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Modelling the reporting discrepancies in bilateral data

In this paper the discrepancies in reported bilateral statistical data ("mirror data") are used to estimate the accuracy of the reporters. The estimated accuracies are to be used to compute optimal combinations of mirror data.

Two models of the discrepancies are presented: (a) unbiased reporting with inaccurate reporters having a large variance, and (b) biased reporting with inaccurate reporters having a large bias (either positive or negative). Estimation methods are least squares regression and maximum likelihood.

A numerical illustration is given, using data of the international trade in services. It is shown how to judge the two models empirically.

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Contents

1 Introduction and notation

The modelling of discrepancies among reports is a common problem in the compilation of macro-economic statistics. See Annex A of Wroe et al. (1999) for a literature review. In general there is some bookkeeping relation between the reported values which does not hold. The solution consists of first estimating the accuracy of the various reports and then finding the optimal adaption of the reported values such that the bookkeeping relation holds, based on these accuracies.

The estimation of the reporting accuracies can be a difficult problem. In this paper we consider a situation where this is relatively easy: bilateral data where each quantity is reported twice. For instance, with international trade data we may have for a particular trade flow the value reported by the exporter and the value reported by the importer ("mirror values"). Other bilateral data are direct foreign investment, foreign debt, and international migration.

Discrepancies in international trade data may have several causes, such as omitting transactions and misidentifying trade partners; see for instance Tsigas et al. (1992), p.298 and Gehlhar (1996), p.6/7 and ITC (2005) and Ferrantino and Wang (2007), p.5. In the current paper we discuss the issue from a general statistical modelling point of view, without considering particular causes. (Of course nothing is better than research into the causes of the discrepancies, for instance by case studies of customs records – and taking measures to increase the accuracy.)

After a short introduction of the notation of the paper, the classical approach to the modelling of the reporting errors is discussed in section 2. Here the reporting errors have a zero mean (unbiased) and different variances. Several estimation methods for these variances are discussed. The hurried reader who is not interested in estimation is advised to skip the subsections (2.2) and further.

In section 3 a different model is discussed and estimated, with biased reporting. Here the reporting errors do not have different variances about a zero mean, but they have different means; either positive or negative.

A fundamental problem of both models is discussed in section 4: the models are identified up to a general parameter shift between the two sides of the mirror (say, the exporters and the importers). A solution is given.

A numerical illustration for both models is given in section 5. The use of the estimates is also shown and the two models are compared empirically using the illustration results.

A conclusion is given in section 6. Technical details and related issues are presented in appendices.

For earlier work on the subject of modelling mirror discrepancies in international trade data, see Tsigas et al. (1992) and the references therein. They discuss the simultaneous estimation of the reporting error parameters using regression analysis, including the fundamental problem of our section 4 (which they do not solve).

In the notation of the model we will use the wording of international trade. Let Y_{ijt} be the logarithm of the unknown true value of the trade flow from country i to country j in year t. The index t might also indicate sectors, or a combination of years and sectors. Let Y_{ijt}^{exp} and Y_{ijt}^{imp} be the value of Y_{ijt} as reported by the exporter i and the importer j, respectively.

The reporting discrepancies are defined as follows:

$$\Delta Y_{ijt} \equiv Y_{ijt}^{\mathrm{exp}} - Y_{ijt}^{\mathrm{imp}} \tag{1.1}$$

Since the logarithm of the flows is used, we have relative discrepancies here[1].

2 Unbiased reporting

2.1 The variance model

In this section we discuss the classical approach to the modelling of reporting errors, where they are stochastic with zero mean and different variances. Then the optimally weighted average of these observations uses the reciprocal of the error variances as weights. In this way, an accurate observation (with a small error variance) has a large weight. See for instance equation (5) of the seminal paper on this subject, Stone et al. (1942).

After a presentation of the model, two alternative estimation methods are presented in subsections 2.2 and 2.3, respectively. In subsection 2.4 the effect of the non-negativity of the variances on the computations is discussed.

We assume that the variances are country specific. Hence:

$$\mathrm{E}\left[Y_{ijt}^{\mathrm{exp}}\right] = Y_{ijt} \tag{2.1}$$

$$\mathrm{E}\left[Y_{ijt}^{\mathrm{imp}}\right] = Y_{ijt} \tag{2.2}$$

$$\mathrm{Var}\left[Y_{ijt}^{\mathrm{exp}}\right] = V_i^{\mathrm{exp}} \tag{2.3}$$

$$\mathrm{Var}\left[Y_{ijt}^{\mathrm{imp}}\right] = V_j^{\mathrm{imp}} \tag{2.4}$$

[1] For a small relative discrepancy between two reported values a and b we have $\Delta Y = \log a - \log b = \log(a/b) \approx a/b - 1 = (a-b)/b \approx (a-b)/a$; see also the computations in table 5.1. Instead of using logs, levels might be used, or something in between (see Box and Cox (1964)), but this has no effect on the formulas in the this paper (except the exp and the log in (5.1) and (5.6)).

for all $i \neq j$. From (2.1) and (2.2) it follows that:

$$\mathrm{E}\left[\Delta Y_{ijt}\right] = 0 \tag{2.5}$$

and hence

$$\mathrm{Var}\left[\Delta Y_{ijt}\right] = \mathrm{E}\left[\left(\Delta Y_{ijt}\right)^2\right] \tag{2.6}$$

If the reporting errors are uncorrelated with each other then we also have:

$$\mathrm{Var}\left[\Delta Y_{ijt}\right] = V_i^{\mathrm{exp}} + V_j^{\mathrm{imp}} \tag{2.7}$$

for all $i \neq j$.

2.2 Least squares regression

The substitution of equation (2.6) into (2.7) gives:

$$\mathrm{E}\left[\left(\Delta Y_{ijt}\right)^2\right] = V_i^{\mathrm{exp}} + V_j^{\mathrm{imp}} \tag{2.8}$$

As a warming up to the more formal modelling in the next subsection, we note that this suggests the following regression model:

$$\left(\Delta Y_{ijt}\right)^2 = V_i^{\mathrm{exp}} + V_j^{\mathrm{imp}} + \varepsilon_{ijt} \tag{2.9}$$

where ε_{ijt} is a regression error term with mean zero. The V_i^{exp} and V_j^{imp} are the unknown coefficients to be estimated.

Of course this model can not be estimated (is not identified): given a least squares estimate, one can always add a constant to all V_i^{exp} and subtract the same constant from all V_j^{imp} without changing the residuals. This is the same as the well-known dummy variables problem: only the differences between the V_i^{exp} and the differences between the V_j^{imp} are identified, but not their levels. Note however that we are not interested in such differences here, but we want to compare mirror data: an export reporting accuracy versus an import reporting accuracy. We will come back to this problem later, in section 4.1 below.

The sum of the squared residuals is:

$$\mathrm{SSQ} \equiv \sum_i \sum_{j \neq i} \sum_t {\varepsilon_{ijt}}^2 = \sum_i \sum_{j \neq i} \sum_t \left(\left(\Delta Y_{ijt}\right)^2 - V_i^{\mathrm{exp}} - V_j^{\mathrm{imp}}\right)^2 \tag{2.10}$$

The first order conditions for a minimum are, for all i:

$$\frac{\partial \mathrm{SSQ}}{\partial V_i^{\mathrm{exp}}} = -2 \sum_{j \neq i} \sum_t \varepsilon_{ijt} = 0 \tag{2.11}$$

and for all j:

$$\frac{\partial \mathrm{SSQ}}{\partial V_j^{\mathrm{imp}}} = -2 \sum_{i \neq j} \sum_t \varepsilon_{ijt} = 0 \tag{2.12}$$

Note that this system is dependent, because the sum over i of the equations (2.11) is the same as the sum over j of the equations (2.12). This is another way of looking at the problem of the solution not being unique.

2.3 Maximum likelihood

In addition to the above assumptions, let the reported values be normally distributed. Then, using the equations (2.5) and (2.7), their discrepancy is distributed as:

$$\Delta Y_{ijt} \sim \mathrm{N}\left(0, V_{ij}\right) \tag{2.13}$$

with

$$V_{ij} \equiv V_i^{\mathrm{exp}} + V_j^{\mathrm{imp}} \tag{2.14}$$

for all $i \neq j$. In appendix A it is shown that

$$\mathrm{Var}[\varepsilon_{ijt}] = (\gamma_2 + 2)\, V_{ij}^2 \tag{2.15}$$

where the ε_{ijt} are the regression error terms implicitly defined by equation (2.9) above[2] and γ_2 is the (excess) kurtosis of the distribution of the discrepancies. The kurtosis of the normal distribution is zero. In such cases, with a constant kurtosis, the error variance is proportional with the V_{ij} and the model is called heteroscedastic. For maximal precision, least squares requires the weighting of the squared residuals with the reciprocal of the error variances. And hence in the first order condition for least squares the residuals themselves are thus weighted. As we shall see, this is indeed the result of maximum likelihood.

Using the formula of the normal density, we get the loglikelihood of the unknown variance parameters, given the observed discrepancies ΔY_{ijt} (and omitting an irrelevant additive constant of $-n \log \sqrt{2\pi}$, where n is the number of observations):

$$\begin{aligned} \log L &= \sum_i \sum_{j \neq i} \sum_t \log \left(\frac{1}{\sqrt{V_{ij}}} \exp \left(-\frac{(\Delta Y_{ijt})^2}{2V_{ij}} \right) \right) \\ &= \sum_i \sum_{j \neq i} \sum_t \left(-\frac{1}{2} \log\left(V_{ij}\right) - \frac{(\Delta Y_{ijt})^2}{2V_{ij}} \right) \end{aligned} \tag{2.16}$$

[2] Of course, the variance of the "true" regression error terms is not the same as the variance of the least squares regression residuals.

Note that this likelihood function is highly nonlinear: the large parentheses in the last member of (2.16) contain the difference between two expressions which go to infinity for small V_{ij}.

The first order condition for a maximum likelihood with respect to V_i^{exp} is:

$$\begin{aligned}\frac{\partial \log L}{\partial V_i^{\text{exp}}} &= \sum_{j\neq i}\sum_t \frac{\partial}{\partial V_{ij}}\left(-\frac{1}{2}\log(V_{ij}) - \frac{(\Delta Y_{ijt})^2}{2V_{ij}}\right)\\ &= \frac{1}{2}\sum_{j\neq i}\sum_t\left(-\frac{1}{V_{ij}} + \frac{(\Delta Y_{ijt})^2}{V_{ij}^2}\right)\\ &= \frac{1}{2}\sum_{j\neq i}\left(\frac{1}{V_{ij}}\sum_t\left(\frac{(\Delta Y_{ijt})^2}{V_{ij}} - 1\right)\right)\\ &= 0\end{aligned} \tag{2.17}$$

For the derivation of the first equal sign in (2.17) we have made use of:

$$\frac{\partial V_{ij}}{\partial V_k^{\text{exp}}} = \begin{cases} 1 & \text{if } i = k \\ 0 & \text{otherwise}\end{cases} \tag{2.18}$$

First order condition (2.17) is repeated here on one line, as follows: for each i we have

$$\sum_{j\neq i}\left(\frac{1}{V_{ij}}\sum_t\left(\frac{(\Delta Y_{ijt})^2}{V_{ij}} - 1\right)\right) = 0 \tag{2.19}$$

As an aid in the interpretation of this equation, note that in the case of a model with only one variance parameter, say $V_{ij} = V^*$, we would have $(1/V^*)^2\sum\sum\left((\Delta Y_{ijt})^2 - V^*\right) = 0$ and the variance estimate would simply be equal to the sample variance:

$$V^* = \frac{\sum\sum(\Delta Y_{ijt})^2}{n} \tag{2.20}$$

where n is the number of observations of ΔY_{ijt}.

The first order condition (2.19) can also be written using the regression error terms ε_{ijt} defined by the least squares regression model (2.9). Then we have for all i:

$$\sum_{j\neq i}\left(\frac{1}{V_{ij}^2}\sum_t \varepsilon_{ijt}\right) = 0 \tag{2.21}$$

Compare with the first order condition for least squares in (2.11) above. Indeed in (2.21) the residuals are weighted with the reciprocal of their variance given in (2.15) above. (In appendix C this is discussed further.)

The first order condition with respect to V_j^{imp} is similar to the first order condition with respect to V_i^{exp}. For all j:

$$\sum_{i\neq j}\left(\frac{1}{V_{ij}^2}\sum_t \varepsilon_{ijt}\right) = 0 \tag{2.22}$$

Finally, note that we have here the same problem as in the above section 2.2 about least squares: the first order conditions (2.21) and (2.22) together do not define a unique solution, for the same reason as in section 2.2. See section 4.1 below.

2.4 Variances are not negative

Since all coefficients in the model are variances, we have the following restrictions for all k:

$$V_k^{\text{exp}} \geq 0 \tag{2.23}$$

$$V_k^{\text{imp}} \geq 0 \tag{2.24}$$

Although all the left hand side data in the regression equation (2.9) are positive, these restrictions might be violated by the least squares regression (section 2.2). Hence we must minimise the sum of squares under these restrictions.

With maximum likelihood (section 2.3), the total variances V_{ij} are always positive, since otherwise the loglikelihood function does not exist. However, the components V_i^{exp} and V_j^{imp} can be negative. Hence in this case they must be restricted too, by (2.23) and (2.24). Moreover we can not have both a V_i^{exp} and a V_j^{imp} equal to zero (with $i \neq j$), because then V_{ij} is zero. We can have more than one zero-variance in the following three cases: only export reporting variances are zero, only import reporting variances are zero, or only the two variances of one country are zero. This seems an odd feature of the variance model.

3 Biased reporting

From the first three lines of table 5.1 on page 11 one might suspect that France reports its exports with a negative bias. Even more so, it seems that Italy reports its exports with a positive bias. Hence we consider a model where the reporting errors do not have different variances about a zero mean, but different means; either positive or negative:

$$\mathrm{E}\left[Y_{ijt}^{\text{exp}}\right] = Y_{ijt} + \mu_i^{\text{exp}} \tag{3.1}$$

$$\mathrm{E}\left[Y_{ijt}^{\text{imp}}\right] = Y_{ijt} + \mu_j^{\text{imp}} \tag{3.2}$$

with

$$\mathrm{Var}\left[Y_{ijt}^{\text{exp}}\right] = \mathrm{Var}\left[Y_{ijt}^{\text{imp}}\right] = V_0 \tag{3.3}$$

Then:

$$\mathrm{E}\left[\Delta Y_{ijt}\right] = \mu_i^{\text{exp}} - \mu_j^{\text{imp}} \tag{3.4}$$

and also, assuming independent reporting errors:

$$\operatorname{Var}[\Delta Y_{ijt}] = 2V_0 \tag{3.5}$$

This suggests the following regression model:

$$\Delta Y_{ijt} = \mu_i^{\text{exp}} - \mu_j^{\text{imp}} + \varepsilon_{ijt} \tag{3.6}$$

Compare with equations (1) and further of Tsigas et al. (1992), who also assumes biased reporting errors.

Model (3.6) is homoscedastic: the regression errors have the same variance. Hence OLS is an efficient method here. Assuming normally distributed errors, OLS is the same as maximum likelihood.

Of course the bias model can also not be estimated, as discussed above for the variance model. Here one can always add a constant to all μ_k^{exp} and all μ_k^{imp} without changing the residuals. See section 4.2 below.

4 The fundamental problem of non-uniqueness and the symmetry axiom

As discussed above, the variance model and the bias model both have a fundamental problem: the optimal solution is not unique.

Note that there is no empirical way out of this, using the observed discrepancies; we have to choose between parameter estimates which have the same fit to the observed discrepancies. Only extra information about the country accuracies can help; for instance from case studies of individual trade transactions.

In the absence of any prior indication that export reporting is relatively accurate (or inaccurate) compared to import reporting, the simplest choice is to assume a generally symmetric situation. This is translated to the two models as shown in the next two subsections.

We compare this symmetry axiom with the non-simultaneous methods such as the GTAP method described in appendix F, or the computations in ITC (2005). These methods use the entire discrepancies in the computation, without decomposing them in the two reporting errors; the latter is only possible with some kind of simultaneity. Hence these method do not have to find a way out of our non-uniqueness problem. However, they are implicitly symmetrical: all exporters together are judged on the same set of discrepancies as are all importers together, and hence on average the two groups get the same result. If all exporters report without error and all importers report with errors (both positive and negative) then this is not detected by non-simultaneous methods; nor by the methods proposed here.

Finally, if there is prior knowledge then this symmetry axiom might be replaced by an empirical rule such as: import data are on average 25% more reliable than export data. (The guiding principle of Statistics Canada's *World Trade Analyzer* is: import data are more reliable than export data; see LeBlanc (2000).)

4.1 The symmetry in the variance model

As we saw in section 2 above, the variance model can not be estimated (is not identified): one can always add a constant to all V_i^{exp} and subtract the same constant from all V_j^{imp} without changing the V_{ij} and hence without changing the regression residuals or the likelihood.

For this model we use the symmetry axiom to propose the following restriction[3] [4]:

$$\sum_k V_k^{\text{exp}} = \sum_k V_k^{\text{imp}} \tag{4.1}$$

If log values are used, and the parameters are relative as discussed in footnote 1 on page 3, then these summations might be weighted with the size of the countries in order to prevent that an arbitrarily small country has the same influence as a large country. We did not use weights in the illustration in section 5 below, since the illustration includes countries which do not differ very much in size.

As an alternative to (4.1), the above mentioned imaginary empirical rule "import data are on average 25% more reliable than export data" might be formulated as: $(1-c)^2 \sum_k V_k^{\text{exp}} = \sum_k V_k^{\text{imp}}$ with $c = 0.25$. We have not applied such a rule.

Also as an alternative for equation (4.1), we experimented with an extra optimisation criterion: choose the solution with largest correlation between the V_k^{exp} and the V_k^{imp}. The idea behind this was: a country has or has not a good national statistical bureau, and this affects both the export reporting and the import reporting. However, this did not give satisfactory results. This might be due to a non-optimal stationary point; see the computer appendix D. Also, this "nested optimisation" is more complicated than the addition of an equality restriction to the optimisation.

[3] Equation (4.1) holds also for the so-called adjusted means, or least squares means. See for instance Searle et al. (1980), equation (4.2) and the next. In a private communication Cyrille Schwellnus suggested a refinement, where the reciprocals of the number of categories in the formula for the adjusted means are replaced by a ratio which depends on the number of trading partners of the country.

[4] The non-negativity restrictions (2.23) and (2.24) might be used to find a unique solution for the variance model. They increase the residual sum of squares, or decrease the likelihood, and it might be that only one combination of binding restrictions gives the smallest increase or decrease, respectively. We have not followed this line of inquiry because it can only work, if at all, if the estimate without the restrictions violates the restrictions. Also there is a computer programming problem here; see the last section of appendix D below.

4.2 The symmetry in the bias model

As discussed above, the bias model is not identified, similar to the variance model. One can always add a constant to all μ_k^{exp} and all μ_k^{imp} without changing the residuals. A restriction is imposed, again motived by symmetry, as with (4.1) above. Think of a single discrepancy from i to j, where $\mu_i^{\text{exp}} + \mu_j^{\text{imp}} = 0$ implies that the truth is halfway between the two reports. This results in a restriction on the overall sum:

$$\sum_k \left(\mu_k^{\text{exp}} + \mu_k^{\text{imp}} \right) = 0 \tag{4.2}$$

A weighted sum might be used here, as with equation (4.1); see the discussion below that equation.

Note that this problem is computationally trivial here, compared with the variance model, since here are no non-negativity restrictions on the parameters. Hence one can easily apply equation (4.2) to estimation results by adding a constant to all coefficient estimates such that the equation holds. We have applied this idea to the table published in Tsigas et al. (1992); see appendix E below.

Finally, as an alternative to (4.2), the above mentioned imaginary empirical rule “import data are on average 25% more reliable than export data” might be formulated as: $\sum_k \left((1-c)\mu_k^{\text{exp}} + \mu_k^{\text{imp}} \right) = 0$ with $c = 0.25$. Again, we have not applied such a rule.

5 A numerical illustration using trade in services

5.1 The data

We illustrate the above methods with table 5.1, which contains reported trade in services between four large European countries. This is the lowest number of countries which leaves some degrees of freedom in the model: $4 \times (4-1) = 12$ observed discrepancies and $2 \times 4 = 8$ reporting error parameters to be estimated, giving 4 degrees of freedom. (With three countries we have $6 - 6 = 0$ degrees of freedom.) The services are the category “Other Commercial Services” (OCS). For instance the first line shows that the trade flow from France to Germany was reported by France as 1.3 billion USD and the same flow was reported by Germany as 4.8 billion USD. The total discrepancy at the bottom of the Δ column is fairly small and we did not adjust the data to make this zero[5].

[5] The total discrepancy is 1.5 billion USD, or 3%. This is surprisingly low considering the individual discrepancies. Issues such as total exports not being equal to total imports are outside the scope of this paper; one might adjust the data prior to modelling.

The table also gives three indicators of the percentage discrepancies. The "Δ logs" is the ΔY_{ijt} in the model formulas. (For example, in the first row we have $\log 1.3 - \log 4.8 = -1.31 = -131\%$[6]. The "mean" in the penultimate column is the mean of the two reported values. (For example $(1.3-4.8)/((1.3+4.8)/2) = -1.15 = -115\%$.) The "$|\Delta|$/import" is the GTAP criterion of accuracy for the trade in goods; see appendix F below. Only three of the 12 discrepancies are below the GTAP 20% threshold, in any of the three percentage columns.

Table 5.1 Reported trade in services, OCS (2002, billion USD)

reporting exporter	reporting importer	reported export	reported import	Δ	|Δ|	Δ logs	Δ / mean	|Δ| / import (GTAP)
						%		
France	Germany	1.3	4.8	−3.5	3.5	−131	−115	73
France	Italy	1.8	3.7	−1.9	1.9	−72	−69	51
France	UK	3.8	3.3	0.5	0.5	14	14	15
Germany	France	4.7	3.6	1.1	1.1	27	27	31
Germany	Italy	1.8	3.6	−1.8	1.8	−69	−67	50
Germany	UK	6.6	3.9	2.7	2.7	53	51	69
Italy	France	3.3	1.5	1.8	1.8	79	75	120
Italy	Germany	3.4	1.4	2.0	2.0	89	83	143
Italy	UK	3.6	1.2	2.4	2.4	110	100	200
UK	France	5.7	4.8	0.9	0.9	17	17	19
UK	Germany	7.5	9.1	−1.6	1.6	−19	−19	18
UK	Italy	2.9	7.0	−4.1	4.1	−88	−83	59
Total		46.4	47.9	−1.5	24.3			

Source: OECD Statistics on International Trade in Services

5.2 Estimates of the variance model and the use of the estimates

Results table 5.2 gives the total reported exports and reported imports per reporter country, the total abs discrepancy and the percentage ratio of these two.

The estimated variance parameters are also given, based on the discrepancy between the log values, according to the method LS (least squares) and ML (maximum likelihood), as discussed in the preceding subsections 2.2 and 2.3, respectively. Each zero estimate is at the lower bound.

It is interesting to compare the estimation results with the columns labelled "rel $|\Delta|$" in table 5.2, which show the sum of the abs discrepancies as a percentage of the export or import. The UK has the largest import discrepancy percentage (67%); this is also larger than the export

[6] Throughout this paper, all numbers which are a difference between logs are presented as percentage by merely multiplying them with 100, without first transforming them with "$\exp(\ldots)-1$".

Table 5.2 Estimates of the variance model

	export reporting					import reporting				
				$\sqrt{V^{\text{exp}}}$					$\sqrt{V^{\text{imp}}}$	
	total	$\lvert\Delta\rvert$	rel $\lvert\Delta\rvert$	LS	ML	total	$\lvert\Delta\rvert$	rel $\lvert\Delta\rvert$	LS	ML
	billion USD		%			billion USD		%		
France	6.9	5.9	86	62	0	9.9	3.8	38	8	0
Germany	13.1	5.6	43	27	40	15.3	7.1	46	71	83
Italy	10.3	6.2	60	81	101	14.3	7.8	55	67	71
UK	16.1	6.6	41	0	17	8.4	5.6	67	38	14
Total	46.4	24.3				47.9	24.3			

discrepancy percentage of Italy (60%). However, both estimates of the import reporting error of the UK (38% and 14%) are much smaller than both export reporting error estimates of Italy (81% and 101%). Hence, in the published export from Italy to the UK the latter's report should have the largest weight.

These estimates can be used according the recipe of Stone et al. (1942), discussed above at page 3. If one of the two variances is estimated as zero then use the "lim" of this formula for that variance approaching zero; this amounts to the use of the report of that country only. This makes the first lines in table 5.1 useless as an example because one of the two partner countries involved has a zero for the relevant estimated variance. For instance France has a zero estimated variance as export reporter, making the first three lines of table 5.1 useless as an example.

Consider then the fifth line of the data table 5.1: the trade flow from Germany to Italy, reported by these countries as 1.8 and 3.6 billion USD, respectively. Using the ML estimate from table 5.2, we have:

$$\exp \frac{\log(1.8)/40^2 + \log(3.6)/71^2}{1/40^2 + 1/71^2} = 2.1 \text{ billion USD} \tag{5.1}$$

5.3 An estimate of the bias model and the use of the estimate

Table 5.3 shows the results for the illustration data for the bias model. The -1.5 and $+1.5$ are (apart from sign) the same as the -1.5 in the Total row in the data table 5.1. The $+2$% and -2% reflect the symmetry equation (4.2).

How are these estimated biases to be used? The recipe of Stone et al. (1942), discussed above at page 3, is not applicable here. We combine the equations (3.1) and (3.2) as follows. For $k = 1, 2$:

$$y_k = Y_{ijt} + \varepsilon_k \tag{5.2}$$

Table 5.3 **Estimate of the bias model**

	export		import	
	$+\Delta$	μ^{exp}	$-\Delta$	μ^{imp}
	billion USD	%	billion USD	%
France	-4.9	-56	-3.8	-22
Germany	2.0	-4	3.1	22
Italy	6.2	75	7.8	52
UK	-4.8	-12	-5.6	-54
Total	-1.5	+2	+1.5	-2

Not all columns add up to the total, due to rounding.

with

$$y_1 \equiv Y_{ijt}^{\text{exp}} - \mu_i^{\text{exp}} \tag{5.3}$$

$$y_2 \equiv Y_{ijt}^{\text{imp}} - \mu_j^{\text{imp}} \tag{5.4}$$

Consider (5.2) as a regression model with one regression coefficient: Y_{ijt}. The least squares estimate of this coefficient is the mean of the y_k. Hence the recipe is: correct the two reports for their estimated bias and then take their unweighted mean, or correct the mean of the reports with the mean bias:

$$\widehat{Y}_{ijt} = \frac{\left(Y_{ijt}^{\text{exp}} - \mu_i^{\text{exp}}\right) + \left(Y_{ijt}^{\text{imp}} - \mu_j^{\text{imp}}\right)}{2} = \frac{Y_{ijt}^{\text{exp}} + Y_{ijt}^{\text{imp}}}{2} - \frac{\mu_i^{\text{exp}} + \mu_j^{\text{imp}}}{2} \tag{5.5}$$

Consider for example again the fifth line of the data table 5.1: the flow from Germany to Italy, reported by these countries as 1.8 and 3.6 billion USD, respectively. The result is:

$$\exp \frac{\left(\log(1.8) + 0.04\right) + \left(\log(3.6) - 0.52\right)}{2} \;=\; 2.0 \text{ billion USD} \tag{5.6}$$

Finally, note that with this model the estimate is not always closest to the most accurate reporter. If a reporter with a positive bias reports nevertheless the lowest value of the two (or a reporter with a negative bias reports the highest value) then this pulls the estimate to that reporter. As an extreme example, imagine that reporter i has no bias at all, while partner j has, say, a positive bias μ but reports μ less than i. Then the best estimate is exactly equal to the report of the reporter j, who is not the accurate reporter.

5.4 Empirical comparison of unbiased versus biased reporting

It is possible to judge empirically the two main models of this paper: the unbiased reporting versus the biased reporting. We give the results for the four country illustration data.

As noted above, the degree to which the models fit the data is not dependent on the chosen way out of the non-uniqueness problem of section 4: this problem consist of the fit being the same for a range of solutions. Thus, residual sum of squares and R^2 values and loglikelihoods are unique even when the parameter estimates are not.

First, as a rough measure, we used a statistical computer program and obtained R^2 values from OLS, by arbitrarily fixing one of the parameters to zero. We ignored here the non-negativity restrictions on the parameters of the variance model, which increases the fit of the variance model. Nevertheless the R^2 values from OLS are 45 and 89 per cent for the variance model of unbiased reporting with equation (2.9) and for the biased reporting model with equation (3.6), respectively. Hence, here the bias model shows a considerably better fit. (Note however that an R^2 for the variance model near 100% would be a questionable result, since here the regression error variance must be about twice the variance of the discrepancy; see equation (2.15) above.)

Also the models are compared on the basis of their likelihoods from the estimates of the previous subsections. We assume normally distributed reporting in both models. See appendix B for details. The result is again a much better fit of the bias model.

Additional support for one of the two models might come from more data, for instance data for more years. We have compared the data over 2002 in table 5.1 with the same data[7] over 2001. The signs of the discrepancies are the same, and the correlation between the discrepancies in the two years is 0.96. This seems to support the bias model.

Finally, one might consider the two reporting models as special cases of a general model which contains the variance formulas (2.3) and (2.4), with (4.1), and the bias formulas (3.1) and (3.2), with (4.2). Then one might apply the standard tests for nested hypotheses. This would tell us for each of the sub models if they can be rejected when considered as a restriction on the general model. However, it would not tell us more about the relative position of the two models.

To be precise, in the variance model we must add a general μ term to the right hand side of (2.1) and (2.2). These μ terms cancel each other in equation (2.5). Then the variance model is obtained from the general model by assuming that all biases are equal, and the bias model is obtained by assuming that all variances are equal.

6 Conclusion and remaining questions

In this paper we have explored models and estimation techniques for the analysis of discrepancies in bilateral data (“mirror values”). Including also the appendices below, this

[7] The discrepancies in 2001 are, in the same order as in table 5.1, and from the same source: -3.7, -1.6, 1.3, 0.9, -2.1, 2.1, 1.5, 1.2, 2.6, 1.1, -2.9, -3.3 billion USD.

seemingly simple problem proves to contain a surprisingly large amount of technical details, both statistical and numerical.

Solving discrepancies between mirror values requires a choice between two models. First: the traditional model for making macro-economic statistics, based on Stone et al. (1942), where the discrepancies are caused by unbiased reporting errors. Second: the model of Tsigas et al. (1992) where the reporting errors have a non-zero bias, either negative or positive.

Although the choice between the two models is not affected by it, both models suffer from the fundamental problem discussed (but not solved) by Tsigas et al. (1992): the estimates of the reporting error parameters are not unique. A way out of this problem is presented for each model, based on the *a priori* symmetry between export reporting and import reporting.

The empirical choice between these models has been discussed. A small illustrative data set on international trade in services fits best to the bias model. A method has been derived how to the use the estimated biases in combining mirror values: first correct the two reports individually for their bias, then take their unweighted mean. We have shortly discussed a general model which encompasses both.

Some methodological questions are still to be answered:

- Are there better methods than the symmetry equations (4.1) and (4.2) to find a way out of the fundamental problem of non-uniqueness?
- Do the first order conditions of the various estimation techniques of the variance model including equation (4.1) have a unique solution, or are there multiple points satisfying the first order conditions (after solving the fundamental problem)?
- What are the standard errors of the estimates in the tables 5.2 and 5.3?

And of course the empirical questions about the most appropriate model for the bilateral data about trade (in goods or in services), investment, aid, migration, etcetera, are still remaining.

Appendix A The proof of equation (2.15)

The proof of equation (2.15) is given here. The following shorthand notations are used: Δ for the discrepancy ΔY_{ijt} (the column "Δ logs" in table 5.1), V for variance V_{ij} and ε for regression error term ε_{ijt}. Then:

$$\mathrm{E}[\Delta] = 0 \tag{A.1}$$

$$\mathrm{Var}[\Delta] = V \equiv \sigma^2 \tag{A.2}$$

$$\mathrm{E}[\varepsilon] = \mathrm{E}\left[\Delta^2 - V\right] = 0 \tag{A.3}$$

Hence:

$$\begin{aligned}\mathrm{Var}[\varepsilon] &= \mathrm{E}\left[\varepsilon^2\right] = \mathrm{E}\left[\left(\Delta^2 - V\right)^2\right] = \mathrm{E}\left[\Delta^4\right] + V^2 - 2V\mathrm{E}\left[\Delta^2\right] \\ &= \mu_4 + V^2 - 2V^2 = \mu_4 - V^2 = \frac{\mu_4}{\sigma^4}V^2 - V^2 \\ &= \left(\frac{\mu_4}{\sigma^4} - 1\right)V^2 = (\gamma_2 + 2)\,V^2\end{aligned} \tag{A.4}$$

where $\gamma_2 \equiv \mu_4/\sigma^4 - 3$ is the (excess) kurtosis of the distribution of the discrepancy Δ and $\mu_4 \equiv \mathrm{E}\left(\Delta^4\right)$.

As an aside: the last member of equation (A.4) implies that the kurtosis γ_2 is never smaller than -2. This is indeed its lower limit and it is easily seen that this limit is reached by any discrete distribution with has a nonzero probability for only two values of the stochast, with equal probabilities. Then $x - \mu = \pm\sigma$ and $\mu_4 \equiv \mathrm{E}\left[(x-\mu)^4\right] = \mathrm{E}\left[\sigma^4\right] = \sigma^4$. An example is the Bernoulli distribution with $p = 1/2$.

Appendix B Comparing the likelihoods of non-nested models

The loglikelihood of regression model with normally i.i.d. regression errors is:

$$\begin{aligned}\log L &= \sum_{k=1}^{n} \log\left(\frac{1}{\sigma}\exp\left(-\frac{\varepsilon_k^2}{2\sigma^2}\right)\right) = -n\log\sigma - \frac{n}{2\sigma^2}\sum_k \frac{\varepsilon_k^2}{n} \\ &= -n\log\sigma - \frac{n}{2\sigma^2}\sigma^2 = -n\log\sigma - n/2\end{aligned} \tag{B.1}$$

where the ε_k are the regression error terms and σ is the standard deviation of these error terms[8]. We have $n = 12$ here. The last member of (B.1) is the concentrated loglikelihood, based on the

[8] Again, for brevity we have omitted the irrelevant term $-n\log\sqrt{2\pi}$. Of course this term must be included in the loglikelihood when comparing with a model which is not based on the normal distribution.

ML estimator of the error variance: $\sigma^2 = \sum \varepsilon_k^2/n$, the mean square of the residuals; it shows that maximum likelihood is in this case the same as least σ, or least squares.

Table B.1 shows the results. The first loglikelihood value can be found in the result file of the GAMS program which goes with this paper, and the second value can easily be computed by hand from the indicated equation and the σ. The last column shows the loglikelihood values relative to the largest value: the bias model[9].

Table B.1 Loglikelihood values

reporting model	loglikelihood			
	formula	σ	value	relative value
variance model (unbiased reporting)	eq (2.16)		−54.8	−5.0
biased reporting	eq (B.1)	38.6 %	−49.8	0

For the comparison of the likelihoods of nonnested models, we use the Likelihood Axiom (or Likelihood Principle) which states, loosely speaking, that all the information about a model, given the data, is contained in the likelihood function. See for instance Edwards (1976), p.31 or Berger and Wolpert (1988). It follows that the probability distribution of the likelihood ratio under a null hypothesis is not relevant. We use a loglikelihood difference of 2 as a benchmark. This is the loglikelihood difference between $\mu = x$ and the familiar significance limits $\mu = x \pm 2\sigma$ when x is a drawing from a normal distribution with known variance σ^2 and unknown mean μ. (The loglikelihood is: $\log \mathrm{L}(\mu) = -(\mu - x)^2/2\sigma^2$ + constant.) See for instance Edwards (1976), p.76, about the "2-unit support limits". Hence here the bias model is significantly more likely than the variance model.

Appendix C Regression analysis when the error variance is related to the regression coeffcients

The first order conditions (2.21) and (2.22) contain a weighted sum of regression errors. Note however that this differs from the minimisation of the weighted least squares

$$\mathrm{SSQ}^{\mathrm{W}} \equiv \sum_i \sum_{j \neq i} \left(\frac{1}{V_{ij}^2} \sum_t \varepsilon_{ijt}{}^2 \right) \tag{C.1}$$

[9] Published loglikelihood values themselves have little meaning; it is usually not easy for the reader to appreciate their value. They depend on such trivia as whether or not the $-n \log \sqrt{2\pi}$ term is included (in the case of the normal distribution) and on the dimension of the stochast: changing this dimension with some factor shifts the loglikelihood with n times the log of this factor and may change the sign of the loglikelihood. However, differences between loglikelihood values (based on the same data, in the same dimension, etcetera) are meaningful; see the last paragraph of this appendix.

That gives the first order condition with respect to V_k^{exp}:

$$\frac{\partial \mathrm{SSQ}^{\mathrm{W}}}{\partial V_k^{\text{exp}}} \tag{C.2}$$

which is differs from the derivative with the weights held fixed:

$$\sum_i \sum_{j \neq i} \left(\frac{1}{V_{ij}^2} \sum_t \frac{\partial \varepsilon_{ijt}{}^2}{\partial V_k^{\text{exp}}} \right) = -2 \sum_{j \neq k} \left(\frac{1}{V_{kj}^2} \sum_t \varepsilon_{kjt} \right) \tag{C.3}$$

Equating the last member of (C.3) to zero gives the first order condition (2.21) above. Compare with Theil (1971), p.245-246 and Amemiya (1973) for an early discussion of this subject. They discuss the minimisation of $\mathrm{SSQ}^{\mathrm{W}}$ where the V_{ij} shown in (C.1) are fixed estimates from OLS residuals.

Appendix D Computer programming

The estimates of the model have been computed with the NLP (nonlinear programming) method in the GAMS computer program, minimising the residual sum of squares, or maximising the loglikelihood.

In all cases we included also the symmetry equation (4.1) or (4.2) in the GAMS model.

As starting values for the solver, we used for all i:

$$V_i^{\text{exp}} = \frac{1}{2} \left(\sum_{j \neq i} (\Delta Y_{ijt})^2 \right) / (4-1) \tag{D.1}$$

and likewise for the V_j^{imp}. Next, the starting values for V_{ij} were computed from the definition (2.14).

The GAMS program code is available at www.cpb.nl, at the web page of this paper.

Finally, a critical note. GAMS was designed for efficiently minimising or maximising an objective function with restrictions. However, we found that it is less tailored to the estimation of statistical models by minimising a sum of squares or maximising a likelihood, compared with statistical and econometric software such as SAS or TSP. In the first place, no distinction is made in GAMS between zeros and missing values if an array is partly filled with data.

Second, GAMS produces without a warning an arbitrary solution of an ill-defined problem. For example the value of two variables for which the squared sum is minimal:

```
Variables X,Y,S;
Equations EQ;
EQ .. S =e= sqr(X+Y);
Model M /all/;
X.L=1; Y.L=2;
Solve M minimizing S using NLP;
```

This can also be the case when searching for the value of two variables for which the sum is zero, as a square system to be solved with MCP:

```
Variables X,Y;
Equations EQ1,EQ2;
EQ1 .. X+Y =e= 0;
EQ2 .. X =e= -Y;
Model M /all/;
X.L=1; Y.L=2;
Solve M using MCP;
```

In a way, this problem is at the heart of this paper; see section 4 above about problems without a unique solution. The model status of the NLP solution may be either "locally optimal" or just "optimal", depending on the solver. (We tested several solvers. The MILES solver detects the non-uniqueness in the MCP model.) Of course this is a problem which requires care in other software too; for instance in SAS/IML, one has to specify that convergence tests must be based only on changes in the parameter space and not on the basis of changes of the function value, or the size of the gradient.

Appendix E Tsigas, Hertel and Binkley, 1992

In Tsigas et al. (1992) a bias model for seven regions is estimated, like our (3.6) above. They use the reverse definition for the discrepancy, compared with our definition of ΔY_{ijt} in equation (1.1) on page 3 above. An intercept term is added and hence they need two identifying country bias restrictions. This gives for our equation (3.6), with the residual term omitted and a general intercept term μ included:

$$-\Delta Y_{ijt} = \mu + \mu_j^{\text{imp}} - \mu_i^{\text{exp}} \tag{E.1}$$

On page 30, at the end of their section 3, they discuss the search for finding two countries which can be used as "base reporters", whose bias is zero. To this end, attention is restricted to the model estimates for which the multiple equality $\mu = \mu_j^{\text{imp}} = \mu_i^{\text{exp}} = 0$ is not rejected statistically[10] for some pair of base reporters i and j. They find 17 such estimates, reported in their table 5 with the $\exp(\ldots)$ function of the μ_i^{exp} and μ_j^{imp} coefficients. (The dashes must be read as ones.) The two reporters in such a base reporter pair have the same bias.

[10] Van Leeuwen and Lejour (2006), p.7 present the same model for other regions. They assumed that this equality holds for $i = j =$ the combination of Belgium plus Luxembourg, and also that this choice has little effect on the outcome, with the large number of regions they have.

Our symmetry equation (4.2) can be applied to this table 5, as follows. Pick any line of the table, and take logs of all coefficients except the estimated intercept term. (The dashes now become zeros.) Then absorb the intercept term into the bias coefficients by subtracting it from all export bias coefficients, or adding it to all import bias coefficients. Finally, add a constant to all bias coefficients such that they add up to zero as in our (4.2). This gives for instance for the USA an export bias of -2.4% and an import bias of $+2.0\%$, for any line in the table. (We might have used a weighted sum of biases here.)

This is shown in formulas as follows. If for example the intercept term μ in (E.1) is absorbed by subtracting it from all export bias coefficients then we have:

$$-\Delta Y_{ijt} = \mu_j^{\mathrm{imp}} - \tilde{\mu}_i^{\mathrm{exp}} \tag{E.2}$$

with

$$\tilde{\mu}_i^{\mathrm{exp}} \equiv \mu_i^{\mathrm{exp}} - \mu \tag{E.3}$$

We generalise our symmetry equation (4.2) by adding weights:

$$\sum_k w_k \left(\mu_k^{\mathrm{imp}} + \mu_k^{\mathrm{exp}}\right) = 0 \tag{E.4}$$

If the weighted sum of all coefficients must be zero then the weighted average must be subtracted from each coefficient. This average, say A, can be written as the average of two averages:

$$\begin{aligned} A &= \frac{1}{2}\left(\frac{1}{\sum w}\sum_j w_j \mu_j^{\mathrm{imp}} + \frac{1}{\sum w}\sum_i w_i \tilde{\mu}_i^{\mathrm{exp}}\right) = \\ &= \frac{1}{2}\left(\frac{1}{\sum w}\sum_j w_j \mu_j^{\mathrm{imp}} + \frac{1}{\sum w}\sum_i w_i \left(\mu_i^{\mathrm{exp}} - \mu\right)\right) = \frac{1}{2}\left(\overline{\mu}^{\mathrm{imp}} + \overline{\mu}^{\mathrm{exp}} - \mu\right) \end{aligned} \tag{E.5}$$

The $\overline{\mu}^{\mathrm{imp}}$ and $\overline{\mu}^{\mathrm{exp}}$ are the weighted averages of the original coeffcients. The new set of coefficients consist of the import coefficients

$$\mu_j^{\mathrm{imp}} - A = \mu_j^{\mathrm{imp}} - \frac{1}{2}\left(\overline{\mu}^{\mathrm{imp}} + \overline{\mu}^{\mathrm{exp}} - \mu\right) \tag{E.6}$$

and the export coefficients

$$\tilde{\mu}_i^{\mathrm{exp}} - A = \mu_i^{\mathrm{exp}} - \mu - \frac{1}{2}\left(\overline{\mu}^{\mathrm{imp}} + \overline{\mu}^{\mathrm{exp}} - \mu\right) = \mu_i^{\mathrm{exp}} - \frac{1}{2}\left(\overline{\mu}^{\mathrm{imp}} + \overline{\mu}^{\mathrm{exp}} + \mu\right) \tag{E.7}$$

It is easily seen that if, alternatively, the intercept term μ is absorbed by adding it to all import bias coefficients then we get the same result, based on $\tilde{\mu}_j^{\mathrm{imp}} \equiv \mu_j^{\mathrm{imp}} + \mu$.

It is also easily seen that any other initial choice of coefficients with the same right hand side of regression equation (E.1) will produce the same result. For instance if the import coeffcients have another base reporter then all μ_j^{imp} are, say, δ larger and μ is δ smaller. Then (E.6) becomes:

$$\left(\mu_j^{\text{imp}} + \delta\right) - \frac{1}{2}\left(\left(\overline{\mu}^{\text{imp}} + \delta\right) + \overline{\mu}^{\text{exp}} - (\mu - \delta)\right) = \mu_j^{\text{imp}} - \frac{1}{2}\left(\overline{\mu}^{\text{imp}} + \overline{\mu}^{\text{exp}} - \mu\right) \tag{E.8}$$

which is the same as (E.6). Equation (E.7) also stays the same:

$$\mu_i^{\text{exp}} - \frac{1}{2}\left(\left(\overline{\mu}^{\text{imp}} + \delta\right) + \overline{\mu}^{\text{exp}} + (\mu - \delta)\right) = \mu_i^{\text{exp}} - \frac{1}{2}\left(\overline{\mu}^{\text{imp}} + \overline{\mu}^{\text{exp}} + \mu\right) \tag{E.9}$$

With a change of export base reporter all μ_i^{exp} are, say, δ larger and μ is also δ larger. This also has no effect.

Appendix F GTAP

The GTAP organisation uses a model-free method of analysing reporting errors in the international trade of goods. The description of the method is taken from Gehlhar (1996), pp.22-23. This method is (implicitly) based on the assumption of unbiased reporting, producing sign-free reliability indicators. An exporting reliability indicator for country i is computed as follows:

$$\frac{\sum_{j \neq i} \; a_{ij} \; \overline{Y}_{ij}^{\text{exp}}}{\sum_{j \neq i} \; \overline{Y}_{ij}^{\text{exp}}} \tag{F.1}$$

where the $\overline{Y}$ (with the dash) are *levels*, not logs. And a similar equation for the imports.

The accuracy switch a_{ij} is defined for both exporters and importers as:

$$a_{ij} = \begin{cases} 1 & \text{if} \quad |\Delta\overline{Y}_{ij}|/\overline{Y}_{ij}^{\text{imp}} \;\leq\; 20\% \\ 0 & \text{otherwise} \end{cases} \tag{F.2}$$

For small discrepancies the rule in equation (F.2) is approximately equal to $|\Delta Y_{ij}| \leq 20\%$. We have omitted the t subscript here. For each bilateral trade flow, say from i to j, the reported value of the most reliable reporter is accepted as the true value, based on a comparison of the reliabilities as computed above. The non-simultaneous nature of this method is mitigated somewhat as follows: “Each reporter is given an opportunity to disregard the value reported by its worst partner.” See Gehlhar (1996), p.22. There is no similar outlier correction for the flattering value reported by its most accurate partner.

It is interesting to compare this method with the results of the variance model in table 5.2 above. Note that the variance model is also sign-free. As noted in section 5.1 above, only three out of our 12 discrepancies are below the 20% relative discrepancy as defined in equation (F.2); see table F.1. We ignore the “worst partner” correction. Italy does not occur in table F.1 and hence it has a zero GTAP reliability for export reporting and for import reporting. This agrees with the results in table 5.2, as does the good result for France and the UK. The result for Germany differs: here it is a bad export reporter, and in table 5.2 it is a bad import reporter.

Table F.1 Discrepancies below 20% in table 5.1

exporter	importer	$\lvert\Delta\rvert$ / import
		%
France	UK	15
UK	France	19
UK	Germany	18

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