## CPB Discussion Paper

## No 74

December 2006

## Schooling inequality and the rise of research ${ }^{\text {a }}$

Bas Straathof ${ }^{\text {b }}$

[^0]CPB Netherlands Bureau for Economic Policy Analysis
Van Stolkweg 14
P.O. Box 80510

2508 GM The Hague, the Netherlands

Telephone $\quad+31703383380$
Telefax $\quad+31703383350$
Internet www.cpb.nl


#### Abstract

English

During the last twenty years the share of researchers in the workforce has been rising in OECD countries. In the same period, the distribution of schooling has become more equal. This paper proposes that the rise in the proportion of researchers is caused by the decline in schooling inequality. In particular, comparative static analysis of a semi-endogenous growth model demonstrates that a rising proportion of researchers can be a steady state phenomenon when schooling inequality is declining over time. This outcome can be accompanied by a rise in the wages of high-skilled labor compared to low-skilled labor.


Keywords: Schooling inequality; Economic growth; Skill premium
JEL classification: O40, I20, J24


#### Abstract

Dutch

Gedurende de laatste twintig jaar is het aandeel van onderzoekers in de beroepsbevolking gestegen in OESO landen. In dezelfde periode is de verdeling van scholing meer gelijk geworden. Dit artikel stelt dat de groei in het aandeel van onderzoekers het gevolg is van de afname in scholingsongelijkheid. Comparatief statische analyse van een semi-endogeen groeimodel toont aan dat het aandeel van onderzoekers structureel kan toenemen indien scholingsongelijkheid afneemt. Deze uitkomst kan gepaard gaan met een toename in de lonen van hoogopgeleiden ten opzichte van de lonen van laagopgeleiden.


Steekwoorden: Scholingsongelijkheid; Economische groei; Scholingspremie

## Contents

Summary ..... 7
1 Introduction ..... 9
2 The decline in schooling inequality ..... 13
3 The model ..... 19
4 Steady state ..... 23
5 Discussion ..... 25
5.1 Skill effect ..... 25
5.2 Wage effect ..... 27
5.3 Overall effect ..... 28
5.4 Empirical evaluation ..... 29
6 Concluding remarks ..... 31
A Endogenous schooling growth ..... 33
B Additional tables ..... 35
References ..... 37

## Summary

The rising proportion of researchers poses a new challenge to modeling economic growth. This paper proposes an explanation for the rise of research that builds on the decline in schooling inequality. The hypothesis that the rise in the proportion of researchers is caused by the decline in schooling inequality is founded on the assumption that schooling matters more for the productivity of researchers, than it matters for the productivity of other workers. A consequence of this assumption is that people with a high level of education will choose to become researchers, while people with a lower level of education will choose to become production workers.

Whether a person will choose a job in production or a job in research depends on which job gives her a higher income. Her income, in turn, depends on her productivity in the job she has chosen, and on the wage rate per unit of output of that job. A change in the distribution of schooling influences both her productivity and her wage per unit of output. Let us focus first on how a change in the distribution of schooling would affect the productivity of a worker who initially is indifferent between research and production because her income is the same in both jobs. A change in the distribution of schooling will give this 'indifferent worker' a comparative advantage in either research or production, depending on how her schooling compares to the schooling of the rest of the workforce. For example, a reduction in schooling inequality might reduce her level of schooling compared to others. The relative decrease in her level of schooling lowers her relative productivity in both jobs, but her loss in productivity is smallest in production. Therefore, she would no longer be indifferent about the type of job, but instead she would prefer a job in production - provided that wages per unit of output remain unchanged. As a result, the proportion of researchers will decrease. I refer to this as the skill effect.

A change in the distribution of schooling not only changes the productivity of the indifferent worker, but it also has consequences for the wage rates of both job types. Even if individual workers would not change jobs, the aggregate labor productivity of each sector would be affected by a change in the distribution of schooling. For example, a decline in schooling inequality could raise the average productivity of production workers more than the productivity of researchers. This would increase the amount of human capital available to the production sector more than the amount available to the research sector, leading to rise in the wages for researchers compared to the wages of production workers. The corresponding change in wage rates makes a job in research more attractive for the marginal worker. I refer to this as the wage effect.

In the example given above, the skill effect and the wage effect were working in opposite directions. This is not a general result. In fact, the analysis shows that the direction of both effects is an empirical, rather than theoretical, matter. Concerning the skill effect, I present evidence indicating that a reduction in schooling inequality indeed makes production more attractive for the indifferent worker. The direction of the wage effect can be inferred from
evidence on wage inequality. The growth of the college wage premium in the United States is consistent with a wage effect working in the opposite direction of the skill effect, leading to a rise in the proportion of researchers.

In his critique on first generation endogenous growth models, Charles Jones has pointed out that the observed rise in the absolute number of researchers should, according to those theories, have led to ever-increasing rates of economic growth - not to the observed constant growth rates (Jones, 1995b). Since then, a number of models have been proposed that are capable of linking both empirical trends. ${ }^{1}$ More recently, Jones (2002) has drawn attention to the fact that not only the absolute number of researchers is rising over time, but also the proportion of researchers in the workforce. This trend is clearly visible in Figure 1.1, which shows that the aggregate number of researchers measured in full-time equivalent units (FTEs) relative to total employment has been rising in the OECD during the last two decades. ${ }^{2}$

The rising proportion of researchers poses a new challenge to modeling economic growth. Responding to this challenge, Jones (2002) shows that a constant growth rate can be consistent with a rising proportion of researchers when there are exponential returns to education. This paper proposes an alternative explanation for the rise of research, which builds on another empirical trend: the decline in schooling inequality. Figure 1.2 displays the evolution of the proportions of the population that have primary, secondary, and tertiary education as the highest attained level of education. Both the rise of tertiary education and the decline of primary education have contributed to a greater equality in educational attainment. ${ }^{3}$

The hypothesis that the rise in the proportion of researchers is caused by the decline in schooling inequality is founded on the assumption that schooling matters more for the productivity of researchers, than it matters for the productivity of other workers. A consequence of this assumption is that people with a high level of education will choose to become researchers, while people with a lower level of education will choose to become production workers. ${ }^{4}$

Whether a person will choose a job in production or a job in research depends on which job gives her a higher income. Her income, in turn, depends on her productivity in the job she has chosen, and on the wage rate per unit of output of that job. A change in the distribution of schooling influences both her productivity and her wage per unit of output. Let us focus first on how a change in the distribution of schooling would affect the productivity of a worker who

[^1]Figure 1.1 Number of researchers (FTE) in percentage of total employment for the OECD ${ }^{\text {a }}$

${ }^{\text {a }}$ Data have been interpolated at the country level; sources: OECD (MSTI), World Bank (WDI)

Figure 1.2 Proportion of population with primary, secondary, or tertiary eduction for the OECD ${ }^{\text {a }}$


[^2]initially is indifferent between research and production because her income is the same in both jobs. ${ }^{5}$ A change in the distribution of schooling will give this 'indifferent worker' a comparative advantage in either research or production, depending on how her schooling compares to the schooling of the rest of the workforce. For example, a reduction in schooling inequality might reduce her level of schooling compared to others. The relative decrease in her level of schooling lowers her relative productivity in both jobs, but her loss in productivity is smallest in production. Therefore, she would no longer be indifferent about the type of job, but instead she would prefer a job in production - provided that wages per unit of output remain unchanged. As a result, the proportion of researchers will decrease. I will refer to this as the skill effect.

A change in the distribution of schooling not only changes the productivity of the indifferent worker, but it also has consequences for the wage rates of both job types. Even if individual workers would not change jobs, the aggregate labor productivity of each sector would be affected by a change in the distribution of schooling. For example, a decline in schooling inequality could raise the average productivity of production workers more than the productivity of researchers. This would increase the amount of human capital available to the production sector more than the amount available to the research sector, leading to rise in the wages for researchers compared to the wages of production workers. The corresponding change in wage rates makes a job in research more attractive for the marginal worker. I will refer to this as the wage effect.

In the example given above, the skill effect and the wage effect were working in opposite directions. This is not a general result. In fact, the analysis shows that the direction of both effects is an empirical, rather than theoretical, matter. Concerning the skill effect, I present evidence indicating that a reduction in schooling inequality indeed makes production more attractive for the indifferent worker. The direction of the wage effect can be inferred from evidence on wage inequality (see below). The growth of the college wage premium in the United States is consistent with a wage effect working in the opposite direction of the skill effect, leading to a rise in the proportion of researchers.

The approach taken in this paper differs in two respects from the approach taken by Jones (2002). The first difference is that in Jones' model all workers have the same level of education. The second difference concerns the effects of growth in the average level of education. In Jones' model, the combination of growth in the average years of schooling and growth in the population perturbs the steady state. Jones, following Bils and Klenow (2000), incorporates Mincerian returns to time spent on education, such that a constant absolute increase in the time spend on education generates exponential growth in human capital. This relation introduces a scale effect into the model, which, in the presence of population growth, causes the proportion of researchers to rise over time. I follow Arnold (1998) and Strulik (2005) in assuming a constant growth rate

[^3]for the average level of education. ${ }^{6}$ Constant exponential growth in education has approximately the same effects as exogenous population growth and does not prevent the economy from being in a steady state. ${ }^{7}$

One of the possible outcomes of the model presented below is that a decline in schooling inequality raises the wage rate of researchers compared to the wage rate of production workers. Similar results can be found in the literature on skill biased technical change ${ }^{8}$ and in the literature on job assignment ${ }^{9}$. The absence of skill biased technological change makes my model more closely related to the assignment models than to the models on skill biased technical change. In particular, the 'composition effect' discussed by Teulings (2005) is comparable to the 'wage effect' mentioned above. Laitner (2000) studies the relation between the distribution of abilities and wage inequality using a model with endogenous education and unbiased - but exogenous - technological change.

The theoretical interest in the relation between the distribution of skills and wage inequality largely stems from the increase in the college wage premium in the United States in the second half of the twentieth century, which coincided with an increase in the supply of college graduates (Katz and Murphy, 1992; Murnane et al., 1995). The focus of the theoretical literature on this single empirical fact has yielded models that reproduce a negative relation between schooling inequality and the wage premium. However, Goldin (1999) has pointed out that the expansion of secondary schooling in the United States between 1910 and 1940 has led to a reduction in the wage premium (see also Goldin and Katz 2000; 2001a; 2001b). The model presented below can reproduce both a positive and a negative relation depending on the level of schooling inequality.

The decline in schooling inequality is estimated by fitting a cumulative distribution function to data on educational attainment. This distribution function, which is also used in the theoretical model, is introduced in section 2 . The section continues with evidence on the decline of schooling inequality. After the basic model has been presented in section 3, its steady state will be solved for in section 4 . Section 5 starts with a discussion of the wage and skill effects and discusses empirical indications for the direction of the two effects. Section 6 summarises the findings.

[^4]| Table 2.1 | Cumulative weighting schemes |  |  |
| :--- | :---: | :---: | :---: |
| Education | $w_{1}$ | $w_{2}$ | $w_{3}$ |
| None | 0 | 0 | 0 |
| $w_{4}$ |  |  |  |
| Primary | $1 / 3$ | $1 / 2$ | $1 / 4$ |
| Secondary | $2 / 3$ | $3 / 4$ | $3 / 4$ |
| Tertiary | 1 | 1 | 1 |

## 2 The decline in schooling inequality

Figure 1.2 shows that the proportion of the population with primary education only has decreased while the proportion with tertiary education has risen. The figure also suggests that the distribution of schooling has become more uniform over time. However, a quantification of this change in the shape of the distribution requires two problems to be solved. First, it is not entirely clear how this distribution actually looks like, because "educational attainment" is an ordinal variable. This problem can be overcome by assigning arbitrary weights to the four levels of educational attainment. The sensitivity of the outcome of the analysis to the choice of weights can be checked by using several weighting schemes. Table 2.1 shows the four weighting schemes used in the estimation procedure. The first scheme, $w_{1}$, increases with the same amount for each advance in the level of education. The second scheme puts a larger weight on primary education, the third scheme on secondary, and the fourth on tertiary education.

A second problem that needs to be taken care of concerns the choice of a statistic that indicates the 'shape' of the distribution. The approach taken here is to fit a functional form to the distribution, such that the properties of distribution are captured by the parameters of the function. ${ }^{10}$ Because distribution function will also be used in the theoretical model, I will make a small detour, discussing its foundations first.

Order all people in economy according to their level of schooling. This level of schooling, denoted by $k$, is one-dimensional, implying that it reflects some general notion of intelligence or capability. People are indexed from 0 to $L$, where $L$ is both the person with highest level of education and the size of the population. The level of schooling of person $i$ depends on his relative ranking, $i / L$, and on the parameters $s$ and $\sigma(s, \sigma>0)$.

$$
\begin{equation*}
k(i)=(\sigma+1) s(i / L)^{\sigma} \tag{2.1}
\end{equation*}
$$

Integration of $k(i)$ over the workforce shows that $s$ is simply the average level of schooling.

$$
\begin{equation*}
\frac{1}{L} \int_{0}^{L}(\sigma+1) s(i / L)^{\sigma} \mathrm{d} i=\left[s \frac{i^{\sigma+1}}{L^{1+\sigma}}\right]_{0}^{L}=s \tag{2.2}
\end{equation*}
$$

[^5]The chosen specification of $k$ has the advantage that changes in $\sigma$ do not affect average schooling. The shape of the distribution of $k$ can be altered by varying $\sigma$ without affecting the mean of the distribution. The cumulative distribution function (cdf) of $k, F(k)$, can be derived in a straightforward manner by solving equation 2.1 for $i / L$. Differentiating $F(k)$ with respect to $k$ yields the marginal distribution function, $f(k)$.

$$
\begin{align*}
& F(k)=\frac{1}{[(\sigma+1) s]^{\frac{1}{\sigma}}} k^{\frac{1}{\sigma}}  \tag{2.3}\\
& f(k)=\frac{1}{\sigma[(\sigma+1) s]^{\frac{1}{\sigma}}} k^{\frac{1-\sigma}{\sigma}} \tag{2.4}
\end{align*}
$$

The domain of both functions runs from $k(0)=0$ to $k(L)=(\sigma+1) s .{ }^{11}$
This implies that a larger $\sigma$ causes the maximum schooling level to increase, even though $s$ is fixed. A rise in $\sigma$ therefore widens the gap between the people without education and the people with the highest level of education. Besides affecting the domain of $k, \sigma$ also affects its variance and coefficient of variation.

$$
\begin{align*}
\operatorname{var}(k) & =\int_{0}^{(\sigma+1) s} k^{2} f(k) \mathrm{d} k-s^{2}=\frac{\sigma s^{2}}{2+\sigma^{-1}}  \tag{2.5}\\
\operatorname{cv}(k) & =\frac{\sqrt{\operatorname{var}(k)}}{s}=\sqrt{\frac{\sigma}{2+\sigma^{-1}}} \tag{2.6}
\end{align*}
$$

Both the variance and coefficient of variation of $k$ are increasing in $\sigma$. These properties make $\sigma$ a reasonably appropriate measure of inequality and whenever I mention 'inequality' below, I will implicitly refer to $\sigma$.

The data on schooling are taken from the dataset compiled by Barro and Lee (2000; 2001). The variables used are the proportion of the population above 25 for which the highest attained level of schooling is primary, secondary, or tertiary education. ${ }^{12}$ The series have been aggregated for 24 OECD countries using data on the size of the population from the World Bank's World Development Indicators database. ${ }^{13}$

Figure 2.1 shows how the shape of the schooling distribution has changed between 1965 and 2000. The dots are the actual data for weighting scheme $w_{1}$, the curve is the estimated cdf. ${ }^{14}$ The proposed distribution function seems to have a reasonably good fit for all eight years. The shape of the cdf has clearly changed over time: the convex curve of 1965 has become linear in 2000, implying a decrease in inequality.

```
\({ }^{11}\) Plugging back \(k(L)\) into equations 2.1 and 2.3 yields the more intuitive expressions \(k(i)=k(L)(i / L)^{\sigma}\) and \(F(k)=(k / k(L))^{1 / \sigma}\).
\({ }^{12}\) The data reported by Barro and Lee for 2000 are projections.
\({ }^{13} \mathrm{~A}\) list of these countries can be found in appendix \(B\).
\({ }^{14}\) Estimates for \(s\) and \(\sigma\) have been obtained by nonlinear estimation of the function \(F(k)=b_{1}\left(k^{1 / b_{2}}\right)\) using STATA's modified Gauss-Newton algorithm. Parameters \(s\) and \(\sigma\) have been recovered through \(s=\left(\left(b_{2}+1\right) b_{1}^{b_{2}}\right)^{-1}\) and \(\sigma=b_{2}\).
```

Figure 2.1 Cumulative distribution of schooling based on uniform weighting scheme $\left(w_{1}\right)^{\text {a }}$


1970


1980


1990


2000


[^6]In order to assess the robustness of this result, the cdf also has been estimated for weighting schemes $w_{2}, w_{3}$, and $w_{4}$. Figures 2.2 and 2.3 show the evolution of $s$ and $\sigma$, respectively. The mean level of schooling has steadily increased over time for all four weighting schemes. This is not very surprising since it is well-known that the average years of schooling has been rising consistently in the last decades. More remarkable is the robust downward trend of $\sigma$. Figure 2.3 shows that the decline in schooling inequality has been at least as pervasive as the rise in the average level of education. The exact estimation results are reported in table 2.2. ${ }^{15}$ Results for individual countries are reported in tables B. 1 and B. 2 of appendix B.

[^7]Figure 2.2 Mean of schooling distribution (s) by weighting scheme ${ }^{\text {a }}$

a Population above 25; 1965-2000; data sources: Barro and Lee (2000), World Bank (WDI)

Figure 2.3 Shape of schooling distribution ( $\sigma$ ) by weighting scheme ${ }^{\text {a }}$


[^8]Table 2.2 Estimation results for $\sigma$ and $s$

| Year | Statistic | $w_{1}$ | $w_{2}$ | $w_{3}$ | $w_{4}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1965 | $\sigma$ | 2.16 | 1.40 | 2.61 | 2.96 |
|  | $S$ | 0.30 | 0.40 | 0.27 | 0.23 |
|  | $R_{a}^{2}$ | 0.99 | 0.99 | 0.99 | 0.98 |
| 1970 | $\sigma$ | 1.98 | 1.28 | 2.39 | 2.72 |
|  | $s$ | 0.32 | 0.42 | 0.29 | 0.24 |
|  | $R_{a}^{2}$ | 0.99 | 0.99 | 1.00 | 0.98 |
| 1975 | $\sigma$ | 1.82 | 1.18 | 2.20 | 2.49 |
|  | $s$ | 0.34 | 0.44 | 0.31 | 0.26 |
|  | $R_{a}^{2}$ | 0.99 | 0.99 | 1.00 | 0.98 |
| 1980 | $\sigma$ | 1.34 | 0.87 | 1.55 | 1.87 |
|  | $s$ | 0.41 | 0.52 | 0.39 | 0.32 |
|  | $R_{a}^{2}$ | 0.99 | 0.98 | 1.00 | 0.97 |
| 1985 | $\sigma$ | 1.26 | 0.82 | 1.46 | 1.76 |
|  | $s$ | 0.43 | 0.53 | 0.41 | 0.34 |
|  | $R_{a}^{2}$ | 0.99 | 0.98 | 1.00 | 0.97 |
| 1990 | $\sigma$ | 1.15 | 0.75 | 1.33 | 1.61 |
|  | $s$ | 0.45 | 0.56 | 0.44 | 0.36 |
|  | $R_{a}^{2}$ | 0.99 | 0.99 | 1.00 | 0.98 |
| 1995 | $\sigma$ | 1.06 | 0.69 | 1.21 | 1.49 |
|  | $s$ | 0.47 | 0.58 | 0.46 | 0.38 |
|  | $R_{a}^{2}$ | 0.99 | 0.99 | 1.00 | 0.98 |
| 2000 | $\sigma$ | 1.00 | 0.65 | 1.14 | 1.40 |
|  | $s$ | 0.49 | 0.60 | 0.48 | 0.40 |
|  | $R_{a}^{2}$ | 1.00 | 0.99 | 0.99 | 0.98 |

Data sources: Barro and Lee (2000); World Bank (WDI)

## 3 The model

Consumers maximise the discounted stream of instantaneous utility using the subjective discount rate $\rho$.

$$
\begin{equation*}
\max _{\left\{C_{t}\right\}_{0}^{\infty}}\left\{\int_{0}^{\infty} \ln \left(C_{t}\right) \exp [-\rho t] \mathrm{d} t\right\} \tag{3.1}
\end{equation*}
$$

Here, instantaneous utility is assumed to equal the $\log$ of a consumption index $C$. Consumers have CES preferences over $n$ symmetric goods.

$$
\begin{equation*}
C=\left(\int_{0}^{n} x_{j}^{\gamma} \mathrm{d} j\right)^{\frac{1}{\gamma}}=n^{\frac{1}{\gamma}} x \tag{3.2}
\end{equation*}
$$

The elasticity of substitution is determined by the parameter $\gamma$.
The production of $x$ requires an amount of human capital equivalent to $H_{x} / n$. Aggregate consumption is therefore a function of $n$ and $H_{x}$.

$$
\begin{equation*}
C=n^{\frac{1-\gamma}{\gamma}} H_{x} \tag{3.3}
\end{equation*}
$$

The flow of new goods depends on the amount of human capital available for research, $H_{n}$.

$$
\begin{equation*}
\dot{n}=H_{n} \tag{3.4}
\end{equation*}
$$

Entry into the research sector is free, meaning that the value of an invention, $v$, equals the wage rate per unit of human capital, $w_{n}$. Research is funded through the savings of consumers, who in return get a share of the profits, $\pi$, that an invention generates. The rate of return on investing in research is $\pi / v$. The Ramsey rule that follows from utility maximisation is therefore given by

$$
\begin{equation*}
\hat{C}=g_{L}+\frac{\pi}{v_{n}}-\rho \tag{3.5}
\end{equation*}
$$

Here, $\hat{C}$ is the growth rate of consumption and $g_{L}$ is the (exogenous) growth rate of the workforce (a hat denotes a growth rate; $g$ is reserved for fixed growth rates).

There are two types of jobs in the economy: research jobs and production jobs. Workers may freely choose which type of job they take, but are assumed to choose the job that gives them the highest income. Education is valuable for both kinds of jobs, but its effects on productivity differ per job. The level of schooling of person $i$ allows either for a production of $h_{x}(i)$ consumption goods or for the invention of $h_{n}(i)$ new product designs. The exact specifications for a worker's productivity are

$$
\begin{align*}
h_{x}(i) & =a k(i)^{\alpha}=a \tilde{s}^{\alpha}(i / L)^{\sigma \alpha}  \tag{3.6}\\
h_{n}(i) & =b k(i)^{\beta}=b \tilde{s}^{\beta}(i / L)^{\sigma \beta}  \tag{3.7}\\
\tilde{s} & \equiv(\sigma+1) s,
\end{align*}
$$

where $a, b>0$ and $\beta>\alpha \geq 0$. The latter condition ensures that the elasticity of output with respect to schooling is higher for researchers than for production workers. With this setup,
relatively highly educated people will end up in research. What remains to be determined is what level of schooling marks the border between production workers and researchers.

The worker that is indifferent between a production job and a research job is indexed $L_{x}$, such that the workers 0 through $L_{x}$ produce consumption goods and the workers $L_{x}$ through $L$ invent new products. The worker that is indifferent between production and research, must earn the same income with both kinds of jobs.

$$
\begin{equation*}
w_{x} h_{x}\left(L_{x}\right)=w_{n} h_{n}\left(L_{x}\right) \tag{3.8}
\end{equation*}
$$

Here, $w_{x}$ is wage rate per unit of output in production, and $w_{n}$ is the wage rate per unit of output in research. After substitution for $h_{x}$ and $h_{n}$, the ratio of the wage rates can be seen to be related to the allocation of labor.

$$
\begin{equation*}
\frac{w_{x}}{w_{n}}=\frac{b \tilde{s}^{\beta}\left(L_{x} / L\right)^{\sigma \beta}}{a \tilde{S}^{\alpha}\left(L_{x} / L\right)^{\sigma \alpha}}=\frac{b}{a} \tilde{S}^{\beta-\alpha}\left(\frac{L_{x}}{L}\right)^{\sigma(\beta-\alpha)} \tag{3.9}
\end{equation*}
$$

The aggregate amounts of human capital can be found by integration over the appropriate range of the labor force. ${ }^{16}$

$$
\begin{align*}
& H_{x}=\int_{0}^{L_{x}} a \tilde{s}^{\alpha}(i / L)^{\sigma \alpha} \mathrm{d} i=\frac{a \tilde{s}^{\alpha}}{\sigma \alpha+1}\left(\frac{L_{x}}{L}\right)^{\sigma \alpha+1} L  \tag{3.10}\\
& H_{n}=\int_{L_{x}}^{L} b \tilde{s}^{\beta}(i / L)^{\sigma \beta} \mathrm{d} i=\frac{b \tilde{s}^{\beta}}{\sigma \beta+1}\left(1-\left(\frac{L_{x}}{L}\right)^{\sigma \beta+1}\right) L \tag{3.11}
\end{align*}
$$

The profit value ratio, $\pi / v$, follows from the zero profit condition in research, $v=w_{n}$, and the part of consumption that is being paid out as dividends.

$$
\begin{equation*}
\frac{\pi}{v}=\frac{(1-\gamma) C}{n w_{n}}=\frac{1-\gamma}{\gamma} \frac{w_{x}}{w_{n}} \frac{H_{x}}{n} \tag{3.12}
\end{equation*}
$$

After substitution for the ratio of wage rates and human capital employed in production, the profit value ratio becomes

$$
\begin{equation*}
\frac{\pi}{v}=\frac{(1-\gamma) b \tilde{s}^{\beta}}{\gamma(\sigma \alpha+1)}\left(\frac{L_{x}}{L}\right)^{\sigma \beta+1} \frac{L}{n} \tag{3.13}
\end{equation*}
$$

Using this last expression the Ramsey rule can be formulated in terms of $L_{x} / L$ and $L / n$.

$$
\begin{equation*}
\hat{C}=\frac{(1-\gamma) b \tilde{s}^{\beta}}{\gamma(\sigma \alpha+1)}\left(\frac{L_{x}}{L}\right)^{\sigma \beta+1} \frac{L}{n}+g_{L}-\rho \tag{3.14}
\end{equation*}
$$

Equation 3.3 yields another expression for the growth rate of consumption.

$$
\begin{equation*}
\hat{C}=\frac{1-\gamma}{\gamma} \hat{n}+\hat{H}_{x} \tag{3.15}
\end{equation*}
$$

The growth rates of $\hat{H}_{x}$ and $n$ can be obtained from 3.10 and 3.4 together with 3.11.

$$
\begin{align*}
\hat{H}_{x} & =\alpha g_{s}+(\sigma \alpha+1) \hat{L}_{x}-\sigma \alpha g_{L}  \tag{3.16}\\
\hat{n} & =\frac{H_{n}}{n}=\frac{b \tilde{s}^{\beta}}{\sigma \beta+1}\left(1-\left(\frac{L_{x}}{L}\right)^{\sigma \beta+1}\right) \frac{L}{n} \tag{3.17}
\end{align*}
$$

[^9]In the first expression, $g_{s}$ is the exogenous growth rate of mean level of schooling (endogenous growth of the mean schooling level is discussed in appendix A). Substitute for $\hat{n}$ and $\hat{H}_{x}$ to get the growth rate of consumption in terms of $L_{x} / L$ and $L / n$.

$$
\begin{equation*}
\hat{C}=\frac{(1-\gamma) b \tilde{s}^{\beta}}{\gamma(\sigma \beta+1)}\left(1-\left(\frac{L_{x}}{L}\right)^{\sigma \beta+1}\right) \frac{L}{n}+\alpha g_{s}+(\sigma \alpha+1) \hat{L}_{x}-\sigma \alpha g_{L} \tag{3.18}
\end{equation*}
$$

Together, equations $3.14,3.17$, and 3.18 provide sufficient information to study the dynamic behavior of the model.

Before we proceed with the analysis of the dynamic properties of the model, let us first rephrase the condensed model formed by equations $3.14,3.17$, and 3.18 in order to reduce its complexity. Define $\Lambda \equiv L_{x} / L$ and $\lambda \equiv s^{\beta} L / n$. It turns out to be that the steady state of the model coincides with constant values for $\Lambda$ and $\lambda$.

$$
\begin{align*}
\hat{C} & =\frac{(1-\gamma) b(\sigma+1)^{\beta}}{\gamma(\sigma \alpha+1)} \Lambda^{\sigma \beta+1} \lambda+g_{L}-\rho  \tag{4.1}\\
\beta g_{s}+g_{L}-\hat{\lambda} & =\frac{b(\sigma+1)^{\beta}}{\sigma \beta+1}\left(1-\Lambda^{\sigma \beta+1}\right) \lambda  \tag{4.2}\\
\hat{C} & =\frac{(1-\gamma) b(\sigma+1)^{\beta}}{\gamma(\sigma \beta+1)}\left(1-\Lambda^{\sigma \beta+1}\right) \lambda+\alpha g_{s}+(\sigma \alpha+1) \hat{\Lambda}+g_{L} \tag{4.3}
\end{align*}
$$

After substituting out $\hat{C}$ and solving for $\hat{\Lambda}$, we obtain a system of two equations in $\Lambda$ and $\lambda$.

$$
\begin{align*}
& \hat{\Lambda}=\frac{(1-\gamma) b(\sigma+1)^{\beta}}{\gamma(\sigma \alpha+1)}\left(\left(\frac{1}{\sigma \alpha+1}+\frac{1}{\sigma \beta+1}\right) \Lambda^{\sigma \beta+1}-\frac{1}{\sigma \beta+1}\right) \lambda-\frac{\alpha g_{s}+\rho}{\sigma \alpha+1}  \tag{4.4}\\
& \hat{\lambda}=\beta g_{s}+g_{L}-\frac{b(\sigma+1)^{\beta}}{\sigma \beta+1}\left(1-\Lambda^{\sigma \beta+1}\right) \lambda \tag{4.5}
\end{align*}
$$

The steady state of this system is characterised by a constant share of production workers in the labor force, $\Lambda$, and a constant $\lambda$. Setting $\hat{\lambda}=0$ in 4.5 and $\hat{\Lambda}=0$ in 4.4 yields the steady state value of $\lambda$ as functions of $\Lambda^{*}$, the steady state value of $\Lambda$ (steady state levels carry a star).

$$
\begin{align*}
& \lambda^{*}=\frac{(\sigma \beta+1)\left(\beta g_{s}+g_{L}\right)}{b(\sigma+1)^{\beta}}\left(1-\Lambda^{* \sigma \beta+1}\right)^{-1}  \tag{4.6}\\
& \lambda^{*}=\frac{\left[(1-\gamma) g_{L}+\gamma \rho+(\gamma \alpha+(1-\gamma) \beta) g_{s}\right](\sigma \alpha+1)}{(1-\gamma) b(\sigma+1)^{\beta}} \Lambda^{*-\sigma \beta-1} \tag{4.7}
\end{align*}
$$

The first expression has been used to simplify the second expression. Equate both expressions for $\lambda^{*}$ to get a solution for $\Lambda^{*}$ and, after substitution of $\Lambda^{*}$, a solution for $\lambda^{*}$ as well.

$$
\begin{align*}
\Lambda^{*} & =\left(\frac{\Theta}{1+\Theta}\right)^{\frac{1}{\sigma \beta+1}}  \tag{4.8}\\
\lambda^{*} & =\frac{(\sigma \beta+1)\left(\beta g_{s}+g_{L}\right)}{b(\sigma+1)^{\beta}}(1+\Theta)  \tag{4.9}\\
\Theta & \equiv \frac{\sigma \alpha+1}{\sigma \beta+1}\left(1+\frac{\gamma\left(\alpha g_{s}+\rho\right)}{(1-\gamma)\left(\beta g_{s}+g_{L}\right)}\right) \tag{4.10}
\end{align*}
$$

The steady state growth rate of consumption can be retrieved either by substituting for $\lambda^{*}$ in equation 4.1 using 4.7 or by substituting for $\lambda^{*}$ in equation 4.3 using 4.6.

$$
\begin{equation*}
g_{C}=\frac{1}{\gamma} g_{L}+\frac{\gamma \alpha+(1-\gamma) \beta}{\gamma} g_{s} \tag{4.11}
\end{equation*}
$$

As was to be expected of a semi-endogenous growth model, the growth rate of consumption in the steady state depends on the growth rate of the population. In addition, consumption growth depends on the growth rate of the mean level of education, such that steady state economic
growth becomes feasible in the absence of population growth. Similar results have been obtained by Arnold (1998) and Strulik (2005). The presence of education-driven growth is also reflected by the fact that $\Lambda^{*}$ is smaller than one if population growth is zero but schooling growth is positive (see equation 4.8 ). Even when the population is fixed, researchers are employed and new products are introduced to the market. This is why the steady state growth rate of consumption is higher than the rate of productivity growth in the production sector as long as $g_{s}>0$ (remember $\beta>\alpha$ ). However, as both population growth and schooling growth are exogenous, the label 'semi-endogenous' is still appropriate.

The solution for $g_{C}$ in 4.11 could also have been found using a shortcut. The steady state growth rates of $\hat{H}_{x}$ and $\hat{n}$ are given by

$$
\begin{align*}
g_{H_{x}} & =\alpha g_{s}+g_{L}  \tag{4.12}\\
g_{n} & =\beta g_{s}+g_{L} . \tag{4.13}
\end{align*}
$$

Applying these growth rates to equation 3.15 immediately yields the steady state growth rate of consumption. Above expressions clearly illustrate that growth in the average level of schooling raises both the productivity of production workers and researchers. By doing so, advances in education affect economic growth in much of the same way as population growth does.

Figure 5.1 Change in the education of worker $i$ due to a change in inequality


## 5 Discussion

The steady state values of $\lambda$ and $\Lambda$ have been derived in the previous section. Equations 4.8, 4.9, and 4.10 show that the steady state values are dependent on $\sigma$ : schooling inequality matters for the amount of research being done as well as the number of product types available for consumption. The fact that $\sigma$ occurs several times in each of these equations indicates that the impact of a change in $\sigma$ is quite complex. Below we will analyse the effects of a change in $\sigma$ on the steady state in two steps.

In the first step it will be shown how $\sigma$ affects the kind of job - production or research - that is preferred by worker $L_{x}$, while keeping the wage rates constant. This effect of $\sigma$ on the labor market is the skill effect mentioned in the introduction. With the second step it is shown how the wage rates will adjust after the skill effect has taken place. The adjustment of the wage rates naturally causes workers to reconsider their job choice. This second effect is the wage effect. The two steps do not reflect the transitional dynamics of the model and are only used to make the comparative static effects of a change in $\sigma$ more insightful.

### 5.1 Skill effect

A change in the shape of the schooling distribution may have a positive or a negative effect on the schooling of person $i$, depending on his ranking. The education of person $i$ will increase in response to a rise in $\sigma$ if the following condition holds:

$$
\begin{equation*}
i / L>\exp \left[\frac{-1}{\sigma+1}\right] \tag{5.1}
\end{equation*}
$$

This condition is obtained by differentiating equation 2.1 with respect to $\sigma$. A graphical representation is given in figure 5.1.

A change in schooling affects the productivity of a worker, both for production and research. A worker will be more inclined to do research if $w_{x} \mathrm{~d} h_{x}<w_{n} \mathrm{~d} h_{n}$ and he will be more inclined to take a production job if $w_{x} \mathrm{~d} h_{x}>w_{n} \mathrm{~d} h_{n}$ (we keep wages fixed for the moment). The change in the relative attractiveness of the jobs can be found by differentiating equations 3.6 and 3.7 with respect to $k(i)$. The condition below marks the level of schooling at which $w_{x} \mathrm{~d} h_{x}=w_{n} \mathrm{~d} h_{n}$.

$$
\begin{equation*}
k(i)=\left(\frac{\alpha a}{\beta b} \frac{w_{x}}{w_{n}}\right)^{\frac{1}{\beta-\alpha}} \tag{5.2}
\end{equation*}
$$

Figure 5.2 Effect of a rise in education on income (constant wages) and domain of schooling for worker $L_{x}$


Figure 5.3 The skill effect: Type of job chosen by worker $L_{x}$ in response to a rise in inequality (constant wages)


A level of schooling that exceeds this value will encourage workers to do research. If the education of a worker is below this value, a marginal increase in schooling will raise the attractiveness of a production job. People with a low level of education benefit from more education because it makes them more productive in their current occupation. Their productivity as a researcher remains very low, causing their wage gap between production and research to widen in stead of diminish. The reverse applies to highly educated production workers. A rise in their level of schooling will reduce the difference between their current income and the income that they would earn in research. The first line in figure 5.2 shows how the attractiveness of a job depends on the education of the worker. The second line is the domain of the schooling level. Only a part of this domain applies to the indifferent worker $L_{x}$.

In general, a change education can either raise or lower the attractiveness of a job in research, depending on the level of schooling. However, there is only one worker that might actually switch jobs: worker $L_{x}$. Can we be more specific about the incentives faced by $L_{x}$ ? Fortunately, we can. Use equation 3.9 to solve for $k\left(L_{x}\right)$ as a function of the ratio of wages and compare the outcome with the schooling level for which $w_{x} \mathrm{~d} h_{x}=w_{n} \mathrm{~d} h_{n}$.

$$
\begin{equation*}
k\left(L_{x}\right)=\left(\frac{a}{b} \frac{w_{x}}{w_{n}}\right)^{\frac{1}{\beta-\alpha}}>\left(\frac{\alpha a}{\beta b} \frac{w_{x}}{w_{n}}\right)^{\frac{1}{\beta-\alpha}} \tag{5.3}
\end{equation*}
$$

This leaves us with the clean result that if the education of worker $L_{x}$ increases, then he will choose to be a researcher; if his education decreases, he will choose a job in production. This is the skill effect: after a change in $\sigma$, worker $L_{x}$ can improve his income by switching jobs because his level of schooling has changed. Figure 5.3 contains a graphical representation of this result. The domain labeled 'Production' is where worker $L_{x}$ chooses a production job; the domain labeled 'Research' is where he chooses to become a researcher.

## Wage effect

So far, we have analysed the effects of a change in inequality keeping wage rates constant. However, a change in inequality is unlikely to leave wage rates unaffected. The underlying reason is that a change in inequality will affect both types of human capital. If $H_{x}$ and $H_{n}$ change, then - in general - there will be over- or under-investment in research. When this happens a change in wages is required to bring the economy back to the steady state.

The complexity of the model makes it impractical to discuss the impact of a change in $\sigma$ on the wages through its effect on $H_{x}$ and $H_{n}$. In stead, I will discuss the change in the wage rates using the ratio of wage bills as this is mathematically more convenient. An expression for the ratio of the wage bills can be derived using the equation for the wage ratio (3.9) in combination with the definitions of human capital $(3.6,3.7)$.

$$
\begin{equation*}
\frac{w_{x} H_{x}}{w_{n} H_{n}}=\frac{\sigma \beta+1}{\sigma \alpha+1} \frac{1}{\left(\left(\frac{L_{x}}{L}\right)^{-\sigma \beta-1}-1\right)} \tag{5.4}
\end{equation*}
$$

The steady state value of the wage bill ratio follows from substituting $L_{x} / L$ with $\Lambda^{*}$, which is given by 4.8.

$$
\begin{equation*}
\left(\frac{w_{x} H_{x}}{w_{n} H_{n}}\right)^{*}=1+\frac{\gamma\left(\alpha g_{s}+\rho\right)}{(1-\gamma)\left(\beta g_{s}+g_{L}\right)} \tag{5.5}
\end{equation*}
$$

The convenient property of the wage bill ratio is that it is independent of $\sigma$ in the steady state. A change in $\sigma$ will therefore only have temporary effects on the wage bill ratio.

If we differentiate the $\log$ of the wage bill ratio in equation 5.4 with respect to $\sigma$ while keeping $L_{x} / L$ constant, we find that the sign of this derivative depends on $L_{x} / L$.

$$
\begin{equation*}
\frac{\mathrm{d}}{\mathrm{~d} \sigma} \ln \left(\frac{w_{x} H_{x}}{w_{n} H_{n}}\right)=\frac{\beta}{\sigma \beta+1}-\frac{\alpha}{\sigma \alpha+1}+\frac{\beta \ln \left[\frac{L_{x}}{L}\right]}{\left(\frac{L_{x}}{L}\right)^{-\sigma \beta-1}-1} \tag{5.6}
\end{equation*}
$$

The precise value of $L_{x} / L$ for which $\frac{\mathrm{d}}{\mathrm{d} \sigma} \ln \left(\frac{w_{x} H_{x}}{w_{n} H_{n}}\right)=0$ can be found by numerically solving the following equation:

$$
\begin{equation*}
\frac{\beta}{\sigma \beta+1}-\frac{\alpha}{\sigma \alpha+1}=\frac{-\beta \ln \left[\frac{L_{x}}{L}\right]}{\left(\frac{L_{x}}{L}\right)^{-\sigma \beta-1}-1} . \tag{5.7}
\end{equation*}
$$

I will label the solution to this equality $\Lambda^{\#}$. There will be at most one solution as both $\beta \ln \left[\frac{L_{x}}{L}\right]$ and $\left(\left(\frac{L_{x}}{L}\right)^{-\sigma \beta-1}-1\right)^{-1}$ are monotonically increasing in $L_{x} / L$. A higher $\sigma$ causes the wage bill ratio to rise above its steady state value if the proportion of production workers is lower than $\Lambda^{\#}$; the wage bill ratio will decline if $L_{x} / L>\Lambda^{\#}$. The first line in figure 5.4 refers to this "capacity effect" of a change in $\sigma$ on the wage bill ratio.

When the wage bill ratio deviates from its steady state value, an adjustment on the labor market needs to take place to reach the steady state again. Suppose a rise in inequality leads to

Figure 5.4 The wage effect: Adjustment of wage ratio to steady state


Figure 5.5 Job choice by worker $L_{x}$ in response to a rise in inequality: skill and wage effects ${ }^{\mathbf{a}}$

${ }^{a} \exp \left[\frac{-1}{\sigma+1}\right]<\Lambda^{\#}$
an increase in the wage bill ratio, then a return to the steady state requires a decrease in the wage bill for production relative to that of research. This can only be accomplished by a drop in $w_{x}$ relative to $w_{n}$. Alternatively, if $L / L_{x}>\Lambda^{\#}$, then the wage rate for researchers is too high relative to the wage rate for production workers. The second line in figure 5.4 shows how wages adjust to a change in $\sigma$.

### 5.3 Overall effect

Above we have first established the effect of a change in $\sigma$ on job choice keeping wage rates constant. Second, we have established the effect of a change in $\sigma$ on the wage rates assuming that the skill effect has already taken place, such that workers have chosen their jobs in accordance with their education. Combining the two effects allows us to analyse the overall comparative static effects of a rise in $\sigma$. The overall comparative static effects are summarised in figure 5.5. The skill effect is shown on the first line, which is identical to figure 5.3. The second line shows the wage effect assuming that the skill effect has already taken place.

The three dashed arrows represent three scenario's for arriving at a new steady state when inequality increases. The leftmost arrow shows the response of worker $L_{x}$ if a large part of the workforce is employed in research. First, worker $L_{x}$ finds out that his level of education is lower, which induces him to take a production job. Second, the new worker $L_{x}$ is confronted with a decline in $w_{x} / w_{n}$ causing him to become a researcher. Which of the two effects is dominant depends on the precise parameter values. The rightmost arrow describes the opposite situation. Worker $L_{x}$ experiences a rise in schooling and decides to do research. The second worker $L_{x}$

Figure 5.6 Job choice by worker $L_{x}$ in response to a decline in inequality: skill and wage effects ${ }^{\text {a }}$

sees a rise in $w_{x} / w_{n}$ and takes a production job.
The middle arrow shows a situation in which the skill and the wage effects work in the same direction. If $\exp \left[\frac{-1}{\sigma+1}\right]<L_{x} / L<\Lambda^{\#}$ like in figure 5.5 , higher inequality will cause the proportion of researchers in the workforce to increase. If $\Lambda^{\#}<L_{x} / L<\exp \left[\frac{-1}{\sigma+1}\right]$, then the shift will be towards production.

### 5.4 Empirical evaluation

Figure 5.6 summarises the effects of a decline in schooling inequality. Not surprisingly, the figure is a 'mirror image' of figure 5.5 as all effects work exactly in the opposite direction. The theoretical analysis does not suggest that a decline in schooling inequality leads to a rise in the proportion of researchers under all circumstances. In fact, the theory does not even fix the directions of the skill and wage effects. Ultimately, the link between schooling inequality and the proportion of researchers in the workforce remains an empirical question.

The direction of the skill effect depends on the threshold value $\exp [-1 /(\sigma+1)]$. The estimates for $\sigma$ from section 2 can be used to compute this threshold. Tables 5.1 and 5.2 show the estimated thresholds for the four weighting schemes and for individual countries, respectively. The estimated thresholds are smaller than the actual share of researchers in the workforce (fig. 1.1), which implies that the skill effect has a negative effect on the proportion of researchers.

Table 5.1 Estimation results for $\exp [-1 /(\sigma+1)]$

| Year | $w_{1}$ | $w_{2}$ | $w_{3}$ | $w_{4}$ |
| :--- | :---: | :---: | :---: | :---: |
| 1965 | 0.73 | 0.66 | 0.76 | 0.78 |
| 1970 | 0.72 | 0.65 | 0.74 | 0.76 |
| 1975 | 0.70 | 0.63 | 0.73 | 0.75 |
| 1980 | 0.65 | 0.59 | 0.68 | 0.71 |
| 1985 | 0.64 | 0.58 | 0.67 | 0.70 |
| 1990 | 0.63 | 0.56 | 0.65 | 0.68 |
| 1995 | 0.62 | 0.55 | 0.64 | 0.67 |
| 2000 | 0.61 | 0.55 | 0.63 | 0.66 |
|  |  |  |  |  |
| Aggregate of OECD countries |  |  |  |  |

## Table 5.2 Estimation results for $\exp [-1 /(\sigma+1)]$ by country

| Country | 1965 | 1970 | 1975 | 1980 | 1985 | 1990 | 1995 | 0000 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| AUS | 0.65 | 0.61 | 0.62 | 0.62 | 0.61 | 0.61 | 0.60 | 0.59 |
| AUT | 0.69 | 0.70 | 0.70 | 0.65 | 0.64 | 0.64 | 0.63 | 0.62 |
| BEL | 0.77 | 0.76 | 0.75 | 0.73 | 0.72 | 0.70 | 0.70 | 0.69 |
| CAN | 0.69 | 0.64 | 0.61 | 0.58 | 0.57 | 0.56 | 0.55 | 0.54 |
| CHE | 0.78 | 0.70 | 0.71 | 0.64 | 0.64 | 0.63 | 0.62 | 0.62 |
| DEU | 0.66 | 0.66 | 0.69 | 0.66 | 0.64 | 0.63 | 0.62 | 0.62 |
| DNK | 0.65 | 0.65 | 0.65 | 0.65 | 0.64 | 0.62 | 0.63 | 0.62 |
| ESP | 0.93 | 0.92 | 0.90 | 0.84 | 0.82 | 0.76 | 0.73 | 0.70 |
| FIN | 0.86 | 0.78 | 0.75 | 0.69 | 0.71 | 0.63 | 0.62 | 0.61 |
| FRA | 0.79 | 0.79 | 0.77 | 0.72 | 0.70 | 0.70 | 0.68 | 0.66 |
| GBR | 0.79 | 0.75 | 0.72 | 0.71 | 0.69 | 0.68 | 0.66 | 0.65 |
| GRC | 0.90 | 0.88 | 0.86 | 0.80 | 0.77 | 0.75 | 0.72 | 0.70 |
| IRL | 0.78 | 0.77 | 0.76 | 0.70 | 0.69 | 0.66 | 0.64 | 0.63 |
| ISL | 0.86 | 0.83 | 0.79 | 0.76 | 0.73 | 0.71 | 0.69 | 0.67 |
| ITA | 0.87 | 0.84 | 0.83 | 0.77 | 0.75 | 0.73 | 0.71 | 0.70 |
| JPN | 0.72 | 0.74 | 0.71 | 0.67 | 0.66 | 0.63 | 0.62 | 0.60 |
| KOR |  |  |  | 0.71 | 0.66 | 0.63 | 0.60 | 0.59 |
| NLD | 0.87 | 0.68 | 0.68 | 0.67 | 0.66 | 0.64 | 0.63 | 0.62 |
| NOR | 0.81 | 0.74 | 0.72 | 0.68 | 0.67 | 0.58 | 0.57 | 0.57 |
| NZL | 0.66 | 0.66 | 0.60 | 0.58 | 0.58 | 0.61 | 0.60 | 0.59 |
| PRT | 0.95 | 0.93 | 0.90 | 0.88 | 0.86 | 0.83 | 0.82 | 0.80 |
| SWE | 0.72 | 0.72 | 0.70 | 0.66 | 0.65 | 0.64 | 0.59 | 0.58 |
| TUR | 0.94 | 0.93 | 0.92 | 0.90 | 0.88 | 0.87 | 0.83 | 0.83 |
| USA | 0.64 | 0.63 | 0.61 | 0.55 | 0.55 | 0.53 | 0.53 | 0.52 |
| Weighting scheme $w 1$ |  |  |  |  |  | 0 |  |  |

With the skill effect working in the opposite direction, the rise of researcher therefore has to stem from the wage effect. One indication for the presence of a wage effect is a change in the ratio of the wage rates for high and low-skilled workers. In fact, wage inequality has been growing substantially during the last decades - a fact that has been well documented in the literature (Katz and Murphy, 1992; Murnane et al., 1995, and others).

## 6 Concluding remarks

During the last forty years the shape of the distribution of schooling has been changing. This has led to a decline in schooling inequality. The theoretical analysis presented in this paper shows that this development can have a variety of effects on the proportion of researchers in the workforce. Whether the decline in schooling inequality has a positive effect on the proportion of researchers therefore becomes an empirical question. The widely observed increase in the college wage premium indicates that the decline in schooling inequality might indeed be responsible for the rise of research.

The analysis has demonstrated that a rising proportion of researchers can be a steady state phenomenon when schooling inequality is declining over time. This result contrasts with the hypothesis of Jones (2002), who suggests that the rise of research is a consequence of advances in the mean level of education. Further evidence is needed in order to be able to discriminate between the two theories.

## Appendix A Endogenous schooling growth

The assumption of section 3 that the average level of schooling grows at an exogenous and constant rate, $\hat{s}=g_{s}>0$, has been made for analytical convenience. However, in real life education is not free and therefore growth in the mean level of education requires growth in resources devoted to education. This appendix discusses two cases for which constant growth in mean schooling is feasible in the steady state.

In order to avoid notational changes in the model, assume that the population, $P$, consists of the normal workforce, $L$, and the part of the population being a teacher or student, $P_{s}$ ( $P=P_{s}+L$ ). Furthermore, suppose that the change in mean level of education is affected by the amount of human capital per capita that is available for education, $H_{S} / P$, and by a discount factor, $\delta$. In particular, the change in mean education is given by $\dot{s}=H_{s} / P-\delta s$. Human capital depends on the people involved in education activities and on their average education, which is assumed to equal that of the population: $H_{s}=s^{\varepsilon} P_{s}, 0 \geq \varepsilon \geq 1$. (Better educated teachers will teach more effectively; better educated students will learn quicker.) Substituting for $H_{s}$ and dividing by $s$ gives an expression for $\hat{s}$.

$$
\begin{equation*}
\hat{s}=s^{\varepsilon-1} \frac{P_{s}}{P}-\delta \tag{A.1}
\end{equation*}
$$

Define $\Lambda_{s} \equiv P_{s} / P$ and take the growth rate of $(\hat{s}+\delta)$ to get

$$
\begin{equation*}
\mathrm{d} \ln (\hat{s}+\delta) / \mathrm{d} t=(\varepsilon-1) \hat{s}+\hat{\Lambda}_{s} . \tag{A.2}
\end{equation*}
$$

This last expression implies that there can be two specifications that allow for a constant and positive growth rate of mean education in the steady state. First, $\varepsilon=1$ in combination with $\hat{P}_{s}=\hat{P}$ yields $g_{s}=\Lambda_{s}^{*}-\delta$. With this specification, growth in education stems entirely from the positive effect of schooling as an input on schooling as an output, while the proportion of people involved in education remains constant. This specification has been proposed by Lucas (1988) and Rosen (1976). See also Arnold (1998) and Strulik (2005).

Second, if $\varepsilon<1, \mathrm{~d} \ln (\hat{s}+\delta) / \mathrm{d} t$ will go to zero as time proceeds. Setting $\mathrm{d} \ln (\hat{s}+\delta) / \mathrm{d} t=0$ yields $\hat{s}=\frac{1}{1-\varepsilon} \hat{\Lambda}_{s}$. Steady state schooling growth can only be positive if the proportion of the population active in education is growing, but for this proportion to grow at a constant rate, the population should grow at a different rate than the workforce. If both $\hat{\Lambda}_{s}$ and $\hat{L}$ are to be constant, the population should grow according to $\hat{P}=g_{\Lambda_{s}} \frac{P_{s}}{L}+g_{L}$.

The results presented above demonstrate that the growth rate of mean education can be positive and constant in the steady state, but only under restrictive assumptions. A more detailed and general treatment of the effects of schooling on economic growth is given by Bils and Klenow (2000).

## Appendix B Additional tables

| Table B. 1 | Estimation results for the mean level of schooling (s) by country and year, uniform weights ( $w_{1}$ ) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Country | 1965 | 1970 | 1975 | 1980 | 1985 | 1990 | 1995 | 2000 |
| AUS | 0.41 | 0.47 | 0.46 | 0.47 | 0.47 | 0.48 | 0.49 | 0.51 |
| AUT | 0.33 | 0.32 | 0.32 | 0.39 | 0.40 | 0.42 | 0.43 | 0.45 |
| BEL | 0.24 | 0.25 | 0.27 | 0.29 | 0.31 | 0.34 | 0.35 | 0.37 |
| CAN | 0.37 | 0.43 | 0.49 | 0.54 | 0.55 | 0.57 | 0.60 | 0.63 |
| CHE | 0.24 | 0.33 | 0.32 | 0.42 | 0.42 | 0.43 | 0.45 | 0.46 |
| DEU | 0.38 | 0.37 | 0.34 | 0.38 | 0.41 | 0.42 | 0.45 | 0.46 |
| DNK | 0.42 | 0.41 | 0.41 | 0.42 | 0.43 | 0.46 | 0.45 | 0.47 |
| ESP | 0.07 | 0.08 | 0.11 | 0.17 | 0.20 | 0.26 | 0.30 | 0.35 |
| FIN | 0.14 | 0.23 | 0.27 | 0.35 | 0.33 | 0.43 | 0.46 | 0.48 |
| FRA | 0.21 | 0.21 | 0.24 | 0.31 | 0.33 | 0.34 | 0.37 | 0.40 |
| GBR | 0.21 | 0.27 | 0.31 | 0.33 | 0.35 | 0.37 | 0.39 | 0.41 |
| GRC | 0.10 | 0.12 | 0.15 | 0.21 | 0.25 | 0.28 | 0.31 | 0.34 |
| IRL | 0.23 | 0.23 | 0.25 | 0.33 | 0.35 | 0.40 | 0.42 | 0.44 |
| ISL | 0.14 | 0.18 | 0.22 | 0.26 | 0.30 | 0.33 | 0.36 | 0.39 |
| ITA | 0.13 | 0.15 | 0.18 | 0.24 | 0.27 | 0.29 | 0.32 | 0.34 |
| JPN | 0.30 | 0.27 | 0.32 | 0.38 | 0.40 | 0.45 | 0.47 | 0.49 |
| KOR |  |  |  | 0.32 | 0.39 | 0.44 | 0.48 | 0.51 |
| NLD | 0.13 | 0.35 | 0.36 | 0.38 | 0.40 | 0.42 | 0.45 | 0.46 |
| NOR | 0.18 | 0.28 | 0.31 | 0.36 | 0.38 | 0.52 | 0.53 | 0.55 |
| NZL | 0.37 | 0.38 | 0.50 | 0.53 | 0.53 | 0.51 | 0.52 | 0.53 |
| PRT | 0.05 | 0.07 | 0.10 | 0.12 | 0.15 | 0.18 | 0.20 | 0.23 |
| SWE | 0.30 | 0.31 | 0.34 | 0.40 | 0.41 | 0.43 | 0.51 | 0.52 |
| TUR | 0.06 | 0.06 | 0.08 | 0.10 | 0.12 | 0.14 | 0.18 | 0.19 |
| USA | 0.43 | 0.45 | 0.48 | 0.58 | 0.59 | 0.63 | 0.64 | 0.65 |
| Data source: Barro and Lee (2000) |  |  |  |  |  |  |  |  |


| Table B. 2 | Estimation results for schooling inequality ( $\sigma$ ) by country and year, uniform weights ( $w_{1}$ ) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Country | 1965 | 1970 | 1975 | 1980 | 1985 | 1990 | 1995 | 2000 |
| AUS | 1.29 | 1.04 | 1.09 | 1.07 | 1.03 | 1.01 | 0.97 | 0.91 |
| AUT | 1.74 | 1.80 | 1.79 | 1.29 | 1.26 | 1.22 | 1.17 | 1.09 |
| BEL | 2.81 | 2.72 | 2.45 | 2.21 | 2.01 | 1.85 | 1.79 | 1.71 |
| CAN | 1.66 | 1.27 | 1.01 | 0.83 | 0.81 | 0.75 | 0.67 | 0.61 |
| CHE | 2.94 | 1.84 | 1.91 | 1.22 | 1.21 | 1.17 | 1.11 | 1.07 |
| DEU | 1.39 | 1.42 | 1.65 | 1.41 | 1.23 | 1.20 | 1.12 | 1.08 |
| DNK | 1.28 | 1.32 | 1.30 | 1.28 | 1.26 | 1.07 | 1.15 | 1.08 |
| ESP | 13.09 | 11.02 | 8.20 | 4.81 | 3.90 | 2.70 | 2.16 | 1.78 |
| FIN | 5.67 | 3.11 | 2.44 | 1.73 | 1.92 | 1.20 | 1.10 | 1.02 |
| FRA | 3.32 | 3.25 | 2.90 | 2.04 | 1.83 | 1.76 | 1.59 | 1.42 |
| GBR | 3.26 | 2.46 | 2.08 | 1.93 | 1.74 | 1.57 | 1.44 | 1.34 |
| GRC | 8.11 | 6.95 | 5.54 | 3.46 | 2.84 | 2.40 | 2.07 | 1.78 |
| IRL | 2.99 | 2.91 | 2.70 | 1.81 | 1.64 | 1.37 | 1.27 | 1.18 |
| ISL | 5.74 | 4.32 | 3.29 | 2.61 | 2.17 | 1.88 | 1.65 | 1.47 |
| ITA | 5.90 | 4.86 | 4.22 | 2.78 | 2.45 | 2.19 | 1.97 | 1.80 |
| JPN | 2.03 | 2.35 | 1.93 | 1.50 | 1.37 | 1.13 | 1.06 | 0.98 |
| KOR |  |  |  | 1.90 | 1.44 | 1.16 | 0.99 | 0.91 |
| NLD | 6.10 | 1.61 | 1.58 | 1.49 | 1.37 | 1.27 | 1.16 | 1.09 |
| NOR | 3.87 | 2.34 | 1.99 | 1.63 | 1.50 | 0.81 | 0.79 | 0.76 |
| NZL | 1.43 | 1.38 | 0.93 | 0.85 | 0.85 | 1.02 | 0.96 | 0.90 |
| PRT | 19.87 | 12.26 | 8.45 | 7.15 | 5.43 | 4.31 | 3.95 | 3.41 |
| SWE | 2.09 | 2.03 | 1.83 | 1.36 | 1.31 | 1.23 | 0.89 | 0.86 |
| TUR | 14.73 | 13.44 | 11.29 | 8.30 | 6.91 | 5.90 | 4.37 | 4.35 |
| USA | 1.24 | 1.13 | 1.05 | 0.68 | 0.66 | 0.58 | 0.56 | 0.54 |
| Data source: Barro and Lee (2000) |  |  |  |  |  |  |  |  |

Table B. 3 Country codes

| AUS | Australia | FIN | Finland | KOR | Republic of Korea |
| :--- | :--- | :--- | :--- | :--- | :--- |
| AUT | Austria | FRA | France | NLD | Netherlands |
| BEL | Belgium | GBR | United Kingdom | NOR | Norway |
| CAN | Canada | GRC | Greece | NZL | New Zealand |
| CHE | Switzerland | IRL | Ireland | PRT | Portugal |
| DEU | Germany | ISL | Iceland | SWE | Sweden |
| DNK | Denmark | ITA | Italy | TUR | Turkey |
| ESP | Spain | JPN | Japan | USA | United States |

## References

Acemoglu, D., 1998, Why do new technologies complement skills? directed technical change and wage inequality, Quarterly Journal of Economics, vol. 113, no. 4, pp. 1055-89.

Acemoglu, D., 2002, Technical change, inequality, and the labor market, Journal of Economic Literature, vol. 40, no. 1, pp. 7-72.

Arnold, L.G., 1998, Growth, welfare, and trade in an integrated model of human-capital accumulation and research, Journal of Macroeconomics, vol. 20, no. 1, pp. 81-105.

Azuma, Y. and H.I. Grossman, 2003, Educational inequality, Labour, vol. 17, no. 3, pp. 317-35.

Barro, R.J. and J.W. Lee, 2000, International data on educational attainment updates and implications, Tech. Rep. 7911, National Bureau of Economic Research.

Barro, R.J. and J.W. Lee, 2001, International data on educational attainment: Updates and implications, Oxford Economic Papers, vol. 53, no. 3, pp. 541-63.

Bils, M. and P.J. Klenow, 2000, Does schooling cause growth?, American Economic Review, vol. 90, no. 5, pp. 1160-83.

Costrell, R.M. and G.C. Loury, 2004, Distribution of ability and earnings in a hierarchical job assignment model, Journal of Political Economy, vol. 112, no. 6, pp. 1322-63.

Dinopoulos, E. and P. Thompson, 1998, Schumpeterian growth without scale effects, Journal of Economic Growth, vol. 3, no. 4, pp. 313-35.

Dupuy, A. and P. Marey, 2005, Shifts and Twists in the Relative Productivity of Skilled Labor, Unpublished manuscript.

Galor, O. and O. Moav, 2000, Ability-biased technological transition, wage inequality, and economic growth, Quarterly Journal of Economics, vol. 115, no. 2, pp. 469-97.

Goldin, C., 1999, Egalitarianism and the returns to education during the great transformation of american education, Journal of Political Economy, vol. 107, no. 6, p. 94.

Goldin, C. and L.F. Katz, 2000, Education and income in the early twentieth century: Evidence from the prairies, Journal of Economic History, vol. 60, no. 3, pp. 782-818.

Goldin, C. and L.F. Katz, 2001a, Decreasing (and then increasing) inequality in america: A tale of two half-centuries, in F. Welch, ed., The causes and consequences of increasing inequality, pp. 37-82, University of Chicago Press, Chicago and London.

Goldin, C. and L.F. Katz, 2001b, The legacy of u.s. educational leadership: Notes on distribution and economic growth in the 20th century, American Economic Review, vol. 91, no. 2, pp. 18-23.

Ha, J. and P. Howitt, 2006, Accounting for trends in productivity and r\&d: A schumpeterian critique of semi-endogenous growth theory, Journal of Money, Credit, and Banking (forthcoming).

Howitt, P., 1999, Steady endogenous growth with population and r\&d inputs growing, Journal of Political Economy, vol. 107, no. 4, pp. 715-30.

Jones, C.I., 1995a, R\&d-based models of economic growth, Journal of Political Economy, vol. 103, no. 4, pp. 759-84.

Jones, C.I., 1995b, Time series tests of endogenous growth models, Quarterly Journal of Economics, vol. 110, no. 2, pp. 495-525.

Jones, C.I., 2002, Sources of u.s. economic growth in a world of ideas, American Economic Review, vol. 92, no. 1, pp. 220-39.

Katz, L.F. and K.M. Murphy, 1992, Changes in relative wages, 1963-1987: Supply and demand factors, Quarterly Journal of Economics, vol. 107, no. 1, pp. 35-78.

Laitner, J., 2000, Earnings within education groups and overall productivity growth, Journal of Political Economy, vol. 108, no. 4, pp. 807-32.

Li, C.W., 2000, Endogenous vs. semi-endogenous growth in a two-r\&d-sector model, Economic Journal, vol. 110, no. 462, pp. C109-22.

Lucas, R.E., 1988, On the mechanics of economic development, Journal of Monetary Economics, vol. 22, no. 1, pp. 3-42.

Murnane, R.J., J.B. Willett and F. Levy, 1995, The growing importance of cognitive skills in wage determination, Review of Economics and Statistics, vol. 77, no. 2, pp. 251-66.

OECD, 2004, Main Science and Technology Indicators (MSTI), Paris.

Peretto, P.F., 1998, Technological change and population growth, Journal of Economic Growth, vol. 3, no. 4, pp. 283-311.

Peretto, P.F. and S. Smulders, 2002, Technological distance, growth and scale effects, Economic Journal, vol. 112, no. 481, pp. 603-24.

Ram, R., 1990, Educational expansion and schooling inequality: International evidence and some implications, Review of Economics and Statistics, vol. 72, no. 2, pp. 266-74.

Rosen, S., 1976, A theory of life earnings, Journal of Political Economy, vol. 84, no. 4, p. 67.

Strulik, H., 2005, The role of human capital and population growth in r\&d-based models of economic growth, Review of International Economics, vol. 13, no. 1, pp. 129-45.

Teulings, C.N., 1995, The wage distribution in a model of the assignment of skills to jobs, Journal of Political Economy, vol. 103, no. 2, pp. 280-315.

Teulings, C.N., 2005, Comparative advantage, relative wages, and the accumulation of human capital, Journal of Political Economy, vol. 113, no. 2, pp. 425-61.

World Bank, 2003, The 2003 World Development Indicators CD-ROM.

Young, A., 1998, Growth without scale effects, Journal of Political Economy, vol. 106, no. 1, pp. 41-63.


[^0]:    ${ }^{\text {a }}$ I would like to thank Lex Borghans, Robin Cowan, Arnaud Dupuy, Robert Dur, Gregory Jolivet, Joan Muysken, Richard Nahuis, Michael Sattinger, Sjak Smulders, Eddy Szirmai, Bas ter Weel, and Adriaan van Zon for comments and discussions. Financial support from the Netherlands Organisation for Scientific Research (NWO) is gratefully acknowledged. Part of the research was done while I was affiliated with UNU-MERIT (University of Maastricht) and ECIS (Eindhoven University of Technology).
    ${ }^{\text {b }}$ CPB Netherlands Bureau for Economic Policy Analysis, P.O. Box 80510, NL-2508GM The Hague, The Netherlands, email: s.m.straathof@cpb.nl.

[^1]:    ${ }^{1}$ See for example Jones (1995a); Young (1998); Peretto (1998); Dinopoulos and Thompson (1998); Howitt (1999); Li (2000); Peretto and Smulders (2002).

    2 The 24 OECD countries are listed in Table B.3. Jones (2002) presents data for G5 countries that show a rising proportion since 1950.
    ${ }^{3}$ The decline in schooling inequality has also been reported by Ram (1990), who finds that there exists an inverse relationship between schooling inequality and the average years of schooling if the average years of schooling in a country exceeds seven. Detailed evidence on the decline in schooling inequality is provided in section 2 of this paper.
    ${ }^{4}$ The analysis put forward in this paper can be generalised by allowing some production jobs to be as sensitive to schooling as research jobs. In this case, some workers with a higher level of education will have production jobs.

[^2]:    a Highest level of education attained; population above 25; data sources: Barro and Lee (2000), World Bank (WDI)

[^3]:    ${ }^{5}$ The existence of such an 'indifferent worker' is guaranteed if the level of schooling is distributed continuously over all workers.

[^4]:    ${ }^{6}$ Appendix A treats endogenous education.
    ${ }^{7}$ Ha and Howitt (2006) observe that the growth rate of R\&D workers has declined threefold since 1953, while productivity growth has remained constant. They argue that semi-endogenous growth models are inconsistent with these observations, but this is not the case if semi-endogenous growth is driven by advances in education. Moreover, they also observe that a sustained fraction of GDP is devoted to R\&D. Equation 5.4 shows that this is true for the model in this paper - even if population growth is zero.
    ${ }^{8}$ Acemoglu (1998; 2002), Galor and Moav (2000), Azuma and Grossman (2003)
    9 Teulings (1995; 2005), Costrell and Loury (2004), Dupuy and Marey (2005)

[^5]:    ${ }^{10}$ An alternative approach is to estimate several moments of the distribution (variance, skewness, etc.). The advantage of such an approach is that no assumptions have to be made about the type of distribution. The disadvantage is that it does not yield a single statistic that summarises the shape of the distribution. This is particularly problematic considering the very small number of observations.

[^6]:    a Population above 25; 1965-2000; data sources: Barro and Lee (2000), World Bank (WDI)

[^7]:    ${ }^{15}$ Considering the extremely small number of observations, I have omitted all of the usual test statistics, except for the adjusted $R^{2}$.

[^8]:    a Population above 25; 1965-2000; data sources: Barro and Lee (2000), World Bank (WDI)

[^9]:    16 The constants of integration are zero because without workers there will be no production.

