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Author(s)	:	Sander Muns
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Variable Capital Utilization

The model outlined in Elbourne et al. (2009) is extended with a feature capturing a varying rate of capital utilization. The utilization rate can be chosen instantaneously by the capital owners. A higher utilization rate induces capital owners to receive a higher rental fee. However, they incur a utilization cost which is increasing and convex in the utilization rate. A higher utilization rate makes capital users more productive at the expense of the higher rental fee. The utilization feature results in less volatility in rental fees as well as less volatility in the real interest rate.

Contents

1 Introduction

The model described in Elbourne et al. (2009) tacitly assumes complete utilization of the two production factors labour and capital. Capital is assumed to be fully utilized each time period. Moreover, each worker works the number of hours he chooses to supply, which means that involuntary unemployment is nonexistent. Although households are allowed to choose the number of hours they work, their effort or effective output during each working hour is constant.

However, a varying rate of capacity utilization for production factors is supported by the empirical evidence (see King and Rebelo (1999)). During an economic boom, firms can increase output without hiring new workers. Such behaviour can be explained if firms face fixed costs in hiring and firing. Consequently, workers probably increase hourly effort during economic booms. This causes the effective level of labour supply to vary over the business cycle.

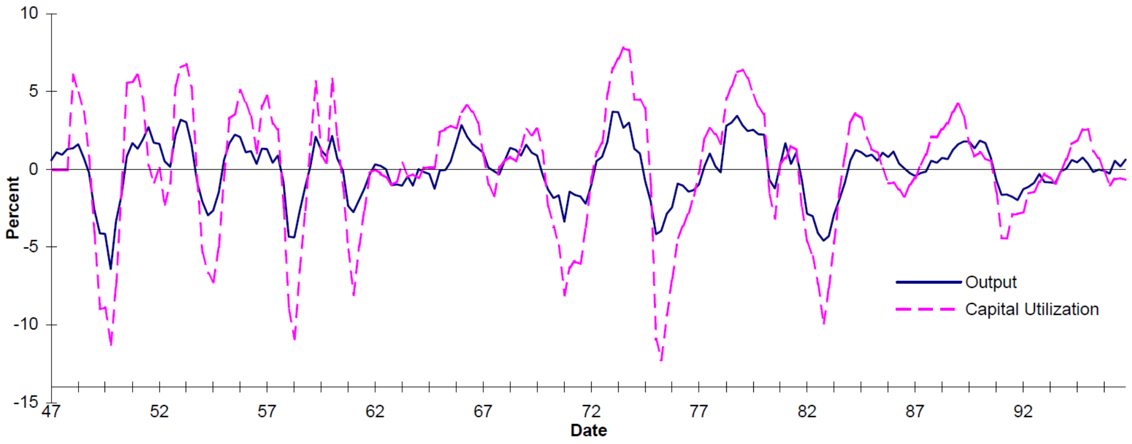
An example of observed variability of capacity utilization in the capital market can be seen during recessions when car sales drop. It is very unlikely that automobile manufacturers respond immediately by selling surplus equipment. As Bresnahan and Ramey (1994) document, they prefer to shut down manufacturing plants temporarily, thereby enabling an easy restart of production when car sales rise again. The level of serviced capital is temporarily decreased while the level of installed capital remains constant.

This memo will focus on capacity utilization in capital. Varying rates of labour effort and labour utilization is beyond the scope of the present study. The next section discusses some of the features of capital utilization. These features are an indication of what we overlook when a varying rate of capital utilization is neglected. Section 3 discusses different methods of modelling variable capital utilization. It turns out that the standard method is not supported by the available empirical evidence. The implementation of capital utilization for the benchmark model described in Elbourne et al. (2009) is outlined in section 4. Some exercises are provided in section 5 to illustrate the consequences of a varying rate of capital utilization. Conclusions are drawn in section 6.

2 Features of capital utilization

A convenient proxy for capital utilization is electricity utilization. A higher capital utilization rate means that capital goods such as machinery and computers are used more intensively, resulting in higher electricity utilization. Figure 2.1 is taken from King and Rebelo (1999).[1]

Figure 2.1 Capital utilization and output.



Capacity utilization is clearly a cyclical phenomenon: it rises during economic booms and declines during recessions. The figure indicates that capital utilization is more volatile than output.

In order to accommodate possible future shocks, firms install slightly more capital than the demand for capital services necessitates. This leads to a gap between serviced capital and installed capital. As a result, firms are able to instantaneously increase the level of serviced capital in the event of a positive technology shock. The level of installed capital on the other hand is fixed in the short run. Therefore, the utilization rate of capital increases when a positive shock occurs.

Empirical evidence tells us that monetary policy shocks only affect inflation with a lag (see Christiano et al. (1999)). Micro founded theoretical models had a hard time producing such responses without a varying rate of capital utilization. The model responses to a monetary policy shock in Christiano et al. (2005) improve upon generating inflation inertia and output persistence if a variable rate of capital utilization is included.

The required magnitude of a technology shock to replicate a given shock to output decreases if the utilization rate of capital is allowed to vary. A positive technology shock induces less

[1] The measure of capital utilization is subject to substantial measurement error (see Shapiro (1989)).

investment when firms have some idle capital, because some of the needed additional capital is obtained by servicing idle capital. The smaller increase in capital demand indicates that a smaller increase in the real interest rate is required in order to clear the capital market.

A variable utilization rate results in differences between productivity growth and technology growth, which causes measurement errors in productivity growth if the varying utilization rate is neglected (see Basu et al. (2006)). Estimated productivity growth exceeds technological growth when technology shocks induce higher utilization rates. As a result, too much variance of the productivity growth is attributed to the variance of technology when the utilization rate is incorrectly assumed to be constant.

The well-known pro-cyclical behaviour of productivity (see e.g. Kydland and Prescott (1990)) may well be the result of variable capacity utilization if productivity is measured by the average output of a particular production factor. Figure 2.1 shows that factors are worked harder during booms; failure to take a varying rate of capital utilization into account is a source of measurement error. Consequently, the observed variability of productivity could be caused, at least in part, by failure to control for a varying utilization rate.

We also know that productivity leads the business cycle, whilst employment lags the business cycle (see e.g. Kydland and Prescott (1990)). Bils and Cho (1994) stipulate that variable capacity utilization allows firms to adjust the utilization rate of the installed capital stock instantaneously in response to a technology shock. As a result, the actual productivity per installed unit of capital leads the cycle.

Greenwood et al. (1988) argue that a varying rate of capital utilization provides the solution to a problem discussed by Barro and King (1984). This problem occurs when an improved technology for newly installed capital initiates new investments. The increase in capital demand induces a higher real interest rate to clear the capital market. If the wealth effect dominates the income effect, households reduce consumption to account for the lower level of discounted wealth. As a result of the substitution effect, households tend to increase labour supply. Since the level of capital is fixed in the short-run, an increase in contemporaneous labour supply results in a higher output, but a lower labour productivity. Following this logic, consumption and labour productivity move counter-cyclically. Implementing a varying rate of capital utilization mitigates the effects of the shock on investment demand, and, hence, the real interest rate. Moreover the level of serviced capital can be adjusted instantaneously. As a result, consumption and labour productivity will move with the cycle rather than counter-cyclically.

Neiss and Pappa (2002) show that real effects of monetary shocks can be generated at relatively low degrees of nominal rigidity when a varying utilization rate is included. A variable capital utilization reduces investment volatility without the need to introduce capital adjustment costs. Variable labour utilization generates persistence because real wages become less sensitive to unanticipated monetary shocks.

In a two-country model with variable capital utilization, Baxter and Farr (2005) show that variable capital utilization significantly reduces the required size of productivity shocks needed to replicate observed output volatility by 20-40%. It has no effect on the required persistence of the shock.[2]

Overall, incorporating a varying rate of capital utilization into macroeconomic models should improve the models' ability to reproduce the correct magnitudes in the responses of investment, the interest rate, the inflation rate and output to shocks. A varying rate of capital utilization will also allow a model to obtain the required timing of a productivity change and to obtain the cyclical properties discussed above. Despite these potential advantages, Smets and Wouters (2007) point out that in the presence of wage stickiness, variable capital utilization is less important than investment adjustment costs.[3]

[2] Baxter and Farr (2005) do not model a varying rate of labour utilization. In an earlier version of the paper, Baxter and Farr (2001) find that variable labour utilization does not reduce the required size or persistence of productivity shocks.

[3] Cooley et al. (1995) also find that a varying rate of capital utilization has a negligible impact on the model in terms of business cycle properties.

3 Modelling variable capital utilization

In order to conceptualise utilization, section 3.1 introduces a formal definition. However, there is no consensus on how to capture the friction for capital utilization in macroeconomic models. Section 3.2 involves an overview of the standard approach, where a higher utilization rate results in a penalty by means of a higher depreciation rate. This is the so-called depreciation-through-use assumption. In section 3.3, alternative approaches are discussed that set the depreciation rate for capital at a constant rate and impose the utilization cost in a different way.

3.1 A definition for utilization

Let K and K_s represent the levels of installed capital and serviced capital, respectively. The utilization rate of capital for period t is given by u_t. The corresponding serviced level of capital K_t^s is given by

$$K_t^s = \frac{u_t}{u_{ss}} K_t = U_t K_t$$

In a steady state $u_t = u_{ss}$ holds, which implies that the level of serviced capital equals the level of installed capital. In fact for any arbitrary positive value for u_{ss}, the capacity level of installed capital is set at K_t/u_{ss}. A unit of capital is assumed to be utilized at u_t of this capacity level.

It is useful to keep in mind that capacity is not a strictly defined concept. For instance, at what number of hours is a grocery store servicing at a utilization level equal to 100%? One can argue that 40 hours are in a standard workweek. Alternatively, a store is allowed to keep its doors open during a larger number of hours. As a consequence, utilization is also not a strictly defined concept. Measuring the relative utilization is an easier task. The ratio u_t/u_{ss} is simply a normalized measure for utilization, being equal to unity in the steady state. Imposing $U_{ss} = 1$ for the normalized utilization level $U_t = u_t/u_{ss}$ is not as restrictive as it might appear.

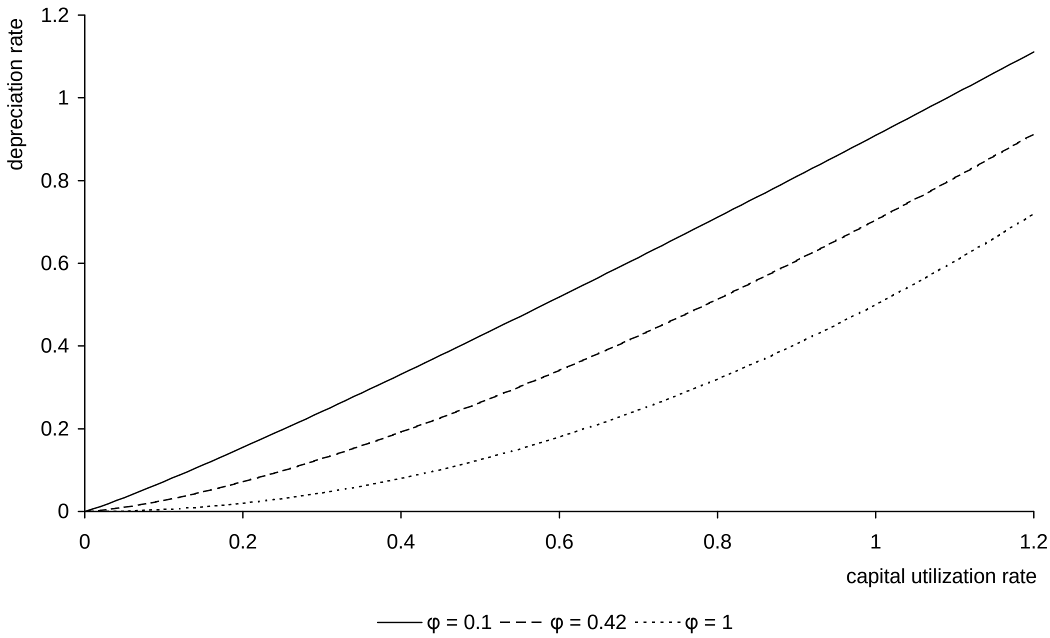
3.2 Depreciation-through-use models

Greenwood et al. (1988) show that the business cycle properties of a model improve when a variable capital utilization rate is included. The level of serviced capital is modelled as a jump variable: $K_t^s = U_t K_t$. Capital owners do not distinguish between existing capital and newly installed capital. Of course, some force must be at work to stop producers from setting the utilization rate U_t at infinity. As is the standard approach in the literature, the depreciation rate increases in the utilization rate. The functional form in Greenwood et al. (1988) is given by the iso-elastic function

$$\delta_t = \frac{U_t^{1+\phi}}{1+\phi}. \tag{3.1}$$

The value $\phi = 0.42$ is chosen because it implies a yearly depreciation rate of 0.1 in a deterministic steady state for the model.[4] Figure 3.1 illustrates the depreciation function graphically for three different elasticities.

Figure 3.1 The depreciation rate as a function of the capital utilization rate.



The labour utilization rate is assumed to be constant in the model, although Greenwood et al. (1988) note that an alternative interpretation of the capital utilization rate is that it reflects the portion of labour involved in production activities, with the remainder being involved in maintenance activities.

Burnside and Eichenbaum (1996) consider the impact of shocks on labour productivity by modelling a varying utilization rate for both production factors labour and capital. Labour utilization is defined as the product of the level of effort, E_t, (chosen by the worker) and the fixed number of shifts, H, per worker. Capital utilization is as in Greenwood et al. (1988):

$$L_t^s = E_t H L_t$$
$$K_t^s = U_t K_t$$

[4] Indeed, this corresponds with a steady state utilization rate of 0.253. It is argued in section 3.1 that the steady state utilization rate can be set at an arbitrary positive level.

The effort level is chosen by the worker and enters the utility function. Observe that the level of employment, L_t, is not an adequate measure for unemployment if further assumptions regarding the dispersion in employment among households are absent. The two different utilization rates are linked by the shift length:

$$U_t = n_t H$$

where n_t is the number of shifts chosen by the firm. Note that the service capital-labour ratio K_t^s / L_t^s is allowed to vary over time. They will nevertheless move together as capital and labour are complements in production. The depreciation function adds a scale parameter to (3.1):

$$\delta_t = \delta_1 U_t^{1+\phi} \tag{3.2}$$

Estimating this functional form on U.S. data reveals $\delta_1 = 0.0195$ and $\phi = 0.56$. The variable capital utilization rate substantially magnifies and propagates the impact of shocks.

A monetary New-Keynesian model is estimated in Neiss and Pappa (2002). It is shown that a variable capital utilization rate reduces the need to incorporate capital adjustment costs. The setup for the variable utilization rates and the depreciation parameters are taken from Burnside and Eichenbaum (1996). However, the number of labour hours per worker, H_t, is allowed to vary instantaneously, and the number of workers, L_t, is predetermined.

Basu and Kimball (1997) investigate why productivity is procyclical. The varying rate of utilization is modelled as in Burnside and Eichenbaum (1996), but H_t denotes the time-varying number of hours per week for each worker:

$$L_t^s = E_t H_t L_t \tag{3.3}$$

$$K_t^s = U_t K_t \tag{3.4}$$

The non parametric form for the depreciation rate slightly differs from the functional form in Burnside and Eichenbaum (1996):

$$\delta_t = \delta_0 + \delta_1 U_t^{1+\phi}$$

Setting $\delta_0 = 0$ as in (3.2) rules out "rust and dust," i.e. Burnside and Eichenbaum (1996) do not allow for a nonzero depreciation rate if capital is unutilized ($U_t = 0$).[5] First, this is undesirable from an empirical point of view. A firm which keeps old-fashioned machinery idle for a large number of years is unlikely to survive in a competitive world. As the economic value has reduced over the idle years, the old-fashioned machinery should be depreciated. Second, by

[5] Although utilization rates are arbitrarily scalable by positive numbers, a utilization rate equal to zero always means that no capital is serviced.

using a real interest rate of 2.5% and a steady state depreciation rate equal to 13%, the rust-and-dust share of depreciation amounts to about 40% in Basu and Kimball (1997). Including rust-and-dust means that the lines in figure 3.1 do not necessarily cross the origin.

A point estimate for the elasticity parameter ϕ equals unity for the rust-and-dust functional form. Basu and Kimball (1997) find a rather large 95% confidence interval for ϕ: $[-0.2, 2]$. Negative values can be excluded by economic reasoning. The depreciation rate would be a concave function in the utilization rate, implying that firms would tend to set the utilization rate at an infinite level. In general, a smaller elasticity ϕ results in a steeper curve for utilization rates below unity (see figure 3.1). Therefore, the relatively small elasticity ($\phi = 0.56$) in Burnside and Eichenbaum (1996) partly compensates for the missing of rust-and-dust.

It is shown that the utilization rate is procyclical. It explains 40-60% of the cyclicality of the Solow residual and it eliminates the evidence for increasing returns to scale.

A two country business cycle model is considered in Baxter and Farr (2005). A common problem is that the required cross-country correlation of productivity shocks to replicate the correlation in output needs to be implausibly large. The introduction of variable capital utilization overcomes the problem. The functional form for the depreciation rate is taken from Basu and Kimball (1997). The model is considered at the values $\phi = 1$ (from Basu and Kimball (1997)), $\phi = 0.10$ (from King and Rebelo (1999)) and $\phi = 0.05$. Overall, the empirical results for $\phi = 0.10$ match the data slightly better than the other two cases. Despite only a modest correlation of the productivity shocks ($\rho = 0.26$), a positive co-movement across countries is now observed for labour inputs as well as for investments.

3.3 Alternative approaches

This section discusses alternative approaches. None of the models relies on the depreciation-through-use assumption. The utilization cost is modelled by (*i*) a utility function where a higher level of capital utilization directly results in a smaller level of utility, or (*ii*) firms are heterogeneous in order to explain different utilization levels, or (*iii*) a higher utilization rate results in a penalty by means of buying a consumption good.

Bils and Cho (1994) stipulate that variable capacity utilization improves the model by allowing it to mimic the stylised fact that productivity leads the business cycle, and employment lags the business cycle (see for example Kydland and Prescott (1990)). Turning to the first empirical observation, labour utilization is modelled as in Basu and Kimball (1997):

$$L_t^s = E_t H_t L_t$$

The four variables are simultaneously determined. The right-hand side variables give disutility to the households. Based on data from textile looms and electricity usage as a proxy for utilization, the number of hours per worker, H_t, affects the utilization rate of capital, but the employment level, L_t, does not. Bils and Cho (1994) remain silent about the empirical impact of effort, E_t, on utilized capital. In the model, it is assumed that if a worker works more hours per week or puts more effort in his work, the utilization rate of the capital he operates also increases:

$$K_t^s = E_t H_t K_t$$

The level of installed capital, K_t, is predetermined. The depreciation rate for capital, δ, is constant (0.025 per quarter). Quadratic adjustment costs are implemented for the (installed) capital-worker ratio (K_t/L_t) in order to generate a friction in the total number of hours worked. This mechanism should be able to replicate the second empirical observation that employment lags the business cycle.

By including the utilization feature the same amount of output variability is generated. Nevertheless, the generated business cycles differ dramatically from actual cycles. Total labour hours are far more procyclical, labour productivity is far more acyclical, and the correlation between labour hours and labour productivity is much stronger in the generated cycles compared with observed cycles.

A model imposing idiosyncratic technological shocks to heterogeneous production firms is in Cooley et al. (1995). The idiosyncratic shocks are ex-ante uniformly distributed. Depending on the actually observed shock, the firm decides whether to operate (and pay a fixed operating cost) or not. This means that each plant is either active or inactive. An inactive firm does not demand factor inputs. In such a way, an inactive stock of capital and labour arises. The values for active and inactive stocks provide measures for the capital utilization rate and the unemployment rate, both along the extensive margin. The model is calibrated for an average rate of capacity utilization equal to 0.82, as measured by the Federal Reserve Board of Governors. Observe that this number is based on a utilization rate along the intensive margin. The quarterly depreciation rate is fixed at 0.0188 such that it represents the investment share in output divided by the capital-output ratio. It appears that the business cycle properties of the model are hardly affected by including a varying rate of capacity utilization.

Fagnart et al. (1999) discuss a model which incorporates heterogeneous monopolistic firms. Each intermediate firm faces idiosyncratic technology shocks. As in Cooley et al. (1995), the distribution of the idiosyncratic shocks is ex-ante the same. Once uncertainty is resolved, firms differ in production and employment decisions. The differences between the contemporaneously chosen level of serviced capital and the predetermined level of installed capital imply a varying rate of capital utilization. The quarterly depreciation rate (0.0188) and the average rate of capacity utilization (0.82) are adopted from Cooley et al. (1995). In contrast, capital utilization is modelled along the intensive margin instead of the extensive margin. Overall, the results differ from Cooley et al. (1995), but are qualitatively similar to Burnside and Eichenbaum (1996).

An imperfect competition model is in Nakajima (2005). The purpose is to show that there is a plausible dynamic model in which preference shocks alone can replicate the actual U.S. business cycles. On aggregate, firms completely rent the predetermined capital stock. During a fraction $U_t(j)$ of period t the capital rent by firm j is active. The firm is inactive during the remaining time in period t. Note that utilization varies here along the extensive margin. The steady state capital utilization rate is set at 0.82 which is taken from Cooley et al. (1995). It turns out that the model is able to replicate the U.S. business cycles rather well.

In a seminal article, Christiano et al. (2005) seek to understand the observed inertial behavior of inflation and persistence in aggregate output. Among other frictions, a variable utilization rate of capital enables the model to replicate the inertia. The quarterly depreciation rate is constant at $\delta = 0.025$. Unlike the depreciation-through-use literature, the cost of utilizing capital, $a(U_t)$, is a direct cost in terms of consumption goods. The resulting aggregate resource constraint is therefore given by[6]

$$C_t + I_t + a(U_t)K_t = Y_t,$$

where $I_t = K_{t+1} - (1-\delta)K_t$. Hence,

$$C_t + (K_{t+1} - K_t) + (\delta + a(U_t))K_t = Y_t$$

The aggregate resource constraint is in the spirit of the depreciation-through-use literature given by:

$$C_t + I_t = Y_t,$$

where $I_t = K_{t+1} - (1-\delta(U_t))K_t$. Aggregate demand has not changed from a conceptual point of view if $\delta(U_t) = \delta + a(U_t)$:

$$C_t + (K_{t+1} - K_t) + \delta(U_t)K_t = Y_t$$

A depreciation-through-use assumption implies that firms are faced with a higher demand for capital goods for higher levels of utilization rates. Instead, the utilization costs in Christiano et al. (2005) are expressed in consumption goods since the cost $a(U_t)K_t$ represents a number of consumption goods. If the cost is modelled in terms of capital goods, the following

[6] Note that the first paragraph in Christiano et al. (2005, p. 13) states that (in adjusted notation): "The increasing, convex function $a(U_t)K_t$ denotes the cost, in units of consumption goods, of setting the utilization rate to U_t." Therefore, the aggregate resource constraint in Christiano et al. (2005, p. 14) is incorrectly given by $C_t + I_t + a(U_t) \leq Y_t$,

counterintuitive response could appear to an expansionary policy shock (see Christiano et al. (2005, p. 27)). An expansionary policy shock stimulates the demand for serviced capital, because households tend to expand consumption. As a result, the serviced capital market clears at a higher rental rate of capital. The marginal product of utilization equals the rental rate of capital. Therefore, the shock pushes the marginal product of utilization upwards. Further, investment increases because borrowing becomes cheaper. The marginal cost of installed capital increases due to the resulting investment adjustment costs. Hence, the marginal cost of utilization is higher if this cost is expressed in terms of capital goods instead of consumption goods. The increase in the marginal cost of utilization could offset the increase in the marginal benefit of utilization, if investment adjustment costs are sufficiently high. As a result, an expansionary shock could lead to a decrease in the utilization rate of capital. Precisely defined conditions for this counterfactual observation are not given. Note that the main difference with the depreciation-through-use models is that the increase in the utilization cost is smaller if the corresponding costs are expressed in consumption goods instead of capital goods.

The capital utilization rate U_t is normalized to unity in the steady state. The cost function, $a(U_t)$, is zero in steady state and has a relatively small elasticity parameter, $\phi = \frac{a''(1)}{a'(1)} = 0.01$. It suffices for a linearized model to define ϕ: equating the marginal product of utilization to the marginal cost of utilization gives $r_t^k = a'(U_t)$, where r_t^k is the rental rate of return for period t. The first order Taylor expansion around the steady state $U_{ss} = 1$ and $r_{ss}^k = r^k$ gives:

$$r^k + \left(r_t^k - r^k\right) = a'(1) + a''(1)\,(U_t - 1)$$

Rewriting and using the steady state relation $r^k = a'(1)$ gives

$$\frac{r_t^k - r^k}{r^k} = \frac{a''(1)}{a'(1)} \left(\frac{U_t - 1}{1}\right)$$

Letting $\hat{x}_t$ denote relative deviations from the steady state in period t:

$$\hat{r}_t^k = \varphi \hat{U}_t$$

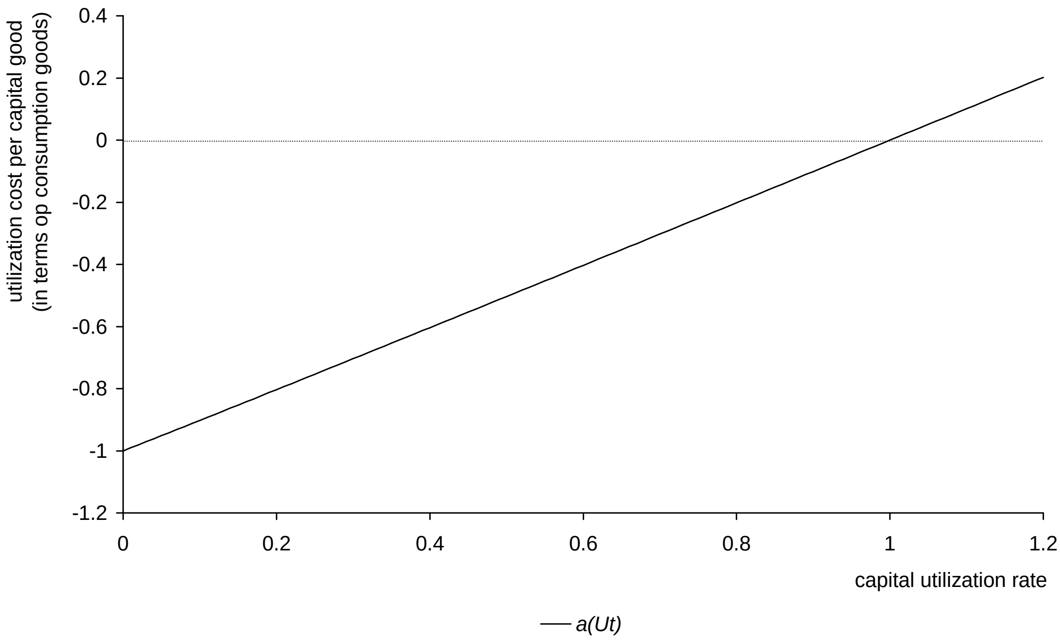
For example, the cost function

$$a(U_t) = cU_t^{1.01} - c$$

satisfies the imposed steady state conditions $a(1) = 0$ and $\frac{a''(1)}{a'(1)} = 0.01$ for $c > 0$. Figure 3.2 depicts this slightly convex function for $c = 1$. The small elasticity results in a stable marginal cost $a'(U_t)$. This implies that a large change in U_t is required to let the marginal cost of utilization match a given change in the marginal product of utilized capital.

The model responses to a monetary policy shock improve upon generating inflation inertia and output persistence if a variable rate of capital utilization is included.

Figure 3.2 utilization costs as a function of the utilization rate ($c = 1$).



Another linearized large scale DSGE model with a variable utilization rate is in Smets and Wouters (2007). The introduction of a large number of frictions raises the question whether each of those frictions are really necessary to model the data. Bayesian estimation gives $\phi = 1.18$, which fits in the 95% confidence interval $[0,2]$ from Basu and Kimball (1997). It turns out that the introduction of variable capacity utilization is less important in the presence of wage stickiness.

Roeger et al. (2008) describe an endogenous growth DSGE model for analysing the costs and benefits of fiscal reforms. Two frictions are imposed in the capital market. The adjustment cost in investments depends in a quadratic manner on the chosen level of investment as well as the corresponding change in investment. Further, a capital utilization cost is imposed on the households:

$$a(U_t) = a_1(U_t - U_{ss,t}) + a_2(U_t - U_{ss,t})^2$$

As in Christiano et al. (2005), this cost represents a cost per unit of capital which is expressed in terms of consumption units. The variable $U_{ss,t}$ denotes a moving average steady state utilization rate:

$$U_{ss,t} = (1 - \rho_U)U_{ss,t-1} + \rho_U U_t$$

However, estimates for ρ_U, a_1, and a_2 are unreported. Further, the impact of the utilization feature on the model responses is not investigated.

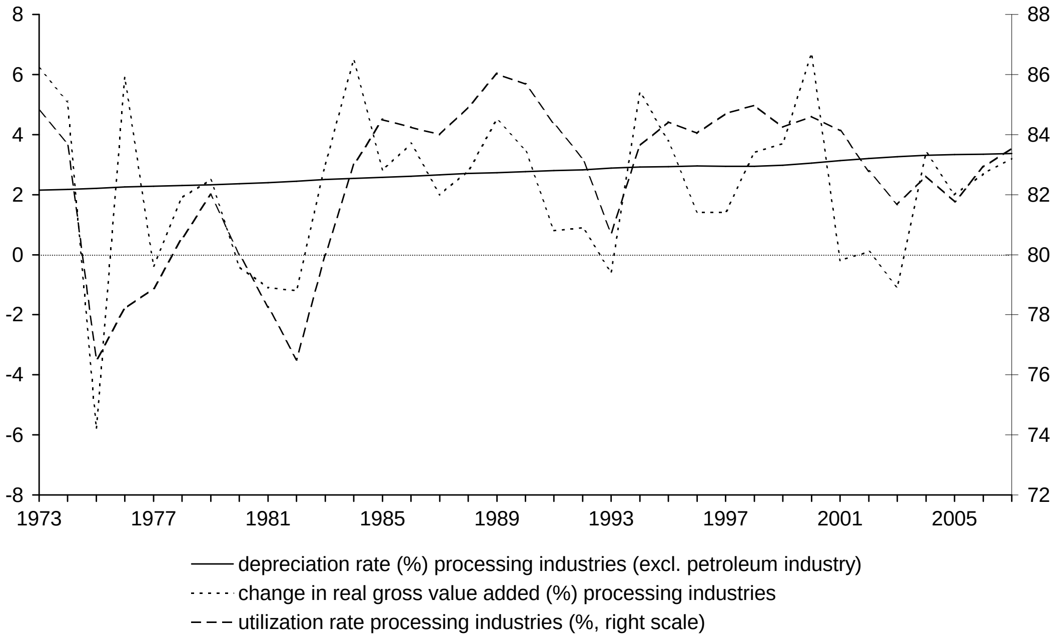
4 Implementation in the model

As noted before, the implementation of the utilization feature is restricted to capital utilization. Section 4.1 motivates why the depreciation-through-use assumption is not adopted. Section 4.2 provides a detailed view of the required changes in the benchmark model, without specifying the utilization cost function in an explicit way. The cost function is described in an explicit way in section 4.3 by imposing some explicit conditions.

4.1 The depreciation rate

The majority of the specialized literature on variable capital utilization imposes a utilization cost in terms of capital goods via the depreciation rate. Since investment adjustment costs are present in the model, it might be sensible to pay attention to the criticism in Christiano et al. (2005) of a utilization cost in terms of capital goods. Moreover, we show that there is some empirical controversy whether depreciation rates are increasing in the utilization rate. As an illustration, figure 4.1 shows the depreciation rate, the capital utilization rate and the change in real gross added value over the period 1973-2007 for Dutch processing industries.[7]

Figure 4.1 Depreciation rate, capital utilization rate and change in gross value added



[7] Source capital utilization rate: Statistics Netherlands, Business survey manufacturing industry
Source depreciation rate and gross value added: Statistics Netherlands, National accounts of the Netherlands 2007

Notice that the utilization rate is on average near the estimate of 0.82 from the Federal Reserve Board of Governors, but is not as cyclical as in figure 2.1. An explanation involves measurement errors in capital utilization. Whilst the depreciation rate is not constant (it slowly increases over time), it does not appear to match the cycle.

Capital depreciation as measured in our data is likely to differ from the actual decrease in the economic value of capital. The depreciation rate is constructed by Statistics Netherlands as a weighted average of the constant depreciation rates of the different types of capital goods. This means that the depreciation rate only varies as a result of changes in the composition of the aggregate capital stock. For instance, an increasing share of computers in the capital stock increases the depreciation rate of capital since computers have a relatively short lifetime.

For short to medium term economic analysis, the slow but steady rise in the depreciation rate used by Statistics Netherlands is irrelevant. Therefore, we keep things simple by assuming a constant depreciation rate of capital, whilst keeping in mind the potential shortcomings.[8]

4.2 Changes in the benchmark model

The implementation of a varying rate of capital necessitates some changes in the benchmark model for the capital owners (the actuarial firms) as well as the capital users (the production firms). As before, the benchmark model is the model outlined in Elbourne et al. (2009). Here, we explain the differences with the benchmark model.

Definition

In section 3.1 it was argued that the utilization rate can be normalized to $U_t = u_t/u_{ss}$. The level of serviced capital in period t is defined as a fraction of the predetermined level of installed capital:

$$K_t^s = U_t K_{t-1} \tag{4.1}$$

where the timing convention from DYNARE is adopted here for the predetermined K_t. It is crucial to note that the level of serviced capital can be instantaneously adjusted by changing the (normalized) utilization rate, U_t. The utilization of capital varies only along the intensive margin.

[8] In Appendix A the statistical relation between the (mismeasured) depreciation rate and the utilization rate is extensively discussed.

Cost Function

As standard in the literature, the capital owners choose the utilization rate.[9] The cost function, $a(U_t)$, specifies the utilization cost per unit of installed capital in period t, K_{t-1}. This standard specification defines the utilization cost as a linear function in the scale of the capital owner. When the installed stock of capital doubles, the utilization cost also doubles, provided that the utilization rate remains unchanged.

Actuarial firms

Actuarial firms own the capital. They benefit from renting the serviced capital to the capital users. Profits decrease by the cost of the chosen utilization rate. The Lagrangian reads:

$$\mathscr{L}\begin{pmatrix} B_t, I_t, K_t, \\ U_t, Z_t, Z_t^f \end{pmatrix} = \sum_{t=s}^{\infty} Q_s \begin{pmatrix} N_s - (1+r_{s-1})N_{s-1} - B_s + (1+r_{s-1}^g)B_{s-1} \\ -q_s Z_s + (q_s + div_s)Z_{s-1} - q\,f_s Z\,f_s + (q\,f_s + div\,f_s)Z\,f_{s-1} \\ +r_s^k U_s K_{s-1} - \left[1 + \Psi\left(\frac{I_s}{K_s} - 1 - \delta\right)\right] I_s \\ -a(U_s)K_{s-1} + \Lambda_s\left[I_s + (1-\delta)K_{s-1} - K_s\right] \end{pmatrix}$$

where

$$Q_t = \frac{1}{(1+r_0)\dots(1+r_{t-1})}$$

denotes the discount rate, $U_t K_{t-1}$ represents the time t level of serviced capital, K_t^s, and expectations are omitted for brevity.

In comparison with the benchmark model the first order conditions with respect to K_t changes, and a new first order equation for the utilization variable, U_t, is introduced. Further the profit conditions now include U_t:

$$\Lambda_t = \frac{1}{1+r_t}\left[r_{t+1}^k U_{t+1} + \left(\frac{I_{t+1}}{K_t}\right)^2 \Psi'\left(\frac{I_{t+1}}{K_t} - \delta\right) - a(U_{t+1}) + \Lambda_{t+1}(1-\delta)\right] \tag{4.2}$$

$$r_t^k = a'(U_t) \tag{4.3}$$

$$\begin{aligned} div_t^{AI} &= N_t - (1+r_{t-1})N_{t-1} - B_t + (1+r_{t-1})B_{t-1} + div_t + div\,f_t \\ &\quad + r_t^k U_t K_{t-1} - \left[1 + \Psi\left(\frac{I_t}{K_t} - 1 - \delta\right)\right] I_t - a(U_t)K_{t-1} \end{aligned} \tag{4.4}$$

$$\begin{aligned} 0 &= N_{t+1} - (1+r_t)N_t - B_{t+1} + (1+r_t)B_t + div_{t+1} + div\,f_{t+1} \\ &\quad + r_{t+1}^k U_{t+1} K_t - \left[1 + \Psi\left(I_{t+1}/K_t - \delta\right)\right] I_{t+1} - a(U_{t+1})K_t \end{aligned} \tag{4.5}$$

[9] In this case a higher rental rate induces capital owners to set the utilization rate at a higher level. It makes no difference if the capital users choose the utilization rate. Still, a higher utilization rate necessitates the capital owners to buy additional goods from the capital users. At the margin, the capital users set the utilization rate such that the rental rate of capital equals the marginal product of utilization. It turns out in equation (4.3) that the first order condition for the utilization rate remains the same. The marginal cost and the marginal product are only reversed from a conceptual point of view.

The preceding four equations replace the three equations (A.13)-(A.15) from the benchmark model. Appendix B outlines how (4.3) changes if the standard depreciation-through-use method is adopted.

Production firms

The capital is used by production firms. Their pricing problem remains unaffected, so the NKPC stays the same. Production firms produce by instantaneously renting serviced capital:

$$Y_t = A_t \, (K_t^s)^{\chi} \, L_t^{1-\chi} \tag{4.6}$$

This replaces equation (A.32) from the benchmark model.

Conditional on a given production level Y_t, firms minimize contemporaneous nominal costs by choosing the level of serviced capital, K_t^s, and the amount of labour, L_t. Still, firms take wages and rental costs as given which are denominated in P_t (CPI prices), whereas domestic production is denominated in $P_{H,t}$ (PPI prices). By defining $g(S_t) = P_t/P_{H,t}$, the following Lagrangian for the cost minimization problem is obtained:

$$\mathscr{L}(K_t^s, L_t) = g(S_t) w_t L_t + g(S_t) r_t^k K_{t-1}^s - \lambda_t \left[Y_t(K_t^s, L_t) - \overline{Y} \right]$$

where λ_t denotes the producer's real marginal cost of an additional unit of output in period t. A higher utilization cost, $a(U_t)$, induces a higher rental rate, r_t^k, by the first order condition (4.3) of the actuarial firm. This in turn raises the real marginal cost.

The predetermined capital stock, K_t, is replaced by the instantaneously determined, K_t^s. This means that the block of equations (A.33)-(A.35) from the benchmark model is replaced by the following equations:

$$g(S_t) w_t = \lambda_t \, (1-\chi) \, A_t \left(\frac{K_t^s}{L_t} \right)^{\chi} \tag{4.7}$$

$$g(S_t) r_t^k = \lambda_t \chi A_t \left(\frac{K_t^s}{L_t} \right)^{\chi - 1} \tag{4.8}$$

$$div_t = \frac{Y_t}{g(S_t)} - w_t L_t - r_t^k K_t^s - \frac{PCosts_t}{g(S_t)} \tag{4.9}$$

Here we are assuming that the associated utilization cost is not paid domestically. More specifically, the costs are denominated in the domestic consumption good (CPI prices) price as in Christiano et al. (2005) rather than in the domestic producer price (PPI prices). Instead of (A.39) from the benchmark model, domestic demand is now given by:

$$\Upsilon_t = C_t + I_t + G_t + \Psi \left(\frac{I_t}{K_{t-1}} - \delta \right) I_t + \xi \, (\Delta F B_t) + a(U_t) K_{t-1} \tag{4.10}$$

Eight equations from the benchmark model ((A.13)-(A.15), (A.32)-(A.35) and (A.39)) are replaced by the ten equations (4.1)-(4.10). In total, two extra variables (K_t^s and U_t) and two extra equations ((4.1) and (4.3)) are included.

4.3 The cost function

First some general properties are imposed on the utilization cost function. Then, the functional form for the cost function is chosen.

General properties for the cost function

It is natural that a higher utilization rate is associated with a higher cost for the capital owner, this means

$$a'(U_t) > 0 \qquad \text{for all } U_t \geq 0 \tag{4.11}$$

Capital markets are assumed to be competitive. Therefore, capital owners consider the marginal product of utilization, r_t^k, as given. The marginal cost of utilizing capital must be increasing in the utilization rate, otherwise they choose an infinite level of utilization. A sufficient condition is that utilization costs are convex in the utilization rate:

$$a''(U_t) \geq 0 \qquad \text{for all } U_t \geq 0 \tag{4.12}$$

Steady state properties for the cost function

Variables without time subscripts denote steady state values. The capital accumulation equation $K_t = I_t + (1-\delta)K_{t-1}$ implies $I = \delta K$. Hence, the first order condition for investment are in the steady state:

$$\Lambda = 1 + \Psi(0) + \Psi'(0)$$

Investment adjustment costs are minimal and zero in the steady state:

$$\Psi(0) = \Psi'(0) = 0$$

Hence, $\Lambda = 1$. Foreign investment firms still choose foreign bond holdings to maximize profits. Therefore, the steady state version of their first order conditions continues to hold (see Elbourne et al. (2009) equations (A.18) and (A.19)):

$$\Lambda f = 1 + \xi'(0) \tag{4.13}$$

$$\Lambda f = \frac{1}{1+r}\left(r^{fo} + \Lambda f\right) \tag{4.14}$$

The first equation defines the marginal cost of holding additional foreign bonds in the steady state. The latter equation equates this cost to the corresponding marginal revenue. Adjustment costs in foreign bond holdings are minimal and zero in the steady state:

$$\xi'(0) = 0$$

By combining the latter three equations:

$$r = r^{fo} \tag{4.15}$$

Then, by the first order condition for K_t in the steady state (equation (4.2)):

$$r^{fo} = r^k U - a(U) - \delta \tag{4.16}$$

From the first order condition for U_t (equation (4.3)) we have

$$r^k = a'(U) \tag{4.17}$$

Equations (4.16) and (4.17) result in the following steady state equation

$$a(U) = a'(U)U - r^{fo} - \delta \tag{4.18}$$

The latter equation cannot be considered as a differential equation. It is imposed in the steady state for any cost function $a(U_t)$, but not necessarily outside the steady state.

It is natural to let $a(U) = 0$ such that domestic demand (4.10) is unaffected in the steady state by the utilization component $a(U)$. Otherwise, steady state profits of the capital owners are affected by the steady state rate of capital utilization. For the benchmark model, it can be shown that the rental rate of capital is the risk free rate raised by a compensation for depreciation:

$$r^k = r + \delta$$

Using $a(U) = 0$, equations (4.15) and (4.16) show that the steady state value for r^k remains unchanged for $U = 1$. Notice that this condition is also imposed in Christiano et al. (2005). Hence, it is useful to normalize the utilization rate by applying the procedure outlined in section 3.1:

$$U_t = \frac{u_t}{u_{ss}}$$

This extra equation contains one extra variable, u_t, and an arbitrary positive constant u_{ss}.

From the previous discussion and equation (4.17) the following conditions are imposed on the function $a(U_t)$:

$$a(1) = 0 \tag{4.19}$$

$$a'(1) = r^k \tag{4.20}$$

A negative side effect for this setup is a negative demand for the consumption good ($a(U_t) < 0$) for below steady state utilization of capital ($U_t < 1 = U_{ss}$). This is not a conceptual problem as long as the total demand of the capital owners is positive. The magnitude of the negative demand components, $a(U_t)K_t$, must be smaller than the demand for investments and investment adjustment costs, $\left[1 + \Psi\left(\frac{I_t}{K_{t-1}} - \delta\right)\right] I_t$.

The functional form

Some literature base the utilization cost function on a quadratic functional form (e.g. Burriel et al. (2009)):

$$a(U_t) = c_0 + c_1 U_t + \frac{c_2}{2} U_t^2$$

Imposing equations (4.19) and (4.20) gives $c_0 = -r^k - c_1$ and $c_2 = r^k - c_1$. This results in

$$a'(U_t) = c_1 + (r^k - c_1)U_t$$
$$a''(U_t) = r^k - c_1$$

By (4.11) and (4.12) we obtain the constraint $0 < c_1 \leq r^k$. The elasticity of the adjustment cost in utilization varies in U_t for this particular setup:

$$\frac{U_t a''(U_t)}{a'(U_t)} = \frac{1}{1 + \frac{c_1}{U_t(r^k - c_1)}}$$

The utilization cost is more elastic for higher utilization rates because $0 < c_1 \leq r^k$. In order to stay away from a varying elasticity and stay closer to the majority of the literature, the frequently used iso-elastic functional form is preferred:

$$a(U_t) = c_0 + \frac{c_2}{1+\phi} U_t^{1+\phi} \tag{4.21}$$

Note that the elasticity is constant for this setup since

$$\frac{U_t a''(U_t)}{a'(U_t)} = \phi$$

for any U_t. Indeed, the iso-elastic setup is adopted from the depreciation-through-use literature. Other literature such as Christiano et al. (2005) and Smets and Wouters (2007) typically use a linearized model. As outlined in section 3.3, only the steady state elasticity for the adjustment cost $a'(U)$ with respect to the utilization rate is needed for such models. Equations (4.19) and (4.20) imply

$$c_0 = -\frac{r^k}{1+\phi}$$
$$c_2 = r^k$$

Substitution in (4.21) gives

$$a(U_t) = \frac{r^k(U_t^{1+\phi} - 1)}{1+\phi}$$

To make fair comparisons with the benchmark model, we set $r^k = 9.5\%$. Further, $u_{ss} = 0.82$ by the estimate in Cooley et al. (1995) which is in turn based on the estimate of the Federal Reserve

Board of Governors.[10] As in Basu and Kimball (1997), who in turn are cited by Baxter and Farr (2005) and Gertler et al. (2007), the baseline elasticity is $\phi = 1$:

$$a(u_t/0.82) = \frac{0.095((u_t/0.82)^2 - 1)}{2}$$

[10] As mentioned before, this is consistent with the data from the Questionnaire Statistics Netherlands.

5 Results

This section assesses the implementation of a varying rate of capital utilization in the benchmark model from Elbourne et al. (2009). The utilization feature changes the dynamics of the model, but not the steady state. As mentioned before, the absence of a utilization feature is equivalent with a utilization cost increasing to infinity when the utilization rate slightly differs from the steady state utilization rate. More formally, this corresponds to $a''(1) = r^k\phi \to \infty$. Indeed, the utilization rate varies more in response to shocks for a smaller positive elasticity, ϕ . Hence, the impact of the friction is larger for smaller $r^k\phi$. To highlight the impact of the utilization feature, a small value is chosen for the elasticity, but within the Basu and Kimball (1997) 95% confidence interval $[0,2]$. The comparison in Baxter and Farr (2005) revealed that the King and Rebelo (1999) estimate $\phi = 0.10$ provides a reasonable model. Therefore, the implemented utilization cost function is given by

$$a(u_t/0.82) = \frac{0.095((u_t/0.82)^{1.1} - 1)}{1.1}$$

The effect of a technology shock and a monetary shock are evaluated because the utilization literature abounds with such shocks. The consequences of a technology shock are discussed in section 5.1. Section 5.2 discusses a foreign nominal interest rate shock.

5.1 Technology shock

The level of technology temporarily increases by 2%. Agents are surprised in period one by this shock.[11] However, they anticipate that this shock lasts for 30 periods because they have perfect foresight. In fact, two permanent shocks are imposed: in period one an unanticipated shock and in period 30 an offsetting anticipated shock. The shock is temporary, so the steady state remains unaltered. First, the dynamics for the benchmark model are discussed. Subsequently, the implications of the utilization feature are explained.

The benchmark model

The unanticipated shock (period 1): The shock directly affects output and productivity. As a consequence, labour demand and capital demand rise, and to clear factor markets, both the real wage and the rental rate of capital increase (see figure 5.1). The real interest rate also increases by a no arbitrage condition for actuarial firms on savings and capital investments. Households

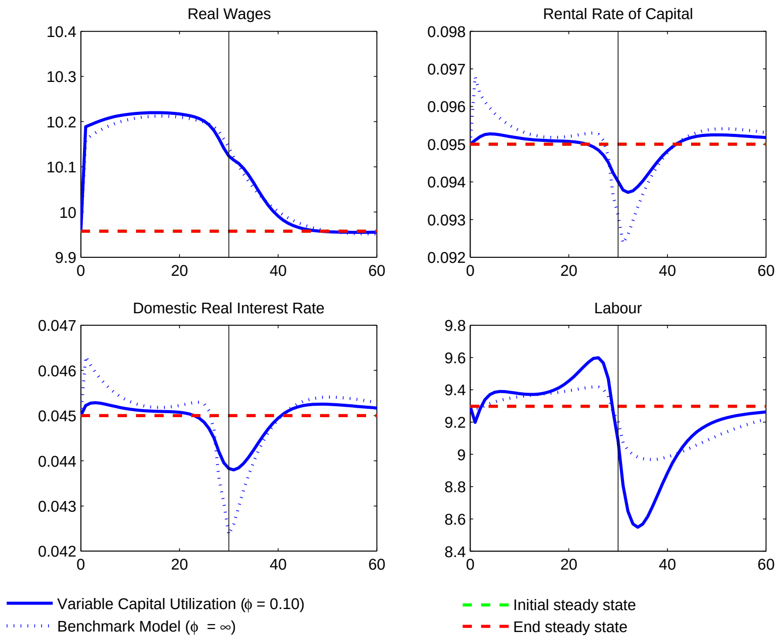
[11] This scenario is rather similar to simulation 2 in Elbourne et al. (2009). Here, we impose a temporary shock of 2% in a model which has an endogenous tax on labour, instead of a permanent 5% shock to a model which has an endogenous tax on consumption.

increase labour supply because they foresee that the increased wage is only temporary. The higher wage results in a higher labour income for the households. As a result, both consumption and savings increase (see figure 5.2).

The actuarial firms make an initial profit because of the unexpected rise in savings as well as the unexpected rise in the rental rate of capital. This profit increases households' wealth. The higher amount of savings induces the actuarial firms to reduce foreign borrowings and raise investments (see figure 5.3). The increase in capital supply mitigates the increase in the rental rate of capital. The higher output results in an increase in production firms' profits and a small downward price correction (see figure 5.4).

The anticipated shock (period 30): Overall, the processes reverse when the shock nears its end. Households start to enjoy the large amount of savings (figure 5.2). Labour supply temporarily decreases below the steady state level (figure 5.2). As a result, output temporarily declines below the steady state level (figure 5.3). The pronounced reduction of goods supply induces a very peaked response in inflation.[12]

Figure 5.1 Temporary increase in technology level



[12] The peak partly boils down to the fact that inflation is a *flow* variable for the stock variable of nominal prices.

Figure 5.2 Temporary increase in technology level (cont.)

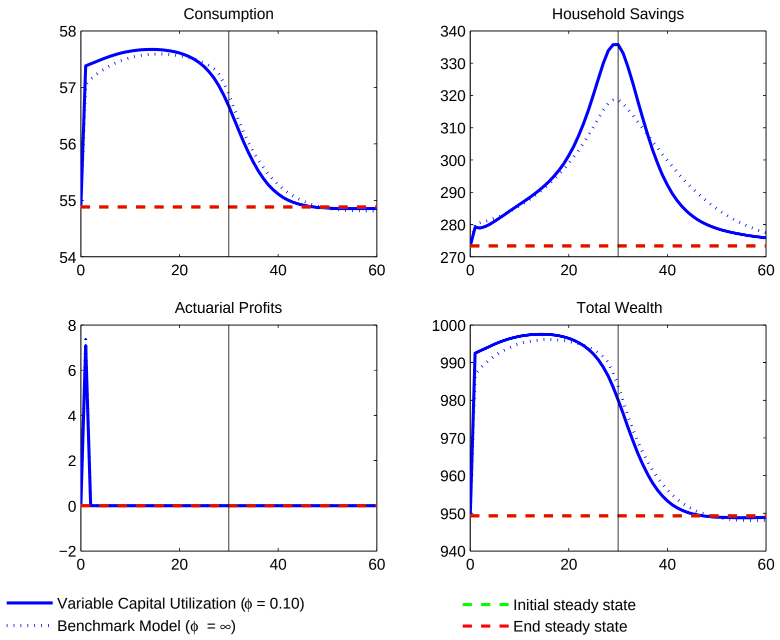


Figure 5.3 Temporary increase in technology level (cont.)

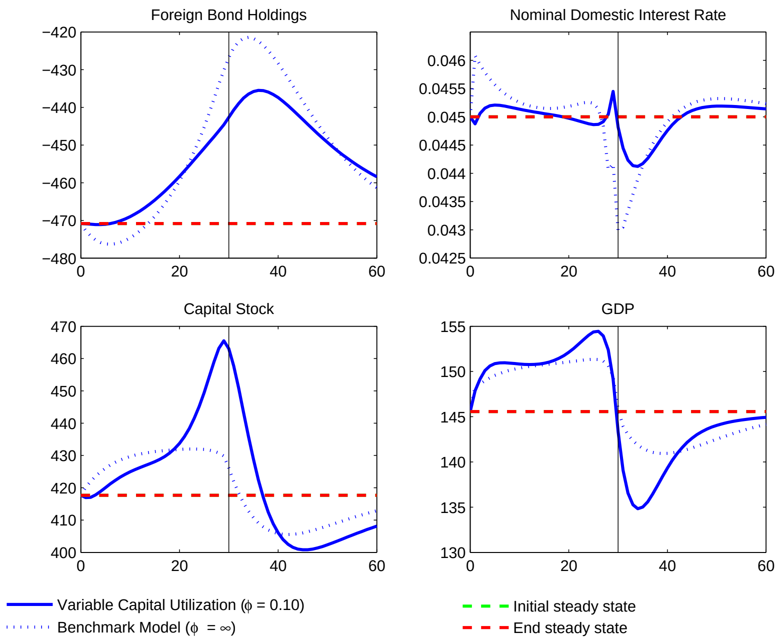
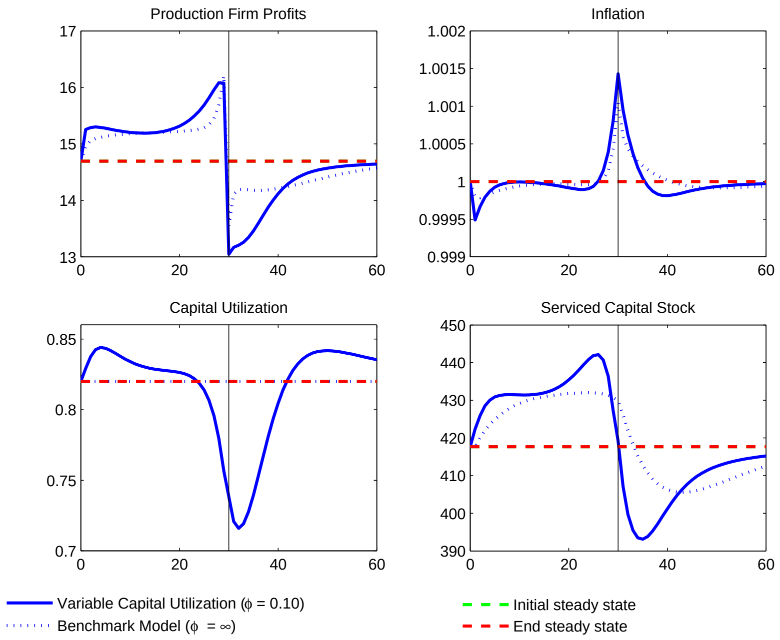


Figure 5.4 Temporary increase in technology level (cont.)



The variable capital utilization model

The unanticipated shock (period 1): The utilization feature provides an additional channel to benefit from the higher level of technology. The actuarial firms are able to supply more serviced capital (figure 5.4) without making additional investments in installed capital (figure 5.3). Therefore, the increase is smaller in both the real rental rate of capital and the real interest rate (figure 5.1). Of course, the resulting level of serviced capital exceeds the level in the absence of the utilization feature. This implies a higher labour productivity, which in turn stimulates the demand for labour by the production firms. Market clearing in the labour market results in a somewhat higher wage rate and a higher labour input (figure 5.1). The higher factor inputs result in a more pronounced increase in output (figure 5.3). Further, households consume more (figure 5.2) because of the higher labour income.

The anticipated shock (period 30): When the technology shock nears its ending, the utilization feature provides a channel for the actuarial firms to reduce the serviced capital (see figure 5.4). Labour productivity, and thereby the labour demanded by the production firms, then decreases even more when the shock matures. Moreover, labour supply decreases more as the households have accumulated more savings from the higher accumulated labour income (figure 5.2). The labour market clears at a relatively low level of labour input, while the wage rate is hardly affected by the friction (figure 5.1). This indicates that the utilization feature affects labour demand and labour supply in a similar downward direction. The lower levels of factor inputs imply that the utilization feature magnifies the decline in output at the maturity of the shock (figure 5.3). Consumption smoothing results in small differences in consumption (figure 5.2) compared to the differences in labour income.

Notably, the direction of the responses is not affected by the utilization feature. Cyclical variables remain cyclical and counter-cyclical variables remain counter-cyclical. Figure 5.4 shows that the feature itself behaves cyclical for this particularly imposed shock. Because foreign bond holdings are less volatile (figure 5.3) adjustment costs are lower, thereby generating slightly less persistence in the model.

A negative technology shock works in exactly the opposite direction. Reversing the story of a positive technology shock, it results in a larger negative response in the serviced level of capital and in output. Again, these effects reverse when the shock matures. Still, the utilization feature generates slightly less persistence in output fluctuations.

5.2 Foreign nominal interest rate shock

The foreign nominal interest unexpectedly increases from 4.5% to 5.0% until period 30. The other exogenous variables, including foreign inflation and foreign output, remain unchanged. Agents are surprised in period one by the shock.[13] As in section 5.1, all agents know that the interest rate will be lowered to 4.5% in period 30. Again, the temporary shock can be regarded as two permanent shocks: in period one an unanticipated shock and in period 30 an offsetting anticipated shock.

The benchmark model

The unanticipated shock (period 1): The shock increases the real return on foreign bonds (see figure 5.5). The shock makes borrowing by the actuarial firms via the foreign investment firms more expensive. By a no-arbitrage condition for actuarial firms, the expected profit of investing in shares of the foreign investment firms equals the expected profit of making capital investments. Buying actuarial notes by the households should also have an equal expected rate of return. Therefore, both the rental rate of capital and the domestic real interest rate adjust in an upward direction.

The higher rental rate of capital induces production firms to substitute labour for capital (figure 5.6). The marginal productivity of labour declines, so the real wage rate declines. The decrease in the real wage offsets the increase in the rental rate of capital such that the real

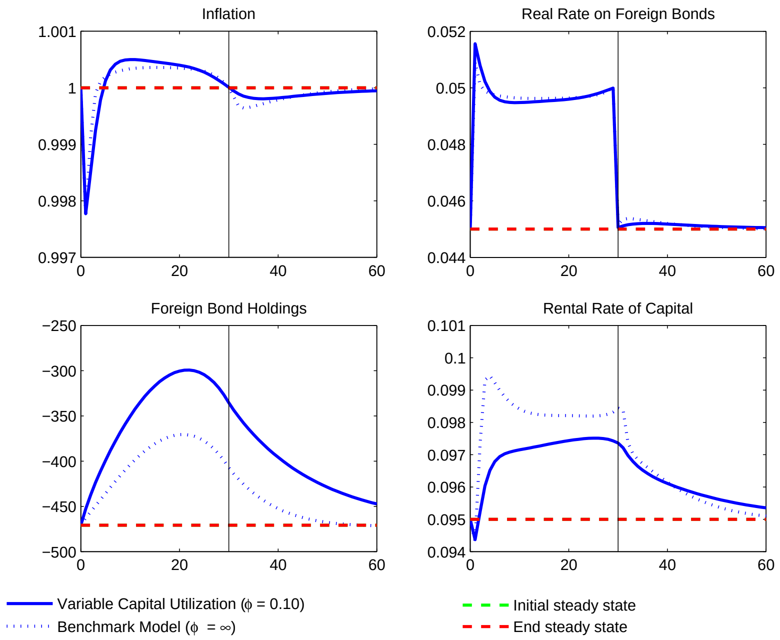
[13] This shock is rather similar to simulation 4 in Elbourne et al. (2009). In that model, the shock lasts for 10 periods.

marginal cost decreases (figure 5.7).[14] Production firms benefit by boosting output, resulting in a small decline in inflation.

Initially, the supply of capital is fixed. Therefore, the lower capital demand results initially in a downward pressure on the rental rate of capital. This implies that the increase in the real interest rate (figure 5.6) is not matched with a similar increase in the rental rate of capital (figure 5.5). Hence, the actuarial firms face an initial loss (figure 5.7), which reduces the total wealth of the shareholders of the actuarial firms, i.e. the wealth of the households (figure 5.8). As a consequence, initial savings are unexpectedly low. The poorer households feel forced to increase the labour supply despite the lower wage, which results in a lower consumption level than in the steady state.

The anticipated shock (period 30): When the period of the interest rate shock approaches its end, foreign borrowing starts to increase again (figure 5.5). Households enjoy the accumulated savings (figure 5.8), so they reduce labour supply even below the steady state value (figure 5.6) resulting in a gradually increasing wage rate (figure 5.7).

Figure 5.5 Temporary increase in the foreign nominal interest rate



[14] The production process is relatively labour intensive as the labour share equals 70%

Figure 5.6 Temporary increase in the foreign nominal interest rate (cont.)

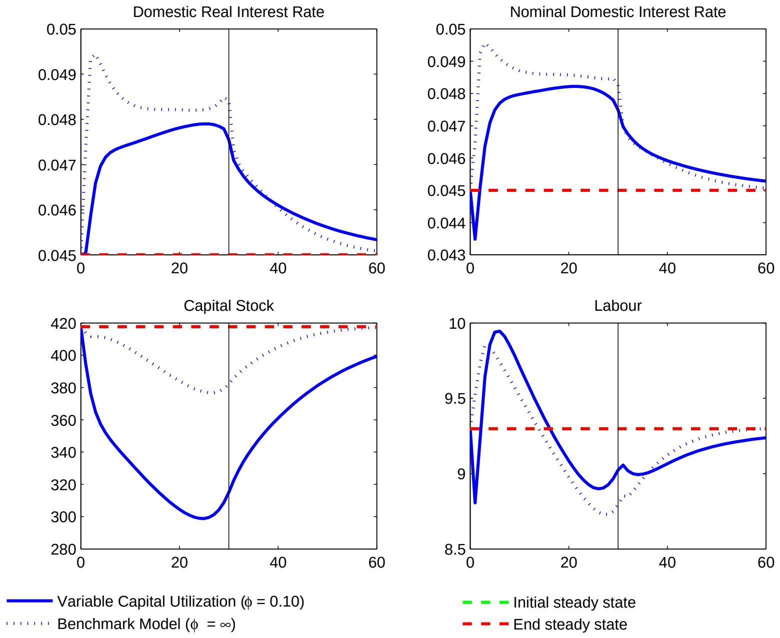


Figure 5.7 Temporary increase in the foreign nominal interest rate (cont.)

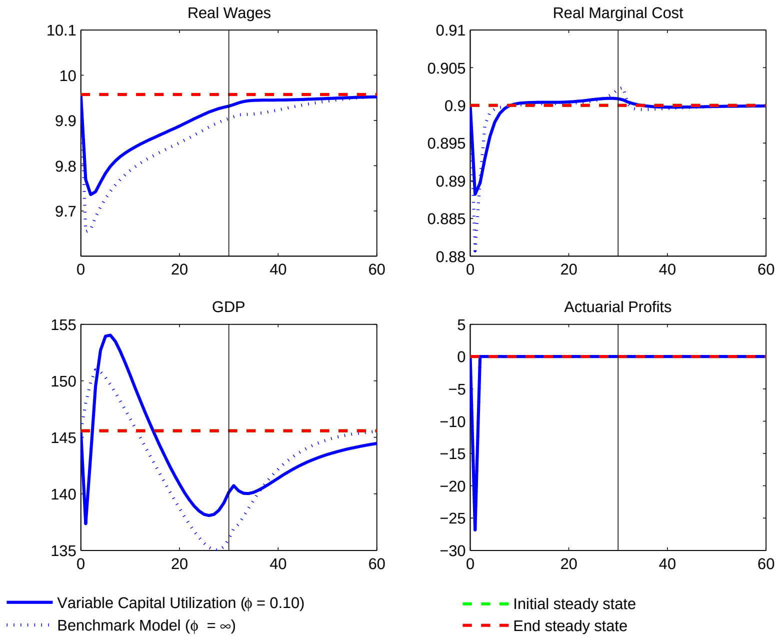


Figure 5.8 Temporary increase in the foreign nominal interest rate (cont.)

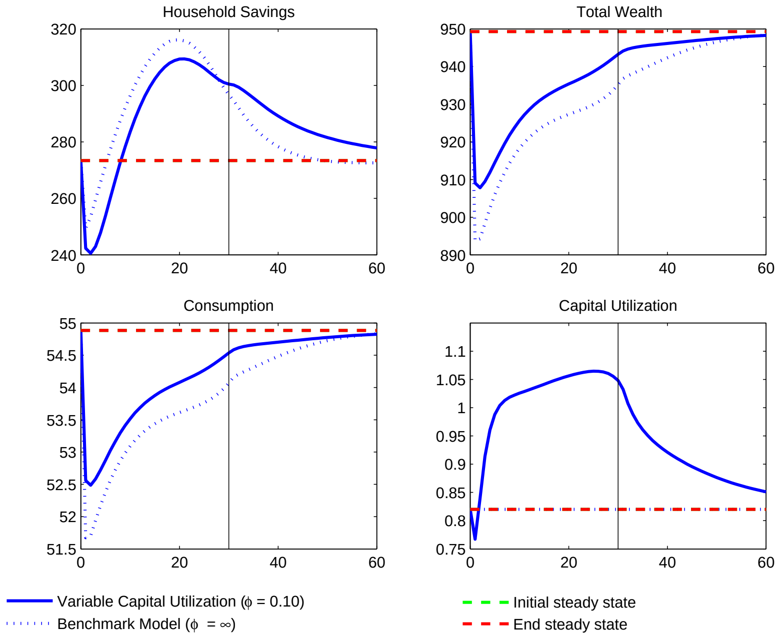
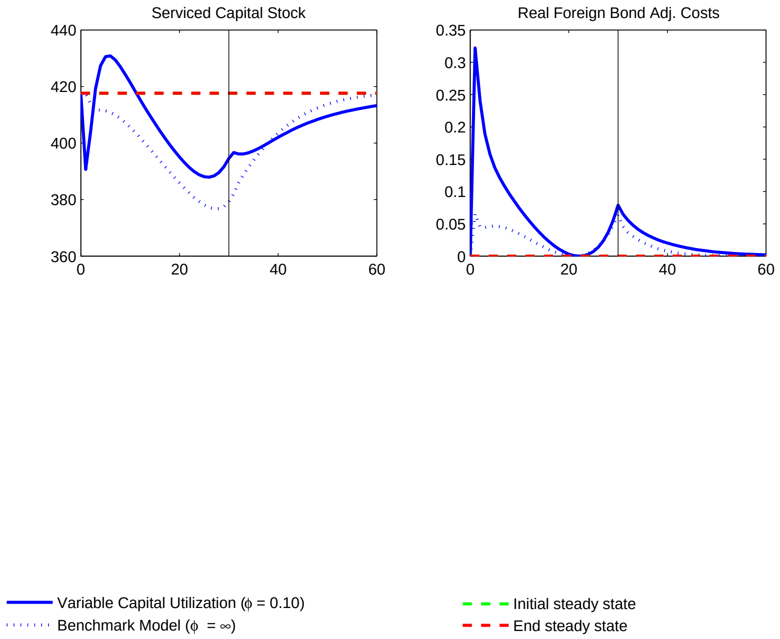


Figure 5.9 Temporary increase in the foreign nominal interest rate (cont.)



The variable capital utilization model

The unanticipated shock (period 1): The drop in investment becomes even more pronounced when variable capital utilization allows the actuarial firms to offset some of the drop in the level of installed capital (figure 5.6). As a result, their need for savings increases even more (figure 5.8). Hence, a smaller increase in the real interest suffices to restore equilibrium in the market for savings (figure 5.5). The impact on both the real interest rate and the rental rate of capital is dampened. The latter follows from the higher supply of serviced capital.

Notably, the utilization rate (figure 5.8) as well as the rental rate of capital (figure 5.5) decrease in the initial period. The actuarial insurance firms face a trade-off in reducing investments or reducing the utilization rate. The first channel is preferred due to the higher real interest rate. However, only the latter channel enables the actuarial firms to *instantaneously* decrease the level of serviced capital in the initial period. The utilization rate is optimal if the corresponding marginal cost, the utilization cost, equals the marginal benefit, the rental rate of capital. Hence, the rental rate of capital decreases initially. By a no arbitrage condition for actuarial firms, the real interest rate hardly changes in the initial period (figure 5.6). This implies that households are not rewarded for increasing labour supply instantaneously, resulting in a lower level of initial labour supply. The initial decrease in the serviced capital stock results in a lower labour demand. The lower labour input (figure 5.6) mitigates the effect on labour productivity, i.e. the real wage. Because the production process is labour intensive, the initial decrease in labour input reverses the initial effect on output. Instead of a benchmark 4% initial increase in output, the utilization feature results in an initial 5% decrease in output.

The main difference between the two models after the initial period can be explained by the fact that the utilization feature provides a channel to circumvent the adverse effects of the higher interest rate on borrowing. The larger drop in investments enables the firm to reduce the relatively expensive foreign borrowing (figure 5.5). Although the installed capital stock is lower (figure 5.6), the serviced capital stock is higher (figure 5.9) due to the higher utilization rate. Therefore, labour productivity and the real wage rate becomes less responsive to the shock (figure 5.6). The impact on consumption is smaller with a varying rate of utilization (figure 5.8) since the real interest rate and the real wage rate turned out to be less responsive to the interest rate shock. The higher factor usage results in a higher peak in output (figure 5.7). Hence, output becomes more volatile due to the varying utilization rate.

The anticipated shock (period 30): The dynamics of the anticipated shock are hardly affected by the utilization feature. Some extra persistence is in the model because the adjustment costs in foreign bond holdings are relatively high (figure 5.9). The utilization rate moves in lockstep with the rental rate of capital (see figures 5.5 and 5.8), which is straightforward from equation (4.3). A temporary decrease in the foreign nominal interest results in opposite effects.

6 Concluding remarks

The benchmark model in Elbourne et al. (2009) assumes that each unit of installed capital is utilized at a constant rate. Empirical evidence suggests that the utilization of capital varies over time. This memorandum has outlined a method to handle a varying rate of capital utilization.

The utilization rate can be chosen instantaneously by the capital owners. A higher utilization rate makes capital users more productive at the expense of a higher rental fee. Capital owners receive a higher rental fee for a higher utilization rate. However, they incur a utilization cost which is increasing and convex in the utilization rate. The cost is defined in terms of consumption goods, instead of the standard way to define the cost in terms of capital goods. This choice is motivated by the fact that our data suggests that a higher utilization rate of capital does not result in a higher depreciation and is in line with the criticism that the depreciation-through-use method could result into a counterintuitive response (see Christiano et al. (2005, p. 27)).

The implication of a varying capacity utilization is assessed for two different types of shocks. In line with the literature, the persistence of the shocks is not the major consequence of the utilization feature. This may be a consequence of the relatively small adjustment costs. Clearly, the resulting dynamics and required magnitudes of the shocks are affected by the utilization feature.

The utilization feature magnifies the overall effects of a given technology shock. The responses become more peaked. This implies that a smaller technology shock is sufficient to obtain the responses from the benchmark model. As suggested by Basu et al. (2006), too much of the variance of productivity growth in the benchmark model is attributed to the variance of technology. On the other hand, the overall impact of the given foreign nominal interest rate shock on the model is smaller if the utilization feature is included. Hence, a larger foreign nominal interest rate shock is needed to obtain the benchmark responses. Or stated differently, the responses are less peaked.

The intuition behind the different impact of the shocks is as follows: each shock has a direct effect on the model as well as an indirect effect via the real interest rate. The utilization feature dampens the impact on the demand for installed capital and, hence, the impact on the real interest rate. This indirect effect mitigates the effect of a foreign nominal interest rate shock. To the contrary, the mitigated response in the real interest rate enables investors to reap the benefits of a positive technology shock even more effectively.

In general, the utilization rate does not necessarily behave in a cyclical way, it is implemented in such a way that it moves in lockstep with the rental rate of capital (see equation (4.3)). This boils down to the exogenously determined nominal interest rate. The utilization rate becomes procyclical if a monetary policy is adopted which is characterized by a higher nominal interest rate for output expansions.

Appendix A Correlation of depreciation and utilization

This appendix shows that there is no statistical relationship between the published depreciation rate and the utilization rate. As argued in the text, the published rate of depreciation is unlikely to be a good description of short-term changes in the depreciation rate caused by higher capital utilization. This is because the published depreciation rate is simply an average of the fixed depreciation rates for the different types of capital good. Changes in the published rate likely only reflect changes in the composition of capital goods employed in the economy and not changes in the actual depreciation rates of particular types of capital . Still, Table A.1 contains correlation coefficients for the three variables considered: the measured depreciation rate, δ_t, the utilization rate, U_t, and the relative change in gross value added, ΔY_t. As the literature suggests, the depreciation rate seems to vary with the utilization rate of capital ($\rho = 0.42$). Leading or lagging depreciation by one period (a year) does not alter this observation. At first sight, one could regard this result as a confirmation for using the depreciation-through-use method. The intuitive result of correlated utilization rates and changes in real gross value added is confirmed by the data ($\rho = 0.58$). Remarkably, the depreciation rate appears to be hardly correlated with the change in real gross value added ($\rho = 0.04$).

Table A.1 Correlation coefficients (Δ denotes a percentage change)

	Complete sample 1974-2007	First half 1973-1990	Second half 1991-2007
$\rho(\delta_t, U_t)$	0.42	0.56	− 0.39
$\rho(\delta_t, \Delta Y_t)$	0.04	0.19	0.00
$\rho(U_t, \Delta Y_t)$	0.58	0.67	0.49

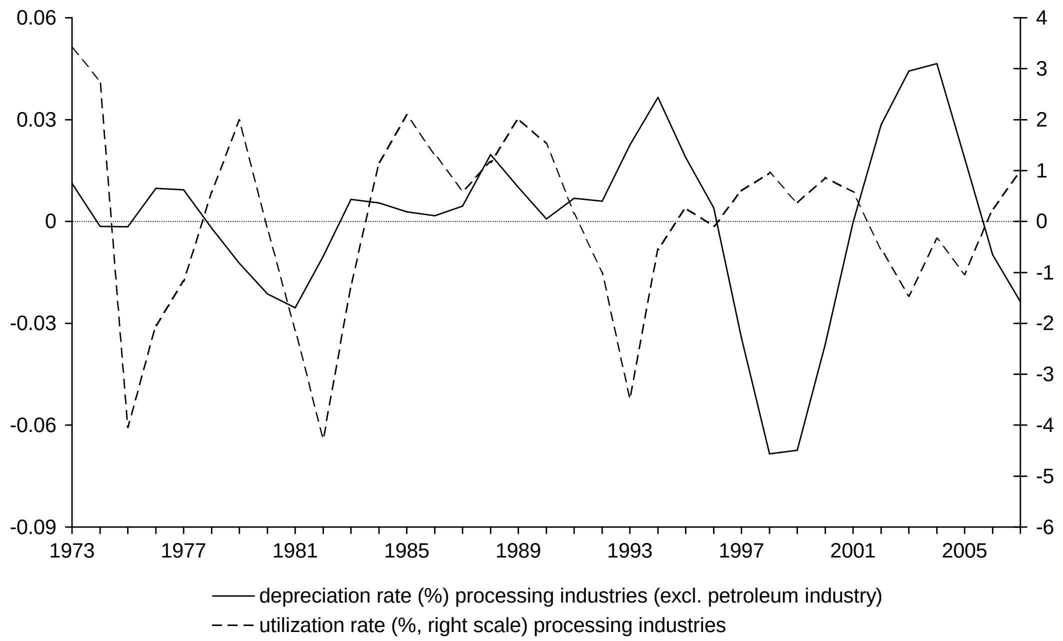
Considering the first half and the second half of the sample separately changes conclusions (see second and third column in Table A.1). It turns out that the correlation between depreciation and utilization is only attributable to the first half of the sample period. In fact, the correlation coefficient is even negative over the years 1991-2007. Again, leading or lagging variables does not change results.

From an economic perspective, it is well known that the share of IT capital in the capital stock has grown over the past decades. First, this implies that the mean economic life of durable goods has decreased. This explains the upward sloping trend of the depreciation rate in figure 4.1. Second, the dynamics in the depreciation rate increasingly depend on the dynamics of technological innovations. For instance, the increasing IT growth during the 1990s resulted both in an accelerated depreciation of IT capital and an increase in gross value added growth, while the utilization rate of capital, in particular computers, remained unaffected. Levy (1994) shows that technological progress leads to accelerated depreciation of producer durable goods and equipment since newer and more advanced technology makes older equipment obsolete. At least for some categories of capital, depreciation is correlated with technology shocks. However, the depreciation rate and the change in gross value added are uncorrelated during the second half of our sample. Therefore, our data does not provide evidence that technological progress, and hence growth in gross value added, leads to accelerated depreciation of capital.

In any case, a straightforward relation between the utilization rate and the depreciation rate cannot be identified from the data. The best explanation for the positive correlation in the first half of the sample are the relatively low troughs in utilization at the first sector, and the relatively high troughs in utilization at the second sector of the first sample. These outliers have a large impact on the correlation coefficient. The order of peaks and troughs in utilization combined with the slowly growing depreciation rate entails a spurious relation between depreciation and utilization. Therefore, depreciation and utilization are regarded as independent variables.

Figure A.1 depicts Hodrick-Prescott filtered yearly data using smoothing parameter $\lambda = 100$. Still, depreciation and utilization are positively correlated during the first half of the sample period ($\rho = 0.31$) and negatively correlated over the second half of the sample ($\rho = -0.61$).

Figure A.1 HP-filtered depreciation rate and HP-filtered capital utilization rate



If depreciation depends positively on capital utilization, then accelerated depreciation must be observed in case of a higher utilization of capital. Therefore, the analysis is extended in figure A.2 and table A.2 to yearly percentage changes.

Figure A.2 Yearly changes (%) in depreciation rate and capital utilization rate

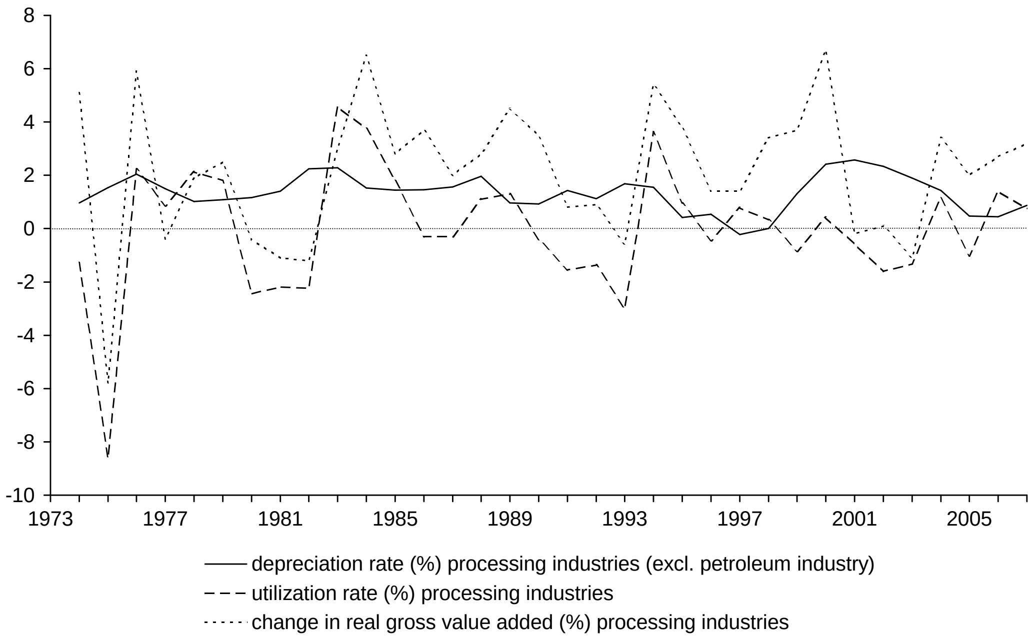


Table A.2 Correlations for HP-filtered data ($\lambda = 100$, Δ denotes a percentage change)

	Complete sample 1974-2007	First half 1973-1990	Second half 1991-2007
$\rho(\Delta\delta_t, \Delta U_t)$	− 0.05	0.13	− 0.28
$\rho(\Delta\delta_t, \Delta Y_t)$	− 0.11	− 0.10	− 0.14
$\rho(\Delta U_t, \Delta Y_t)$	0.76	0.78	0.72

The change in depreciation rate appears to be negatively correlated with the change in utilization rate ($\rho = -0.05$). Lagged changes in depreciation are positively correlated with changes in utilization ($\rho = 0.23$), whilst leading changes in depreciation provides a negative correlation coefficient ($\rho = -0.36$). The changes in utilization rate and gross value added are still correlated ($\rho = 0.76$). Again, a completely different picture emerges when the complete sample 1973-2007 is split up in a first sample 1973-1990 and a second sample 1991-2007. The negative contemporaneous correlation between the changes in the depreciation rate and the utilization rate is completely attributable to the second half of the sample period.

Appendix B First order condition for the depreciation-through-use method

The Lagrangian for the depreciation-through-use method is given by

$$\mathscr{L}\left(\begin{array}{c} B_t, I_t, K_t, \\ U_t, Z_t, Z_t^f \end{array}\right) = \sum_{t=s}^{\infty} Q_t \left(\begin{array}{l} N_t - (1+r_{s-1})N_{s-1} - B_t + (1+r_{s-1}^g)B_{s-1} \\ -q_s Z_s + (q_t + div_s)Z_{s-1} - q f_s Z f_t + (q f_s + div f_s) Z f_{s-1} \\ +r_s^k U_s K_{s-1} - \left[1 + \Psi\left(\frac{I_s}{K_s} - 1 - \delta\right)\right] I_s \\ +\Lambda_t \left[I_s + (1 - a(U_t))K_{s-1} - K_s\right] \end{array}\right)$$

Then, the first order condition with respect to U_t shows that the marginal cost of capital Λ_t affects the utilization rate:

$$r_t^k = \Lambda_t a'(U_t) \tag{B.1}$$

If the marginal cost of capital, Λ_t, is more responsive to shocks then the rental rate of capital, r_t^k and the utilization cost is convex in the utilization, then higher rental rates are not associated with higher utilization rates. This is especially relevant if investment adjustment costs are high. This is in line with the remark in Christiano et al. (2005) that a positive monetary shock might result in a lower utilization rate in the presence of high investment adjustment costs $\Psi(.)$.[15]

[15] We tested for the potential counterintuitive response in utilization, but we did not observe such behaviour if (B.1) was implemented and a large investment adjustment cost ($cp = 10^9$) was imposed. An explanation may involve that the investment adjustment costs depend on I_t and K_t, instead of I_{t-1} and I_t as in Christiano et al. (2005)

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