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Hourglass models of world-wide problems such as climate change

A simple model of "hourglass" problems is presented. For such problems, the benefit of a national policy measure is propagated to all countries through one single world-wide variable. The prime example is the effect of the reduction of CO_2 emission on the world climate. Five optimal solutions are given, for various situations and points of view, followed by a comparison with the outcome of permit trading.

1 Introduction

In this short paper[1] national policy measures are discussed whose benefit is propagated to all countries through one single world-wide variable. Let the benefits (possibly in the form of evaded damage) for a single country i be as follows:

$$b_i = f_i(Z) \tag{1.1}$$

with

$$Z = g\left(\sum x_j\right) \tag{1.2}$$

The $f_i()$ and $g()$ are functions. The x_j is the policy measure taken by country j. The Z is like the narrow passage at the middle of an hourglass: all effects of the x_j on the b_i run through Z. (The function $f_i(Z)$ may include the satisfaction of country i with evaded damage in other countries.) The costs of the policy measure for country i are given by a cost function $c_i(x_i)$. The Z is a public good which is privately produced by its consumers.

Particularly relevant is the case where the countries vary in size and nature. The prime example is the effect of the reduction of CO_2 emission on the world climate. Other examples are the effect of strategic petroleum reserves on the world oil price (see Table 2 in Mulder et al., 2007) and the effect of university research on the pool of public scientific knowledge.

Substitution of (1.2) into (1.1) gives:

$$b_i = F_i\left(\sum x_j\right) \tag{1.3}$$

with

$$F_i(.) \equiv f_i\left(g(.)\right) \tag{1.4}$$

It is assumed that all F_i are concave (diminishing marginal benefits) and all c_i are convex (increasing marginal costs); at least one of the two must be strictly so, making sure that the second order condition is satified for the maximum problems below.

Part of the discussion below is also found in Finus (2001) and Finus (2002)[2]. See Anthoff et al. (2009) for a recent discussion of equity in the sharing of costs and benefits.

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[2] Beware of a difference in notation: Finus maximizes the profit from the emission of CO_2. His benefit from emission β is our costs of emission reduction c and his damage of emission ϕ is our benefit of emission reduction b.

2 The optimum for one country

The first order condition for a maximum profit for country i, conditional on the x_j of the other countries, is:

$$\frac{\partial\ b_i - c_i\,(x_i)}{\partial x_i} = F_i'\left(\sum x\right) - c_i' = 0 \tag{2.1}$$

or

$$F_i'\left(\sum x\right) = c_i' \tag{2.2}$$

The left-hand side of (2.2) is the marginal benefit of the policy measure, for instance in dollars per ton CO_2 reduction. A non-negative solution for x_i exists only if, with $x_i = 0$, the left-hand side of (2.2) is at least as large as the right-hand side:

$$F_i'\,(\ldots + x_{i-1} + 0 + x_{i+1} + \ldots) \geq c_i'(0) \tag{2.3}$$

Otherwise the other countries already do so much that they have lowered the left-hand side of (2.3) below the right-hand side.

A unique Nash equilibrium exists if the simultaneous system of the equations (2.2) for all i has a unique solution. If the c_i' are constants (i.e., the c_i are not *strictly* convex) then there is in general no Nash equilibrium since then each country has its own optimum value for $\sum x_j$.

3 A large country

Let there be a large country ℓ, consisting of n identical provinces $i = 1,\ldots,n$ with the size of an ordinary country and with identical x_i. Then the benefits are $nb_i = n\,F_i(\sum x)$ and the costs are $nc_i\,(x_i) = nc_i\,(x_\ell/n)$. The summation $\sum x$ runs over over all countries including this large country; see (1.3). The first order condition for an optimum is:

$$\frac{\partial nF_i\,(\sum x) - nc_i\,(x_\ell/n)}{\partial x_\ell} = nF_i' - (n/n)c_i'\,(x_\ell/n) = 0 \tag{3.1}$$

or

$$nF_i' = c_i'\,(x_i) \tag{3.2}$$

Being a large country is advantageous: the marginal benefits increase with a factor n, while the marginal costs do not; each province benefits from the effort of the other provinces. The optimal x_ℓ is larger than n times the optimal x_i.

4 A treaty with uniform proportional action

Instead of a large country, let us consider a treaty between countries. Here is a simple treaty:

$$x_i = \alpha s_i \quad \forall j \in T \tag{4.1}$$

where T is the set of signatories of the treaty and α is positive constant. If T contains all countries, we have a world-wide treaty. The s_i is the size of country i in some metric. For instance, if the treaty equates the percentage reduction of CO_2 emission of the signatories then s_i is the level of CO_2 emission of country i.

The optimum value of α for country i, being a signatory, satisfies:

$$\frac{\partial\, b_i - c_i\left(\alpha s_i\right)}{\partial \alpha} = \left(\sum_{j \in T} s_j\right) F_i' - c_i' s_i = 0 \tag{4.2}$$

or

$$\frac{\sum_{j \in T} s_j}{s_i} F_i' = c_i' \tag{4.3}$$

Compared with (2.2), the marginal benefit is upscaled with the size increase due to the treaty and consequently the treaty increases the optimal x_i.

The combined profit of all signitaries is $\sum_{i \in T} b_i - c_i(x_i)$ and hence the treaty-wide optimal α satisfies the following first order conditions, where the T is omitted for brevity:

$$\frac{\partial \sum_i \left[b_i - c_i\left(\alpha s_i\right)\right]}{\partial \alpha} = \sum_i \left[\left(\sum s_j\right) F_i' - c_i' s_i\right] = 0 \tag{4.4}$$

or

$$\sum F_i' = \frac{\sum c_i' s_i}{\sum s_i} \tag{4.5}$$

Compared with (2.2), the marginal benefit is upscaled to the sum of the national marginal benefits and the marginal costs are the weighted average of the national marginal costs.

5 A treaty with full cooperation

Consider a treaty not restricted by (4.1) above. The first order conditions for a treaty-wide optimum are as follows, where again all summations run over T:

$$\frac{\partial \sum_j \left[b_j - c_j\left(x_j\right)\right]}{\partial x_i} = \sum F_j' - c_i' = 0 \tag{5.1}$$

or

$$\sum F_j' = c_i' \quad \forall i \in T \tag{5.2}$$

This solution equates the national marginal costs to each other, as it should. The solution is reached if each country satisfies (5.2), taking into account the benefits of all others. Finus calls this “full cooperation”. The countries behave as if they are one large country.

Solution (5.2) equals (4.3) for a country i which has an average marginal benefit per unit of size:

$$\frac{F_i'}{s_i} = \frac{\sum F_j'}{\sum s_j} \tag{5.3}$$

This is easily seen when substituting $(\sum s)F_i'/s_i = \sum F'$ into (4.3).

6 A treaty between countries compared with one large country

With the treaty of the previous section, the combined signatories act as if they are one large country. With the treaty of section 4 this is not the case, due to restriction (4.1). However, in the symmetric case, where the signatories are like the provinces of the large country in section 3, the solutions (3.2), (4.3), (4.5) and (5.2) are the same.

Of course, in the case of a treaty the countries still have their national profit $b_i - c_i = F_i(\sum x) - c_i$ to consider. For instance they might argue in favour of (4.3) rather than (4.5). Also, it might be profitable not to sign the treaty. In particular, the condition (2.3) above might not hold; for instance if a country's F_i' function is relatively small (with a small country) or c_i' is relatively large.

7 Tradable permits

Let tradable permissions *not* to act be issued. There is a market for these permits. The demand for permits comes from countries who do not do enough, i.e. they have $x_i < \underline{x}_i$ where $\underline{x}_i$ is a fixed minimum. If x_i is below this minimum then the gap must be filled by buying permits from the countries with $x_i > \underline{x}_i$. The permit price p clears the market:

$$\sum (x_i - \underline{x}_i) = 0 \tag{7.1}$$

Consider the ex post maximum profit for a country i, taking into account the reaction of the other countries such that the market is always cleared. The first order condition for this maximum is:

$$\frac{\partial\, F_i(\sum \underline{x}) - c_i\,(x_i) + (x_i - \underline{x}_i)p}{\partial x_i} = -c_i' + p + (x_i - \underline{x}_i)\,\frac{\partial p}{\partial x_i} = 0 \tag{7.2}$$

or

$$p + (x_i - \underline{x}_i)\,\frac{\partial p}{\partial x_i} = c_i' \tag{7.3}$$

With perfect competition we have

$$\frac{\partial p}{\partial x_i} = 0 \tag{7.4}$$

and (7.3) becomes the least cost solution, equating the marginal costs to each other:

$$p = c_i' \quad \forall i \in T \tag{7.5}$$

Together with (7.1), this gives one equation more than the number of countries, and the same number of variables (the p and the x_i). See Finus (2002), p.61 who states that this result holds only "provided that there is perfect competition!". If the "cap" $\sum \underline{x}$ is choosen such that $\sum F'(\sum \underline{x})$ is equal to the value of p in the solution of (7.5) with (7.1), then we have the same outcome as (5.2).

For the case without perfect competition, see for instance the seminal paper Hahn (1984) and the recent paper Wirl (2009) .

8 Conclusions

We have discussed models where the benefits of all national policy measures are propagated to all countries through one single world-wide variable: "hourglass models". The prime example is the effect of the reduction of CO_2 emission on the world climate.

The optimal policy measure for one country is derived. It does not depend on the benefit of the other countries.

A large country has a large marginal benefit (dollar benefit per dollar cost), and hence a large optimal policy measure.

When some countries sign a treaty to equate their policy measures in some way (such as a uniform percentage reduction of CO_2 emission) then the value of this uniform percentage which is optimal for one signatory country does not depend on the benefit of any other country, signatory or not. However, the treaty increases the signatories' marginal benefits of the measure. It can be profitable for a particular country not to sign the treaty.

The treaty-wide optimum value for this percentage is derived.

A treaty-wide optimum not restricted by such a percentage rule is defined. Using the example of CO_2, in this optimum (a) the signatories' marginal costs per ton CO_2 reduction are equated to each other and (b) they are equal to the treaty-wide marginal benefit per ton CO_2 reduction. The signatories behave as if they are one large country. Again, it can be profitable for a particular country not to sign the treaty.

The outcome (a) is the same as the trade in permits with perfect competion on the permit market.

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