CPB Memorandum

CPB Netherlands Bureau for Economic Policy Analysis

Sector	:	Labour market and welfare state
Unit/Project	:	Life cycle model
Author(s)	:	Sijmen Duineveld
Number	:	237
Date	:	10 February 2010

Introducing home production in a life-cycle model

In this paper we will analyze the effects of the introduction of home production in the life-cycle model as developed by Van Erp en De Hek (2009). To introduce home-production we have added an extra nest in the utility function: the consumption good in the model of Van Erp and De Hek is substituted with a composite consumption good that consists of goods directly purchased at the market and goods that are produced at home. For the latter goods we also assume that inputs have to be bought in the market. With this adjusted model, we analyze what the effects on labour supply, wealth and consumption patterns are. Surprisingly, the introduction of home production results in more accumulation of wealth.

1 Introduction

This memo explores the extension of the life-cycle model, as developed by Van Erp and De Hek (2009), with home production. More specifically, we will analyze whether the inclusion of home production adds important dynamics to the model. Of special interest are the effects of a tax increase on both models. This policy analysis is interesting, because in a model with home production substitution between the untaxed home production and the taxed market production sector is possible. This is also likely to affect retirement behaviour.

The memo will start with a brief summary of the life-cycle model of Van Erp and De Hek (2009). Then two issues will be discussed that arise with the introduction of home production. First, the valuation of home production will be addressed; second, ageing and home production will be reviewed. Thereafter the model including home production will be described. After this model review the differences between the two models will be discussed. This will be done for the base scenario and a scenario where taxes increase. Finally conclusions will be drawn.

2 Life-cycle model by Van Erp and De Hek

In this section a simplified version of the life-cycle model by De Hek and Van Erp will be presented. This summary should give a impression of the essential mechanisms of the model[1]. The model is a structural life-cycle model of an representative agent that maximizes utility with two decision variables: consumption versus saving and labour supply versus leisure. The agent has certainty about the future. The utility function includes consumption, leisure and a bequest. The model has the following institutions: private pension arrangements, state pensions and taxes. The model uses heterogeneity in the ageing process of agents which is represented by an extra time cost of working as the agent ages. The heterogeneity in the speed of the ageing process is used to simulate different retirement ages. The distribution of the speed of ageing is used to match observed retirement ages.

In a simple representation of the model (where bequest is omitted) finite lifetime utility is given by:

$$U_{c,v} = \sum_{t=s}^{T} \left(\frac{1}{1+\rho} \right)^{t-s} \xi_{t,s} u(c_t, v_t) \tag{2.1}$$

with

[1] For a full review of the model, see Van Erp and De Hek (2009).

$$u(c_t, v_t) = \frac{1}{1-\frac{1}{\gamma}} x(c_t, v_t)^{1-\frac{1}{\gamma}} \qquad \gamma > 0, \gamma \neq 1 \tag{2.2}$$

where ρ is the rate of time preference at which future utility is discounted and $\xi_{t,s}$, denotes the probability that the individual of age s will be alive at time t. The parameter γ is the intertemporal elasticity of substitution of the composite good, x. In each period the utility derived from consumption and leisure is described by a CES-function in which σ is the elasticity of intratemporal substitution between consumption and leisure:

$$x(c_t, v_t) = \left[\mu_c^{\frac{1}{\sigma}} c_t^{1-\frac{1}{\sigma}} + \mu_v^{\frac{1}{\sigma}} v_t^{1-\frac{1}{\sigma}} \right]^{\frac{1}{1-\frac{1}{\sigma}}} \qquad \sigma > 0, \sigma \neq 1 \tag{2.3}$$

where μ_c and μ_v are measures of the relative weights of consumption and leisure in the utility function. Other forms of this utility function are used by French (2005) and Gustman and Steinmeier (2005).

Utility is maximized subject to a time and budget constraint. The time constraint is given by

$$v_t + l_t + d_t l_t = l^{\max} \tag{2.4}$$

where l_t represents hours of work. The term $d_t l_t$ represents the extra time that recovery from work consumes, which depends on time worked and the variable time cost of working, d_t. This last parameter is the most crucial aspect of the model since it represents the ageing process. The increasing value of this variable represents a decreasing health condition which increases the time to recover from work. It should be noted though, as Van Erp and De Hek (2009) do, that this has the same effect as a change in preferences, where both the relative weight and the minimum amount of leisure are adjusted. The model uses a minimum labour supply constraint:

$$l_t \geq l^{\min} \tag{2.5}$$

The budget constrain is given by:

$$a_t = (1 + r - \tau_a) a_{t-1} + (w_t l_t + aow_t + pb_t) - pp_t - tax_t - (1 + \tau_c) p_t c_t \tag{2.6}$$

where a_t is end of period wealth, τ_a, the tax rate on wealth, w_t, the wage rate, τ_c , the consumption tax rate. The pension system is reflected in the terms aow_t and pb_t, where the first refers to state pensions and the second to private pensions. A borrowing constraint is imposed as:

$$a_{t-1} \geq 0 \text{ for } t \geq s \tag{2.7}$$

Later the institutional aspects such as pensions and taxation have to be modelled.

3 The valuation of home production

To model the substitution between market consumption and home produced goods a value has to be assigned to home production. This is not easily done with home production since no external price of household goods is established as they are not traded. To attach a value to home production two things need to be specified: a production technology and the (relative) price of the output. The production function determines output quantity. The value thereof is established with the elasticity of substitution between the market good and the household good.

3.1 Home production function

This section consists of two parts. In the first part some general empirical results are reviewed. In the second part, different forms of production functions are reviewed.

3.1.1 Empirical results on home production function

In most papers on home production strong assumptions are made on the functional form of the production function without much empirical justification. Therefore little is known about the nature of household production functions. One exception is the paper by Fitzgerald, Swenson and Wicks (1996). In this paper the functional form of the production function is a translog, which is very flexible[2]. A translog includes, for instance, a Cobb-Douglas specification. Three type of inputs were used in the analysis: female labour, male labour and capital. They estimate the translog function for various types of home production (for instance cleaning and cooking).

Their main results are:

- hours of female and male are significant;
- capital is not significant;
- constant returns-to-scale;
- different types of household work have different production functions;
- a Cobb-Douglas production function with male and female labour input and capital is rejected.

[2] See Christensen, Jorgenson and Lau (1973) for a review of the translog function.

The results should be interpreted with some caution though, since the measurement techniques for output are subject to considerable errors. And, with respect to the last result, a different Cobb-Douglas production function may not be rejected.

The paper by Fitzgerald, Swenson and Wicks (1996) is interesting for other reasons too. They find that education and age are not important for the determination of productivity at home[3]. The result with respect to education may be especially interesting when heterogeneity in skill level (education) is introduced in the model, since the skill level apparently does not affect productivity at home. With the results of this paper in mind different types of production functions are reviewed.

3.1.2 Types of production functions

In the literature three types of production functions for household production are used. These are functions of (1) labour; (2) labour and capital; (3) labour and market goods (other than capital). In theory a combination of (2) and (3) is also possible, but this has not been used in practice (to my knowlegde). Furthermore the following functional forms are used: (a) linear; (b) concave; (c) Cobb-Douglas; and (d) Constant Elasticity of Substitution (CES).

Output as a function of labour

In the literature three functional forms for the production functions with labour input can be identified: linear (De Ree and Alessie, 2008), Cobb-Douglas (with male and female labour input) and concave (Piggott and Whalley, 1998 and Rupert et al., 1995). The Cobb-Douglas with two agents can be ruled out here, since our model has only one representative agent. For modelling purposes a linear production function may pose some difficulties as it will result in a corner solution as either working at home or working for the market is more productive. This doesn't seem realistic, since a vast majority of working age people carry out at least some household tasks.
A production function with decreasing marginal productivity seems more appropriate. The most simply form is used by both Piggott and Whalley (1998) and Rupert, et al. (1995), which is:

$$s(h) = h^{\alpha} \tag{3.1}$$

where s is output, h labour input and α a parameter. However, this production function is in contradiction with the empirical research by Fitzgerald, et al., which found that household production should exhibit constant returns-to-scale. In addition this type of production function

[3] Regarding age it is unclear whether this test included both a linear and a quadratic term of age, so some caution is appropriate.

makes the analytical derivation of the effect of wages on leisure and consumption much more complicated since the average price is not equal to the marginal price.

Output as function of labour and capital

Home production functions that include capital and labour are mostly used in dynamic macroeconomic models used for business cycle analysis and static models used for tax analysis. Incorporation of capital in a life-cycle model significantly complicates the model, since the evolution of capital has to modelled too, but may add little to the model results. This may therefore not be worth the effort, especially since Fitzgerald et al. found capital to be insignificant in household production.

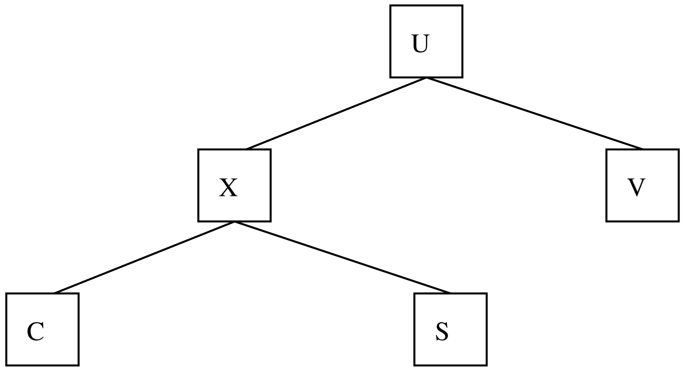
Output as a function of labour input and market good inputs

The advantage of this type of production function is that it can combine a decreasing marginal productivity of labour with constant-returns-to-scale without the complexity that modelling capital requires. In addition, this solution does not add much restrictions to fit the model to empirical data (for instance, Graham and Green (1984) estimate a Cobb-Douglas home production function with male and female labour supply and intermediate inputs), since data on household expenses do not distinguish goods that are directly consumed (market consumption goods) from goods that are used as inputs in home production (intermediate goods). The only requirement is that the sum of market consumption goods and intermediate goods match patterns of total expenses. One limitation of this solution is that domestic output can not be stored, since it has to be consumed within the same period as it is produced. However, this disadvantage does not weigh up to the advantages, so this production function is the preferred model. Within this class of models, the most flexible functional form seems most convenient. Therefore the CES production function is chosen.

3.2 Utility function with home production

In the literature on household production we find various ways to incorporate home produced goods in the utility function. In the standard utility function (in models without home-production), utility is derived from consumption and leisure. To limit the number of deviations from the model by Van Erp and De Hek we have chosen to add an extra nest to the utility function. So the consumption good of the model of Van Erp and De Hek is replaced with a composite consumption good, that consists of the home produced good, *S*, and goods bought at the market, *C*. The new nesting structure is depicted in Figure 3.1.

Figure 3.1 The nested utility function



Thus, the utility function at the highest level of the original model remains unchanged (the symbol c in equation 2.3 is replaced with x.)

$$u(x,v)=\left[\mu_x^{\frac{1}{\sigma}}x^{1-\frac{1}{\sigma}}+\mu_v^{\frac{1}{\sigma}}v^{1-\frac{1}{\sigma}}\right]^{\frac{1}{1-\frac{1}{\sigma}}} \tag{3.2}$$

with the addition of nest of the composite good X :

$$x(c,s)=\left[\psi_c^{\frac{1}{\phi}}c^{1-\frac{1}{\phi}}+\psi_s^{\frac{1}{\phi}}s^{1-\frac{1}{\phi}}\right]^{\frac{1}{1-\frac{1}{\phi}}} \tag{3.3}$$

4 Ageing and home production

One of the most important aspects of the model is the ageing process. In the model without home production ageing is modelled with an increasing time cost of working. With the introduction of home production the question rises how ageing should affect this activity. One might assume that health effects are similar for both home production and market work. This makes the observed shift from working in the market to working at home after retirement less obvious, although retirement from the market job would still occur at some point due to the restriction on the minimum hours worked.

When the alternative way to model the ageing process - a preference shift towards leisure - is considered, a similar problem arises, since the shift in preferences would affect home production and market work in a similar way. Therefore the way ageing is modelled will be addressed in more detail. Basically there are three ways to model ageing effects: a shift in preferences, increasing disutility of work and lower productivity. Let us review each of them.

4.1 A shift in preferences

A very direct way to model retirement behaviour is a shift in preferences towards leisure as the agents ages. The advantage is that the results are going to be clear-cut: a substitution from work (in the home or for the market) to leisure. And given the fixed cost of working at some point a (discrete) shift from market work to home production will occur. The disadvantage is that this exogenous shift in preferences is not easy to justify. Furthermore, there will be little flexibility to fit the model to the observed labour supply to the market or household sector, since the shift in preferences will equally affect market and domestic work.

An example of this shift in preferences is used by Gustman and Steinmeier (2005). They model ageing with an increase in the relative value of leisure versus consumption. Utility is given by:

$$u(c,s,l) = \left[\frac{1}{\alpha} c^{\alpha} + h \frac{1}{\gamma} l^{\gamma} \right] \tag{4.1}$$

where c is consumption and l is leisure. The term h is increasing as the individual ages and finds work more difficult. As noted this may leave the preference between working in the market and working at home unaffected, because the effect on market and household consumption is equal.

4.2 Increasing disutility work

As in the current life-cycle model, another way to model ageing is to increase the disutility from work with ageing. One way to do so is to put a time surcharge on each hour worked. This could then represent the time to recover from work. The advantage of this way of modelling is that the intuition is stronger, in the sense that it can more easily be thought of as a change in health. Furthermore, if some health parameter is included in the model little adjustments would be required[4].

4.3 Lower productivity

As one gets older, productivity may also decline which will eventually induce retirement. The mechanism behind this is that the benefits of working will, at some point, be smaller than the fixed cost of working and consequently result in retirement. There are some problems with this solution. First of all, wages in the market do not tend to decrease with age, even as the pension

[4] This may be a logical extension of the model since the health status plays an important role in retirement behaviour. Furthermore, there is a correlation between health status and income levels, which makes health an important variable in a model with heterogeneity in income levels.

age is approached. Thus, even if productivity does decline an employee is not confronted with this declining productivity, since his wage is unaffected. Second, productivity in the household is not found to be decreasing with age (Fitzgerald et al., 1996).

Based on these arguments the second option is chosen for this model. That means that the time surcharge on working is increasing with age. The model will allow for an ageing effect that differs proportionally in home production and market work. This difference is justified by the absence of external pressure to perform at home, which makes it possible to work at a convenient work pace (that is to 'consume' the time surcharge of ageing while working at home, ie. have as many breaks as one wants).

5 The model including home production

5.1 Equations

The model by Van Erp and De Hek is adjusted in the following ways. In the utility function the consumption good, *c*, is replaced by the composite consumption good, *x*.

$$U_{c,v} = \sum_{t=s}^{T} \left(\frac{1}{1+\rho} \right)^{t-s} \left[\xi_{t,s} u(x_t, v_t)^{1-\frac{1}{\gamma}} + (\xi_{t,s} - \xi_{t+1,s}) v(a_t) \right] \tag{5.1}$$

The utility is discounted at rate ρ and consists of two components. The first is consumption of the composite consumption good, *x*, and leisure, *v* in a CES nest (see equation 5.2). This utility is weighted by the probability of survival of an individual of age *s* at time *t*, $\xi_{t,s}$. The second component is the bequest, *v(a)*. The utility depends on the mortality rate in year *t*: $\xi_{t,s} - \xi_{t+1,s}$. The utility of a bequest is used to ensure that the agent holds assets when he dies, as is empirically observed.

The CES nest of composite consumption, *x*, and leisure, *v*, is given by:

$$u_t(x_t, v_t) = \left[\mu_x^{\frac{1}{\sigma}} x_t^{1-\frac{1}{\sigma}} + \mu_v^{\frac{1}{\sigma}} v_t^{1-\frac{1}{\sigma}} \right]^{\frac{1}{1-\frac{1}{\sigma}}} \tag{5.2}$$

Composite consumption itself also consists of CES nest, and is defined as:

$$x_t(c_t, s_t) = \left[\psi_c^{\frac{1}{\theta}} c_t^{1-\frac{1}{\theta}} + \psi_s^{\frac{1}{\theta}} s_t^{1-\frac{1}{\theta}} \right]^{\frac{1}{1-\frac{1}{\theta}}} \tag{5.3}$$

This equation determines the value attached to market consumption, c, and household consumption, s. The elasticity of consumption is the parameter θ and the parameters ψ_c and ψ_s measure the relative weights. The agent maximizes this nested (lifetime) utility function, subject to a budget and a time constraint. The agent uses his budget on either consumption or savings and can spend his time working for the market, working at home or enjoying leisure time.

The home production function itself is given by:

$$s_t\left(h_t, k_t\right) = \left[\eta^{\frac{1}{\pi}}(bh_t)^{1-\frac{1}{\pi}} + (1-\eta)^{\frac{1}{\pi}} k_t^{1-\frac{1}{\pi}}\right]^{\frac{1}{1-\frac{1}{\pi}}} \tag{5.4}$$

where s is output in home produced goods, η is the weight of labour input and consequently $1-\eta$ is the weight of market inputs in the production function, b is a scalar for the productivity of labour, h is labour input and k is goods input, and π the elasticity of substitution between labour input and intermediate good inputs.

In the time constraint the household labour supply, h, has to be included. As mentioned before the time devoted to household production will also include a time surcharge that increases with age after 50. The time constraint is given by:

$$l^{\max} = v_t + \left(1+d_t\right)l_t + (1+\delta d_t)h_t \tag{5.5}$$

As can be seen in this equation the time surcharge of household production is scaled by the parameter δ, which will be smaller than one, meaning that ageing will affect home production less than working in the market.

The budget constraint is given by:

$$a_t = (1+r^h)a_{t-1} + \left(w_t l_t + aow_t + pb_t\right) - pp_t - (1+\tau_c)p_t c_t - (1+\tau_c)q_t k_t \tag{5.6}$$

where the last term is added to the original model, where q_t is the price of market inputs and τ_c is the consumption tax rate. This tax rate also applies to the intermediate goods, k_t, used as input in home production.

5.2 Parameter settings

In this model the parameters are set such that the labour supply and labour supply elasticity are roughly equal to those of the model by Van Erp and De Hek, which were fit to empirical studies. The starting point are the parameters values of the model of Van Erp and De Hek, and some additional parameters. Only two parameters of the model of Van Erp and De Hek are changed: the parameters of the preference for (composite) consumption and leisure. The preference for (composite) consumption, μ_x,is increased from 0.6 to 0.75 and subsequently the preference for leisure, μ_v, is reduced from 0.4 to 0.25 (so they sum up to 1). The intuition behind the higher preference for composite consumption is that consumption has a broader definition in the model with home production, since consumption includes home produced goods. Therefore the preference for consumption has to increase to obtain a similar market labour supply and labour supply elasticity. Table 5.1 shows the new parameters, and the two altered parameters.

Table 5.1 Value of parameters

Symbol	Description	Value
θ	Substitution elasticity market goods and home produced goods	2
ψ_C	Preference for market consumption	0.75
ψ_S	Preference for home produced goods	0.25
π	Substitution elasticity between intermediate goods and domestic labour supply	0.75
η	Weight of domestic labour supply	0.5
δ	Scalar for ageing effect on domestic work	1
b	Productivity scalar for labour in home production	7
μ_x	Preference for (composite) consumption	0.75
μ_v	Preference for leisure	0.25

6 Results of base scenario

In this chapter the properties of the model with the parameters given in table 5.1 will be analyzed and compared with the model without home production. In some of the figures plots are made for several cohorts to get more insight in the distribution of the results over the different groups. Each group or cohort differs with respect to the speed of their ageing process (different values for d in equation 5.5). The figures will show groups with large, medium and small costs of ageing.

In the previous paragraph it was mentioned that the labour supply elasticities of the two models should be matched. In figures 6.1 and 6.2 we see that the labour supply elasticity is slightly higher in the model with home production and that, especially near retirement, the variation in the labour supply elasticities increases somewhat more. The sharp increase and drop in the labour supply elasticity around retirement can be explained by the very low (or zero) labour supply at retirement in the baseline simulation: any increase in hours worked, due to a higher wage rate, will result in a (infinitely) large labour supply elasticity. When the labour supply at older ages drops to zero (as a result of increased ageing costs) and stays zero, this results in labour supply elasticity of zero as well.

Figure 6.1 Labour supply without home prod.

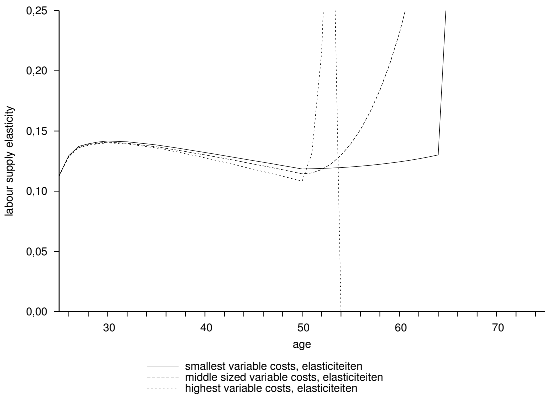
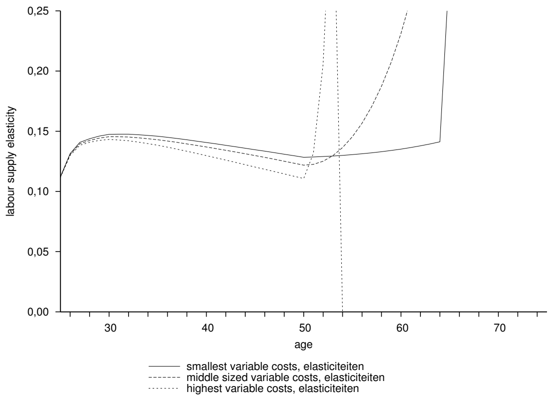


Figure 6.2 Labour supply elasticity with home prod.



The market labour supply was set such that it roughly matches that in the original model as we can see in figure 6.3 with labour supply slightly higher in the model without home production. The domestic labour supply is shown in figure 6.4.

Figure 6.3 Average market labour supply

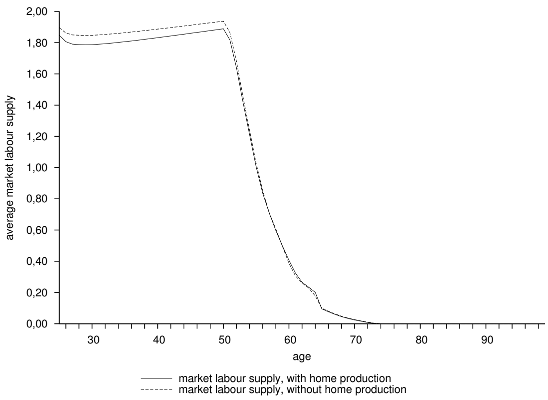
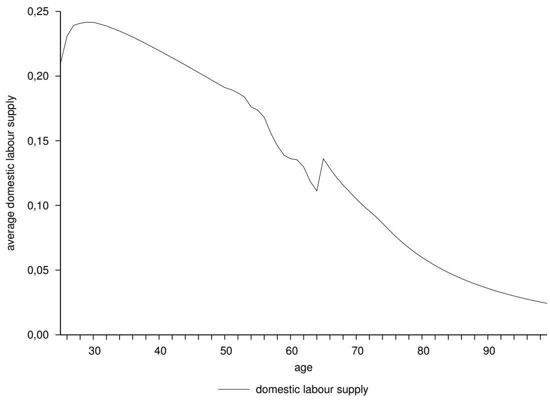


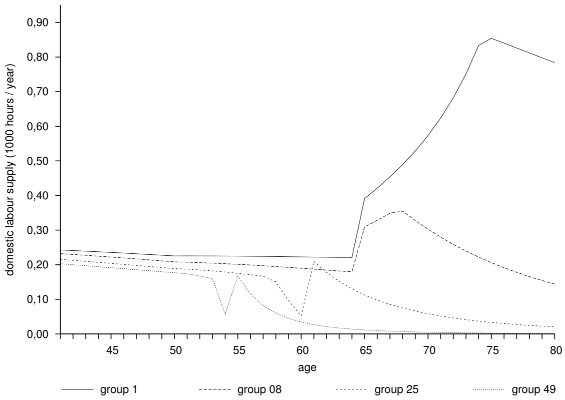
Figure 6.4 Average domestic labour supply



The profiles of domestic labour supply for different individuals in figure 6.5 show a very diverse picture. The cohorts with high variable costs (group 49) have a slight drop in their domestic labour supply just before retirement, when the minimum hours constraint for market labour supply is binding. This means that the agent ages, but still prefers to keep working the minimum hours instead of not working at all, even though he would rather work less than the minimum hours. As a result the opportunity cost of domestic labour supply increases. Immediately after retirement the domestic labour supply rises only slightly above its pre-retirement level. This is only short-lived too as the decreasing health condition increases the opportunity cost of domestic labour supply rapidly.

For the groups with low variable costs (groups 1 and 8) domestic labour supply increases significantly around the retirement age with an especially large increase for the groups with very low variable costs (group 1). The reason for this increase is that the wage rate in the second job (after compulsory retirement at 65 years) steadily declines. So as long as the health has not deteriorated too much, the agent will increase his domestic labour supply as the opportunity cost declines. However, at some point the deterioration in health will dominate the decrease in wages, and domestic labour supply will start to decline.

Figure 6.5 Profiles domestic labour supply



The slightly lower market labour supply results in a slightly lower labour income as we can see in figure 6.6. The maximum wealth level is higher in the model with domestic production as we can see in figure 6.7.

Figure 6.6 Average labour income

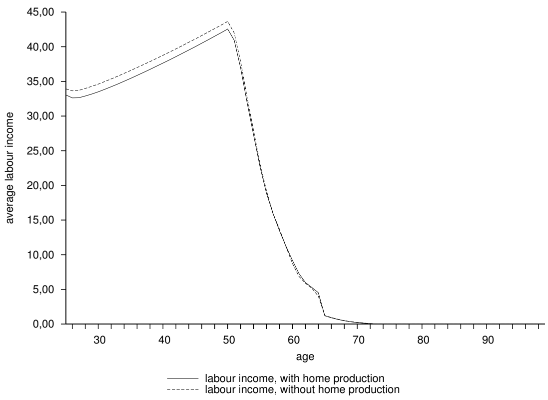
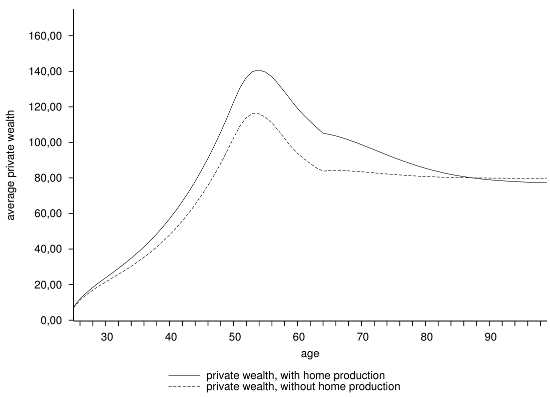


Figure 6.7 Average private wealth



Although wealth is higher for all groups, the increase in average wealth is most of all due to the higher wealth of the groups with high variable costs (group 25 and most of all group 49) as we can see in figures 6.8 and 6.9. The lower opportunity costs for house production for groups with low variable costs (group 1 and 8) as just discussed also shows up in the wealth profiles. These individuals increase their consumption of household goods and so need less markets good to sustain the same level of composite goods, hence they need less wealth after retirement.

Figure 6.8 Profiles wealth without home prod.

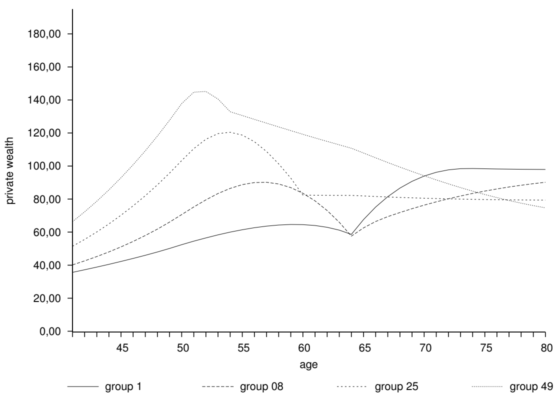
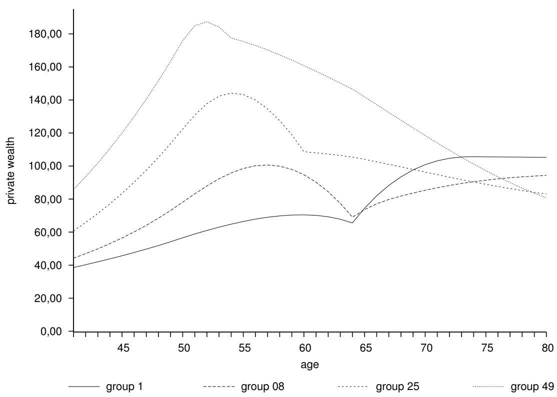
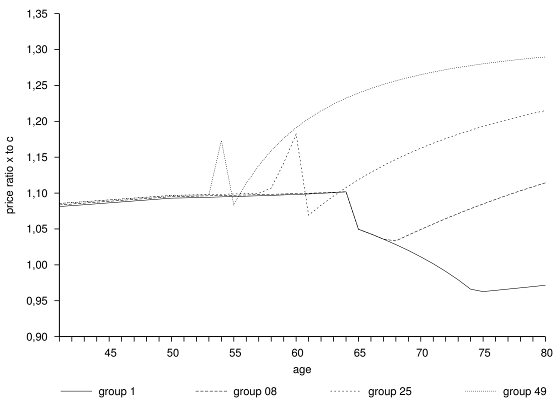


Figure 6.9 Profiles wealth with home prod.



The pronounced increase in wealth by groups with high variable costs can partly be explained by the increase in (shadow) price of home produced goods after retirement shown in figure 6.10. The cause of the increase in the price is the declining productivity at home for the groups with high variable costs (which dominates the reduction in price for the groups with low variable costs). This means that even though wages are (very) low little home production will be carried out, so little home produced goods will be consumed. It may thus be concluded that the price of composite consumption is relatively low early in life, which makes the agent do more domestic work early in life. The reduction in expenses is saved and spent on market goods later in life.

Figure 6.10 Price ratio composite good and market good



This picture also emerges when expenses (sum of market consumption and intermediate goods) are considered. The effects on average expenses and average composite consumption[5] are shown in figure 6.11 and 6.12, respectively. From these figures we can see that expenses and composite consumption fall more and faster between 50 and early 60s in the model without home production. This is mostly due to the sharper reduction in consumption of groups with high variable costs in the model without home production. This is visible from a close examination of group 49 in figures 6.13 and 6.14. For this group expenses falls much more in the model without home production (approximately from 28 to 20) than with home production (approximately from 26 to 21).

If we compare expenses (figure 6.11) and composite consumption (figure 6.12) for the model with home production we see that composite consumption is higher prior to retirement, but declines more rapidly than expenses when agents start retiring. The reason for this is the increase in the price of composite goods relative to market goods, as we have seen in figure 6.10.

Figure 6.11 Average expenses

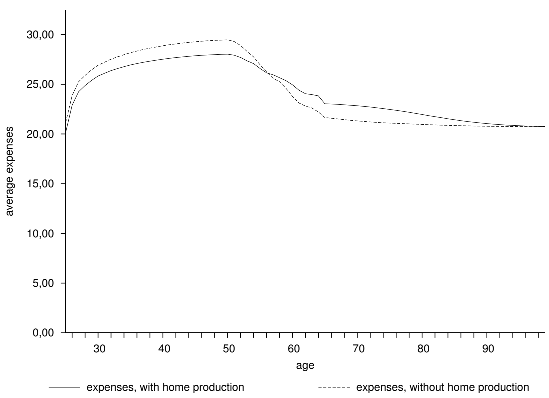
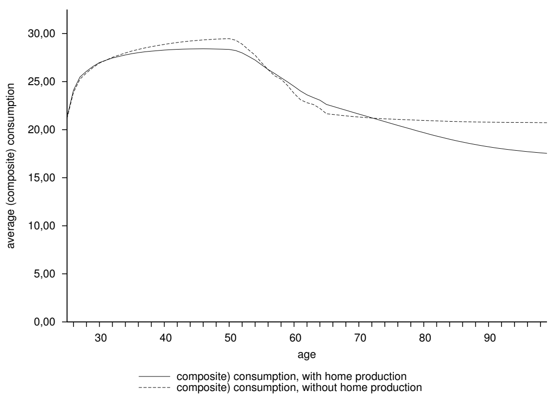


Figure 6.12 Average composite consumption



[5] Consumption consists solely of market consumption in the model without home production, so expenses and composite consumption are the same in this model.

Figure 6.13 Profiles expenses without home prod.

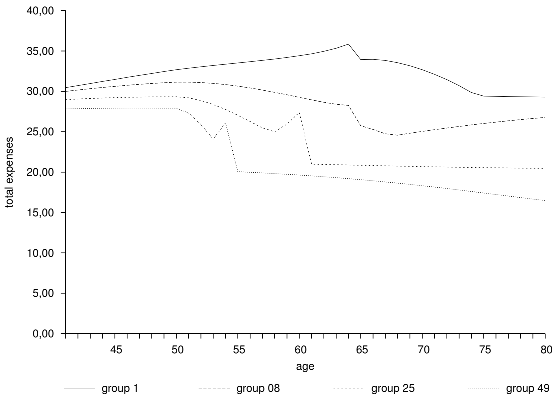
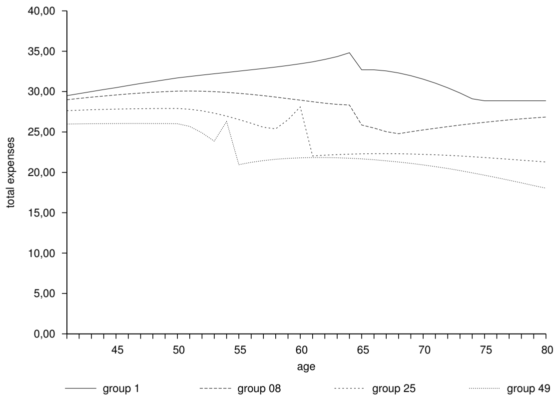
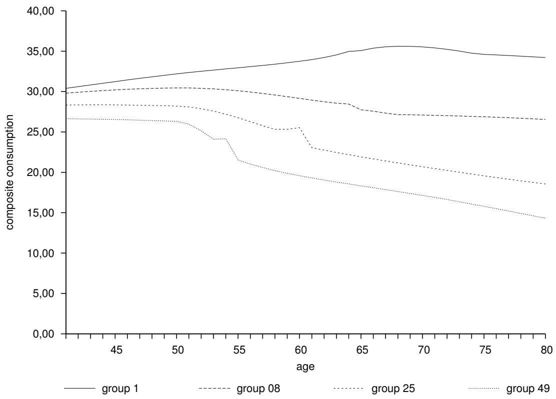


Figure 6.14 Profiles expenses with home prod.



The increase in the relative price of home produced goods is also reflected in the difference between figures 6.14 and 6.15. Although the composite consumption profiles are much smoother for all groups, expenses stabilize more or less after retirement while composite consumption keeps falling, especially for the groups with high variable costs.

Figure 6.15 Profiles composite consumption



7 Results of increase in labour tax

In this section the results of an increase in the labour tax of 1% for the two different models will be compared. It should be noted that this is an uncompensated increase in the tax rate, so it includes wealth effects. The effect on market labour supply is shown in figure 7.1. As expected the reduction in hours worked is similar for the two models, since the labour supply elasticity is similar. Actually the effect is slightly stronger before the age of 50 in the model with home production as the labour supply elasticity is slightly higher in that model. The participation rate shows a sharp drop at the age of 57 as one group decides to retire earlier, but this is not a structural difference but just the result of the discrete choices of groups in the model.

Figure 7.1 **Change in total market labour supply**

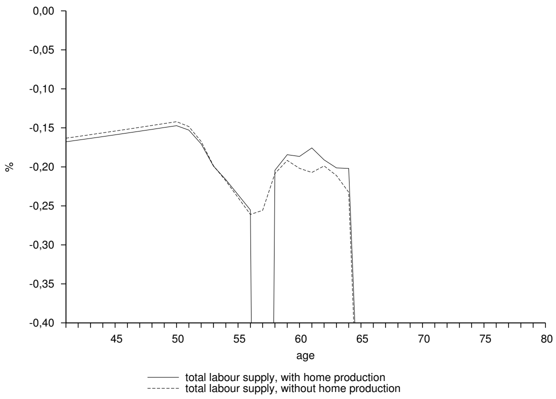
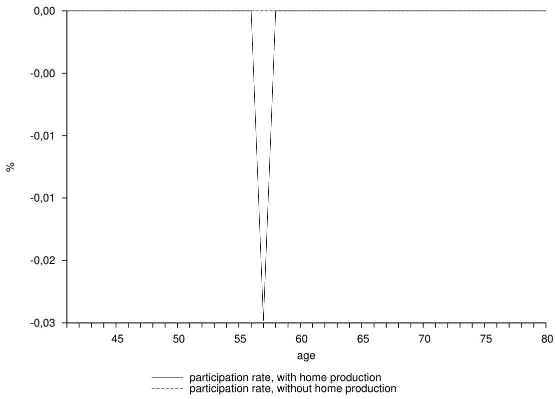


Figure 7.2 **Change in participation rate**



The response in domestic labour supply is shown in figure 7.3. Domestic labour supply increases at all ages as labour supply is shifted from the taxed to the untaxed activity. The percentage change is relatively constant until the mid-50s, after which it falls. The main reason for the constancy of the increase in domestic labour supply is that, until the early 50s, the effect is largely driven by the substitution from market labour, which is also more or less constant until 50.

The reduction in the increase in domestic labour supply after 53 is caused by the (partial) retirement. With (partial) retirement the tax advantage of domestic labour supply becomes smaller, because less time is devoted to working in the market. Of course, the spike in domestic labour supply at 57 is the group which retires earlier and increases it's domestic labour supply as the opportunity cost of home production declines.

The effect on private wealth in figure 7.4 shows that wealth is affected less in the model with home production. The reason is that there is the opportunity to substitute away from the taxed activity, and so relatively less income is needed to consume the same amount of the composite good. The difference in the paths from 56 years onwards is driven by the change in the participation rate of the group that retires earlier. If the participation rate would not be affected, than the paths would continue almost parallel to each other.

Figure 7.3 Change in total domestic labour supply

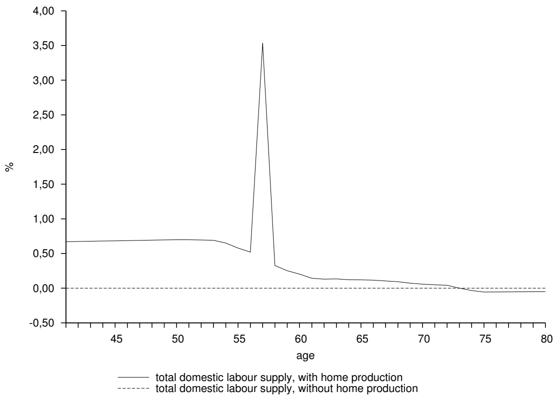
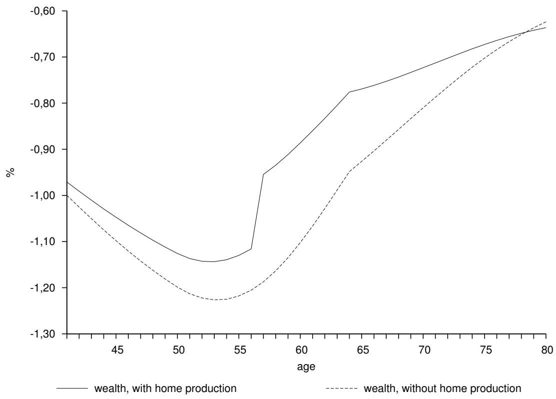


Figure 7.4 Change in total private wealth



As expected the paths for expenses are also little different between the two models as we can see in figure 7.5. The figure for composite consumption is rather different though, as it is much less affected in the model with home production. This is what we would expect since in the model with home production a substitution away from the taxed market labour supply to the untaxed domestic labour supply takes place, which results in a less strong effect on composite consumption.

Figure 7.5 Change in total expenses

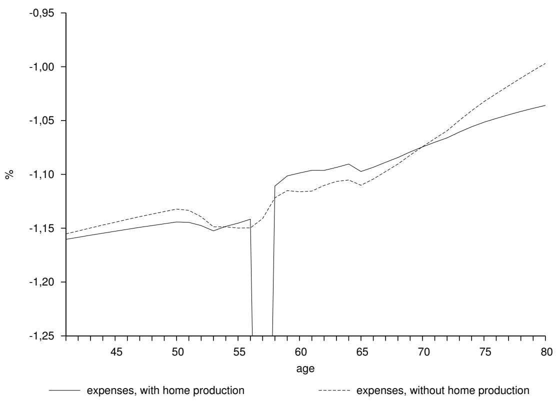
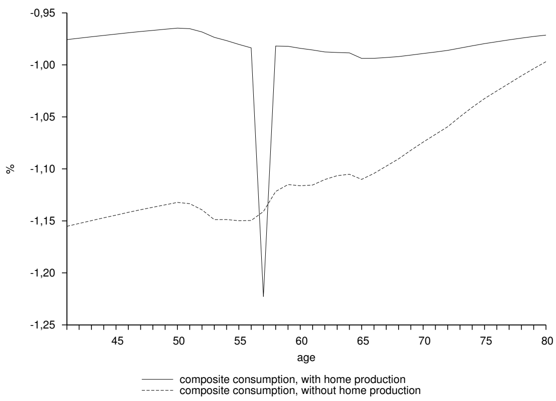


Figure 7.6 Change in total composite cons.



To summarize we see that the effect of a tax increase on labour supply is roughly equal for both models, while the effect on composite consumption is less pronounced in the case with home production which also results in less reduction of wealth.

8 Conclusion

When the labour supply elasticities of the models with and without home production are set roughly equal, the effects on labour supply are very similar as they should be. However, as we have seen in the base scenario wealth and consumption can be altered significantly with the introduction of home production. Especially surprising is that agents save more in the model with home production, while one might have expected that the opportunity of home production reduces the incentives to save for consumption at old age, as the retired agents may be able to produce their own products at lower monetary costs. But as we have seen, the effect is exactly the opposite: the people that age fast and will not produce at home at old age and hence prefer to save more at young ages for consumption at old ages.

It is worth to note though that this result may depend on the model, because ageing affected home production as much as it did working for the market. This contradicts with results from empirical research that have shown that market labour supply is reduced around retirement while domestic labour supply is increased after retirement. Unfortunately we have not been able to simulate the model with such parameter settings ($\delta < 1$). Carrying out simulations like that are therefore a logical extension of this research.

Another issue that may deserve attention in future work is the time constraint. Because the time constraint of the original model left little room for household labour supply, the importance of domestic labour supply in total utility was restricted. More available time can mitigate this, but requires a full recalibration. The results of a model with this alteration should be interesting.

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