Documentation of CORTAX

CORTAX is applied in Bettendorf et al. (2006), a simulation study on the economic and welfare implications of reforms in corporate income taxation. This technical documentation of the model consists of the derivation and listing of the equations of the model and a justification of the calibration.
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1 Technical description of the model

This memo documents version 7 of the model, which is used in in Bettendorf et al. (2006). The first chapter documents the derivation of the equations. The calibration of the model is described in chapter 2. Section 1.1 derives the first-order conditions for consumption and labour supply from utility-maximising households. Section 1.2 derives from profit maximisation, the demand for labour, capital, location specific capital, intermediate inputs and financial assets for domestic and multinational firms. Taxes on corporate income, labour income, consumption and wealth are introduced when appropriate. The tax revenues have to meet the government expenditures on consumption, transfers and debt, see section 1.3. The market equilibria and the linkages with the Rest of the World are presented in section 1.4. Section 1.5 presents the solution procedure.

Notation follows some simple rules. Upper case symbols are used for aggregated values whereas lower case characters are reserved for per capita variables (in terms of the young generation in the country of origin). In the case of variables with two dimensions, the first index refers to the country which owns the resource (residence), whereas the second index denotes the using country (destination). Time subscripts and country indices are dropped in the exposition whenever this is possible.

The rates of return on bonds ($\hat{r}_{wb}$) and equities ($\hat{r}_{we}$) are assumed fixed. The considered countries are small in the sense that they can import (or export) capital from the Rest of the World (ROW) without affecting the world interest rates. In other words, the net supply of capital by the ROW is perfectly elastic. Multinationals are assumed to operate only in the other ‘small’ countries, but not in the ROW (and vice versa). The ROW block does not need to be fully modelled. International capital and good flows are restricted by the current account for each country.

1.1 Households

The overlapping generations framework follows the standard Diamond model (see Heijdra and Van der Ploeg (2002), Chapter 17). An individual is assumed to live for two periods: a working period and a retirement period. In deviation from the standard Diamond model, we assume that each period consist of $T$ years. To keep the model tractable, we make a few simplifying assumptions. First, the consumption share in income is assumed constant when young and when old (i.e. within each period of T years). Since all income components grow at the annual rate $g_a$, consumption when young and when old grow at the same rate $g_a$. Second, young individuals supply the same amount of labour each year, independent of productivity growth. Old individuals do not work and thus have only non-labour income. In sum, households have to
choose consumption paths \( c_s^n (1 + g_a)^s \) and \( c_s^o (1 + g_o)^s \) for the \( s = 0, \ldots, T - 1 \) years in both periods.

Both young and old individuals hold assets in bonds and equities.

### 1.1.2 Consumption and labour supply

#### 1.1.1 Population

The generation sizes are denoted by \( N^y \) and \( N^o \), respectively. Total population \( N = N^y + N^o \) might differ over countries but the population growth rate \( g_n \) is set identical for both countries since we focus on the steady state. This implies \( N^y = (1 + g_a)^y N^o \). The relative population size is written as

\[
\omega_n(i, j) \equiv N^y(i) / N^y(j)
\]  

(1.1)

#### 1.1.2 Consumption and labour supply

Labour supply has to be a constant fraction of the time endowment. Therefore, we have to specify felicity \( v \) such that labour supply is constant even if productivity is growing. One option is to assume log-utility in consumption combined with a unit elasticity of intertemporal substitution, cf. Heijdra and Van der Ploeg (2002). A more flexible approach, which we will adopt here, is to assume that the value of leisure is growing at the productivity growth rate \( g_a \).

\[
v^y(\tau) = \begin{cases} 
\left[ c^y(\tau) \frac{\tau}{\tau} + \alpha_l \left( A_l(\tau) \hat{l}(\tau) \right) \frac{\tau}{\tau} \right]^{\alpha_l} & \text{if } \sigma_l \neq 1 \\
\left[ c^y(\tau) \frac{\tau}{\tau} \rho^u_T \left( A_l(\tau) \hat{l}(\tau) \right) \frac{\tau}{\tau} \right]^{\alpha_l} & \text{if } \sigma_l = 1
\end{cases}
\]  

(1.2)

where \( c^y, o \) is consumption of goods, \( \hat{l} \) is leisure, \( l = 1 - \hat{l} \) is labour supply, \( \alpha_l \) is the weight of leisure and \( \sigma_l \) is the intratemporal substitution elasticity between consumption and leisure.\(^1\) We assume that both consumption per capita and \( A_l \) grow at rate \( g_a \). This implies that

\[
v^y(\tau + 1) = (1 + g_a) v^y(\tau).
\]

Equation (1.2) is combined with a similar expression for the ‘old’ generation, with the restriction that \( \hat{l} = 1 \), in:

\[
U(t) = \frac{1}{1 - 1/\sigma_u} \left[ \sum_{t=0}^{T-1} v^o(t + \tau)^{1 - 1/\sigma_o} \rho_u \sum_{t=0}^{T-1} v^o(t + T + \tau)^{1 - 1/\sigma_o} \right] + \frac{\rho_o}{\rho_a} \sum_{t=0}^{T-1} \left( \frac{1 + g_a}{\rho_a} \right)^{T - t - 1}
\]

\[
= \frac{1}{1 - 1/\sigma_u} \left[ v^o(t)^{1 - 1/\sigma_o} + \frac{\rho_o}{\rho_a} v^o(t + T)^{1 - 1/\sigma_o} \right] \sum_{t=0}^{T-1} \left( \frac{1 + g_a}{\rho_a} \right)^{T - t - 1}
\]  

(1.3)

Wage income equals \( w (1 - \tau_l) l \), where \( w \) denotes the gross wage rate, \( \tau_l \) is the tax rate on labour and \( \hat{w} = w (1 - \tau_l) \) the after tax wage rate. When young, total income, consisting of wage

\(^1\) Sørensen (2001b) models labour supply differently by considering imperfect competition on the labour market. Unions with monopoly power set the wage rate and working hours by maximizing its members’ expected consumer surplus from work. Since the wage rate exceeds the market-clearing level, a fraction of the workers gets involuntary unemployed.
The wealth of the young generation accumulates as

\[ s_w(t, t + \tau) = (1 - \tau)w(t)l + tr^\tau(t) - (1 + \tau)\bar{c}(t) \]  \hspace{1cm} (1.4) \\
s_w(t, t + \tau) = (1 + g_a)\bar{s}_w(t, t) \hspace{1cm} 0 \leq \tau < T

and similar for old households:

\[ s_o(t, t + T) = \pi^o(t + T) + tr^o(t + T) - (1 + \tau)\bar{c}(t + T) \]  \hspace{1cm} (1.5) \\
s_o(t, t + T + \tau) = (1 + g_a)^\tau \bar{s}_o(t, t + T) \hspace{1cm} 0 \leq \tau < T

Households accumulate wealth (assets \( a \)) according to:

\[ a(t, t + \tau) = \rho_o a(t, t + \tau - 1) + s_w(t, t + \tau), \quad 0 \leq \tau < T, \quad a(t, t - 1) = 0 \]
\[ a(t, t + \tau) = \rho_o a(t, t + \tau - 1) + s_o(t, t + \tau), \quad T \leq \tau < 2T, \quad a(t, t + 2T - 1) = 0 \]

The wealth of the young generation accumulates as

\[ a(t, t + \tau) = s_w(t, t) \sum_{j=0}^{\tau} \rho_i^{\tau-j}(1 + g_a)^j, \quad \tau = 0, \ldots, T - 1 \]
\[ a(t, t + T - 1) = s_w(t, t) \sum_{j=0}^{T-1} \rho_i^{T-1-j}(1 + g_a)^j = s_w(t, t) \rho_i^T - (1 + g_a)^T \rho_i - (1 + g_a) \]  \hspace{1cm} (1.6)

Similarly, the wealth of the old generation accumulates as:

\[ a(t, t + T + \tau) = \rho_o a(t, t + T + \tau - 1) + s_o(t, t + T + \tau) \]
\[ = \rho_i\rho_o^{\tau+1} a(t, t + T - 1) + s_o(t, t + T) \sum_{j=0}^{\tau} \rho_i^{T-1-j}(1 + g_a)^j, \quad \tau = 0, \ldots, T - 1 \]

For wealth in the final year (at age \( T \)), this implies:

\[ a(t, t + 2T - 1) = \rho_i\rho_o^T a(t, t + T - 1) + s_o(t, t + T) \sum_{j=0}^{T-1} \rho_i^{T-1-j}(1 + g_a)^j \]
\[ = \rho_i\rho_o^T a(t, t + T - 1) + s_o(t, t + T) \frac{\rho_i^T - (1 + g_a)^T}{\rho_i - (1 + g_a)} \]  \hspace{1cm} (1.7)

Combine (1.6) and (1.7) with \( a(t, t + 2T - 1) = 0 \) and obtain the lifetime budget restriction:

\[ s_w(t, t) = \frac{s_o(t, t + T)}{\rho_i^T} \]
\[ \tilde{w}(t)l + tr^\tau(t) - (1 + \tau)\bar{c}(t) = -\frac{(\pi^o(t + T) + tr^o(t + T) - (1 + \tau)\bar{c}(t + T))}{\rho_i^T} \]  \hspace{1cm} (1.8)

We assume that location specific capital (a fixed factor) is owned by the old generation.
Using the constant-growth assumption, we can write the budget equation for period $t$ as:

$$\bar{w}(t)l + tr^y(t) - (1 + \tau_c)c^y(t) = -\left(\frac{1 + g_{u}}{\rho_u}\right)^T \left[\pi^u(t) + tr^\sigma(t) - (1 + \tau_c)c^\sigma(t)\right]$$  \hspace{1cm} (1.9)$$

Maximizing (1.3) subject to (1.8) yields the first order conditions for $c^y, \dot{l}$ and $c^\sigma$, where $\lambda_u$ denotes the Lagrange multiplier for the budget constraint:\footnote{Strictly speaking, a non-negative restriction on labour supply should be added. However, this restriction is normally not binding in this case.}

$$v^y(0)^{-1/\sigma_u} \sum_{x=0}^{T-1} \left(\frac{1 + g_u}{\rho_u}\right)^x = \lambda_u(1 + \tau_c)$$  \hspace{1cm} (1.10)$$

$$\alpha_A(0)^{1-1/\sigma_u}v^y(0)^{-1/\sigma_u} \sum_{x=0}^{T-1} \left(\frac{1 + g_u}{\rho_u}\right)^x = \lambda_u\bar{w}(0)$$  \hspace{1cm} (1.11)$$

$$\frac{\rho_o}{\rho_u} v^o(T)^{-1/\sigma_o} \sum_{x=0}^{T-1} \left(\frac{1 + g_u}{\rho_u}\right)^x = \lambda_o(1 + \tau_c)$$  \hspace{1cm} (1.12)$$

The first order conditions (1.10) and (1.11) imply that the marginal rate of substitution between consumption and leisure when young should equal the net wage rate:\footnote{In the gams-program we fix $A(0) = 1$ in the base year.}

$$\dot{l} = \left(\frac{\alpha A(1 + \tau_c)}{\bar{w}}\right)^{0/\sigma} c^y$$  \hspace{1cm} (1.13)$$

The first and third equation imply the Euler equation:

$$\left(\frac{v^y(0)}{v^o(T)}\right)^{1/\sigma_u-1/\sigma_o} \left(\frac{c^y(0)}{c^o(T)}\right)^{-1/\sigma_o} = \rho_o \left(\frac{\rho_o}{\rho_u}\right)^T$$  \hspace{1cm} (1.14)$$

### 1.1.3 Portfolio

The portfolio consists of bonds and stocks, which are perceived as imperfect substitutes. Bonds of different origin, yielding the same net interest rate ($\rho_h$), are considered perfect substitutes. The same holds for domestic and foreign equities. Total wealth $a$ is specified as a CES-composite of aggregate bonds $b$ and equities $e$:

$$a = \left[\alpha_e \frac{1}{\sigma_e} b^{\sigma_e+1} + (1 - \alpha_e) \frac{1}{\sigma_b} e^{\sigma_b+1}\right]^{\frac{1}{\sigma_b+\sigma_e}}$$  \hspace{1cm} (1.15)$$

where $\alpha_e$ is a taste parameter and $\sigma_e$ the substitution elasticity between bonds and stocks. The total (after tax) return on the portfolio satisfies:

$$\rho_a a = \rho_b b + \rho_e e$$  \hspace{1cm} (1.16)$$
where $\rho_x$ denotes the gross after tax rate of return on asset composite $x$ ($x = \{s, b, e\}$). The optimal portfolio composition is found by maximizing (1.16) subject to (1.15), where the Lagrange multiplier is seen to equal the total rate of return $\rho_s$. The first-order conditions imply:

$$b = \left(\frac{\rho_b}{\rho_s}\right)^{\sigma_s} \alpha_s a$$ (1.17)

$$e = \left(\frac{\rho_e}{\rho_s}\right)^{\sigma_s} (1 - \alpha_s) a$$ (1.18)

$$\rho_s = \left[\alpha_s \rho_b^{\sigma_s + 1} + (1 - \alpha_s) \rho_e^{\sigma_e + 1}\right]^{\frac{1}{\sigma_s + 1}}$$ (1.19)

In the general case holds that $b + e \leq a$, meaning that a fraction of wealth is lost in making the aggregate.

**Proof.** From (1.17)-(1.19), one can derive that

$$\frac{b + e}{a} = \frac{\alpha_s \hat{\rho} \sigma_s + (1 - \alpha_s)}{\Gamma}$$

with $\hat{\rho} \equiv \rho_b / \rho_e$

$$\Gamma \equiv \left[\alpha_s \hat{\rho}^{\sigma_s + 1} + (1 - \alpha_s)\right]^{\frac{1}{\sigma_s + 1}}$$

After some manipulations, one gets

$$\frac{\partial (b + e)}{a} = \frac{\alpha_s \hat{\rho} \sigma_s^{-1}}{\Gamma} - \frac{\alpha_s \hat{\rho} \sigma_s + (1 - \alpha_s)}{\Gamma^{2}}\sigma_s \Gamma^{-\sigma_s} \alpha_s \hat{\rho} \sigma_s$$

implying that

$$\hat{\rho} = 1 \Rightarrow \frac{\partial (b + e) / a}{\partial \hat{\rho}} = 0 \Rightarrow \frac{b + e}{a} = 1$$

$$\hat{\rho} < 1 \Rightarrow \frac{\partial (b + e) / a}{\partial \hat{\rho}} < 1$$


1.1.4 Taxation of portfolio income

Capital income is assumed to be only taxed in the country of residence. Tax authorities have full information about these income flows. Dividends, capital gains and interest income from bonds are taxed at the rate $\tau_d$, $\tau_g$ and $\tau_b$, respectively.

The after-tax rate of return on bonds is then by definition equal to:

$$\rho_b = 1 + r_b = 1 + \hat{r}_b (1 - \tau_b)$$ (1.20)

where $\hat{r}_b$ is the world rate of return on bonds. The net return on equity $r_e = \rho_e - 1$ will be derived below in equation (1.53).
1.1.5 Aggregate consumption, wealth and savings (in a given period)

Aggregate consumption (per capita young) grows at rate $g_a$ if and only if

c(t) = (1 + g_a)c(t - 1, t - 1) \quad \text{We have already assumed that the consumption profile for each young person is} \quad c(t) = (1 + g_a)c(t - 1, t - 1) \quad \text{This implies that in a given period all young persons (of every birth year) consume the same amount} \quad c(t) \quad \text{and every old person consumes} \quad c^o(t) \quad \text{In per capita terms, total consumption equals:} \quad ^5

\begin{equation}
c(t) = \sum_s \omega^s(t - s) \left[ c^{o}(t - T - s, t) + \frac{c^{o}(t - T - s, t)}{(1 + g_a)^T} \right]
\end{equation}

Aggregate wealth is less straightforward, as it is not uniform across generations. Observe from equation (1.6) that for each household of the young generation holds:

\begin{equation}
a(t - i, t) = s_n^y(t - i, t) \left[ \frac{\rho^t + 1 - (1 + g_a)^t + 1}{\rho_t - (1 + g_a)} \right]
\end{equation}

where we define $\theta \equiv \rho_t/(1 + g_a)$.

Similarly, for the old generation, using $a(t - i, t) = (1 + g_a)^{-i} a(t, t + i)$ in equation (1.7) implies:

\begin{equation}
a(t - T - i, t) = (1 + g_a)^{-T + i} a(t, t + T + i)
\end{equation}

Total wealth AS is the summation over all young and old cohorts:

\begin{equation}
AS(t) = \mathcal{N} \sum_{i=0}^{T-1} \left[ \omega^i(-i) a(t - i, t) + \omega^i(-i) \frac{a(t - T - i, t)}{(1 + g_a)^T} \right]
\end{equation}

5 The assumptions on population in section 1.1.1 imply:

$\mathcal{N}^i(-s) = \sum (1 + g_a)^{-s} \mathcal{N}^i, \quad i = o, y$

$\omega^s(-s) = \sum (1 + g_a)^{-s} \sum \omega^s(-s) = 1$

$\omega^s(-s) = \mathcal{N}^i(-s) \mathcal{N}^o(-T - s) \mathcal{N}^o$
such that in per capita terms:

$$as(t) = [\bar{w}(t)l + tr^\gamma(t) - c^\gamma(t)]\chi(g_a,g_n,\rho_s)$$ \hspace{1cm} (1.23)

where $\chi$ is a (country-specific) parameter, depending on population growth, productivity growth and the return on savings. This parameter does not depend on time, unlike $N^0$ and $s^\gamma$. This implies that aggregate savings including interest income are:

$$S(t) = AS(t) - AS(t-1) = AS(t) \left[ 1 - \frac{1}{(1+g_n)(1+g_a)} \right]$$ \hspace{1cm} (1.24)

which equals zero if productivity and population growth are both zero.

**Saving rate** As an additional piece of evidence, we might use the saving rate to calibrate the model. One common definition of the saving rate is savings as fraction of households disposable income. Savings are defined in equation (1.24). Disposable income is the sum over income of young and old generations. Young households earn wage income, receive transfers and build up assets for which they get interest income. Old households receive profit-income, transfers and interest income. Note that only the interest income variates between households. This implies that:

$$Y_d(t) = N^0 \sum_{i=0}^{T-1} \omega^t(-i) [\bar{w}(t)l + tr^\gamma(t) + (\rho_s - 1)a(t-i,t-1)]$$
$$+ N^0 \sum_{i=0}^{T-1} \omega^t(-i) [\pi^o(t) + tr^o(t) + (\rho_s - 1)a(t-T-i,t-1)]$$
$$= N^0 \left[ \bar{w}(t)l + tr^\gamma(t) + \frac{\pi^o(t) + tr^o(t)}{(1+g_n)^T} \right] + (\rho_s - 1)AS(t-1)$$ \hspace{1cm} (1.25)

Combined with equations (1.23) and (1.24) we obtain the saving rate:

$$\omega_{sy} = \frac{S(t)}{Y_d(t)} = \frac{\left[ 1 - \frac{1}{(1+g_n)(1+g_a)} \right] \chi(g_a,g_n,\rho_s) \left[ \bar{w}l + tr^\gamma - (1 - \tau_c)c^\gamma \right]}{\bar{w}l + tr^\gamma + \frac{\pi^o + tr^o}{(1+g_n)^T} + (\rho_s - 1)\chi(g_a,g_n,\rho_s) \left[ \bar{w}l + tr^\gamma - (1 - \tau_c)c^\gamma \right]}$$ \hspace{1cm} (1.26)

**Compensating variation** In simulations, we compare the welfare impact of tax changes by calculating the compensating variation ($cv$). The $cv$ is calculated as the change in transfers to young households required to compensate the change in welfare. The system of equations used to calculate $cv$ consists of the definition of welfare in (1.2) and (1.3), the optimal response of labour and consumption in (1.13) and (1.14), and the budget equation (1.9).

The interpretation of the $cv$ is hampered by the fact that a welfare gain is represented by a negative compensation. In the output tables we overcome this by reporting the $cv_{gain} \equiv -100 \frac{cv}{y}$, where $y$ is GDP in the base case.
1.2 Firms

Three types of firms are active in each country: pure domestic firms, headquarters of multinationals and subsidiaries of foreign multinationals. Firm’s types are represented by the superscripts \(d\), \(m\) and \(f\), respectively. Each country is endowed with a stock of a fixed factor, named ‘location specific capital’. Its size is assumed proportional to the generation size \(N\) to avoid that productivity differentials would arise from differences in country size (cfr. Sørensen (2001a), p. 7). To be precise, this factor is called fixed since its supply is perfectly inelastic. An individual firm can choose the amount of this factor optimally. In equilibrium, the fixed factor is paid its marginal productivity. The three firm types are successively discussed in the following paragraphs. The last paragraph describes corporate taxation.

1.2.1 Domestic firms

The marginal investor maximises the present value of the representative firm, which is equal to the discounted stream of dividends. As will be clear shortly, the discount rate of investors residing in different countries differs due to varying tax rates on capital income. It implies that the present value differs between investors. To single out a unique investor, we assume that the marginal investor is domestic.

The gross return on equities in period \(t\) consists of dividends and capital gains:

\[
\hat{r}_{we} V^d_t = \text{Div}^d_t + V^d_{t+1} - V^d_t
\]

where \(\hat{r}_{we}\) is the world rate of return on equity, \(V^d_t\) is the value of the firm and \(\text{Div}^d_t\) the distributed profits. The net return on equity \(r_e(i, j) = \rho_e(i, j) - 1\) follows from subtracting personal taxes:

\[
r_e(i, j) V^d_t(i, j) = \hat{r}_{we}(j) V^d_t(i, j) - \tau_d(i) \text{Div}^d_t(i, j) - \tau_g(i) \left( V^d_{t+1}(i, j) - V^d_t(i, j) \right)
\]

\[
\Rightarrow r_e(i, j) V^d_t(j) = (1 - \tau_d(i)) \text{Div}^d_t(j) + (1 - \tau_g(i)) \left( V^d_{t+1}(j) - V^d_t(j) \right)
\]

The second line follows from the assumption that each investor irrespective of its residence country receives the same dividend and capital gain per share, with \(\text{Div}^d(j) = \sum \text{Div}^d(i, j)\) and \(V^d(j) = \sum V^d(i, j)\). This equation shows that investors who face different tax rates will value firms differently. In principle, investors who require the lowest net return are willing to pay the most for an equity. Under the assumption that the marginal investor is domestic \((i = j)\), recursive substitution of (1.28) shows that the value of the firm equals the sum over the present value of the dividends:

\[
V^d_t(j) = \sum_{s=t}^{\infty} \Lambda(j) \text{Div}^d_s(j) R_s(j) \quad \text{with} \quad R_s(j) \equiv \frac{1}{(1 + \hat{r}_e(j))^{s-t+1}}
\]

(1.29)
\[ \bar{r}_e(j) \equiv \frac{r_e(j, j)}{1 - \tau_g(j)} \]
\[ \Lambda(j) \equiv \frac{1 - \tau_d(j)}{1 - \tau_g(j)} \]

where \( \bar{r}_e \) represents the discount rate relevant for firm’s decisions. From here onward, we drop the country index, since both the firm and the marginal investor reside in country \( j \).

**Production function** Maximization of the firm’s value requires that an expression for the dividends is derived.\(^6\) The first key ingredient is the production function. For the representative domestic firm it is specified as:

\[ Y^d = A^d \left( VA^d \right)^{\alpha^d} \quad \text{with} \quad 0 < \alpha_e < 1 \]  
(1.30)

\[ A^d = \left( A_0 \omega^d N^\gamma \right)^{1-\alpha^d} \]

where \( Y^d \) denotes total output, \( A^d \) the output contribution of the fixed factor, \( VA^d \) value-added and \( \alpha^d \) the share of value-added in production. The exogenous fraction of the fixed factor that is in use by domestic corporations is denoted by \( \omega^d \). Value-added is a CES-function of employment \( L^d \) and capital \( K^d \):

\[ VA^d = A_0 \left[ \alpha^d_v \left( L^d \right)^{\sigma_v \alpha^d_v / \sigma_v - 1} + \alpha^d_k \left( K^d \right)^{\sigma_v \alpha^d_k / \sigma_v - 1} \right]^{\sigma_v / \sigma_v - 1} \]  
(1.31)

where \( \alpha^d_v \) is a share parameter and \( \sigma^d_v \) is the substitution elasticity between labour and capital. The total factor productivity (TFP) level \( A_0 \) serves two purposes: it facilitates the calibration of GDP and it allows for the introduction of productivity growth. We assume that TFP is uniform within a country across the three firm-types. In addition, we assume that its growth rate \( g^a \) is uniform across countries. We impose steady growth with \( g^k = g^y \) and employment growth equal to population growth \( g^n \). Equations (1.30) and (1.31) implies \( g^y = g^va = (1 + g^a)(1 + g^n) - 1 \).\(^7\)

Marginal productivities are derived as:

\[ \frac{\partial Y^d}{\partial L^d} = \left( \alpha^d_v \frac{Y^d}{VA^d} \right) \alpha^d_v A_0^{1-\sigma^d_v} \left( \frac{VA^d}{L^d} \right)^{1/\sigma^d_v} \]  
(1.32)

\[ \frac{\partial Y^d}{\partial K^d} = \left( \alpha^d_k \frac{Y^d}{VA^d} \right) \alpha^d_k A_0^{1-\sigma^d_k} \left( \frac{VA^d}{K^d} \right)^{1/\sigma^d_k} \]  
(1.33)

**Debt or equity financing** The second ingredient for the expression of dividends is the determination of the debt ratio. Investment can be financed by issuing bonds or by retaining

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\(^6\) The following analysis can be found in e.g. Salinger and Summers (1983).

\(^7\) The growth rate \( g^y \) applies on the steady growth path to \( Y, w, L, D, I, \Pi, \hat{\Pi}, Div \).
profits (issuing new shares is not considered). The gross world rates of return on bonds and equities are denoted by $\hat{r}_{wb}$ and $\hat{r}_{we}$, respectively. First, an interior solution for the financing mix is obtained by assuming that both debt and equity financing are extremely costly at the corner:

$$c_i^j(d_i^j) = \chi_0 (1 - d_i^j)^{-(1 - \epsilon_b)} (d_i^j)^{-\epsilon_b} - c_{i,b,0}, \quad i = d, m, f \quad \text{with} \; \chi_0, \epsilon_b > 0 \quad (1.34)$$

where $d_i^j$ is the firm’s debt-asset ratio and $c_i^j$ is the cost of financial distress per unit of capital. This cost represents the output which is lost as financial decisions distract managers from productive activities. The partial derivative of this cost w.r.t. the debt-ratio is:

$$\frac{\partial c_i^j}{\partial d_i^j} = \left[ (1 - \epsilon_b) (1 - d_i^j)^{-1} - \epsilon_b (d_i^j)^{-1} \right] \chi_0 (1 - d_i^j)^{-(1 - \epsilon_b)} (d_i^j)^{-\epsilon_b}$$

$$= \left[ (1 - \epsilon_b) (1 - d_i^j)^{-1} - \epsilon_b (d_i^j)^{-1} \right] (c_i^j + c_{i,b,0}) \quad (1.35)$$

This implies that the cost function has its minimum at:

$$d_{i,b,min} = \epsilon_b \quad \text{with} \; c_i^j(d_{i,b,min}) = \chi_0 (1 - \epsilon_b)^{-(1 - \epsilon_b)} (\epsilon_b)^{-\epsilon_b} - c_i^j \quad (1.36)$$

When $c_{i,0}$ is used to set this minimum equal to zero, we can calibrate the actual debt-asset ratio with the parameters $\epsilon_b$ and $\chi_0$. The sensitivity of the cost function w.r.t. changes in the debt-ratio will depend on both parameters, but mainly on $\chi_0$, whereas $\epsilon_b$ first of all represents the debt rate at which financial distress costs are minimized.

**Dividends** The tax base $\hat{\Pi}^d$ of corporate taxation is defined as:

$$\hat{\Pi}^d = Y^d - wL^d - \left( \beta_b d_i^j \hat{r}_{wb} + c_i^j \right) K^d - \delta D^d \quad (1.37)$$

where $\beta_b$ is the deductible fraction of interest payments, $\delta$ the depreciation rate of capital for tax purposes and $D^d$ the stock of depreciation allowances. When exponential depreciation is allowed for tax purposes, the accumulation of depreciation rights is similarly specified as the accumulation of physical capital:

- fiscal: $D_{t+1}^d = I_t^d + (1 - \delta)D_t^d$ \quad (1.37)
- economic: $K_{t+1}^d = I_t^d + (1 - \delta)K_t^d$ \quad (1.38)
The first order condition of investment gives

$$\frac{\partial Y^d}{\partial L^d} = w$$

(1.41)

With the CRS production function and perfect competition, each production factor is paid its after tax marginal return. This will also hold for the fixed production factor (\(\omega^dN^y\)):

$$\Pi^d = \left(1 - \tau^d_x\right)\frac{\partial Y^d}{\partial (\omega^dN^y)}\omega^dN^y = \left(1 - \tau^d_x\right)(1 - \alpha^d)Y^d$$

(1.42)

The optimal debt ratio has to satisfy the condition:

$$\left[\frac{\partial c^d_{b,j}}{\partial d^d_{b,j}}(1 - \tau^d_x) + \hat{r}_{wb}\left(1 - \tau^d_x\beta_b\right) + 1\right]R_t = R_{t-1} \Rightarrow$$

$$\chi_0 \left(1 - \epsilon_b\right)\left(1 - d^d_{b,j}\right)^{-1} - \epsilon_b\left(d^d_{b,j}\right)^{-1} = \hat{r}_c - \hat{r}_{wb}\left(1 - \tau^d_x\beta_b\right)$$

(1.43)

Since debt normally carries the lowest financing cost (\(\hat{r}_c > \hat{r}_{wb}\left(1 - \tau^d_x\beta_b\right)\)), condition (1.43) generally implies that \(d^d_{b,j} > \epsilon_b\) and \(\partial c^d_{b,j}/\partial d^d_{b,j} > 0\). The first order condition of investment gives

$$\lambda^d + \mu^d = \Lambda$$

(1.44)

10 The specification (1.37) yields a similar optimal condition for capital as in Sørensen (2001b). One could favour a change of the time index for investment into \(t + 1\).

11 A difference with Sørensen (2001b) can be noted. The value of the depreciation allowances in OECDTAX (see (67), in our notation) is

$$\delta D = \frac{\delta (\delta + r)}{\delta + r}K$$

Since \(D\) and \(K\) grow at rate \(g\) in the steady state, (1.37)-(1.38) imply

$$\delta D = \frac{\delta (\delta + g)}{\delta + g}K$$

Normally holds that \(r > g\), and \(\delta > \delta\), implying that the tax allowance in OECDTAX is larger (for a given \(K\)).

12 Instead of \(\hat{r}_c\), Sørensen (2001b) uses \(\hat{r}_c\) in the equivalent of (1.43). Normally, \(\hat{r}_c < \hat{r}_c\) holds.
The condition for the state variable $D^d$ is:

$$\left[\Lambda \tau_d^d \delta_t + (1 - \delta_t) \lambda_{t+1}^d \right] R_t = \lambda_t^d R_{t-1} \quad \text{(1.45)}$$

At the steady growth path the shadow value $\lambda^d$ is constant at the value:

$$\lambda^d = \frac{\Lambda \tau_d^d \delta_t}{\bar{r}_c + \delta_t} \quad \text{(1.46)}$$

which is the present value of the stream of depreciation allowances for one unit of capital.

Finally, the first order condition for capital can be derived as:

$$\left[ \Lambda \left( \frac{\partial Y^d_t}{\partial K^d_t} - c^d_{t,0} \right) (1 - \tau_d^d) - \Lambda \hat{\kappa}_{w,b} d^d_{t,0} \left(1 - \tau_d^d \beta_b \right) - \Lambda d_{t,1}^d + \mu_{t+1}^d (1 - \delta_t) \right] R_t = - \left( \Lambda d_{t,1}^d - \mu_t^d \right) R_{t-1} \quad \text{(1.47)}$$

Use (1.44) to simplify this expression to:

$$\frac{\partial Y^d_t}{\partial K^d_t} = c^d \quad \text{(1.48)}$$

where we define the user cost of capital stock $c^d$ and the marginal cost of finance $r^d$ as:

$$c^d \equiv \frac{r^d + \delta_t - \lambda^d (\bar{r}_c + \delta_t)}{\Lambda (1 - \tau_d^d)} \quad \text{(1.49)}$$

$$r^d \equiv d^d_{t,0} \hat{\kappa}_{w,b} \left(1 - \tau_d^d \beta_b \right) + (1 - d^d_{t,0}) \bar{r}_c + c^d_{t,0} (1 - \tau_d^d) \quad \text{(1.50)}$$

The value of the firm is shown to be equal to the sum of the values of the physical and the accounting stock of capital, see Salinger and Summers (1983, eq. (14)):

$$V^d = \Lambda (1 - d^d_{t,0}) K^d + \lambda^d (D^d - K^d) \quad \text{(1.51)}$$

**Proof.** Multiplying (1.47) with $K^d_t$ gives

$$\left[ \Lambda \left( \frac{\partial Y^d_t}{\partial K^d_t} - c^d_{t,0} \right) (1 - \tau_d^d) - \Lambda \hat{\kappa}_{w,b} d^d_{t,0} \left(1 - \tau_d^d \beta_b \right) - \Lambda d_{t,1}^d + \mu_{t+1}^d (1 - \delta_t) \right] K_t^d R_t + \left( \Lambda d_{t,1}^d - \mu_t^d \right) K_t^d R_{t-1} = 0$$

Substituting (1.38) and (1.44) gives

$$\Lambda \left[ Y^d_{t,0} - w_t L^d_t - c^d_{t,0} K^d_t \right] (1 - \tau_d^d) - \Pi_t - \hat{\kappa}_{w,b} d^d_{t,0} \left(1 - \tau_d^d \beta_b \right) K_t^d - d_{t,1}^d K_t^d \right] R_t + \mu_{t+1}^d K_{t+1}^d R_t - \left( \Lambda - \lambda_{t+1}^d \right) L_t^d R_t + \left( \Lambda d_{t,1}^d - \mu_t^d \right) K_t^d R_{t-1} = 0$$

13 Notice that the only difference with the corresponding condition (69) in Sørensen (2001b) concerns the effect of depreciation allowances. Whereas the discount rate $r^d$ is assumed for depreciation allowances in OECDTAX, the rate $\bar{r}_c$ applies in our dynamic context.
Similarly, substitution of (1.37) and (1.45) yields
\[
A \left[ \left( \frac{Y^d}{w_t} - \omega_t K^d \right) \left( 1 - \varepsilon_t \right) - \Omega_t - \hat{w}_{bd} d_{b,t} \left( 1 - \varepsilon_t \right) \right] K_t^d - \left( d_t^d + \hat{d}_{b,t+1} \right) K_t^{d+1} - d_t^d K_t^d - \left( \Lambda d_{b,t+1}^d - \mu_{b,t+1}^d \right) K_{t+1}^{d+1} + \left( \Lambda d_{b,t+1}^d - \mu_{b,t+1}^d \right) K_{t+1}^d R_{t+1} + \left( \Lambda d_{b,t+1}^d - \mu_{b,t+1}^d \right) K_t^{d+1} R_{t+1} + \left( \Lambda d_{b,t+1}^d - \mu_{b,t+1}^d \right) K_{t+1}^d R_t + \left( \Lambda d_{b,t+1}^d - \mu_{b,t+1}^d \right) \Lambda d_{b,t+1}^d (1 + \hat{e}_t) \Lambda d^d d_t R_t = 0
\]
In view of (1.39), this is written as
\[
\Lambda D_i^d d_t R_t + \Lambda \left( 1 - d_{b,t+1}^d \right) K_{t+1}^d + \lambda_{i+1}^d \left( D_t^{d+1} - K_t^{d+1} \right) R_{t+1} = \left[ \Lambda \left( 1 - d_{b,t}^d \right) K_t^d + \lambda_i^d \left( D_t^d - K_t^d \right) \right] R_{t-1}
\]
Recursive substitution finally shows that
\[
V_t^d = \sum_{s=t}^{\infty} \Lambda D_i^d d_t R_s = \Lambda (1 - d_{b,s}^d) K_s^d + \lambda_s^d (D_s^d - K_s^d)
\]

As dividends grow at rate \( g_y \) (1 + \( g_a \)) (1 + \( g_a \)) at the steady growth path, an alternative expression is easily found:
\[
V^d = \frac{\Lambda D_i^d}{\hat{r}_c - g_y} \quad \text{with} \quad \hat{r}_c > g_y
\] (1.52)
Furthermore, \( \Delta V^d = g_y V^d \) in the steady state implies \( D_i^d / V^d = \hat{r}_w - g_y \) in view of (1.27).
Substitution in (1.28) yields:
\[
r_c(i, j) = (1 - \varepsilon_t(i)) (\hat{r}_w^e(j) - g_y) + (1 - \varepsilon_t(i)) g_y
\] (1.53)
\[
\hat{r}_c (j) = \Lambda (j) \hat{r}_w^e (j) + (1 - \Lambda (j)) g_y
\] (1.54)

### 1.2.2 Multinational parent company

The domestic operations of the multinationals are analogously specified:
\[
Y^m = A^m (VA^m)^{\alpha_m^m} \quad \text{with} \quad 0 < \alpha_m^m < 1
\] (1.55)
where \( Y^m \) denotes total output, \( A^m \) the output contribution of the fixed factor, and \( VA^m \) value-added. Multinationals hold fraction \( \omega^m = 1 - \omega^d \) of the fixed factor. Value-added is a CES-function of employment \( L^m \) and capital \( K^m \):
\[
A^m = \left( A_0 \omega^m N^m \right)^{1 - \alpha_m^m} \quad (1.56)
\]
\[
VA^m = A_0 \left[ \alpha_m^m (L^m)^{\alpha_m^m - 1} + \alpha_m^m (K^m)^{\alpha_m^m - 1} \right]^{\alpha_m^m - 1} \quad (1.57)
\]
Marginal productivities are similar to (1.32) and (1.33). When the corporation’s debt-asset ratio \( d_{b}^m \) deviates from \( \varepsilon_t \), it has to pay financial distress costs, cf. (1.34). The marginal cost of finance is defined as:
\[
r^m = d_{b}^m (1 - \varepsilon_t) \hat{r}_{wb} + (1 - d_{b}^m) \hat{r}_c + (1 - \varepsilon_t) \hat{e}_b^m
\] (1.58)
The parent company supplies $Q(j)$ units as an input to its foreign subsidiary $j$. When the tax rate on profits differs between both countries, transfer pricing might be attractive to shift taxable profits between the jurisdictions (Sørensen (2001b), p. 24). However, charging a different price than the real cost (i.e. $p_q \neq 1$) involves a type of organizational costs. The cost arising from a distorted transfer price is assumed to be:

$$c_q = \frac{|p_q - 1|^{1+\epsilon_q}}{1 + \epsilon_q} \quad \text{with } \epsilon_q > 0$$  \hspace{1cm} (1.59)

$$\Rightarrow \frac{\partial c_q}{\partial p_q} = \text{sign}(p_q - 1)|p_q - 1|^{\epsilon_q}$$

The corporate tax base is given by

$$\hat{\Pi}^m = Y^m - wL^m + \sum_{j \neq i} (p_q(j)Q(j) - (b^m_d \hat{r}_{wb} + c^m_b))K^d - \delta D^m$$ \hspace{1cm} (1.60)

The dividends originating from domestic operations are:

$$\text{Div}^m_{it} = Y^m_{it} - w_iL^m_{it} + \sum_{j \neq i} (p_q(jQ(j) - (d^m_{ib} \hat{r}_{wb} + c^m_{ib}))K^d_{it} - \hat{\Pi}^m_{it} - t^m_{it} + d^m_{ib+1}K^m_{ib+1} - d^m_{ib}K^m_{ib}$$ \hspace{1cm} (1.61)

The optimal decisions of multinationals follow from the maximization of its total value, which is described in the following paragraph.

### 1.2.3 Multinational subsidiaries

Production of the subsidiary in country $j$ is given by:

$$Y^f(j) = A^f(j) A^q_0 Q(j)^{\alpha_q} VA^f(j)^{\alpha_f^q} \quad \text{with } 0 < \alpha_q + \alpha_f^q < 1$$  \hspace{1cm} (1.62)

$$A^f = \left(A_0 \omega^f N^v\right)^{1-\alpha_f^v - \alpha_q}$$ \hspace{1cm} (1.63)

$$VA^f(j) = A_0 \left[ \alpha_f^q \left(L^f(j)\right)^{\alpha_f^l - 1} + \alpha_f^l \left(K^f(j)\right)^{\alpha_f^k - 1} \right]^{\alpha_f^{l-1}}$$ \hspace{1cm} (1.64)

where $Y^f$ denotes total output, $A^f$ the output contribution of the fixed factor, $Q$ the intermediate input and $VA^f$ value-added.

The equity of the subsidiary is assumed to be completely provided by its parent, implying that the equity cost equals the opportunity cost in the parent’s country ($\hat{r}_e(i)$). The multinational finances the remaining fraction of the capital stock by issuing bonds at the cost $\hat{r}_{wb}$. The subsidiary’s marginal cost of finance is written as

$$r^f(j) \equiv d^f_b(j)(1 - \tau_e(j)) \hat{r}_{wb} + \left(1 - d^f_b(j)\right) \hat{r}_e(i) + \left(1 - \tau^f(j)\right) c^f_b(j)$$ \hspace{1cm} (1.65)
where financial distress costs $c^f$ are defined in equation (1.34). Its tax base is defined according to the foreign jurisdiction:

$$
\hat{\Pi}^f(j) = Y^f(j) - w(j)L^f(j) - p_q(j)Q(j) - (\beta_b(j)d^f_b(j)\hat{r}_{w,b} + c^f_b(j))K^f(j) - \delta D^f(j) \tag{1.66}
$$

Remaining profits flowing to the parent company follow as

$$
Div^{mf}_i(j) = Y_i^f(j) - w_i(j)L_i^f(j) - p_q(j)Q(j) - (d^f_{b,t}(j)\hat{r}_{w,b} + c^f_{b,t}(j))K^f_{t+1}(j) - d^f_{b,t+1}(j)K^f_{t+1}(j) - d^f_{b,t}(j)K^f_{t+1}(j) \tag{1.67}
$$

**Profit maximization** The multinational maximizes the value

$$
V^m_i = \sum_{t=i}^{\infty} ADiv^m_t R_s = \sum_{t=i}^{\infty} \Lambda \left[ Div^{mm}_t + \sum_{j \neq i} Div^{mf}_t(j) \right] R_s \tag{1.68}
$$

The optimal factor demands and debt ratio are derived similarly as for the domestic firms. For labor:

$$
w = \alpha^m_w \left( \frac{Y^m}{VA^m} \right) \alpha^m_w A_0^{1-1/\sigma} \left( \frac{VA^m}{L^m} \right)^{1/\sigma} \tag{1.69}
$$

$$
w(j) = \alpha^f_w \left( \frac{Y^f(j)}{VA^f(j)} \right) \alpha^f_w A_0^{1-1/\sigma} \left( \frac{VA^f(j)}{L^f(j)} \right)^{1/\sigma} \tag{1.70}
$$

For investment:

$$
\lambda^m = \frac{\Lambda \tau^m \delta_i}{\hat{r}_e + \delta} \tag{1.71}
$$

$$
\lambda^f(j) = \frac{\Lambda \tau^f(j) \delta_i}{\hat{r}_e(i) + \delta} \tag{1.72}
$$

For capital:

$$
c^m = \frac{r^m + \delta_k - \lambda^m (\hat{r}_e + \delta_k)}{\Lambda (1 - \tau^m_i)} = \alpha^m_c \left( \frac{Y^m}{VA^m} \right) \alpha^m_c A_0^{1-1/\sigma} \left( \frac{VA^m}{K^m} \right)^{1/\sigma} \tag{1.73}
$$

$$
c^f(j) = \frac{r^f(j) + \delta_k - \lambda^f(j) (\hat{r}_e(i) + \delta_k)}{\Lambda (1 - \tau^f(j))} = \alpha^f_c \left( \frac{Y^f(j)}{VA^f(j)} \right) \alpha^f_c A_0^{1-1/\sigma} \left( \frac{VA^f(j)}{K^f(j)} \right)^{1/\sigma} \tag{1.74}
$$

For the fixed factor:

$$
\Pi^m = (1 - \alpha^m_c) (1 - \tau^m_i) Y^m \tag{1.75}
$$

$$
\Pi^f(j) = (1 - \alpha^f_c) (1 - \tau^f(j)) Y^f(j) \tag{1.76}
$$

---

14 Pure profits of foreign subsidiaries are assumed to accrue to the old generation living in the parent country.
For the debt ratio:

\[
\frac{(1 - \theta_b)(1 - d^i_{b,f})^{-1} - \theta_b (d^f_{b,f})^{-1}}{(1 - d^f_{b,f})^{-1 - \theta_b} (d^i_{b,f})^{-\theta_b}} = \frac{\bar{r}_e - \bar{r}_{m,b} (1 - \tau^i_{x})}{1 - \tau^i_{x}}, \quad i = m, f
\]  

(1.77)

In addition, the expressions for intermediate inputs and corresponding transfer prices are derived as

\[
\alpha \frac{Y^f(j)}{Q(j)} 
\begin{cases} 
1 - \tau^f_x(j) = p_q(j) \left( \tau^m_x - \tau^f_x(j) \right) + (1 + c_q(j)) (1 - \tau^m_x) & j \neq i \\
\frac{\partial c_q(j)}{\partial p_q(j)} (1 - \tau^m_x) = \tau^f_x(j) - \tau^m_x & (1.79)
\end{cases}
\]

(1.78)

From the last condition follows that the multinational shifts profits to the jurisdiction with the lowest tax rate, since \( \tau^f_x(j) > (\tau^m_x) \).

The first order conditions also imply that the value of the multinational equals the value of the stocks it owns:

\[
V^m = \Lambda (1 - d^m_b) K^m + \lambda^m (D^m - K^m) + \sum_{j \neq i} \left[ \Lambda (1 - d^f_b(j)) K^f(j) + \lambda^f(j) (D^f(j) - K^f(j)) \right]
\]  

(1.80)

**Proof.** Proceeding along the lines followed in the derivation of (1.51), one can show that

\[
\sum_{j = t}^\infty \Lambda \left[ \text{Div}^m_q - (1 - \tau^m_x) \sum_{j \neq t} (p_{q,x}(j) - 1 - c_{q,x}(j)) Q_x(j) \right] R_s = \Lambda (1 - d^m_b) K^m + \lambda^m (D^m - K^m)
\]

\[
\sum_{j = t}^\infty \Lambda \left[ \text{Div}^f_q(j) + (1 - \tau^m_x) (p_{q,x}(j) - 1 - c_{q,x}(j)) Q_x(j) \right] R_s = \Lambda (1 - d^f_b(j)) K^f(j) + \lambda^f(j) (D^f(j) - K^f(j))
\]

Using (1.68) completes the proof.  

In the steady state, these expressions reduce to

\[
V^{mm} \equiv \frac{\Lambda \text{Div}^{mm}_q}{\bar{r}_e - g_s}
\]

\[
= \Lambda (1 - d^m_b) K^m + \lambda^m (D^m - K^m) + \frac{\Lambda (1 - \tau^m_x)}{\bar{r}_e - g_s} \sum_{j \neq t} (p_q(j) - 1 - c_q(j)) Q(j)
\]

\[
V^{mf}(j) \equiv \frac{\Lambda \text{Div}^{mf}(j)}{\bar{r}_e(i) - g_s}
\]

\[
= \Lambda \left( 1 - d^f_b(j) \right) K^f(j) + \lambda^f(j) \left( D^f(j) - K^f(j) \right) - \frac{\Lambda (1 - \tau^m_x) (p_q(j) - 1 - c_q(j)) Q(j)}{\bar{r}_e(i) - g_s}
\]

\[
\text{FDI}(i,j) = \left( 1 - d^f_b(i,j) \right) K^f(i,j)
\]  

(1.83)
1.2.4 Aggregate production

Gross domestic product is defined as the sum of production of all firms in a country corrected for the value of intermediate inputs in foreign subsidiaries:

\[ Y(i) \equiv Y^d(i) + Y^m(i) + \sum_{j \neq i} \left( Y^f(j, i) - p_q(j, i)Q(j, i) \right) \]  

(1.84)

1.2.5 Taxation of corporate income

We assume that corporate income is only taxed in the source country:

\[ \tau^d_d(i) = \tau^m_m(i) = \tau^f_f(i, j) = \tau^\pi(i) \]  

(1.85)

The focus on this pure regime can be motivated by the observation in Devereux (2004) that ‘Although in many countries the legal basis of taxation is on a residence basis, in practice the vast bulk of the international taxation of company equity income is on a source basis’. He also states that only little revenue is raised in the residence country.

A system that follows the source principle is equivalent to the exemption system from Sørensen (2001b) in the absence of an equalization tax and the full exemption of foreign source income (i.e. \( D^E = 0 \) and \( D^{ee} = 1 \) in (100)). These latter two features are indeed of minor importance in practice. All countries operating this system exempt almost all foreign source income from home country tax (\( D^{ee} > 0.95 \)), whereas the equalization tax only exists in Finland and France. The exemption system is applied at all flows within the EU, except by Greece, Ireland and the UK.

For calibration as well as for output purposes, the effective tax rate is calculated. An effective tax rate is defined as the relative difference between pre and post tax capital costs. The effective marginal tax rate (\( \tau^e_x \)) is relevant for marginal investment decisions. In our model the effective marginal tax rate equals:

\[ \tau^e_x = \frac{c^x - (c^x | \tau^\pi = 0)}{c^x} , \quad x = d, m, f \]  

(1.86)

where \( c^x \) is defined in (1.49), (1.73) and (1.74).

1.3 Government

Tax bases regarding dividends and capital gains are aggregated over the firm types as:

\[ D_{\text{iv}}(i, j) = D_{\text{iv}}^d(i, j) + D_{\text{iv}}^m(i, j) = (\tilde{r}_{we} - g_y)E(i, j) \]  

(1.87)

\[ \Delta V(i, j) = \Delta V^d(i, j) + \Delta V^m(i, j) = g_yE(i, j) \]  

(1.88)

with \( E(i, j) = e(i, j)N^\alpha(i) \). Government consumption is assumed a fixed fraction \( \omega_g \) of GDP (defined in 1.84). The government debt \( D^e \) is assumed to be a fixed fraction of GDP.
1.4 Market Equilibria

In the following we will express variables in per capita terms (denoted by lower case symbols), using as general rule:

$$x(i, j) = \frac{X(i, j)}{N^y(i)}$$

where the denominator refers to the population in the country of origin. The government budget becomes:

$$\tau_g(i) w(i) t(i) N^y(i) + \tau_f(i) [c^7(i) N^y(i) + c^9(i) N^o(i)] +$$

$$\tau_g(i) [\bar{\pi} D(i) + \Pi^m(i) + \sum_{j \neq i} \Pi^f(j, i)] +$$

$$\tau_d(i) \sum_j DiV(i, j) + \tau_e(i) \sum_j \Delta V(i, j) + \tau_r(i) \sum_f \tau_{w,b} B(i, j)$$

$$= \omega_b(i) y(i) + tr(i) N^y(i) + tr(i) N^o(i) + \tau_{w,b} d(i) y(i) N^y(i)$$

### 1.4.1 Good markets

The total capital stock in country $i$ is obtained by taking the sum over all active firms:

$$K(i) = K^d(i) + K^m(i) + \sum_{j \neq i} K^f(j, i)$$

The sum of the financial distress costs is abbreviated as

$$c_b(i) K(i) = c^d_b(i) K^d(i) + c^m_b(i) K^m(i) + \sum_{j \neq i} c^f_b(j, i) K^f(j, i)$$

Equilibrium on the goods market in each country requires (including a time subscript):

$$Y_t(i) = C_t(i) + C^r(i) + \sum_{j \neq i} (1 - \delta_k - c_b(i)) K_t(i) + \sum_{j \neq i} (1 + c_q(i, j)) Q_t(i, j) +$$

$$\omega_b(i) Y_t(i) + EX_t(i) + (s(i) - b(i) - e(i)) N^y_t(i)$$

---

$^{15}$ Total pure profits per capita are defined as $\pi = \pi^d + \pi^m + \sum_{j \neq i} \pi^f(j)$. Note that $\pi^o$ in (1.8) is now rewritten as $\pi^o = \frac{\Pi^o}{\Pi^m} = \pi^o (1 + g_y)^r$. 

19
where $EX$ denotes total net exports of the final good, i.e. exclusive of $Q$ (note that gross bilateral exports are undetermined). The last term at the right-hand side represents the resources which are lost in making the saving composite. In per capita terms the steady state equation for good market equilibrium, noting that both population and productivity grow, becomes:

$$y(i) = c^y(i) + \frac{c^d(i)}{(1 + g_n)^t} + k(i) (1 + g_y) - (1 - \delta_k - c_b(i))k(i) + \sum_{j \neq i} (1 + c_q(i, j))q(i, j) + \omega_k(i)y(i) + ex(i) + s(i) - b(i) - e(i)$$  \hspace{1cm} (1.91)

1.4.2 Factor markets

Since domestic and foreign assets are assumed perfect substitutes, net foreign holdings of bonds ($b_w$) and equities ($e_w$) follow from equilibrium on each asset market (Notice that $N^r_{t-1} = N^r_t$):

$$b(i)N^o(i) + b_w(i)N^y(i) = d^b_d(i)K^d(i) + d^m_m(i)K^m(i) + \sum_{j \neq i} d^f_h(j, i)K^f(j, i) + d^g_g(i)Y(i)$$  \hspace{1cm} (1.92)

$$e(i)N^o(i) + e_w(i)N^y(i) = V^d(i) + V^m(i)$$  \hspace{1cm} (1.93)

When a country wants to issue more bonds than it holds, foreigners are willing to hold the excess amount ($b_w > 0$) at the given world interest rate. Analogously, domestic residents own part of the foreign firms when $e_w < 0$. Labour supply should equal total demand for labour, or:

$$l^d(i) + l^m(i) + \sum_{j \neq i} l^f(j, i, i)\omega_k(j, i, i) = l(i)$$  \hspace{1cm} (1.94)

**Extension:** In the basic version the world interest rates are exogenous. In an extended version, the interest rates on bonds and equity are endogenized by postulating a simple reduced form. For each asset, a linear relation between the world interest rate and net capital demand of the EU is specified:

$$\hat{r}_{wx} = \frac{r_x(i)N^o(i)}{\sum y(i)N^y(i)} + \gamma_x, \hspace{1cm} x = b, e$$  \hspace{1cm} (1.95)

1.4.3 Balance of Payments

Net foreign assets are defined as the value of the assets a country owns minus the total value of all assets issued by that country:

$$FA(i) = [B(i) + E(i)] - d_k(i)Y(i)$$

$$- \left[ V^d(i) + d^b_d(i)K^d(i) + V^m_m(i) + d^m_m(i)K^m(i) + \sum_{j \neq i} (V^mf(j, i) + d^f_h(j, i, i)K^f(j, i)) \right]$$

Using equilibrium on the capital market in (1.92) and (1.93), one can derive an alternative expression for the net foreign assets:

$$FA(i) = - [B_w(i) + E_w(i)] + \sum_{j \neq i} [V^mf(i, j) - V^mf(j, i)]$$
The Current Account equals the Trade Balance plus net foreign earnings on bonds, equities and FDI:

\[ CA(i) = -\hat{r}_{wb}B_w(i) - \hat{r}_{we}E_w(i) + \sum_{j \neq i} \left[ \hat{r}_{we}V^{mf}(i, j) + \Pi^{f}(i, j) - \hat{r}_{we}V^{mf}(j, i) - \Pi^{f}(j, i) \right] + EX(i) + \sum_{j \neq i} [p_{q}(i, j)Q(i, j) - p_{q}(j, i)Q(j, i)] \]

In view of the Balance of Payments definition \( FA_{t+1} = (1 + g_y)FA_t = CA_t + FA_t \) one gets:

\[-(\hat{r}_{wb} - g_y)B_w(i) - (\hat{r}_{we} - g_y)E_w(i) + \sum_{j \neq i} \left[ (\hat{r}_{we} - g_y)V^{mf}(i, j) + \Pi^{f}(i, j) - (\hat{r}_{we} - g_y)V^{mf}(j, i) - \Pi^{f}(j, i) \right] + EX(i) + \sum_{j \neq i} [p_{q}(i, j)Q(i, j) - p_{q}(j, i)Q(j, i)] = 0 \]

The per capita expression is easily obtained:

\[-(\hat{r}_{wb} - g_y)B_w(i) - (\hat{r}_{we} - g_y)c_w(i) + \sum_{j \neq i} \left[ (\hat{r}_{we} - g_y)V^{mf}(i, j) + \pi^{f}(i, j) + p_{q}(i, j)q(i, j) \right] - \sum_{j \neq i} \left[ (\hat{r}_{we} - g_y)V^{mf}(j, i) + \pi^{f}(j, i) + p_{q}(j, i)q(j, i) \right] a_h(j, i) + ex(i) = 0 \] (1.96)

where \( a_h(j, i) \equiv N^0(j)/N^0(i) \) is a short-cut for the relative population sizes.

### 1.5 Solution method

The model is implemented in GAMS.\(^\text{16}\). It is solved as a Constrained Nonlinear System, for which the number of equations has to equal the number of variables.\(^\text{17}\) The price of the good is taken as the numeraire. Due to Walras law, one of the equations is redundant. In the GAMS-program the balance of payments condition (1.96) is dropped but checked afterwards.

---

\(^\text{16}\) Knowledge of the brief GAMS tutorial is sufficient for understanding the computer program.

\(^\text{17}\) Technical documentation can be found in www.gams.com/docs/pdf/cns.pdf or in www.gams.com/solvers/conopt.pdf (Appendix A13.2). This method does not allow that variables are at their bounds in the solution.
2 Calibration

This chapter starts with a discussion of the data sources. The first section discusses the calibration of the household block. Section 2.2 motivates the choices in the calibration of the production block. Section 2.3 presents the figures for government expenditures and revenues used in the model.

The main data sources in the calibration are:

NA National accounts, OECD, July 2005
  • Table 1: GDP, expenditure approach
  • Table 3: GDP, income approach
  • Table 5: Population and employment
  • Table 12: Simplified general government accounts

The NA-data are converted in Euro’s using PPP’s. These PPP’s are provided by the Groningen Growth and Development Centre (see www.ggdc.nl) based on OECD-data, see www.oecd.org/std/ppp.

RS Revenue Statistics 1965-2004, OECD

FDI International Direct Investment Statistics Yearbook 2003, OECD


GGDC Groningen Growth and Development Centre, Total Economy Database, 2005

We have used 2002 as benchmark year, which is the final date for which all data is available. Only in a few exceptional cases, which will be be mentioned below, we deviate from this base year.

2.1 Households

2.1.1 Population

Total population and population growth are obtained from UN. Total population \( N \) is defined as the sum of all groups older than 20 years. Population growth \( g_n \) is defined as the growth of \( N \) in 2000, see equation (2.1) below, which implies \( g_n \approx 0.5\% \). Given the Diamond OLG-structure of the model, we divide the population in two cohorts of age 20-60 and of age 60-100, such that the cohort length \( T = 40 \).
**Discussion**  Observations on population dynamics do not fully match with two model features. First, a steady state growth is imposed. Second, the age of death is known with certainty. All persons know that they will live exactly $2 \times T$ years. Two approaches are possible to calibrate the growth rate $g_n$. The first approach interprets $g_n$ as the (annual) net population growth rate, which can be easily calculated as:

$$g_n = \frac{N(t)}{N(t-1)} - 1 \quad (2.1)$$

The second approach is based on the population structure in the steady state:

$$N^\gamma = (1 + g_n)^T N^\nu \quad (2.2)$$

with $N = N^\gamma + N^\nu$. The annual rate $g_n$ is now computed as:

$$g_n = \left( \frac{N^\gamma}{N^\nu} \right)^{1/T} - 1 \quad (2.3)$$

The two alternative measures are calculated for 22 countries and presented for 2000 in Figure 2.1. To reproduce the population structure observed in 2000, a high value for $g_n$ results from (2.3). Applying (2.1) yields the moderate growth rates experienced during the last years. We prefer the first approach. This choice has as drawback that the simulated relative size of the young generation will be smaller than observed.

Finally, since we have constrained the population growth rate $g_n$ the same in all countries (in the steady state), we have to calculate an average over the countries. The EU-averages, weighted with the population sizes $N$, are given in Table 2.1.

<table>
<thead>
<tr>
<th></th>
<th>Weighted averages of $g_n$ under the two calibration approaches</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(2.3)</td>
</tr>
<tr>
<td>EU15</td>
<td>2.36%</td>
</tr>
<tr>
<td>EU15+CEE3</td>
<td>2.44%</td>
</tr>
</tbody>
</table>

**2.1.2 Labor supply**

Labor supply is calculated for 2000 as the actual hours worked as fraction of potential hours:

$$L^s(i) = \frac{h(i)}{2500 \times LF}$$

where

---

The calculations are performed in spreadsheet WPP2000_1950-2000_TAXBEN.xlsx. The raw data are found in the range A16-AA1139; the calculations in AC16-AL1139. The figure and table for 2000 are made from AT1118-AW1137.
Figure 2.1 Calibration of $g_n$ in 2000

<table>
<thead>
<tr>
<th>Country</th>
<th>Based on Age Structure</th>
<th>Net Growth Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poland</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hungary</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Czech Republic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>United Kingdom</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Portugal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Netherlands</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Luxembourg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ireland</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Greece</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td></td>
<td></td>
</tr>
<tr>
<td>France</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Finland</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Denmark</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Belgium</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Austria</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$E$ number of employees and self-employed. Source: NA, T5: employment in persons

$h$ hours worked per person. Source: GGDC

$LF$ labor force. Source: UN, population 15-64

Potential hours is defined as the labor-force times a maximum of 2500 hours per year. This leads to a labor-supply measure between 0.38 for France and 0.51 in the Czech Republic. The implied parameter for the preference-for-leisure $\alpha_l$ ranges between 0.87 in Sweden and 1.55 in Italy, which is well in line with Auerbach and Kotlikoff (1987), see Table 2.3.

Evers et al. (2005) has surveyed the empirical literature on the responsiveness of labor supply to changes in the wage rate. Based on a meta analysis they conclude that the uncompensated wage elasticity ($\varepsilon_L = d\ln l / d\ln w$) is about 0.1 for man and 0.5 for woman. Our simulated wage elasticity for all workers, using the partial model for household behavior, fits nicely in this range, see Figure 2.2.

2.1.3 Consumption, savings and disposable income

Disposable income depends on after-tax labor income, capital income and transfers, see equation (1.25). These determinants of income are discussed below in the sections on firms and government. For given disposable income and by fixing parameters on preferences (like the rate of time preference, the inter- and intratemporal substitution elasticities, see Table 2.2) consumption in both periods is determined by the model. The relation of the resulting aggregate
consumption (c_model) with "Household final consumption expenditure" from \textbf{NA, T1} (c_na), both as shares of GDP, is shown in figure 2.3. The correlation between the model outcome with the data is 0.67. The weighted mean of aggregate consumption (as share of GDP) is slightly lower in the model (0.53) than in the data (0.58).

We fix the rate of time preference at 1.01 such that $\rho_u < \rho_i (i), \ \forall i$. We fix $\rho_o = 1$, as it implies equal weight on felicity of both generations. We fix the intertemporal elasticity of substitution $\sigma_u = 0.5$ in line with CPB (2004). This combination of parameters implies that the saving rate (8% on average) is slightly higher than the saving rate from OECD$^{19}$ (7%). Note that we do not want to match the country variation in saving rates, as these are likely to fluctuate significantly over time. Moreover, the implied net foreign asset position of $-12\%$-GDP (EU-average) approximates the average for the European countries $-7\%$, as given by Lane and Milesi-Ferretti (2006).

\textbf{Discussion} Data on household consumption is obtained from \textbf{NA, T1}: "Household final consumption expenditure". This matches in the model with aggregate consumption $C (i)$ defined as:

\[ C (i) = N^y (i) c^y (i) + N^o (i) c^o (i) \]  

\[ (2.4) \]

A second piece of evidence are the saving rates, which are available for most, but not all countries, from OECD (2005), see footnote 19.

Household consumption is in the model determined by the budget equation (1.8) and the Euler equation (1.14). We have considered three ways to determine the consumption profile of households. The first route is to assume that the consumption profile should match both the budget equation (1.8) and equation (2.4). The preference parameter $\rho_o (i)$ can then be used to match this consumption profile in the Euler equation. The second route is to assume that the consumption profile should match the budget equation and the country-specific saving rates, where again the Euler equation solves for $\rho_o$. The drawback of both alternatives is that the resulting preferences parameter $\rho_o$ is very heterogeneous. In addition, the first route would mismatch the saving rates and the second route would mismatch aggregate consumption. We prefer a third route, where we avoid the extreme variation in preferences by fixing $\rho_u = 1.01$ and $\rho_o = 1$ at the expense that the country-variation in consumption cannot be exactly reproduced.

### 2.1.4 Portfolio

We fix $\alpha_s = 0.7$ and $\sigma_s = 4$. The resulting share of bonds in households portfolio is about 0.69 without much variation between countries. This share is in line with the available evidence for the US: Poterba and Samwick (2003, Table 4) show that US-households hold 34.7% of their wealth as equity, 50.6% as bonds or interest bearing accounts and 14.7% as other financial
assets. Chambers and Schlagenhauf (2002) show that US households hold 52.4% as cash, 19.5% as bonds and 28% as equity. Following Sørensen (2001b), we assume that households can relatively easily substitute between both assets, by fixing $\sigma_s = 4$.

In the last three years, the real interest rate, defined as the 30-years real bonds (obtained from www.aft.gouv.fr/IMG/xls/0512_rendements_indexees.xls) have declined from 2.7% in January 2003 to 1.6% in November 2005, see Figure 2.4. We fix $\hat{r}_{wb}$ at 2%, slightly below the average of 2.2% for 2003-2005. The risk premium too has fallen in recent years, which has led us to choose a relatively modest 2% such that $\hat{r}_{we} = 4%$. What matters for households portfolio decisions is of course the after-tax return on bonds and equity. The taxation of portfolio will be discussed in section 2.3.

2.1.5 Extension with endogenous interest rates

For the extended version, we need to calibrate the parameters of (1.95). Table III.2 in European Commission (2004) reports that one additional GDP point of debt raises real interest rates by 1 basis point. Therefore, we fix $\gamma_x = 0.01 (x = b, e)$. The constant term $\gamma_x$ is used to reproduce the gross interest rates of the base case ($\check{r}_{wx}$).

Figure 2.4 Real interest rate
Table 2.2 Calibrating household behavior – parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Determined by</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population growth</td>
<td>( g_n )</td>
<td>0.005 ( g_n = \frac{\sum N(i)}{\sum N_{-1}(i)} - 1 )</td>
</tr>
<tr>
<td>Employment</td>
<td>( \alpha_l )</td>
<td>(0.88, 1.55) ( l(i) )</td>
</tr>
<tr>
<td>Consumption</td>
<td>( \rho_o )</td>
<td>1.00</td>
</tr>
<tr>
<td>Portfolio</td>
<td>( \alpha_c )</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>( \sigma_p )</td>
<td>4.00</td>
</tr>
<tr>
<td></td>
<td>( \tilde{r}_{wb} )</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>( \tilde{r}_{wc} )</td>
<td>4.00</td>
</tr>
<tr>
<td></td>
<td>( \rho_l )</td>
<td>(1.013, 1.024) ( 1.19 )</td>
</tr>
<tr>
<td></td>
<td>( \rho_b )</td>
<td>(1.008, 1.020) ( 1.20 )</td>
</tr>
<tr>
<td></td>
<td>( \rho_c )</td>
<td>(1.024, 1.040) ( 1.53 )</td>
</tr>
</tbody>
</table>

Table 2.3 Parameter values in the literature

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(M)(^a)</th>
<th>(1)(^b)</th>
<th>(2)(^b)</th>
<th>(3)(^b)</th>
<th>(4)(^b)</th>
<th>(5)(^b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Technological growth</td>
<td>1.5%</td>
<td>1.5%</td>
<td>2%</td>
<td>1.37%</td>
<td>3%</td>
<td>1.5%</td>
</tr>
<tr>
<td>Rate of time preference</td>
<td>1.0%</td>
<td>1.0%</td>
<td>2%</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
</tr>
<tr>
<td>Intertemporal elasticity of substitution</td>
<td>0.5</td>
<td>0.5</td>
<td>0.25</td>
<td>1.00</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>Intratemporal elasticity of substitution</td>
<td>1.0</td>
<td>0.3</td>
<td>0.71</td>
<td>-</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>Preference for leisure</td>
<td>0.88-1.56</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td>Elasticity of substitution between L and K</td>
<td>0.7</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
<td>1.0</td>
</tr>
<tr>
<td>Debt-capital ratio (firms)</td>
<td>0.45-0.55</td>
<td>0.5</td>
<td>0.44</td>
<td>0.14-0.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Economic depreciation of capital</td>
<td>5.0%</td>
<td>11.5%</td>
<td>10.00%</td>
<td>1.0-8.9%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fiscal depreciation of capital</td>
<td>5.0-15.0%</td>
<td>5.5%</td>
<td>14.47%</td>
<td>7.0-17.9%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Real return on bonds</td>
<td>2%</td>
<td>2.5%</td>
<td>3%</td>
<td>1.1%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk premium for equity</td>
<td>2%</td>
<td>2.0%</td>
<td>4%</td>
<td>6.2-11.8%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^a\) (M) parameter value in CORTAX;  
\(^b\) Source: (1) CPB (2004); (2) Broer (1999), Appendix 3; (3) Dietz and Keuschnigg (2003), Table 1; (4) Goulder and Summers (1987), Table 2; (5) Auerbach and Kotlikoff (1987), Chapter 4.

### 2.2 Firms

**Wage rate** We correct the compensation of employees \( W \) (taken from NA, T2) for wages imputed to self-employed \( WSE \):

\[
WSE = \frac{ES}{EE}W
\]  
(2.5)
where the number of self-employed (ES) and dependent employment (EE) are taken from NA, T5. This expression is a simplification of Carey and Tchilinguirian (2000, eq. (10)). The implied labor share is

\[ \omega_L = \frac{W + WSE}{Y - TS} \]  

(2.6)

where \( Y \) is GDP and \( TS \) is "Taxes less subsidies on production and imports" from NA, T3. Implausible low values are corrected by putting a minimum on \( \omega_L > 0.6 \), which is binding for IRL and CZE. The resulting wage share is shown in Figure 2.5. The labor-income share determines the wage rate \( w(i) = \omega_L(i) y(i)/l(i) \) and is a key factor in the calibration of the capital-share in the production function \( \alpha_k(i) \). The latter guarantees that the marginal productivity of capital equals the cost of capital, see (1.49).

---

**GDP and productivity**  
Gross Domestic Product is taken from NA, T1 and is assumed to be the sum of production of domestic and multinational firms, see equation (1.84). We have no information on the division of GDP over the three elements. We therefore have to fix the parameters \( \omega_d \) (at 0.7 as in Sørensen (2001b)) and \( \omega^m = 1 - \omega_d \), but we can use information on FDI-stocks to determine \( \omega^f \) (see below).

Two production function parameters are country specific (but uniform across firms within each country), namely the capital share \( \alpha_k \) and total factor productivity \( A_0 \). The remaining
parameters are assumed to be uniform, both across countries and across firm types, see Table 2.5. Aggregate GDP is matched in the model by variation in the parameter $A_0(i)$. The country-specific productivity parameter $A_0(i)$ ranges from 0.44 in the Czech Republic to 0.94 in Belgium/Luxembourg and the UK.\(^\text{20}\)

We assume that productivity grows at rate $g_a = 1.5\%$ in all countries. Together with the population growth of 0.5\%, it implies that GDP grows at rate $g_y = 2.0\%$.

**Discussion** Chirinko (2002) surveys the empirical literature on the substitution elasticity between labor and capital in the US. First, he points at the wide range of estimates. Estimates range from less than 0.3 using aggregate investment data, 0.25-0.5 using firm-level panel data, to 0.4-0.9 with cointegration estimates on capital and its user cost. In a recent study for the US, Antrás (2004) argues that after controlling for biased technological change, $\sigma_v$ is likely to be considerably below one, and may even be lower than 0.5. In a cross-section for 28 manufacturing sectors in 34 countries, Claro (2003) concludes that $\sigma_v$ is generally close to, but significantly different from one. In a panel regression for 82 countries, Duffy and Papageorgiou (2000) observes that $\sigma_v$ is higher in rich countries and might even exceed one.

The Joint Committee on Taxation in the US has developed two CGE models for the analysis of taxation, named the MEG model and the OLG model. They set $\sigma_v = 1$ in the MEG model, but add that the substitution elasticity between capital and labor is likely to be overstated relative to existing estimates of the substitution between capital and labor, see Joint Committee on Taxation (2003, p.42). For the OLG model, they use $\sigma_v = 0.5$ for the private business sector.

As most empirical studies point at an elasticity of substitution well below unity, we fix $\sigma_v = 0.7$, where we take into account that our modelling horizon is the long run, which likely facilitates the substitution between labor and capital.

Chirinko (2002) shows that the analysis of tax policy is very sensitive to the assumption on $\sigma_v$. Following his suggestion, we undertake tax policy analysis with alternative parameter values.

**Capital and investment** The capital stock follows from the data on GDP, employment and the wage share and on assumptions about the depreciation rate, the rental rate and others. The resulting capital-output ratio fluctuates between 2.5 (PRT) and 4.5 (IRL).

In the steady state, investment has to compensate for capital depreciation and economic growth. Compared to the investment rate from the National Accounts, calculated as Gross Capital Formation as share of GDP (both from **NA, T1**), the resulting share in the model exceeds the observed share. In the model, the investment share fluctuates between 18\% (PRT) and 32\% (IRL).

\(^{20}\) Only the variation, not the absolute value of $A_0$ is informative. To be clear, a value of $A_0 = 1$ does not act as benchmark.
The FDI stocks data for most OECD countries originate from FDI. The data set covers 29 OECD-countries and around 70 partner countries for 1980 until 2002. From this source two types of data have been used: outward and inward position or stocks. Moreover remaining EU25 countries are obtained from EUROSTAT. From that source we have downloaded the corresponding total direct investments, positions abroad and in the reporting economy respectively. For further details, we refer to the upcoming CPB memorandum by Van Leeuwen and Lejour on ‘FDI by country and sector’.

We avoid negative values in FDI-stocks (which are present in the data) by assuming that 
\[ f d_{i,j} > (1 - d_{i,j}) k_{i,j} l_{i,j}^{low} \]
where the debt share \( d \) and the capital-labor ratio \( k/l \) are calculated in earlier steps of the calibration and where \( l_{i,j}^{low} = 10^{-7} \) is the lower bound of employment in subsidiaries. A second adjustment we have made is for the FDI of Belgium/Luxembourg, for which only inward FDI is incompletely available. Without correction, the inward FDI would be much higher in BLU (49%GDP) than in NLD (34%GDP) and the share of labor in domestic firms would be very low (27% in BLU, versus 73% in NLD). We have halved the original FDI-stocks in BLU, such that the share of inward FDI (25%GDP) and the domestic labor shares (77%) are similar to that of the Netherlands. The resulting total inward and outward FDI-stocks, both as shares of GDP, are shown in Figure 2.6.

**Figure 2.6  Inward and outward FDI**

![Figure showing inward and outward FDI stocks](image)
Recall the production function for subsidiaries, with \( VA^f \) defined in (1.64):

\[
Y^f(j) = (\omega^f N^f)^{1-\alpha_q} A_0^{1-\alpha^f} Q(j)^{\alpha_q} VA^f(j)^{\alpha^f} \quad \text{with } 0 < \alpha_q + \alpha^f < 1 \tag{2.7}
\]

with \( 1 - \alpha^f - \alpha_q = 0.025 \). We use the parameter \( \omega^f \), measuring the fixed input in production, to calibrate the FDI-stocks.

**Tax elasticity of FDI** From a meta study of 25 studies containing 371 estimates, De Mooij and Ederveen (2003) report a typical semi-elasticity of FDI with respect to the average tax rate of \(-2.4\) (see De Mooij (2005, Table 2)).\(^{21}\) Figure 2.7 shows the impact of a (series of) unilateral reduction in the statutory tax rate on a country’s inward FDI in a simulation with only the production-side of the model. Our simulated semi-elasticity is smaller than the mean value of the meta study, but fits in the range of estimated values, see De Mooij and Ederveen (2003, fig. 4.1).\(^{22}\)

**Figure 2.7** Response of FDI to 1\%-point reduction in the CIT-rate

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\(^{21}\) In a recent unpublished update of their paper they report a somewhat higher semi-elasticity of \(-4.3\). Their study, however, also includes empirical estimates for US-statelevel data, which might overestimate the semi-elasticity for FDI-responses between countries.

\(^{22}\) The semi-elasticity is calculated in a partial model for production, with exogenous labour supply but endogenous wages. Two ways to improve the tax-sensitivity of FDI is to increase the capital intensity of subsidiaries by assuming that \( \alpha^f > \alpha^d \), or to reduce the debt ratio of multinationals relative to domestic firms \( \epsilon^m < \epsilon^d \). However, we do not have any empirical support for both adjustments.
Transfer pricing  Clausing (2003) estimates the effect on transfer prices of intrafirm trade of US multinationals. She finds that a 10% point lower tax rate in the host country reduces the intrafirm import price by 3 to 5%. The intrafirm export price of the affiliate increases by a similar amount. However, Swenson (2001, p. 17) reports a much smaller impact. A 10% point reduction in the foreign corporate tax rate increases the import price of affiliated US firms by only 0.048%.

Evidence on the elasticity of profit shifting is given by Huizinga and Laeven (2006), who find a macro semi-elasticity of reported profits with respect to the top statutory tax rate of 1.43 in Europe. In the model multinationals can only shift profits by applying transfer prices to intra-firm flows of intermediates. To capture the more general practice of profit shifting, we model transfer prices more sensitive to tax rate differentials by fixing $\varepsilon = 1$. Figure 2.8 shows the impact of a unilateral tax reduction on the exporting transfer price. The figure reveals that a 1%-point reduction in the corporate tax rate leads to a 1.6% increase in profits shifted via transfer pricing.

Debt-asset ratio  We rely on ECB-figures to calibrate our debt-asset ratio. ECB (2004) shows in chart 5 that the debt-equity ratio has fluctuated from 90% in 1995 via 50% in 1999 to 80% in 2002 and 2003. The implied debt-asset ratios, assuming that debt+equity=assets, has fluctuated between 47% in 1995 via 33% in 1999 to 44% in 2002 and 2003.

The second piece of information we use in the calibration of the portfolio cost function is the
### Table 2.4 Corporate taxation and debt policy\(^a\)

<table>
<thead>
<tr>
<th>Study</th>
<th>Sample</th>
<th>Semi-elasticity</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graham et al. (1998)(^b)</td>
<td>pool of large US firms, 1981-1992 (p. 153)</td>
<td>0.028</td>
<td></td>
</tr>
<tr>
<td>Graham (1999)(^c)</td>
<td>pool of large US firms, 1980-1994 estimation in levels (Table 3a) estimation in differences (Table 7A)</td>
<td>0.061</td>
<td>0.024</td>
</tr>
<tr>
<td>Gordon and Lee (2001)</td>
<td>all US firms, 1954-1995 pooled estimation (Fig. 3) time-series estimates (Table 5)</td>
<td>0.3-0.45</td>
<td>0.362</td>
</tr>
<tr>
<td>Desai et al. (2004)(^d)</td>
<td>affiliates of US multinationals in more than 150 countries in 1982, 1989 &amp; 1994 total borrowing (Table II(1)) external borrowing (Table III(5))</td>
<td>0.265</td>
<td>0.246</td>
</tr>
</tbody>
</table>

\(^a\) Auerbach (2002, section 3.3.2) and Graham (2003, section 1.3) give a review of the literature but without mentioning a single estimated value.

\(^b\) Dietz and Keuschnigg (2003) use a value of 0.36, while referring to Graham et al. (1998).

\(^c\) The figure for Graham (1999) is calculated as follows:

\[
\frac{d(d)}{d(\tau_c)} = \frac{d(d)}{d(tax\ benefit)} \frac{d(tax\ benefit)}{d(\tau_c)} = \frac{d(d)}{d(tax\ benefit)} (1 - \tau_c)
\]

where \(tax\ benefit\) is defined in (3) and \(\tau_c\) denotes the (before-financing) marginal federal tax rate. The first term is taken from the Tables of estimates, while the average \(\tau_c\) is found in Table 2B. Notice that the personal tax rates and the effective corporate state tax (p. 155) are assumed constant.

\(^d\) Desai et al. (2004) report that the implied (full) elasticity of external borrowing equals 0.19, while the elasticity of parent borrowing is larger at 0.35 (p. 15).

The tax elasticity of the debt-ratio. Table 2.4 surveys the literature on the impact of tax changes (or tax differentials) on the debt-ratio of firms. The semi-elasticities range from 0.02 to 0.45.

The debt-asset ratio in our model is determined mainly by equations (1.34) and (1.43) for domestic firms (and similar equations for multinationals), implying that the parameters \(\varepsilon_b\) and \(\chi_0\) crucially determine the debt-asset ratio and its tax-elasticity. We fix \(\varepsilon_b = 0.35\) and \(\chi_0 = 0.025\) such that the debt-equity ratio in our model ranges between 45% in IRL to 55% in CZE, GRC, with a mean of 51%. The simulated semi-elasticity (using the partial model of firm behaviour) of 0.35 (fluctuating between 0.22 in IRL and 0.38 in DEU, ITA) falls nicely in the range of semi-elasticities.

The calibration of the production-parameters are summarised in Table 2.5.

\[^{23}\text{A higher value of } \varepsilon_b \text{ would imply higher values for the calibrated debt-asset ratios. A higher value of } \chi_0 \text{ would increase the tax elasticity and enlarge the range of simulated ratios.}\]
Table 2.5 Calibration of the production block

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
<th>determined by (or used to match)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_L$</td>
<td>(0.60, 0.77)</td>
<td>(2.6)</td>
</tr>
<tr>
<td>$\alpha_{vk}$</td>
<td>(0.27, 0.60)</td>
<td>(1.49)</td>
</tr>
<tr>
<td>$A_o$</td>
<td>(0.43, 0.94)</td>
<td>(1.84)</td>
</tr>
<tr>
<td>$\sigma^d$</td>
<td>0.70</td>
<td>Sørensen (2001b)</td>
</tr>
<tr>
<td>$\omega_d$</td>
<td></td>
<td>(1.62)</td>
</tr>
<tr>
<td>$\alpha_{dm}$</td>
<td>0.975</td>
<td></td>
</tr>
<tr>
<td>$\alpha_v$</td>
<td>0.875</td>
<td></td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>0.70</td>
<td></td>
</tr>
<tr>
<td>$\delta_k$</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>Debt-equity financing</td>
<td>$\varepsilon_b$</td>
<td>0.35</td>
</tr>
<tr>
<td>Transfer pricing</td>
<td>$\chi_o$</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon_q$</td>
<td>1.0</td>
</tr>
</tbody>
</table>

2.3 Government: expenditures and taxes

The government expenditures consist of the following components:

1. Government consumption, represented as share of GDP as $\omega_c$: calculated from NA, T1 as "Final consumption expenditure" minus "Household final consumption expenditure".
2. Government investment, also included in $\omega_c$: calculated from the government account, NA, T12, as "Gross capital formation".
3. Interest payments on government debt is calculated as $\hat{r}_{wb} \bar{d}_g$, where government debt as ratio from GDP is taken from the OECD Economic Outlook 77.
4. Transfers to households, $N^y tr^y + N^o tr^o$: calculated as the residual of the government budget equation. We assume that 43% of the transfers accrue to the old generation.

To get an impression of the division of transfers between young and old generations, we rely on Table C of European Commission (2005). The EC distinguishes 8 types of social protection expenditures (both in cash and in kind), which we assign to the old and young generations. The ‘old age’ expenditures are completely assigned to the old generation. ‘Sickness’, ‘Survivors’, ‘Housing’ and ‘Other’ are assigned 50-50 to both. The young generation receives benefits for ‘Disability’, ‘Family’ and ‘Unemployment’. The resulting observations for 14 EU-countries implies a share of old-age transfers between 43% for IRL and 71% for ITA (see data/eurostat/transfers.xls). Our division of transfers is at the bottom tail of this distribution.

Taxing corporate income $\tau_\pi$ and $\delta$ Key parameters of the corporate income tax are the legal tax rate $\tau_\pi$ and the fiscal depreciation rate $\delta$ measuring the broadness of the tax base. The legal
tax rates are taken from IFS, except for CZE, HUN and POL (source: Finkenzeller and Spengel (2004)) and DNK (source: Nexia International (2005)). Starting point in the calibration of the tax base is the METR as calculated for debt and equity financing by Devereux et al. (2002). We assume that 25% of the new investment projects are financed with debt and 75% with equity. This is lower than the actual debt-equity mix (40,60) in order to ensure reasonable (depreciation) allowances. In the calibration, we set $\delta_t$ such that this METR is reproduced, but we restrict the fiscal depreciation rate to

$$\delta_t \in [\delta_k, \delta_k + 0.10]$$

Imposing the maximum value is motivated as follows. When one allows for generous depreciation allowances (large difference between $\delta_t$ and $\delta_k$), simulating a reduction in the corporate tax rate might result in an increase in the cost of capital and a reduction of the capital stock (i.e. the taxation paradox). We avoid this undesirable outcome by restricting the value of $\delta_t$.

Figure 2.9 shows that countries with a high tax rate have generally a smaller tax base.

A drawback of this calibration procedure is that the simulated tax revenues correlate poorly ($-0.3$) with the observed tax revenues (as share of GDP), as shown in Figure 2.10. The mean revenues are lower in the model on average (2.6% versus 3.0%). In an alternative procedure, we calibrated $\delta_t$ on the observed corporate tax revenues. However, this resulted in implausible high values for $\delta_t$, that would give rise to a negative response of the capital stock to a reduction in the corporate tax rate.

**Taxing household wealth** The tax rates on income from dividends, interest and capital gains are taken from Ernst and Young (2000) as given in Lorié (2000). Some countries adopt an

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24 Devereux et al. (2004) calculates average METR’s using a 35% weight on debt and 65% weight on equity.

25 The parameter $\delta_t$ should be interpreted broader than the fiscal depreciation rate. It captures all allowances (besides interest expenditures) that are assumed to be proportional to the capital stock.

26 This restriction is binding for Greece (at the upper bound) and Germany and Ireland (at the lower bound).

27 Only in Greece, the small tax base still implies the taxation paradox, where a tax increase raises investments.

28 Especially for Germany, the calibration choice matters: it has a highly distortive tax system in terms of the METR, but one of the lowest tax revenues. Gaëtan Nicodème (European Commission) pointed at the following explanations. First, about 85% of companies in Germany do not pay corporate taxes. They are rather in the forms of KG and the like and their owners are liable to personal income tax instead. Second, some data do not contain the trade tax (or gewerbesteuer), which is a (deductible) regional tax that comes in addition to the CIT rate of 25%. For example, if you look at the structure of taxation systems publications, this is the difference between table A.2.2.1_G which is corporate income tax (DE is 0.9% of GDP in 2004) and table C.3.1.1_G which is taxes on income of corporations (DE is 2.2% in 2004). The difference being mainly the gewerbesteuer. The drop after the tax reform is due to the possibility for companies to deduct the taxes they paid previously on retained earnings from the taxes they now pay on distributed dividends. There is also the fact that most companies had losses to carry forward from bad earlier years.

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36
imputation system where corporate taxes are (partly) imputed from the dividend tax. These imputation rates are taken from Sørensen (2001b). We have subtracted from the original dividend tax rate the product of the imputation rate and the corporate income tax rate. We
assume that income from assets is only taxed in the country of residence, such that
\[ \tau_b(i, j) = \tau_d(i, j) = \tau_g(i, j) = 0, \quad i \neq j. \]

**Tax rate on consumption** \((\tau_c)\) The tax rate on consumption is calculated as:

\[ \tau_c = \frac{RS_{5110} + RS_{5121}}{C - (RS_{5110} + RS_{5121})} \quad (2.8) \]

where \(C\) is household consumption (including taxes). The tax revenue data come from OECD Revenue Statistics, with

- 5110 General taxes on goods and services
- 5121 Excise taxes

Expression (2.8) is a simplification of Mendoza et al. (1994, (5)) and Carey and Tchilinguirian (2000, (14)). Figure 2.11 shows that the resulting tax rates follow closely the tax rates calculated by Carey and Tchilinguirian (2000, Table 4).

**Labor income tax** The average tax rate on labor income is calculated as

\[ \tau_l = \frac{RS_{1100} + c + RS_{2000} + RS_{3000}}{W + WSE} \quad (2.9) \]

with
\( \omega_k \)  labor income share, as calculated in (2.6)

\( W \)  compensation of employees

\( WSE \)  imputed wage sum of self-employed, see (2.5)

1100  Taxes on income, profit and capital gains of individuals or households

2000  Total social security contributions

3000  Taxes on payroll and workforce

This expression is similar to Mendoza et al. (1994, (7)) and Carey and Tchilinguirian (2000, (12)). Figure 2.12 compares our calculated tax rates for 2002 with the corresponding numbers for 1991-1997 in Carey and Tchilinguirian (2000, Table 4).

**Figure 2.12 Effective tax rate on labor income**
References


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### A Additional tables with model variables and equations

#### Table A.1 List of parameters and exogenous variables

<table>
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<th>Symbol</th>
<th>Description</th>
<th>GAMS</th>
<th>Dimension</th>
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</thead>
<tbody>
<tr>
<td>$\alpha_q$</td>
<td>share parameter intermediate inputs</td>
<td>alfa_q</td>
<td>m</td>
</tr>
<tr>
<td>$\alpha_l$</td>
<td>weight of leisure in utility</td>
<td>alfa_f</td>
<td>1</td>
</tr>
<tr>
<td>$\alpha_s$</td>
<td>share parameter in savings composite</td>
<td>alfa_s</td>
<td>m</td>
</tr>
<tr>
<td>$\alpha_{v,k}$</td>
<td>share parameter of value-added</td>
<td>alfa_v</td>
<td>m+3</td>
</tr>
<tr>
<td>$\alpha_{v,k}$</td>
<td>share parameter capital in value-added</td>
<td>alfa_vk</td>
<td>m+3</td>
</tr>
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<td>$\delta_k$</td>
<td>depreciation rate of capital</td>
<td>delta_k</td>
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<tr>
<td>$\epsilon_b$</td>
<td>scale parameter in marginal distress</td>
<td>eps_b</td>
<td>m</td>
</tr>
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<td>$\epsilon_q$</td>
<td>elasticity of marginal cost of distorting transfer prices</td>
<td>eps_q</td>
<td>m</td>
</tr>
<tr>
<td>$\rho_o$</td>
<td>utility weight of old generation</td>
<td>rho_o</td>
<td>m</td>
</tr>
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<td>rate of time preference</td>
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<td>intratemporal substitution elasticity</td>
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</tr>
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</tr>
<tr>
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<td>chi_0</td>
<td>m</td>
</tr>
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<td>share in fixed factor of domestic firms</td>
<td>omega_d</td>
<td>m</td>
</tr>
<tr>
<td>$\omega_m$</td>
<td>share in fixed factor of multinationals</td>
<td>omega_m</td>
<td>m+m</td>
</tr>
<tr>
<td>$\omega_n$</td>
<td>relative population size</td>
<td>omega_n</td>
<td>m+m</td>
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<td>$\omega_t$</td>
<td>share of age cohort in young population</td>
<td>share_t</td>
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<td>productivity level</td>
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<td>m</td>
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<td>productivity growth rate</td>
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<td>population growth rate</td>
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<td>$N^s, N^o$</td>
<td>size of young/old generation</td>
<td>N</td>
<td>m+2</td>
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<td>$\hat{r}_{wb}$</td>
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<td>$\hat{r}_{we}$</td>
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<tr>
<td>$T$</td>
<td>cohort length</td>
<td>T</td>
<td>1</td>
</tr>
</tbody>
</table>

1 superscripts $d, m, f$ refer to domestic firms, multinationals and foreign subsidiaries.

2 m denotes the number of countries.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>GAMS</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>τ_k</td>
<td>tax rate on household consumption</td>
<td>tau_c</td>
<td>m</td>
</tr>
<tr>
<td>τ_l</td>
<td>tax rate on labour income</td>
<td>tau_l</td>
<td>m</td>
</tr>
<tr>
<td>tr</td>
<td>lump-sum transfers</td>
<td>tr</td>
<td>m×2</td>
</tr>
<tr>
<td>τ_p</td>
<td>tax rate on corporate income</td>
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<td>m</td>
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<tr>
<td>τ_br</td>
<td>total tax rate by residence country on interest income</td>
<td>tau_br</td>
<td>m×m</td>
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<tr>
<td>τ_bs</td>
<td>total tax rate by source country on interest income</td>
<td>tau_bs</td>
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<tr>
<td>τ_dr</td>
<td>total tax rate by residence country on dividends</td>
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</tr>
<tr>
<td>τ_ds</td>
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<td>tau_ds</td>
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<td>imputation rate on dividends</td>
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<tr>
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<td>tau_gr</td>
<td>m×m</td>
</tr>
<tr>
<td>τ_gs</td>
<td>total tax rate by source country on capital gains</td>
<td>tau_gs</td>
<td>m×m</td>
</tr>
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<td>β_b</td>
<td>deductible fraction of corporate interest payment</td>
<td>beta_b</td>
<td>m</td>
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<td>δ_t</td>
<td>depreciation rate of capital for tax purposes</td>
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</tr>
<tr>
<td>Λ</td>
<td>ratio between tax factor on dividends and tax factor on capital gains</td>
<td>theta_e</td>
<td>m</td>
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<td>share of government consumption in GDP</td>
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<td>Symbol</td>
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<td>--------</td>
<td>-------------------------------------------------</td>
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<td>-----------</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>gross after tax return on bond composite</td>
<td>rho_b</td>
<td>m</td>
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<tr>
<td>$\rho_e$</td>
<td>gross after tax return on equity composite</td>
<td>rho_e</td>
<td>m</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>gross after tax interest rate on total savings</td>
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<td>m</td>
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### Table A.5  Equations used to calibrate

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