned}$$

$$\begin{aligned} Y_{ah}(j)+S_{heh}(j) - X_{ah}(j) =& Y_{th}(j) + P^b_u(j) + Y_{wa}(j) + r_b S^s_{ah}(j-1,t-1)\\ &- \left[C_{onp}(j) + L_{iak}(j) + L_{iuk}(j) + L_{i65}(j) + L_{iap}(j) + \tau_{liki}S^s_{ah}(j-1,t-1) + L_{iov}(j) + P^b_p(j)\right]\\ =& \left[Y_{th}(j) + P^b_u(j) + Y_{wa}(j) - \left(L_{iak}(j) + L_{iuk}(j) + L_{i65}(j) + L_{iap}(j) + L_{iov}(j) + P^b_p(j)\right)\right]\\ &+ [r_b + (1-q_{sbh})r_{isk} - \tau_{liki}]\, S^s_{ah}(j-1,t-1) - C_{onp}(j) \qquad (2.19) \end{aligned}$$

$$
\begin{aligned}
S^s_{ah}(j) &= S^s_{ah}(j-1,t-1) + S_{heh}(j) + Y_{ah}(j) - X_{ah}(j) + G_{en}(j) \\
&= \frac{1}{1-q_{bdh}(j)} \left(S^s_{ah}(j-1,t-1) + S_{heh}(j) + Y_{ah}(j) - X_{ah}(j)\right) \\
&= \frac{1}{1-q_{bdh}(j)} [S^s_{ah}(j-1,t-1) + \\
&Y_{th}(j) + P^b_u(j) + Y_{wa}(j) - \Big(L_{iak}(j) + L_{iuk}(j) + L_{i65}(j) + L_{iap}(j) + L_{iov}(j) + P^b_p(j)\Big) \\
&+ [r_b + (1-q_{sbh})r_{isk} - \tau_{liki}]\, S^s_{ah}(j-1,t-1) - C_{onp}(j)]
\end{aligned} \tag{2.20}
$$

$$
\begin{aligned}
s^s_{ah}(j) &= \frac{1}{1-q_{bdh}(j)} \Bigg[s^s_{ah}(j-1,t-1)\frac{p(t-1)}{p}\,[1 + r_b + (1-q_{sbh})r_{isk} - \tau_{liki}] \\
&+ \Big[y_{th}(j) + P^b_u(j) + y_{wa}(j) - \Big(l_{iak}(j) + l_{iuk}(j) + l_{i65}(j) + l_{iap}(j) + l_{iov}(j) + P^b_p(j)\Big)\Big] - c_{onp}(j) \Bigg] \\
&= \frac{1}{1-q_{bdh}(j)} \left[(1+r_{hr})s^s_{ah}(j-1,t-1) + y(j) - c_{onp}(j)\right] \text{ with}
\end{aligned} \tag{2.21}
$$

$$
r_{hr} = \frac{p(t-1)}{p}\,[1 + r_b + (1-q_{sbh})r_{isk} - \tau_{liki}] - 1 \tag{2.22}
$$

$$
\begin{aligned}
y(j) &= y_{th}(j) + p^b_u(j) + y_{wa}(j) - \\
&\Big(l_{iak}(j) + l_{iuk}(j) + l_{i65}(j) + l_{iap}(j) + l_{iov}(j) + p^b_p(j)\Big)
\end{aligned} \tag{2.23}
$$

$$
c_{onp}(j) = (1 + \tau_{co}(j))\, c_{nh}(j) \tag{2.24}
$$

2.1.3 The consumer problem

The consumer maximizes expected utility

$$
EU(j,t) = c_{nh}(j,t)^{[\gamma-1]/\gamma} + \sum_{i=1}^{n_T-j} \left\{ c_{nh}(j+i,t+i)^{[\gamma-1]/\gamma}[1+\beta]^{-i} \prod_{l=0}^{i-1} \zeta(j+l,t+l) \right\} \tag{2.25}
$$

$$
\zeta(j,t) = 1 - q_{bdh}(j,t)
$$

with respect to consumption, conditional on budget equation

$$
s^s_{ah}(j) = \frac{1}{1-q_{bdh}(j)} \left[(1+r_{hr})s^s_{ah}(j-1,t-1) + y(j) - c_{onp}(j)\right] \tag{2.26}
$$

$$
s^s_{ah}(j_e) = 0
$$

Lagrangian

$$L = c_{nh}(t)^{[\gamma-1]/\gamma} + \sum_{i=1}^{n_T-j}\left(c_{nh}(t+i)^{[\gamma-1]/\gamma}[1+\beta]^{-i}\prod_{l=0}^{i-1}\zeta(t+l)\right)$$
$$+\lambda(t)\left(s_{ah}^{s}(t) - \frac{1}{\zeta(t)}\left[(1+r_{hr}(t))s_{ah}^{s}(t-1)+y(t)-c_{onp}(t)\right]\right)$$
$$+\sum_{i=1}^{n_T-j}\lambda(t+i)[1+\beta]^{-i}\prod_{l=0}^{i-1}\zeta(t+l)\,(s_{ah}^{s}(t+i)$$
$$-\frac{1}{\zeta(t+i)}\left[(1+r_{hr}(t+i))s_{ah}^{s}(t+i-1)+y(t+i)-c_{onp}(t+i)\right]\Big) \tag{2.27}$$

First order conditions are:

$$L_{c_{nh}(t+i)} = 0; L_{s_{ah}{}^{s}(t+i)} = 0; i = 0..n_T - j \tag{2.28}$$

$$L_{c_{nh}(t+i)} = \frac{\gamma-1}{\gamma}c_{nh}(t+i)^{-1/\gamma}[1+\beta]^{-i}\prod_{l=0}^{i-1}\zeta(t+l) \tag{2.29}$$
$$+\lambda(t+i)[1+\beta]^{-i}\prod_{l=0}^{i-1}\zeta(t+l)\frac{1+\tau_{co}(t+i)}{\zeta(t+i)} = 0$$

$$L_{s_{ah}{}^{s}(t+i)} = \lambda(t+i)[1+\beta]^{-i}\prod_{l=0}^{i-1}\zeta(t+l) \tag{2.30}$$
$$-\lambda(t+i+1)[1+\beta]^{-i-1}\prod_{l=0}^{i}\zeta(t+l)\frac{1}{\zeta(t+i+1)}(1+r_{hr}(t+i+1)) = 0$$

We derive from this first order conditions:

$$\frac{\gamma-1}{\gamma}c_{nh}(t+i)^{-1/\gamma} + \lambda(t+i)\frac{1+\tau_{co}(t+i)}{\zeta(t+i)} = 0 \tag{2.31}$$

$$\lambda(t+i) - \lambda(t+i+1)[1+\beta]^{-1}\frac{\zeta(t+i)}{\zeta(t+i+1)}(1+r_{hr}(t+i+1)) = 0 \tag{2.32}$$

$$\left(\frac{c_{nh}(t+i)}{c_{nh}(t+i+1)}\right)^{-1/\gamma} = \frac{1+r_{hr}(t+i+1)}{1+\beta}\,\frac{1+\tau_{co}(t+i)}{1+\tau_{co}(t+i+1)} \tag{2.33}$$

$$c_{nh}(t+i) = \left(\frac{1+\beta}{1+r_{hr}(t+i+1)}\,\frac{1+\tau_{co}(t+i+1)}{1+\tau_{co}(t+i)}\right)^{\gamma} c_{nh}(t+i+1) \tag{2.34}$$

Consumption at the age of j_e follows from the no bequest assumption

$$S_{ah}^{s}(j_e) = S_{ah}^{s}(j_e-1,t-1) + S_{heh}(j_e) + G_{en}(j_e) + Y_{ah}(j_e) - X_{ah}(j_e) = 0$$
$$G_{en}(j_e) = q_{bdh}(j_e)S_{ah}^{s}(j_e) = 0$$
$$X_{ah}(j_e) = C_{nh}(j_e) + T_{ah}(j_e)$$
$$C_{nh}() = S_{ah}^{s}(j_e-1,t-1) + S_{heh}(j_e) + Y_{ah}(j_e) - T_{ah}(j_e) \tag{2.35}$$

2.1.4 Remarks

Utility can be written in difference equation format

$$EU(j,t) = c_{nh}(j,t)^{[\gamma-1]/\gamma} + [1+\beta]^{-1}\zeta(j,t)EU(j+1,t+1)$$

In case of equilibrium growth holds

$$EU^*(j,t) = EU^*(j,t-1)\frac{p_{ro}}{p_{ro}(t-1)} \tag{2.36}$$

2.2 Firm behavior

Assume a constant profit tax rate and a constant discount rate for convenience. Consider an economy in which firms produce finished goods with capital and labour. Labour productivity differs among people. Labour in efficiency units is perfectly substitutable

$$y_{ge} = F\left(k^s_{ae}(t-1), l_{ee}\right) \tag{2.37}$$

$$\text{with } l_{ee} = \sum_h l_{ee}(h) = \sum_h l_{de}(h) l^{ci}_{sh}(h) p_{ro}$$

The budget restriction of the firm reads as

$$pi_{ge} = py_{ge} - 10^{-3} p_{lde} l_{de} - r_b S_{be}(t-1) - V_{pdb} - D_{iv} - G_{asb} - V_{mid} - V_{mip} + \Delta S_{be} \tag{2.38}$$

$$\begin{aligned} \text{with } p_{lde} l_{de} &= \sum_h p_{lde}(h) l_{de}(h) \\ &= \sum_h \frac{p_{lde}(h)}{l^{ci}_{sh}(h) p_{ro}} l_{de}(h) l^{ci}_{sh}(h) p_{ro} \\ &= \sum_h p_{lee}(h) l_{ee}(h) \\ &= p_{le} l_{ee} \end{aligned}$$

p_{lde} is the average wage rate. The wage rate in efficiency units $p_{le}(h)$ is the same for all h because labour in efficiency units is homogeneous. We assume that in each period a fraction b_0 of the principal of the debt is repaid, and a fraction b_1 of new investment is financed with new debt. Debt payment, therefore, equals $(r_b + b_0) S_{be}(t-1)$, and total debt evolves as

$$S_{be} = (1-b_0) S_{be}(t-1) + b_1 pi_{ge}$$

Taxes are paid over taxable revenue

$$V_{pdb} = \tau_{vpb}\left[py_{ge} - 10^{-3} p_{le} l_{ee} - r_b S_{be}(t-1) - A_f - V_{mip}\right] \tag{2.39}$$

with A_f being the fiscal depreciation allowance. Fiscal depreciation is based on the historical cost price of investment and is geometric with a fiscal depreciation rate v.[1] That means that, in period t, the firm is allowed to deduct $v(1-v)^{\tau-1}p(t-\tau)i_{ge}(t-\tau)$ for the investment purchased in period $t-\tau$, for all $\tau \geq 1$.

$$A_f = \sum_{\tau=1}^{\infty} v(1-v)^{\tau-1} p(-\tau) i_{ge}(-\tau) \tag{2.40}$$

In our simulation model we will write this as

$$A_f = vA^s(-1) \tag{2.41}$$
$$A^s = (1-v)A^s(-1) + p i_{ge}$$

We will assume that depreciation according to the national accounts equals technical scrap

$$I_{de} = \delta_s K^s_{ae}(t-1) \frac{p}{p(t-1)} \tag{2.42}$$

Reordering of the budget equation gives

$$\begin{aligned} D_{iv} &= p y_{ge} - p_{le} l_{ee} - V_{pdb} - (1-b_1) p i_{ge} - H - (r_b + b_0) S_{be}(t-1) \\ &= (1-\tau_{vpb}) \left[p y_{ge} - 10^{-3} p_{le} l_{ee} \right] - (1-b_1) p i_{ge} + \tau_{vpb} A_f - H - \left[(1-\tau_{vpb}) r_b + b_0 \right] S_{be}(t-1) \end{aligned} \tag{2.43}$$

The variable H contains payments to the government other than taxes and dividends.

$$H = G_{asb} + V_{mid} + (1-\tau_{vpb}) V_{mip}$$

Arbitrage on the capital market leads to

$$r_s S^s_{se} = \Delta S^s_{se}(t+1) + D_{iv}(t+1). \tag{2.44}$$

Expanding forward results in

$$S^s_{se} = \sum_{j=1}^{\infty} D_{iv}(t+j) \left(1 + r_s\right)^{-j}. \tag{2.45}$$

The firm maximizes the value of the firm given the capital accumulation equation and the production function, which leads to the Lagrangian

$$\begin{aligned} L = \sum_{j=1}^{\infty} & \left(D_{iv}(t+j) \right. \\ & - q(t+j) [k^s_{ae}(t+j) - (1-\delta_s) k^s_{ae}(t+j-1) - i_{ge}(t+j)]. \\ & \left. - \lambda(t+j) \left[S_{be}(t+j) - (1-b_0) S_{be}(t+j-1) - b_1 p(t+j) i_{ge}(t+j) \right] \right) (1+r_s)^{-j} \end{aligned} \tag{2.46}$$

[1] Fiscal depreciation may be linear or degressive. Fiscal depreciation equal to a fixed percentage of the book value is allowed if the original investment becomes less productive with age. Since we assume that physical depreciation is exponential, a degressive fiscal depreciation scheme indeed seems most appropriate.

The Lagrangian contains the present value of the depreciation allowance as one of its determinants. The discounted value of the fiscal depreciations can be split up into depreciation on the existing capital stock at time t, AF_{t}, and the discounted value of depreciation on new investments:

$$\sum_{j=1}^{\infty}(1+r_s)^{-j}A_f(t+j)=\sum_{j=1}^{\infty}\sum_{\tau=1}^{\infty}v(1-v)^{\tau-1}i_{ge}(t+j-\tau)p(t+j-\tau)(1+r_s)^{-j}. \tag{2.47}$$

We split this expression into two components, AA an AB, with AA equal to

$$\begin{aligned}
AA &= \sum_{j=1}^{\infty}\sum_{\tau=1}^{j}v(1-v)^{\tau-1}i_{ge}(t+j-\tau)p(t+j-\tau)(1+r_s)^{-j} \\
&= \sum_{\tau=1}^{\infty}\sum_{j=\tau}^{\infty}v(1-v)^{\tau-1}i_{ge}(t+j-\tau)p(t+j-\tau)(1+r_s)^{-j} \\
&= \sum_{\tau=1}^{\infty}\sum_{u=0}^{\infty}v(1-v)^{\tau-1}i_{ge}(t+u)p(t+u)(1+r_s)^{-u-\tau} \\
&= \sum_{u=0}^{\infty}\sum_{\tau=1}^{\infty}v(1-v)^{\tau-1}i_{ge}(t+u)p(t+u)(1+r_s)^{-u-\tau} \\
&= \sum_{u=0}^{\infty}\frac{v}{r_s+v}i_{ge}(t+u)p(t+u)(1+r_s)^{-u}
\end{aligned}$$

and AB equal to

$$\begin{aligned}
AB &= \sum_{j=1}^{\infty}\sum_{\tau=j+1}^{\infty}v(1-v)^{\tau-1}i_{ge}(t+j-\tau)p(t+j-\tau)(1+r_s)^{-j} \\
&= \sum_{j=1}^{\infty}\sum_{u=1}^{\infty}v(1-v)^{u+j-1}i_{ge}(t-u)p(t-u)(1+r_s)^{-j} \\
&= \sum_{u=1}^{\infty}(1-v)^{u}\sum_{j=1}^{\infty}v(1-v)^{j-1}i_{ge}(t-u)p(t-u)(1+r_s)^{-j} \\
&= \sum_{u=1}^{\infty}(1-v)^{u}\frac{v}{r_s+v}i_{ge}(t-u)p(t-u)
\end{aligned} \tag{2.48}$$

Therefore,

$$\sum_{j=1}^{\infty}\left(\frac{1}{1+r_s}\right)^j A_f(t+j) = \sum_{j=0}^{\infty}\frac{\nu}{r_s+\nu} i_{ge}(t+j)p(t+j)(1+r_s)^{-j} + \sum_{j=1}^{\infty}(1-\nu)^j\frac{\upsilon}{r_s+\nu} i_{ge}(t-j)p(t-j) = \sum_{j=1}^{\infty}\frac{\nu}{r_s+\nu} i_{ge}(t+j)p(t+j)(1+r_s)^{-j} + \sum_{j=0}^{\infty}(1-\nu)^j\frac{\upsilon}{r_s+\nu} i_{ge}(t-j)p(t-j) = \sum_{j=1}^{\infty}\frac{\nu}{r_s+\nu} i_{ge}(t+j)p(t+j)(1+r_s)^{-j} + AF_t \tag{2.49}$$

where $AF(t)$ equals the depreciation allowance on investments installed up to time t:

$$AF(t) = \sum_{j=0}^{\infty}(1-\nu)^j\frac{\upsilon}{r_s+\nu} i_{ge}(t-j)p(t-j). \tag{2.50}$$

The value of $AF(t)$ is given and therefore does not affect the optimization problem. Substitution of equation 2.43 and 2.49 into the Lagrangian 2.46 results in

$$L = \sum_{j=1}^{\infty}\Big((1-\tau_{vpb})p(t+j)\left[F\left\{k_{ae}^s(t+j-1), l_{ee}\right\} - 10^{-3}p_{le}l_{ee}\right] - (1-\tau_{vpb}\frac{\nu}{r_s+\nu} - b_1)i_{ge}(t+j)p(t+j) - H(t+j) - \left[(1-\tau_{vpb})r_b+b_0\right]S_{be}(t+j-1) - q(t+j)[k_{ae}^s(t+j) - (1-\delta_s)k_{ae}^s(t+j-1) - i_{ge}(t+j)] + \lambda(t+j)\left[S_{be}(t+j) - (1-b_0)S_{be}(t+j-1) - b_1 p(t+j)i_{ge}(t+j)\right]\Big)(1+r_s)^{-j} + \tau_{vpb}AF(t)$$

First order conditions for an optimum are :

$$\iota.\ L_{l_{ee}} = 0\ ;\ \iota\iota.\ L_{k_{ae}{}^s} = 0\quad;\quad \iota\iota\iota.\ L_{i_{ge}} = 0\quad;\quad \iota\upsilon.\ L_{S_{be}} = 0 \tag{2.51}$$

$$\iota.\ F_{l_{ee}} = 10^{-3}\frac{p_{le}}{p} \tag{2.52}$$

$$\iota\iota.\ q = \frac{\left(1-\tau_{vpb}\right)p(t+1)F_{k_{ae}{}^s}(t+1) + q(t+1)(1-\delta_s)}{1+r_s}, \tag{2.53}$$

$$\iota\iota\iota.\ q = \left(1 - b_1 - \frac{\tau_{vpb}\nu}{\nu+r_s}\right)p + \lambda b_1 p, \tag{2.54}$$

$$\iota\upsilon.\ \lambda(t) = \frac{\left[(1-\tau_{vpb})r_b+b_0\right] + (1-b_0)\lambda(t+1)}{(1+r_s)} \tag{2.55}$$

This last condition can be simplified in case the interest rate and tax rate are constant.

$$\lambda = \frac{(1-\tau_{vpb})r_b + b_0}{r_s + b_0} \tag{2.56}$$

Substitution into the marginal cost of capital leads to

$$q = \left[1 - \frac{\tau_{vpb}\nu}{\nu + r_s} - b_1 + \frac{(1-\tau_{vpb})r_b + b_0}{r_s + b_0} b_1\right] p, \tag{2.57}$$

The first order condition for the marginal product of capital becomes

$$\begin{aligned}
&(1-\tau_{vpb})p(t+1)F_{k^s_{ae}}(t+1) = q(1+r_s) - q(t+1)(1-\delta_s) \qquad (2.58)\\
&= \left(\frac{q}{q(t+1)}(1+r_s) - (1-\delta_s)\right) q(t+1)\\
&= \left(\frac{q}{q(t+1)}(1+r_s) - (1-\delta_s)\right) \left[1 - \frac{\tau_{vpb}\nu}{\nu + r_s} - b_1 + \frac{(1-\tau_{vpb})r_b + b_0}{r_s + b_0} b_1\right] p(t+1),
\end{aligned}$$

or

$$\begin{aligned}
(1-\tau_{vpb})p(t+1)F_{k_{ae}{}^s}(t+1) = &\left(\frac{p}{p(t+1)}(1+r_s) - (1-\delta_s)\right) \times \qquad (2.59)\\
&\times \left[1 - \frac{\tau_{vpb}\nu}{\nu + r_s} - b_1 + \frac{(1-\tau_{vpb})r_b + b_0}{r_s + b_0} b_1\right] p(t+1)
\end{aligned}$$

Define p_{k} as

$$p_{k_{ae}{}^s} = \frac{1}{1-\tau_{vpb}} \left(\frac{p(t-1)}{p}(1+r_s) - (1-\delta_s)\right) \times \tag{2.60}$$

$$\left[1 - \frac{\tau_{vpb}\nu}{\nu + r_s} - b_1 + \frac{(1-\tau_{vpb})r_b + b_0}{r_s + b_0} b_1\right] p \tag{2.61}$$

Then the following holds

$$F_{k_{ae-1}{}^s} = \frac{p_{k_{ae}}}{p} \tag{2.62}$$

The marginal product of capital is determined by exogenous variables only. So, we can use the marginal productivity equation to determine the capital stock. Labour supply is exogenous. The marginal productivity equation of labour and the production function determine the production level and the wage rate. Assume

$$y_{ge} = \left(\kappa k^s_{ae}(t-1)^{\frac{\sigma-1}{\sigma}} + \alpha l_{ee}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{2.63}$$

For the marginal productivity relations holds

$$F_{k^s_{ae}}(t+1) = \kappa \left(\frac{y_{ge}(t+1)}{k^s_{ae}}\right)^{\frac{1}{\sigma}} ; F_{l_{ee}} = \alpha \left(\frac{y_{ge}}{l_{ee}}\right)^{\frac{1}{\sigma}} \tag{2.64}$$

leading to the factor demand relations

$$l_{ee} = \alpha^{\sigma} \left(\frac{10^{-3} p_{ywee}}{p} \right)^{-\sigma} y_{ge} \tag{2.65}$$

$$k^{s}_{ae}(t) = \kappa^{\sigma} \left(\frac{p_{k_{ae}}(t+1)}{p(t+1)} \right)^{-\sigma} y_{ge}(t+1) \tag{2.66}$$

We derive now an expression for the prices. Substitution of the factor demand relations into the production function gives

$$\begin{aligned} y_{ge} &= \left(\kappa k^{s}_{ae}(t-1)^{\frac{\sigma-1}{\sigma}} + \alpha l_{ee}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \qquad (2.67) \\ &= \left(\kappa^{\sigma} \left(\frac{p_{k_{ae}}}{p} \right)^{-\sigma+1} y_{ge}^{\frac{\sigma-1}{\sigma}} + \alpha^{\sigma} \left(10^{-3} \frac{p_{l_{ee}}}{p} \right)^{-\sigma+1} y_{ge}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

Dividing both sides by production and bringing the price to the left side gives

$$p = \left(\kappa^{\sigma} p_{k_{ae}}^{-\sigma+1} + \alpha^{\sigma} \left(10^{-3} p_{l_{ee}} \right)^{-\sigma+1} \right)^{\frac{1}{\sigma-1}} \tag{2.68}$$

Prices are a weighted average of capital costs and wage costs. This factor price frontier is an implicit expression of the wage rate as a function of the exogenous price of final goods , the interest rate and tax parameters.

Different age cohorts have different wages due to differences in productivity, leading to

$$p_{lde}(h) = p_{lee} p_{ro} l^{ci}_{sh}(h) \tag{2.69}$$

We now derive a relation between the capital stock and the value of the firm. We make use of the homogeneity of the production function

$$y_{ge} = k^{s}_{ae}(t-1) F_{k^{s}_{ae-1}} + l_{ee} F_{l_{ee}} \tag{2.70}$$

$$= k^{s}_{ae}(t-1) \frac{p_{k_{ae}}}{p} + 10^{-3} l_{ee} \frac{p_{l_{ee}}}{p} \tag{2.71}$$

After some substitutions the first order conditions 2.53 and 2.55 lead to the following

$$
\begin{aligned}
(1+r_s)\left(q(t)k^s_{ae}(t)-\lambda(t)S_{be}(t)\right) = &\left(1-\tau_{vpb}\right)p(t+1)F_{k^s_{ae}}(t+1)k^s_{ae}(t)+ \\
&+q(t+1)(1-\delta_s)k^s_{ae}(t)-\left[(1-\tau_{vpb})r_b+b_0\right]S_{be}(t)-(1-b_0)S_{be}(t)\lambda(t+1) \\
&=\left(1-\tau_{vpb}\right)p(t+1)\left[y_{ge}(t+1)-F_{l_{ee}}(t+1)l_{ee}(t+1)\right]-\left[(1-\tau_{vpb})r_b+b_0\right]S_{be}(t) \\
&+q(t+1)\left[k^s_{ae}(t+1)-i_{ge}(t+1)\right]-\left[S_{be}(t+1)-b_1p(t+1)i_{ge}(t+1)\right]\lambda(t+1) \\
&=\left(1-\tau_{vpb}\right)\left[p(t+1)y_{ge}(t+1)-10^{-3}p_{le}(t+1)l_{ee}(t+1)\right]-\left[(1-\tau_{vpb})r_b+b_0\right]S_{be}(t) \\
&-\left[\left(1-b_1-\frac{\tau_{vpb}v}{v+r_s}\right)+\lambda(t+1)b_1\right]p(t+1)i_{ge}(t+1)+q(t+1)k^s_{ae}(t+1) \\
&-\left[S_{be}(t+1)-b_1p(t+1)i_{ge}(t+1)\right]\lambda(t+1) \\
&=\left(1-\tau_{vpb}\right)\left[p(t+1)y_{ge}(t+1)-10^{-3}p_{le}(t+1)l_{ee}(t+1)\right]-\left[(1-\tau_{vpb})r_b+b_0\right]S_{be}(t) \\
&-\left(1-b_1-\frac{\tau_{vpb}v}{v+r_s}\right)p(t+1)i_{ge}(t+1)+q(t+1)k^s_{ae}(t+1)-\lambda(t+1)S_{be}(t+1) \\
&=D_{iv}(t+1)-\tau_{vpb}\left(A_f(t+1)-\frac{v}{v+r_s}p(t+1)i_{ge}(t+1)\right)+H(t+1) \\
&+q(t+1)k^s_{ae}(t+1)-\lambda(t+1)S_{be}(t+1),
\end{aligned}
\tag{2.72}
$$

Forward solution leads to the conclusion that the value of the capital stock minus the value of debt equals the discounted value of the dividend payments (the value of the firm) minus the depreciation allowance on investments installed up to time t and an arbitrary constant

$$
qk^s_{ae}-\lambda S_{be}=S^s_{se}-\tau_{vpb}AF+\sum_{j=1}^{\infty}H(t+j)\left(1+r_s\right)^{-j}
\tag{2.73}
$$

So for the value of the firm we have

$$
\begin{aligned}
S^s_{se}&=qk^s_{ae}-\lambda S_{be}+\tau_{vpb}AF-\sum_{j=1}^{\infty}H(t+j)\left(1+r_s\right)^{-j} \\
&=\left[1-\frac{\tau_{vpb}v}{v+r_s}-b_1+\frac{(1-\tau_{vpb})r_b+b_0}{r_s+b_0}b_1\right]pk^s_{ae}-\frac{(1-\tau_{vpb})r_b+b_0}{r_s+b_0}S_{be}+\tau_{vpb}AF-K^s_h
\end{aligned}
\tag{2.74}
$$

$$
\begin{aligned}
AF(t) = &\ \textstyle\sum_{j=0}^{\infty}(1-v)^j\frac{v}{r_s+v}i_{ge}(t-j)p(t-j) \\
= &\ \textstyle\frac{v}{r_s+v}i_{ge}(t)p(t)+\frac{1-v}{r_s+v}\sum_{j=1}^{\infty}v(1-v)^{j-1}i_{ge}(t-j)p(t-j) \\
= &\ \textstyle\frac{v}{r_s+v}i_{ge}(t)p(t)+\frac{1-v}{r_s+v}A_f(t)
\end{aligned}
\tag{2.75}
$$

$$
\begin{aligned}
K_h^s(t) = \quad & \textstyle\sum_{j=1}^{\infty} H(t+j)\,(1+r_s)^{-j} \\
= \quad & H(t+1)\,(1+r_s)^{-1} + \textstyle\sum_{j=2}^{\infty} H(t+j)\,(1+r_s)^{-j} \\
= \quad & H(t+1)\,(1+r_s)^{-1} + \textstyle\sum_{i=1}^{\infty} H(t+1+i)\,(1+r_s)^{-i-1} \\
= \quad & H(t+1)\,(1+r_s)^{-1} + K_h^s(t+1)\,(1+r_s)^{-1}
\end{aligned}
\tag{2.76}
$$

$$
\begin{aligned}
S_{se}^s = & \left[1 - \frac{\tau_{vpb} v}{v + r_s} - b_1 + \frac{(1-\tau_{vpb}) r_b + b_0}{r_s + b_0} b_1\right] p k_{ae}^s + \\
& - \frac{(1-\tau_{vpb}) r_b + b_0}{r_s + b_0} S_{be} + \tau_{vpb} AF - K_h^s \\
= & \left[1 - \frac{\tau_{vpb} v}{v + r_s} - b_1 + \frac{(1-\tau_{vpb}) r_b + b_0}{r_s + b_0} b_1\right] p k_{ae}^s - \frac{(1-\tau_{vpb}) r_b + b_0}{r_s + b_0} S_{be} \\
& + \tau_{vpb} \left(\frac{v}{r_s + v} i_{ge}(t) p(t) + \frac{1 - v}{r_s + v} A_f(t)\right) - K_h^s
\end{aligned}
\tag{2.77}
$$

3 Welfare analysis

In this section welfare is discussed using the utility function of section 2.1.3 with consumption as the only argument. However, labour supply has been modelled, too, but not consistently with optimal behaviour. We intend to integrate labour supply and consumer behaviour later on. The welfare measure of this section is only a first rough approximation and is not been used.

Financial wealth equals

$$s^s_{ah}(t+i-1) = \frac{\zeta(t+i)}{1+r_{hr}(t+i)} s^s_{ah}(t+i) + \frac{1}{1+r_{hr}(t+i)} \left[c_{onp}(t+i) - y(t+i)\right] \tag{3.1}$$

$$\begin{aligned}
s^s_{ah}(t-1) &= \frac{1}{1+r_{hr}(t)} \left[\zeta(t) s^s_{ah}(t) - y(t) + c_{onp}(t)\right] \\
&= \frac{\zeta(t)}{1+r_{hr}(t)} s^s_{ah}(t) + \frac{1}{1+r_{hr}(t)} \left[c_{onp}(t - y(t))\right] \\
&= \sum_{i=1}^{n_T-j} \left[\left(\prod_{l=0}^{i-1} \frac{\zeta(t+l)}{1+r_{hr}(t+l)} \right) \frac{1}{1+r_{hr}(t+i)} \left[c_{onp}(t+i) - y(t+i)\right] \right] && (3.2) \\
&+ \frac{1}{1+r_{hr}(t)} \left[c_{onp}(t - y(t))\right] && (3.3)
\end{aligned}$$

The discounted value of consumption equals total wealth

$$\begin{aligned}
c_{onp}(t) + (1+r_{hr}(t)) \sum_{i=1}^{n_T-j} \left[\left(\prod_{l=0}^{i-1} \frac{\zeta(t+l)}{1+r_{hr}(t+l)} \right) \frac{1}{1+r_{hr}(t+i)} c_{onp}(t+i) \right] &= && (3.4) \\
= (1+r_{hr}(t)) s^s_{ah}(t-1) + h(t) \text{ with} \\
h(t) = y(t) + (1+r_{hr}(t)) \sum_{i=1}^{n_T-j} \left[\left(\prod_{l=0}^{i-1} \frac{\zeta(t+l)}{1+r_{hr}(t+l)} \right) \frac{1}{1+r_{hr}(t+i)} y(t+i) \right]
\end{aligned}$$

with h human wealth.

$$\begin{aligned}
h(t) &= y(t) + \sum_{i=1}^{n_T-j} \left[\left(\prod_{l=0}^{i-1} \frac{\zeta(t+l)}{1+r_{hr}(t+l)} \right) \frac{1+r_{hr}(t)}{1+r_{hr}(t+i)} y(t+i) \right] && (3.5) \\
&= y(t) + \left[\frac{\zeta(t)}{1+r_{hr}(t)} \frac{1+r_{hr}(t)}{1+r_{hr}(t+1)} y(t+1) \right] + \sum_{i=2}^{n_T} \left[\left(\prod_{l=0}^{i-1} \frac{\zeta(t+l)}{1+r_{hr}(t+l)} \right) \frac{1+r_{hr}(t)}{1+r_{hr}(t+i)} y(t+i) \right] \\
h(t+1) &= y(t+1) + \sum_{i=1}^{n_T-j-1} \left[\left(\prod_{l=0}^{i-1} \frac{\zeta(t+1+l)}{1+r_{hr}(t+1+l)} \right) \frac{1+r_{hr}(t+1)}{1+r_{hr}(t+1+i)} y(t+1+i) \right] \\
&= y(t+1) + \sum_{j=2}^{n_T-j} \left[\left(\prod_{k=1}^{j-1} \frac{\zeta(t+k)}{1+r_{hr}(t+k)} \right) \frac{1+r_{hr}(t+1)}{1+r_{hr}(t+j)} y(t+j) \right] \\
&= y(t+1) + \frac{1+r_{hr}(t+1)}{\zeta(t)} \sum_{j=2}^{n_T-j} \left[\left(\prod_{k=0}^{j-1} \frac{\zeta(t+k)}{1+r_{hr}(t+k)} \right) \frac{1+r_{hr}(t)}{1+r_{hr}(t+j)} y(t+j) \right]
\end{aligned}$$

Human wealth can be written as a difference equation

$$h(j,t) = y(j,t) + \frac{1 - q_{bdh}(j,t)}{1 + r_{hr}(t+1)} h(j+1,t+1),\ j \in \{j_w, .., j_e - 1\} \tag{3.6}$$

$$h(j_e,t) = y(j_e,t)$$

$$y(j) = y_{th}(j) + p_u^b(j) + y_{wa}(j) - \left(l_{iak}(j) + l_{iuk}(j) + l_{i65}(j) + l_{iap}(j) + l_{iov}(j) + p_p^b(j)\right)$$

$$ev(j,t) = \frac{EU(j,t)^n - EU(j,t)^0}{EU(j,t)^o} \left((1 + r_{hr}(t)) s_{ah}^s(j-1,t-1) + h(j,t)\right)^o \frac{1}{d_{ish}(t)},\ j \in \{j_w, .., j_e - 1\} \tag{3.7}$$

with '^{o}' indicating a base run value and '^{n}' a current value. The with discount factor scaled equivalent variations make equivalent variations of children and unborn comparable with those of adults. We define equivalent variations for children and unborn as

$$ev(j,t) = ev(j_w, t + j_w - j),\ j < j_w \tag{3.8}$$

The equivalent variations of the different cohorts will be aggregated using cohort sizes to one overall welfare measure. This measure does not evaluate improvements of the Pareto-efficiency of the economy. Improvements of Pareto efficiency occur when all generation experience gains. Policy measures often make some groups worse off. To evaluate improvements of the Pareto efficiency of policy measures one can introduce a Lump Sum Redistribution Authority (Auerbach and Kotlikoff (1988) , pp56)

4 Calibration

4.1 Firms

We determine the wage rate in efficiency units firstly

$$
\begin{aligned}
l_{de} p_{lde} &= \sum_j l_{de}(j) p_{lde}(j) \\
&= \sum_j l_{de}(j) p_{lee} p_{ro} l_{sh}^{ci}(j) \\
&= p_{lee} p_{ro} \frac{\sum_j l_{de}(j) l_{sh}^{ci}(j)}{l_{de}} l_{de}
\end{aligned}
\tag{4.1}
$$

The wage rate in efficiency units can be calculated as

$$
p_{lee} = \frac{p_{lde}}{p_{ro} l_{sh}^{ci}} \tag{4.2}
$$

with

$$
l_{sh}^{ci} = \frac{\sum_j l_{de}(j) l_{sh}^{ci}(j)}{l_{de}} \tag{4.3}
$$

The coefficients of the production function can be calibrated with the labour demand and factor price frontier

$$
\begin{aligned}
\alpha &= 10^{-3} \frac{p_{lee}}{p} \left(\frac{y_{ge}}{l_{ee}} \right)^{-\frac{1}{\sigma}} \\
\kappa &= \frac{p_{kae}}{p} \left(\frac{y_{ge}}{k_{ae}} \right)^{-\frac{1}{\sigma}}
\end{aligned}
\tag{4.4}
$$

The capital stock follows from

$$
\begin{aligned}
k_{ae}^{s}(t-1) &= \frac{i_{ge}(t)}{\delta_s + \frac{l_{ee}(t+1)}{l_{ee}(t)} - 1} \\
k_{ae}^{s}(t) &= l_{ee}(t+1) \left(\frac{\kappa}{\alpha} \right)^{\sigma} \left(\frac{p_{k_{ae}}(t+1)}{10^{-3} p_{l_{ee}}(t+1)} \right)^{-\sigma} \\
\delta_s &= \frac{i_{de}(t)}{k_{ae}^{s}(t-1)}
\end{aligned}
\tag{4.5}
$$

The risk premium has been calibrated in such a way that the wage costs plus capital costs equals gross value added

$$
r_{isk} \mid y_{ge} p = 10^{-3} l_{de} p_{lde} + k_{ae}^{s}(t-1) p_{k_{ae}} \tag{4.6}
$$

4.2 Consumption

$$c_{onp}(t)+\sum_{i=1}^{n_T-j}\left[\left(\prod_{l=0}^{i-1}\frac{\zeta(t+l)}{1+r_{hr}(t+l)}\right)\frac{1+r_{hr}(t)}{1+r_{hr}(t+i)}c_{onp}(t+i)\right]= \tag{4.7}$$

$$=(1+r_{hr}(t))s^s_{ah}(t-1)+h(t)$$

$$c_{onp}(t)+\sum_{i=1}^{n_T-j}\gamma(t+i-1)c_{onp}(t+i)=(1+r_{hr}(t))s^s_{ah}(t-1)+h(t)$$

$$h(t)=y(t)+(1+r_{hr}(t))\sum_{i=1}^{n_T}\left[\left(\prod_{l=0}^{i-1}\frac{\zeta(t+l)}{1+r_{hr}(t+l)}\right)\frac{1}{1+r_{hr}(t+i)}y(t+i)\right] \tag{4.8}$$

$$c_{nh}(t)=\left(\frac{1+\beta}{1+r_{hr}(t+1)}\frac{1+\tau_{co}(t+1)}{1+\tau_{co}}\right)^{\gamma}c_{nh}(t+1) \tag{4.9}$$

$$c_{onp}=(1+\tau_{co})c_{nh}, \tag{4.10}$$

$$c_{onp}(t)=\left(\tfrac{1+\beta}{1+r_{hr}(t+1)}\right)^{\gamma}\left(\tfrac{1+\tau_{co}(t+1)}{1+\tau_{co}}\right)^{\gamma-1}c_{onp}(t+1)$$

$$=\left[\prod_{l=1}^{i}\left(\tfrac{1+\beta}{1+r_{hr}(t+l)}\right)^{\gamma}\left(\tfrac{1+\tau_{co}(t+l)}{1+\tau_{co}(t+l-1)}\right)^{\gamma-1}\right]c_{onp}(t+i) \tag{4.11}$$

$$=\alpha(t+i)c_{onp}(t+i)$$

$$c_{onp}(t)=\left[1+\sum_{i=1}^{n_T-j}\frac{\gamma(t+i-1)}{\alpha(t+i)}\right]^{-1}\left[(1+r_{hr}(t))s^s_{ah}(t-1)+h(t)\right] \tag{4.12}$$

$$\alpha(t+i)=\left[\prod_{l=1}^{i}\left(\frac{1+\beta}{1+r_{hr}(t+l)}\right)^{\gamma}\left(\frac{1+\tau_{co}(t+l)}{1+\tau_{co}(t+l-1)}\right)^{\gamma-1}\right] \tag{4.13}$$

$$\gamma(t+i-1)=\frac{1+r_{hr}(t)}{1+r_{hr}(t+i)}\left(\prod_{l=0}^{i-1}\frac{\zeta(t+l)}{1+r_{hr}(t+l)}\right)$$

$$=\frac{\zeta(t)}{\zeta(t+i)}\left(\prod_{l=1}^{i}\frac{\zeta(t+l)}{1+r_{hr}(t+l)}\right)$$

In case of a constant consumption tax the following holds

$$\frac{\gamma(t+i-1)}{\alpha(t+i)}=\frac{\zeta(t)}{\zeta(t+i)}\left(\prod_{l=1}^{i}\frac{\zeta(t+l)}{1+r_{hr}(t+l)}\right)\left[\prod_{l=1}^{i}\left(\frac{1+\beta}{1+r_{hr}(t+l)}\right)^{-\gamma}\left(\frac{1+\tau_{co}(t+l)}{1+\tau_{co}(t+l-1)}\right)^{-\gamma+1}\right] \tag{4.14}$$

$$=(1+\beta)^{-\gamma i}\frac{\zeta(t)}{\zeta(t+i)}\prod_{l=1}^{i}\zeta(t+l)\prod_{l=1}^{i}(1+r_{hr}(t+l))^{1+\gamma}$$

$$=\chi_1(t+i)\chi_2(t+i)\chi_3(t+i)=\chi(t+i)$$

$$\chi_3(t+i) = \prod_{l=1}^{i} (1+r_{hr}(t+l))^{1+\gamma} \tag{4.15}$$
$$= (1+r_{hr}(t+i))^{1+\gamma}\,\chi_3(t+i-1)$$
$$\chi_3(t) = 1$$

$$\chi_2(t+i) = \frac{\zeta(t)}{\zeta(t+i)} \prod_{l=1}^{i} \zeta(t+l) \tag{4.16}$$
$$= \zeta(t+i-1)\chi_2(t+i-1)$$
$$\chi_2(t) = 1$$

$$\chi_1(t+i) = (1+\beta)^{-\gamma i}$$
$$= \exp\left[-\gamma i \ln(1+\beta)\right]$$

So we have written consumption as

$$c_{onp}(t) = \left[1 + \sum_{i=1}^{n_T - j} (1+\beta)^{-\gamma i}\,\chi_2(t+i)\chi_3(t+i)\right]^{-1} \left[(1+r_{hr}(t))s^{s}_{ah}(t-1) + h(t)\right] \tag{4.17}$$

We use the time preference parameter β to calibrate the macro-consumption (consumption aggregated over the cohorts). The intertemporal substitution elasticity $-\gamma$ is fixed at 0.5. Total financial wealth (wealth aggregated over the cohorts) is calibrated using total taxes on financial wealth and the effective tax rate,which is fixed at 0.7 percent

$$S^{s}_{ah}(t-1) = \frac{L_{iki}(t)}{\tau_{liki}(t)} \tag{4.18}$$

The steady state values of the distribution of financial wealth over the cohorts is used to distribute total wealth over the cohorts in the base year. This distribution influences the calibrated value β and the steady state distribution. This implies the necessity of several iterations. This procedure is similar to Hans Fehr's method.

Several researchers use information on the actual distribution data, for instance Kotlikoff uses information of the actual wealth distribution. Due to inconsistency of this distribution with the parameters of his model, he get adjustment processes to the steady state distribution. The Dream team from Denmark uses the consumption distribution and equation 4.17 to obtain the wealth distribution. This procedure also leads to adjustment processes, because the income development in the past does not need to be consistent with the expected income development used in the model.

4.3 Government

The right hand side variables are exogenous in the calibration year. The tax rates are calibrated as follows.

Labour income tax rate

$$\tau_{liak} = \frac{L_{iak}}{Y_{wa} - P_p^b} - \tau_{pp}$$

Transfer income tax rate

$$\tau_{liuk} = \frac{L_{iuk}}{B_{ijs} + W_{klh}} - \tau_{pp}$$

Tax rate on pension benefits

$$\tau_{li65} = \frac{L_{i65}}{A_{owo} + P_u^b}$$

Consumption tax rate

$$\tau_{co} = \frac{\frac{T_{co}}{C_{onp}}}{1 - \frac{T_{co}}{C_{onp}}}$$

The tax rate on capital income τ_{liki} is fixed at 0.7 percent.

4.4 Pension funds

premium

The total pension premium rate τ_{plb}^{i} is exogenous in the calibration year. The base premium τ_{plb1}^{i} is calculated with the model. The catching up premium results in

$$\tau_{plb2}^{f} = \tau_{plb}^{f} - \tau_{plb1}^{f} \tag{4.19}$$

The base premium can be calculated because the income expectations are known due to our assumption of steady state productivity growth and a constant cost of capital. This results in correct expectations of the required actuarial reserves for workers, which have to be accrued and in correct expectations of the discounted value of the premium base.

Accrual rate

$$o_{pb}^{ci} = \frac{P_u^f(j,t)}{d_j(j_r - 1, t-1)G(j_r - 1, t-1)\left(\frac{p(t)}{p(t-1)}\right)^{p^{ci}}\left(\frac{p_{ro}(t)}{p_{ro}(t-1)}\right)^{p_{ro}^{ci}}\left(\frac{p_{ywee}(t)p(t-1)}{p_{ywee}(t-1)p(t)}\right)^{p_l^{ci}}}, \; j = j_r \tag{4.20}$$

5 Symbols

a_{kws}	children's assistance
a_{owo}	public pensions
b	surplus on the commodity account of the current account
b_a	population size
b_{ap}	pensioners
b_{aw}	workers
b_b	births
b_{blr}	surplus on the total account of the current account
b_{ijn}	auxiliary variable social assistance
b_{ijs}	social assistance
b_{str}	target replacement rate (pensions relative to wages) in the DC system
S_{vf}	annual savings foreign sector
b_{bpf}	gross domestic product, factor prices
b_{bpr}	gross domestic product, market prices
b_d	deaths
b_e	emigrants
b_{ff}	fertility
b_i	labour supply
b_{ijs}	social assistance
b_l	immigrants
blG	labour supply per gender group, $G = f, m$
b_{lh}	total labour supply
b_{lI}	labour supply per gender group, $I = f, m$
b_{ni}	gross national income
c_{ap}	aggregate public consumption
c_{lt}	the discounted value of the premium base
c_{lth}	auxiliary variable for calculating the discounted value of the premium base
c_{na}	net total domestic expenditure on consumption
c_{nh}	net individual expenditure on consumption
c_{np}	material public consumption
c_{onp}	gross individual expenditure on consumption
c_{ont}	total consumption (private plus government)

c_{wp} auxiliary variable for calculating the discounted value of the premium rights of pensioners

d_{eov} depreciation. government

d_{eto} total domestic depreciation investments

d_{fns} public outlays on defence

d_{gb} public depreciation, investments in buildings

d_{if} public depreciation, investments in infrastructure

d_{isc} discount factor, net profits government

d_{ish} discount factor, households

d_{isp} discount factor, pension funds

d_{iv} dividend

d_j passed working years

d_{jt} future working years

d_{nba} asset holdings of the Central Bank

d_r^d discounted value survival rates workers

d_{rw} discounted value of one guilder pension

d_{sh} public depreciation investments in schools

d_{tot} aggregate public depreciation investments

f_{acb} government ownership of shares issued by private firms

f_{acr} government holdings of bonds

f_{ine} new shares purchased by the government

f_{ntk} government deficit

f_{ran} franchise for pension premium payments

g_{asb} public revenues from natural gas

g_{en} intra-generational transfer (life insurance)

g_{pp}^b premium base for public pension premiums

i_{bgb} gross public investments in buildings

i_{bif} gross public investments in infrastructure

i_{bsh} gross public investments in schools

i_{de} depreciation investments of the domestic industry

i_{ge} gross investments of the domestic industry

i_{kto} aggregate government income

i_{na} total net domestic investments, exclusive clearing of land for building

i_{ne} net investments of the domestic industry

i_n	investment returns
i_{ngb}	net public investments in buildings
i_{nif}	net public investments in infrastructure
i_{nov}	gross aggregate public investments
i_{nsh}	net public investments in schools
i_{nsu}	indirect taxes minus subsidies (NR data)
i_{ntc}	total gross domestic investments, exclusive clearing of land for building
i_{nto}	net aggregate public investments
k_{aa}	total domestic capital stock
k_{ae}^{s}	capital stock of enterprises
k_{ggb}	public buildings
k_{gga}^{s}	stock natural gas
k_{gif}	public infrastructure
k_{gsh}	public schools
k_{gto}	aggregate public capital stock
k_h	claim government on firms (valuation by firms)
k_{h2}	claim government on firms (valuation by the government)
ξ	survival rates corrected for immigration and emigration
l_{da}	total domestic labour demand
l_{dab}	total employment, without price effects
l_{de}	labour demand by the domestic industry
l_{dp}	labour demand by the government
l_{ee}	employment in efficiency units in the domestic industries
l_{i65}	taxes paid by 65+ on their AOW
l_{iak}	labour income tax
l_{iap}	private pension tax
l_{iki}	capital income tax
l_{iov}	other income taxes
l_{ito}	aggregate income taxes and premiums
l_{iuk}	taxes on social benefits
l_{sh}^{ci}	productivity profile
m_{pk}	marginal product of capital
m_{pl}	marginal product of labour
n_{bes}	national expenditures

n_{grv} clearing of land for building

n_{mfc} price of goods

o_{ndw} public outlays on education

o_{pbb} accrual rate of pensions in the final wage system

o_{pbs} public outlays on public administration

o_{pdc} accrual rate pensions in the DC-system

o_{pmi} accrual rate pensions in the average wage system

o_{vbu} public transfers to foreigners

o_{vhs} government debt

o_{vin} other income

o_{vvm} net government's wealth

p_{in} indexation factor

p_n^i average pension payments as a percentage of the last earned income, $i = b, f, a, d$

p_p^i pension premium, $i = b, f, a, d$

p_r^i acquired rights of pensioners and workers, $i = b, f, a, d$

p_t^i required actuarial reserves for workers which has to be accrued, $i = f, a$

p_u^i pension benefits, $i = b, f, a, d$

p_w^i required actuarial reserves for workers, $i = f, a$

p_w^d discounted value pension target

p_{cht} public asset holdings of grounds leased to the private sector

p_{gef} participation rate

p_{rf} profits of the domestic industry

p_{ro} general labour productivity measures, applying uniformly to all types of labour in the economy

q points to a quote; suffix indicates the numerator of the quote

q_{asp}^b target replacement rate private pensions

q_{ka} coverage of pension entitlements

q_{ka}^s desired coverage of pension entitlements

q_{kao} lower bound of the coverage of pension entitlements

q_{kab} upper bound of the coverage of pension entitlements

q_{np} actual level private pensions

r_b nominal interest rate

r_{br} real interest rate

r_{ee}^s firms' reserves

r_{eg}^{s}	net discounted claim of government on firms
r_{geb}	profit of government buildings
r_h	rate of return, households
r_{hr}	real rate of return, households
r_{inf}	profit of infrastructure
r_p	rate of return, pension funds
r_{pe}	effective rate of return of pension funds, nominal
r_{pr}	real rate of return of pension funds
r_s	rate of return on shares
r_{sch}	profit of schools
r_{utg}	public interest payments
s_a	total assets in the Netherlands
s_{ae}	aggregate change of firms assets
s_{ab}^{i}	change in pension fund assets through emigration, $i = b, f, a, d$
s_{abh}	change in household's assets through emigration
s_{ac}	inflation correction
s_{af}	aggregate foreign asset holdings
s_{ah}	aggregate individual asset holdings
s_{am}^{i}	change assets pension funds through immigration, $i = b, f, a, d$
s_{amh}	change in households' assets through immigration
s_{be}	bonds issued by firms
s_{bf}	bonds owned by foreigners
s_{bg}	bonds issued by the government net of bonds owned by the government
s_{bh}	bonds owned by households
s_{bhn}	bonds owned by households; low interest rate
s_{bhr}	bonds owned by households; high interest rate
s_{bp}	bonds owned by pension funds
s_{gnr}	seigniorage
s_{he}	revaluation of assets
s_{heh}	revaluation of assets owned by households
s_{hef}	revaluation of assets owned by foreigners
s_{hep}	revaluation of assets owned by pension funds
s_{lo}	gross wages of employees
s_{se}	total shares issued by domestic private firms

s_{sf}	shares owned by foreigners
s_{sh}	shares owned by households
s_{sp}	shares owned by pension funds
s_{ub}	elasticity of substitution between capital and labour in production
s_{ubs}	public outlays on subsidies
s_{vh}	household's savings
s_{vp}	savings private pension funds
t_{ah}	total individual payments on taxes and premiums
t_{ax}	aggregate public revenue from taxes and premiums
t_{co}	consumption tax
τ_{kd}^{b}	actuarial fair pension premium
τ_{pe}	actual pension premium rate
τ_{wp}	employer premiums
τ_{plb}^{i}	pension premium in the average and final wage systems, $i = b, f, a, d$
τ_{plb1}^{i}	basic pension premium in the average and final wage systems, $i = b, f, a, d$
τ_{plb2}^{i}	catching up pension premium in the average and final wage systems, $i = b, f, a, d$
τ_{plg}	average pension premium
τ_{pp}	public pension premium
u	expected utility
u_{ito}	aggregate government expenditure
v	fiscal depreciation of firms
v_{m}^{i}	total assets, pension funds, $i = b, f, a, d$
v_{mb}^{i}	total assets, pension funds at the start of a year, $i = b, f, a, d$
v_{mib}	government dividend income
v_{mid}	annual income of the Central Bank
v_{mip}	annual income from the lease of public grounds
v_{mir}	government interest income
v_{mit}	aggregate government capital income
v_{pdb}	profit tax
v_{rt1}	EMU deficit, exclusive of depreciation
v_{rtk}	EMU deficit
w_{aoz}	disability transfer
w_{klh}	public unemployment transfer
x_{ah}	aggregate individual expenditure

x_{np} total public outlays minus interest payments and adjustment
y_{ah} aggregate gross individual income
y_{ge} gross domestic product of the private industry
y_{gp} gross domestic product of the government
y_{na} net domestic product
y_{ne} net domestic product of the private industry
y_{th} individual transfer income
y_{we} wage sum, enterprises
y_{wa} household labour income
y_{wp} wages government
y_{zh} household capital income
y_{zg} government capital income
y_{zp} private pension fund capital income
z_{bh} individual interest income
z_{bp} interest income, pension funds
z_{br} current account incomes
z_{org} public outlays on health care
z_{sh} household dividend income
z_{sp} pension fund dividend income

6 Total accounts

Table 6.1 The total accounts: circular flow

	house-holds	pension sector	capital	government services	government taxes	government production	firms	foreign sector	Σ
Goods	$-C_{onp}$		$-I_{ntc}$	$-C_{ap}$	I_{nsu}	Y_{gp}	Y_{ge}	$-B$	0
Depreciation			D_{eto}			$-D_{tot}$	$-I_{de}$		0
Net investments			I_{na}		N_{grv}	$-I_{nto}$	$-I_{ne}$		0
Transfers	Y_{th}			$-Y_{th}$					0
Labour income	Y_{wa}					$-Y_{wp}$	$-Y_{we}$		0
Private pensions	P_u	$-P_u$							0
Non-labour income	Y_{zh}	Y_{zp}		$-R_{utg}$	Y_{zg}		$-Y_{ze}$	Z_{br}	0
Income taxes	$-L_{ito}$				L_{ito}				0
Profit tax					V_{pdb}		$-V_{pdb}$		0
Private pension premiums	$-P_p$	P_p							0
Transfers to foreigners				$-O_{vbu}$				O_{vbu}	0
Savings(-)/shortage(+)	$-S_{vh}$	$-S_{vp}$			V_{rtk}		S_{be}	$-S_{vf}$	0
Σ	0	0	0		0		0	0	0

Table 6.2 The total accounts: changes in total assets

	households	pension sector	government	firms	foreign sector	Σ
Savings	S_{vh}	S_{vp}	$-V_{rtk}$	$-S_{be}$	S_{vf}	0
Asset changes by emigration	$-S_{abh}$	$-S_{abp}$			S_{ab}	0
Asset changes by immigration	S_{amh}	S_{amp}			$-S_{am}$	0
Revaluation of assets	S_{heh}	S_{hep}		$-S_{be}$	S_{hef}	0
Change of financial wealth	$-S_{ah}$	V_m^b	V_{rtk}	S_e	$-S_{af}$	0
Σ	0	0	0	0	0	0

Table 6.3 The total accounts: portfolio changes

	households	pension sector	government	firms	foreign sector	Σ
Change of financial wealth	S_{ah}	$-V_m^b$	$-V_{rtk}$	$-S_e$	S_{af}	0
Change of shares	$-S_{sh}$	$-S_{sp}$	$-F_{acb}$	S_{se}	$-S_{sf}$	0
Change of bonds	$-S_{bh}$	$-S_{bp}$	S_{bg}	S_{sb}	$-S_{sb}$	0
Σ	0	0	0	0	0	0

Table 6.4 The total accounts: portfolio

	households	pension sector	capital	government	firms	foreign sector	Σ
Bonds	$-S^s_{bh}$	$-S^s_{bp}$		S^s_{bg}	S^s_{be}	$-S^s_{sb}$	0
Shares	$-S^s_{sh}$	$-S^s_{sp}$		$-F^s_{acb}$	S^s_{se}	$-S^s_{sf}$	0
Capital goods			K^s_a	$-K^s_{gto}$	$-K^s_{ae}$		0
Reserves/provisions government			R^s_{eg}	$-K^s_{h2}$	K^s_h		0
Reserves/provisions firms			R^s_{ee}		$-R^s_{ee}$		0
Total assets	S^s_{ah}	V^{bs}_m	$-S^s_a$	O^s_{vvm}		S^s_{af}	0
Σ	0		0	0	0	0	0

7 Imputed income to adults from government transfers to children

(This section has not been implemented at this moment) Children ($j < j_w$) get children's assistance and also disability allowance. This income has to transferred to adults. Otherwise, government measures could not be judged on their welfare implications because only adults only make independent decisions in the model. In case income of the children goes to the parents the transfer relation reads as

$$Y^o_{th}(j,t) = \frac{1}{b^s_{ah}(j-1,t-1)} \sum_{\tau=0}^{19} \frac{b_{ff}(j-\tau,t-\tau)}{b_{bh}(t-\tau)} b^s_{ah}(\tau-1,t-1) Y_{th}(\tau,t), \; j \in \{16,..,69\} \quad (7.1)$$

because the model assumes women get children between the age of 16 and 50. To make a distribution over adults only we assume that holds

$$Y^o_{th}(j,t) = \frac{1}{b^s_{ah}(j-1,t-1)} \sum_{\tau=0}^{19} \frac{b_{ff}(j-\tau,t-\tau)}{\sum_{i=\max\{16+\tau,j_w\}}^{50+\tau} b_{ff}(i-\tau,t-\tau)} b^s_{ah}(\tau-1,t-1) Y_{th}(\tau,t) \quad (7.2)$$

$$j \in \{j_w,..,69\}$$

This equation can be implemented by introducing

$$
\begin{aligned}
b_{ff}^{h}(0,t) &= \sum_{i=j_w}^{50} b_{ff}(i,t) \\
b_{ff}^{h}(1,t) &= \sum_{i=19}^{50} b_{ff}(i,t-1) = b_{ff}^{h}(0,t-1) + b_{ff}(19,t-1) \\
b_{ff}^{h}(2,t) &= b_{ff}^{h}(1,t-1) + b_{ff}(18,t-2) \\
b_{ff}^{h}(3,t) &= b_{ff}^{h}(2,t-1) + b_{ff}(17,t-3) \\
b_{ff}^{h}(4,t) &= b_{ff}^{h}(3,t-1) + b_{ff}(16,t-4) \\
b_{ff}^{h}(\tau,t) &= b_{ff}^{h}(\tau-1,t-1), \tau \in \{5,..,19\}
\end{aligned}
$$

and

$$q_{bff}(j,0,t) = \frac{b_{ff}(j,t)}{b_{ff}^{h}(0,t)},\ j \in \{16,..,50\} \tag{7.3}$$

$$q_{bff}(j,\tau,t) = q_{bff}(j-1,\tau-1,t-1), \{(j,\tau)\} \in \{(17,1),..,(69,19)\} \tag{7.4}$$

Using these additional variables we can calculate the imputed transfer income for $j \in \{\ j_w,..,69\}$

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