CPB Discussion Paper

No 130

Welfare analysis in transport networks

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ISBN 978-90-5833-418-3

Abstract in English

Should one calculate user benefits from changes in door-to-door journeys or from changes in the use of separate links of the network? Quite often, the second approach is deemed wrong, as consumers are supposed to demand journeys, not parts of journeys. However, we show that for a quite general economic model and under fairly general assumptions regarding the network, both approaches are equivalent. The cost-benefit analysis practitioner can exploit this result. The links approach reveals on what part of the networks user benefits and/or losses are generated. This additional piece of information might help to optimize the project design.

Keywords: Cost-benefit analysis, transport networks

JEL codes: D61, H54, R42

Abstract in Dutch

Moet men welvaartsbaten afmeten aan veranderingen in deur-tot-deur verplaatsingen of aan veranderingen in het gebruik van afzonderlijke delen van het netwerk? De tweede benadering wordt doorgaans als fout bestempeld omdat verondersteld wordt dat de vraag van consumenten betrekking heeft op integrale verplaatsingen, niet op afzonderlijke onderdelen van een verplaatsing. We laten echter zien dat, voor een tamelijk algemeen economisch model en onder tamelijk algemene veronderstellingen ten aanzien van de eigenschappen van het netwerk, beide benaderingen tot een identieke uitkomst leiden. In de KBA-praktijk kan men hier handig gebruik van maken. De tweede benadering laat immers zien op welke delen van een netwerk de winsten of verliezen voor gebruikers ontstaan. Deze informatie kan men aanwenden om het ontwerp van het project te optimaliseren.

Steekwoorden: Kosten-batenanalyse, transport netwerken

Contents

Summary

Should one calculate user benefits on the basis of the change in door-to-door journeys or derive them from the change in the use of each and every separate link of the networks affected? Economists mostly favour the first approach since consumers think in terms of door-to-door journeys. However, since policy measures typically involve a change in the user costs of parts of the networks, one is tempted to use the second approach.

To investigate this question, we first develop a concise general equilibrium model elaborating on Kidokoro (2004, 2006). His model consists of consumers, producers, a government, a network and a congestion externality. We extend his model in two directions. First, we replace his elementary network of two routes between two nodes by a full-fledged network consisting of an undetermined number of links between an undetermined number of nodes for an undetermined number of modalities. Second, we replace his representative consumers living at a particular node by heterogeneous consumers living 'somewhere'.

In this model, the 'door-to-door journeys' are called routes, the separate parts of the networks are called links. Utility is defined as a function of the consumption of routes, not links, as it should be. Nevertheless, it turns out that in equilibrium both the demand for routes and the demand for links and the costs of using the routes and the costs of using the links all plays a role. For measuring welfare we follow the standard procedure of first deriving the expression for indirect utility. We show that indirect utility might be either cast in terms of equilibrium prices for the use of routes or cast in terms of equilibrium prices for the use of links. Then we proceed by deriving the rule-of-a-half (ROH) as an approximation for the change in the user benefits. We show that the ROH can both be evaluated in terms of the use of routes and in terms of the use of links. The two approaches yield exactly the same outcome.

This result implies that the choice for either approach does not depend on theory but on practical considerations. The link approach might give less precise outcomes in case of a new link. In that case, we propose a step-by-step approach for the new link and the most affected competing links. The routes approach lacks precision as well. Since the number of routes is sheer endless, routes always have to be aggregated into origin-destination-matrices. However, the conditions that allow for aggregation are hardly ever being met. From a real-life case, a cost-benefit analysis for a high-speed rail link between Amsterdam and Brussels, we showed that aggregation of journeys which are not perfect substitutes does indeed yield considerable measurement errors. Hence, we recommend the links approach as a complement, or even as a substitute, for the OD-matrix approach.

1 Introduction[1]

Network projects raise many questions for the cost-benefit analysis practitioner. One of the hardest nuts to crack regards the unit of analysis: should one calculate user benefits on the basis of the change in door-to-door journeys or derive them from the change in the use of each and every separate link of the networks affected? Economists mostly favor the first approach since consumers think in terms of door-to-door journeys. However, since policy measures typically involve a change in the user costs of parts of the networks, one is tempted to use the second approach. A number of questions are ultimately related to this same issue. Should a change in the out-of-pocket costs of car travel be treated as a welfare benefit? And what if the change in out-of-pocket costs is due to modal shift? And in case of modal shift, should one use the value of travel time (VOT) of the old or the new transport mode? We will show that the journey, or route approach and the link approach are theoretically equivalent: whether one derives the change in welfare from a change in the demand for routes or from a change in the demand for links, the outcome will be the same.

To proof this, we develop a concise general equilibrium model elaborating on Kidokoro (2004, 2006). His model consists of consumers, producers, a government, a network and a congestion externality. We extend his model in two directions. First, we replace his elementary network of two routes between two nodes by a full-fledged network consisting of an undetermined number of links between an undetermined number of nodes for an undetermined number of modalities. Second, we replace his representative consumers living at a particular node by heterogeneous consumers living 'somewhere'. Kidokoro shows that one ought to measure user benefits from changes in the demand for each of the two routes separately, rather than from the change in the average cost of travel between the node of origin and the node of destination. If both routes are perfect substitutes one might settle for the origin-destination approach, but Kidokoro sees no surplus value in doing that. We carry the analysis one step further by considering an undetermined number of links on each route between a node of origin and a node of destination. We argue that one might calculate user benefits both on the basis of the use of the routes and on the basis of the use of the separate links.

Sugden (1979) pointed already to this way of making welfare assessments of transport networks more down to earth, but in day-to-day practice his suggestion was not followed. Textbook models invariably focus on door-to-door journeys between origins and destinations, the OD-matrix approach. This applies to both the older literature (e.g. Jones, 1977) and the more recent (e.g. Jara-Díaz, 2007). The same is true for CBA guidelines. Mackie et al (2003, Note 6, p. 7) even write explicitly: "User benefits should be calculated on a matrix basis and not a link basis". Unfortunately, they do not explain why. Their recommendation might stem from

[1] We benefited from numerous discussions over the last few years with quite some CBA practitioners. We thank Carel Eijgenraam, Toon van der Hoorn, Gerard de Jong, Sytze Rienstra, Bas Turpijn, Erik Verhoef, Nol Verster and Peter Zwaneveld for their encouragements and comments on earlier versions.

practical considerations. Possibly they favor the OD-matrix approach as a way to deal with problems that arise if the project at hand introduces a new link in the network. Therefore, after having established the theoretical equivalence of the two main approaches, we will reflect on a number of practical issues one encounters in applying the theory.

In section 2, we will first model the network. We define routes as a combination of adjacent links. This is a wider concept of routes than usually employed in the sense that one particular route might comprise links that belong to different transport modes. We will establish the precise relation between the use of routes and the use of links and we will make fairly general assumptions about the relation between the costs of using the routes and the costs of using the links. Then we include the network in a general equilibrium model. Utility is a function of the 'consumption of routes', not links. Nevertheless, in equilibrium both the demand for routes and the demand for links and the costs of using the routes and the costs of using the links all play a role.

Section 3 deals with welfare measurement. We follow the standard procedure of first deriving the expression for indirect utility, which might be either in terms of equilibrium prices for the use of routes or in terms of equilibrium prices for the use of links. By employing Roy's identity, this yields a measure for the change in welfare as the sum of the change in income plus the changes in demand induced by changes in prices. Again, the changes in demand could be either the demand for routes induced by the changes in the user cost of routes or the demand for links induced by the changes in the user cost of links. Then we proceed by deriving the rule-of-a-half (ROH) as an approximation for the change in the user benefits. We show that the ROH can both be evaluated in terms of the use of routes and in terms of the use of links. The two approaches yield exactly the same outcome. This implies that the choice for either approach does not depend on theory but on practical considerations.

In section 4, we explore some practical issues. Regarding the link approach, we note that the approximation by the ROH might be particularly imprecise in case of a new link. In the guise of Nellthorp and Hyman (2001) and Kato et al (2003) we advocate a step-by-step method for calculating user benefits on such a link. This requires some extra work. Regarding the route approach, we observe that no practitioner can ever calculate welfare changes on each and every route, because of the sheer endless number of routes. Therefore, one has to aggregate routes into a manageable number, e.g. by reducing the network to a synthetic OD-matrix, before starting the welfare measurement. We discuss the limitations of this approach slightly more extensively than did Kidokoro (2004). We illustrate it with a real-life case of a CBA for a new high speed rail track, where overlooking the limitations of the OD-matrix approach led to serious errors. The conclusions are summarized in section 5.

2 Model and network

We present a model building on Kidokoro (2004). Trips are the economic goods, consumed and produced, of which equilibrium prices come about on markets. Congestion makes that a gap arises between the private and social costs of a trip. This externality is modeled on the production side of the economy. Government intervenes in the markets for trips with indirect taxes, such as tolls, and with investment in the capacity of the transportation network. Though welfare analysis of these investments can be done in the context of partial equilibrium the full general equilibrium model developed in this section provides a complete reference. It will serve to demonstrate the equivalence between general equilibrium analysis and cost benefit analysis.

As the goods involved are trips over routes and links of a transportation network we elaborate on this first.

2.1 Network

We consider a transportation network comprised of the actually available infrastructure for different modes of transportation. This abstract network is a graph G characterised by a set V of vertices, or nodes, and a set A of arcs, or the direct connections between the nodes, called links. A link is typically a road segment but it is also the service of a bus company between two successive stops of a given line. A link may even be a path for pedestrians between a parking lot and a railway station. A common representation of graph $G=(V,A)$ is by means of its adjacency matrix which indicates for each pair of nodes whether they are adjacent or not, and if so, by how many distinct links. Though there will not be a link between each arbitrary pair of nodes we do assume that all nodes can be reached from all other nodes: the graph is connected. Even an island that cannot be reached by car will be serviced by boat or plane.

A route is a set of connected links. A route is also a set of adjacent nodes. The links of a network themselves are routes too. A given network has an infinite number of routes. Simple routes are those without cycles: a link is at most once in a route. A simple graph is defined by the absence of multiple links and the absence of self-loops. A complete graph has a link between each pair of nodes.

Let N be the number of nodes i of a given network. Let the network have a finite set of links l. If the network were simple and complete it would have exactly $N(N-1)$ links. The number of simple routes r over the network is always finite too. It must be realized that this number soon becomes enormous. A simple and connected network with $N=10$ has some 10 million simple routes. However, in an economic context with cost minimisation, most of these routes will never be eligible for use.

The economic goods in relation to the network are trips, or journeys, over this network: movements from one node to another via a route r. Let x_r^R be the number of trips over such a specific route r of the network in a predetermined period. Use of a route implies the use of one or more links in the same period. The next flow variable is x_l^L , representing the number of movements over a specific link l .

Let M be the matrix indicating from which links a route is composed. Element M_{lr} is 1 when link l is part of route r and is 0 otherwise. The relation between link and route usage is as follows: total link use is equal to the sum of the use of those routes it is part of.[2]

$$x_l^L = \sum_r M_{lr} x_r^R \tag{2.1}$$

Next consider the cost of trips over links, c_l^L , and over routes, c_r^R . For some components of these costs addition over links may be straightforward. Think of distance, or time. However this additivity is less obvious for the valuation of time travelled. Nevertheless such a linear relation is often assumed (see for instance Wohl en Hendrickson, 1985, p.49): the cost of a route is the sum of the cost of the composing links.

$$c_r^R = \sum_l c_l^L M_{lr} \tag{2.2}$$

With the above definitions and relations the economic model can be developed, as we will do below. The point of departure is a general and multi-modal network. Equations (2.1) and (2.2) will be the recurring relations between the routes and the links of the given network. And all information required of the structure of the network is contained in the route-composition matrix M . Observe that in vector notation equation (2.2) implies a matrix pre-multiplication, making use of the convention among economists that prices and costs are represented by row-vectors.

$$x^L = Mx^R \qquad \text{en} \qquad c^R = c^L M$$

Origin and destination pairs (OD)

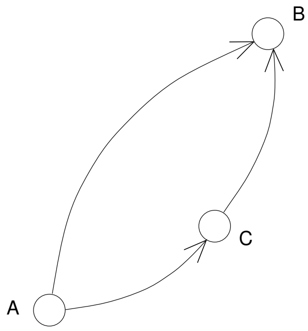
As an extension of the notation consider x_{ijk}^R , the number of trips from node i, an origin, to node j , a destination, over a certain route k. OD-pairs play an important role in traffic models and hence we sometimes employ the *ijk* notation in stead of the more general index r for routes.

Example

Finally consider as an example the following minimal network of 3 nodes, 3 links and 4 routes. This deviates from the classical example of two nodes and multiple links.

[2] For similar modelling see for instance Van Dender (2004).

Figure 2.1 Network with 3 links and 4 routes



The nodes are A, B and C. The links are the one-way connections AB, AC and CB. The four routes are AB, AC, AC + CB and CB. The third route is composed of two connecting links. This can also be seen in the third column of the route-composition matrix.

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \tag{2.3}$$

The one-way-traffic makes that there are just three OD-pairs: (A,B), (A,C) and (C,B). The OD-pair (A,B) knows two alternative routes, which are AB and AC + CB.

2.2 Consumers

At the heart of the deliberations of the consumers are routes: will I make a nice tour on my motorcycle today or visit my aunt, long overdue, by car? Routes, or rather trips over routes, are the arguments in the utility functions and are what is demanded by the consumers. Links are mere intermediate goods, necessary for the production of routes. Links are final consumption when considered as a simple route.

Kidokoro (2004) introduces N different locations *i* as the nodes of a network. He associates with each location *i* a collection of consumers, modelled as a single, representative, consumer *i*. This agrees with the practice of transportation modelling. We generalize by considering a set of heterogeneous households *h*, not necessarily associated with a given location. These different categories of households can make use of each part of the network.

Utility of the *h*-th consumer, U_h, is determined by consumption of a composite good z_h and consumption of a vector x_h^R of trips x_{hr}^R.[3] These trips are characterised by nodes *i* and *j*, and a

[3] Superscripts are mainly used to distinguish between Routes and Links. They are avoided for partial derivatives.

route *k* that connects them. Most of the time we denote this by a route *r*. Substitutability and complementarity of trips is contained in the utility function which is assumed to be concave.

$$U_h = U_h(z_h, x_h^R) \text{ with } x_h^R = (\dots, x_{hr}^R, \dots) \tag{2.4}$$

Income y_h consists of the value of the factors of production in the hands of the consumer (endowment of resources) $\overline{y}_h$. In addition there may be a transfer T_h. For ease of exposition we assume the existence of only a single productive factor that corresponds one-on-one with the composite good. Let p_h^R be the vector of the individual route prices p_{hr}^R pertaining to the consumers. These prices are normalised with the price for the singe factor. This gives the following budget constraint.

$$z_h + p_h^R x_h^R = y_h = \overline{y}_h + T_h \quad \text{where} \quad p_h^R x_h^R = \sum_r p_{hr}^R x_{hr}^R \tag{2.5}$$

For an interior solution, $x_{hr}^R > 0$, we will have that relative trip prices equal the marginal utilities. This amounts to standard micro-economics applied to trips as commodities.

$$\frac{\partial U_h}{\partial x_{hr}^R} = \lambda_h p_{hr}^R \text{ and } \frac{\partial U_h}{\partial z_h} = \lambda_h \quad \text{where } \lambda_h \text{ is the marginal utility of income} \tag{2.6}$$

Wardrop principle as a special case

A special case arises with the Wardrop-principle (Wardrop, 1952), where the functional form of the utility function is such that alternative routes *k* between given locations *i* and *j* are taken to be perfect substitutes. Consumers, given this specification, only care about reaching a given destination from a given point of departure. This can be accomplished by having consumption of trips over the different routes between *i* and *j* additively in the utility function. This implies that the marginal utilities of all routes 1 and 2 from *i* to *j* are always equal. The important consequences of this special case for transportation modelling are discussed below.

$$U_h = U_h(z_h, \dots, x_{hij1}^R + x_{hij2}^R + \dots, \dots) \text{ implying } \frac{\partial U_h}{\partial x_{hij1}^R} = \frac{\partial U_h}{\partial x_{hij2}^R} \tag{2.7}$$

2.3 Producers and congestion

The producer model first of all concerns links as they are the appropriate level of supply of transportation services and thereby to identify the costs. Consider therefore production of trips over a single link *l*. Let the total resource cost C_l^L of producing the total flow X_l^L of link use be a function of this flow and of a capacity parameter I_l. As these total costs contain monetized time cost, which may differ per individual consumer *h*, individual components will be included.[4]

Producers simply charge the consumers their individual average link use costs $c_{hl}^L = c_{hl}^L(X_l^L, I_l)$. They make no profits, nor losses. Further, observe the simplification that link costs are independent of the flows and capacities of other links.

$$C_l^L(X_l^L, I_l) = \sum_h c_{hl}^L(X_l^L, I_l) x_{hl}^L \quad \text{with} \quad X_l^L = \sum_h x_{hl}^L \tag{2.8}$$

Average link costs increase in both total and individual use. Costs decrease with more capacity.

$$\frac{\partial c_{hl}^L}{\partial x_{hl}^L} = \frac{\partial c_{hl}^L}{\partial X_l^L} > 0, \quad \frac{\partial c_{hl}^L}{\partial I_l} < 0 \tag{2.9}$$

There will be a gap between the marginal social cost mc_{hl} of producing trips x_{hl}^L and the average cost per unit charged to the private consumer. Let the congestion externality per unit use t_{hl}^{ext} be defined as that difference. And since the source of an additional car on a road segment will not matter for the degree of congestion it causes on that link, the externality will be equal for all consumers: t_l^{ext}. This result was already contained in the assumption of having the total flow of trips as argument in the cost function. Below subscript *h'* is a help index for summation.

$$mc_{hl} = \frac{\partial C_l^L}{\partial x_{hl}^L} = c_{hl}^L(X_l^L, I_l) + \sum_{h'} \frac{\partial c_{h'l}^L}{\partial x_{hl}^L} x_{h'l}^L \tag{2.10}$$

$$t_{hl}^{ext} = mc_{hl} - c_{hl}^L = \sum_{h'} \frac{\partial c_{h'l}^L}{\partial x_{hl}^L} x_{h'l}^L = \sum_{h'} \frac{\partial c_{h'l}^L}{\partial X_l^L} x_{h'l}^L = t_l^{ext} > 0 \tag{2.11}$$

For longer routes, covering several links, the assumption is that link costs just can be added.

$$c_{hr}^R = \sum_l c_{hl}^L M_{lr} \quad \text{(in vector notation: } c_h^R = c_h^L M \text{)} \tag{2.12}$$

2.4 Government

Government intervenes in the transportation markets with indirect taxes t_l^{ind} per unit link use, possibly to counter congestion. Consumer prices then consist of the individual average link cost plus this tax, taken equal for all.

[4] Individual time preferences are modelled as costs. Thus it can be avoided to introduce time as a separate commodity.

$$p_{hl}^{L} = c_{hl}^{L}(X_{l}^{L}, I_{l}) + t_{l}^{ind} \quad (\, p_{hl}^{L} = mc_{hl} - t_{l}^{ext} + t_{l}^{ind} \,) \tag{2.13}$$

Route prices then contain all the tolls on the composing links.

$$p_{hr}^{R} = \sum_{l} p_{hl}^{L} M_{lr} = \sum_{l} (c_{hl}^{L} + t_{l}^{ind}) M_{lr} \quad (\, p_{h}^{R} = p_{h}^{L} M \,) \tag{2.14}$$

Indirect tax revenue is lump sum redistributed. Finally, the total outlays on public investment I are added to the government account. Thus the description of the accounting of the model is completed.

$$\sum_{h} \sum_{l} t_{l}^{ind} x_{hl}^{L} = \sum_{h} T_{h} + I \tag{2.15}$$

2.5 Equilibrium

Consistent accounting is a prerequisite for a general equilibrium model. Next is the optimality of the decisions of the agents given prices. For the consumers this has been described above. For production the model is just too simple to consider optimality. Government is exogenous. Finally markets must clear and on these market equilibrium prices, endogenous variables of the model, must come about.

We have modelled markets, and hence prices, for link and route use. Both play a role. Though consumer preferences are formulated in terms of routes, links are the intermediate goods needed to produce them. The model is based on fairly general assumptions: no restrictions on the network have been imposed, nor has a specific functional form of utility been chosen. With this general framework we will examine the practical implications for welfare measurement.

Wardrop

In recognition of its practical importance, we return briefly to the specific model of perfect substitutability of the alternative routes of a given origin-destination pair (*i,j*). To satisfy optimality condition (2.5) the endogenous route prices must be equal for the different routes between *i* and *j*. This is the Wardrop-principle.

$$\frac{\partial U_{h}}{\partial x_{hij1}^{R}} = \frac{\partial U_{h}}{\partial x_{hij2}^{R}} \quad \Leftrightarrow \quad p_{hij1}^{R} = p_{hij2}^{R} \tag{2.16}$$

The number of trips over the alternative routes will adjust such that its costs will be equal. This holds for those routes actually used. Alternative routes with higher costs will not be used.

Congestion makes that there is indeed scope for adjustment. When costs no longer can adjust corner solution will arise. Corner solutions will be seen to have practical consequences.

3 Welfare measurement

Based on the general equilibrium framework described above, we will derive expressions for welfare change. We will do so in terms of routes as well as links. Next we will show that Kidokoro's 2004 decompositions of welfare change still hold. These decompositions help in establishing exactly what the welfare effects are, and what are not, following investment in a transportation network. The discussion of welfare measurement, in particular of the rule of half, prepares for the part on practical issues.

3.1 Welfare analysis in a distorted economy

The economy of our model has two distortions,[5] congestion and indirect taxes. When the latter cancelled the former they are corrective, Pigouvian, taxes.

$$p_{hl}^{L} - mc_{hl} = c_{hl}^{L} + t_{l}^{ind} - (c_{hl}^{L} + t_{l}^{ext}) = t_{l}^{ind} - t_{l}^{ext} \tag{3.1}$$

Consider a linear social welfare function W with welfare weights α_h. With the weights set to the inverse of the marginal utilities of income, e.g. $\alpha_h = 1/\lambda_h$, the welfare function represents the principle of 'a dollar is a dollar'. Moreover, with these weights the social optimum can be decentralized as the equilibrium outcome of the economy, also in the case of our distorted economy.[6]

$$W = \sum_{h} \alpha_h U_h(z_h, x_h^R) \quad \text{with} \quad \alpha_h = 1/\lambda_h \tag{3.2}$$

Using budget equations (2.4) and (2.14) and the fundamental relation between the costs of routes and link use, equation (3.2) is rewritten. Social welfare is broken down in overall consumer surplus (CS), producer surplus (PS) and net government income (BS).[7] [8]

$$W = \{ \sum_{h} (\alpha_h U_h(z_h, x_h^R) - y_h) \} + \{ \sum_{h} \overline{y}_h \} + \{ \sum_{h} \sum_{l} t_l^L x_{hl}^L - I \} \tag{3.3}$$

Producer surplus (PS) consists of the value of endowments and profit. The latter is zero. However, when changes are involved, marginal cost, and hence the congestion externality, will be visible again. Net government income (BS) equals total transfers. The above breakdown

[5] 'A distortion is an inequality of the marginal rate of substitution and the marginal rate of transformation between a pair of commodities', Bhagwati (1971).

[6] With quasi-linear utility the welfare weights are one and inconsequential.

[7] See, for instance, Ginsburgh & Keyzer, 1997, paragraph 2.4.2.

[8] The breakdown gives overall consumer surplus. Consumer surplus at market level, possibly better recognized, will appear shortly.

shows that working with consumer and producer surplus implies that either indirect taxes or transfers need to be included.

Public investment projects are modelled as a vector of changes of the capacity parameters: dI_l. They are scaled such that they coincide with investment costs. The projects are financed with lump sum transfers from the consumers so that the government budget remains balanced. Since exogenous dI_l are the triggers of change it figures to first consider price changes that change demand that subsequently change the level of welfare. This chain of causality requires a reformulation of social welfare in terms of indirect utility.[9]

First take indirect utility as a function of income and route prices as an alternative for eq. (3.2).

$$W = \sum_h \alpha_h V_h(p_h^R, y_h) \qquad \text{with} \quad \alpha_h = 1/\lambda_h \tag{3.4}$$

We now derive an expression for welfare change by taking the total differential of W and applying Roy's identity. The investment is assumed to only involve a single link *l*.

$$\begin{aligned} dW &= \sum_h \alpha_h \left(\sum_r \frac{\partial V_h}{\partial p_{hr}^R} \frac{dp_{hr}^R}{dI_l} dI_l + \frac{\partial V_i}{\partial y_h} \frac{dy_h}{dI_l} dI_l \right) \\ &= \sum_h \sum_r \left(-x_{hr}^R(p_h^R, y_h) dp_{hr}^R + dy_h \right) \end{aligned} \tag{3.5}$$

Next indirect utility can be taken as a function of income and link prices to reconsider (3.2).

$$W = \sum_h \alpha_h V_h(p_h^L, y_h) \qquad \text{with} \quad \alpha_h = 1/\lambda_h \tag{3.6}$$

Again applying Roy's identity leads to an identical expression in the demand of link use.

$$dW = \sum_h \sum_l \left(-x_{hl}^L(p_h^L, y_h) dp_{hl}^L + dy_h \right) \tag{3.7}$$

Direct substitution of the relations between routes and links, (2.1) and (2.13), in eq. (3.5) leads to the same result. The important practical significance of this result will be discussed below.

Expressions (3.5) and (3.7) deserve some special attention as what follows is based upon them.

[9] One can also work with the expenditure function and define the welfare measures of compensating and equivalent variation .

First of all, we are still in the realm of general equilibrium. This can be seen from the use of the Marshallian demand functions $x_{hr}^{R}(p_h^R, y_h)$, or $x_{hl}^{L}(p_h^L, y_h)$, which has as arguments income and the prices on all markets.

Secondly, we see that the welfare weights have disappeared in a natural way. Therefore it is unnecessary restrictive to avoid them beforehand by choosing a quasi-linear utility function, as Kidokoro does.

Next, it is relevant to point out that welfare change, by definition, is a net concept. In the world of CBA of infrastructure projects however, there is an emphasis on benefits, with costs taken as given. Here we consider changes. The change in CS, PS and government income we denote as the change of total benefits dTB. The change in total costs, dTC, are the investment expenditures in the capacities of all links, dI. Below a distinction will be made between a link with investment and all other links.

$$dW = dTB - dTC = dTB - dI \quad \text{with} \quad dI = \sum_l dI_l \tag{3.8}$$

A last preparatory comment concerns the consumer surplus visible in equations (3.5) and (3.7) which is now at market level. Lower consumer prices will lead to a higher surplus. The equations can be represented as below. The distortions will reappear in the decompositions that follow.

$$dW \quad = \sum_h \sum_r dCS_{hr}^R + \sum_h dy_h \quad = \sum_h \sum_l dCS_{hl}^L + \sum_h dy_h \tag{3.9}$$

3.2 Decompositions of welfare change

Investment in the capacity of part of a network will change the entire pattern of movements over that network. Moreover, travel cost will not only change for those links and routes with investment but also for all other links and routes. With all these effects it matters crucially to determine which of these, or not, contribute to welfare change. Kidokoro (2004) presents to this purpose three decompositions of welfare change. They involve the loss of welfare as a consequence of the distortions, the deadweight loss. We will establish that the decompositions still hold in the context of a general network, with the assumption that link costs are only a function of own capacity and its use.

Total benefits consist of, the changes in: i) consumer surplus of all links plus indirect tax revenue, or ii) consumer surplus of the link with investment plus indirect taxes on that link plus the net deadweight loss of all other links, or iii) total cost saving of the link with investment plus the net deadweight loss of all links. We will now derive these decompositions.

Changes in income depend on the size of the investment and the effect on indirect tax revenue. The latter will change because the pattern of trips changes. Summation over consumers makes the lump sum redistribution disappear.

$$\sum_h dy_h = \sum_h \sum_l t_l^{ind} dx_{hl}^L - dI \tag{3.10}$$

Substitution in (3.7) and (3.8) gives Kidokoro's first result: total benefits, $dW + dI$, are the changes in the consumer surplus of all links plus the change in indirect tax revenue.

$$dTB = \sum_h \sum_l dCS_{hl}^L + \sum_h \sum_l t_l^{ind} dx_{hl}^L \tag{3.11}$$

For the second result changes in quantities and prices are written as functions of the investments at the root of the changes. Here a distinction is made between the link with investment in its capacity: l^0 and all other links l". Also, for summation over all other links we use l".
Finally, the set of all links, that is including the one with investment, is still denoted by the simple subscript l. An investment on a single link can affect the capacity of a host of routes.

We aim to derive an expression for the change in travel cost of a given link l, given the investment in another link l^0. Take the differential of price equation (2.12).[10] We assume that the tariffs of the indirect taxes do not change.

$$dp_{hl}^L = \frac{\partial c_{hl}^L(X_l^L, I_l)}{\partial X_l^L} \frac{dX_l^L}{dI_{l^0}} dI_{l^0} + \frac{\partial c_{hl}^L(X_l^L, I_l)}{\partial I_{l^0}} \frac{dI_l}{dI_{l^0}} dI_{l^0} \tag{3.12}$$

The second term on the right-hand side of the equation is zero for all links but those with investment. This is because costs of links have been assumed to be independent.

$$dp_{hl"}^L = \frac{\partial c_{hl"}^L(X_{l"}^L, I_{l"})}{\partial X_{l"}^L} \frac{dX_{l"}^L}{dI_{l^0}} dI_{l^0} \tag{3.13}$$

The price change multiplied with the number of trips and subsequently summed over all consumers makes the externality on the other link l" appear. See definition (2.10).

$$\sum_h x_{hl"}^L dp_{hl"}^L = \sum_h x_{hl"}^L \frac{\partial c_{hl"}^L(X_{l"}^L, I_{l"})}{\partial X_{l"}^L} \frac{dX_{l"}^L}{dI_{l^0}} dI_{l^0} = t_{l"}^{ext} \frac{dX_{l"}^L}{dI_{l^0}} dI_{l^0} \tag{3.14}$$

[10] This is the crucial step. See page 303 of Kidokoro (2004).

Substitution of this equation in (3.7), using (3.10) and rearranging gives:

$$dTB = \sum_h dCS_{hl^0} + t_{l^0}^{ind} dX_{l^0}^L + \sum_{l''} (t_{l''}^{ind} - t_{l''}^{ext}) dX_{l''} \tag{3.15}$$

The change in total benefits following an investment in a single link l^0 consists of the change in the consumer surplus of that link, plus the change of indirect tax revenue of that same link, plus the change in the net deadweight losses of all other links. The deadweight loss is caused by both the externality and the indirect tax. This is Kidokoro's second welfare decomposition.

The third result comes about by a comparable derivation which does use the second term on the right hand of equation (3.12). This decomposition emphasizes the distortions in all links. It shows the distinction between first best and second best positions.

$$dTB = -\frac{\partial C_{l^0}^L}{\partial I_{l^0}} X_{l^0}^L dI_{l^0} + \sum_l (t_l^{ind} - t_l^{ext}) dX_l^L \tag{3.16}$$

The first term on the right-hand side denotes the cost saving for all traffic, i.e. not just increase, over the link with investment. In the absence of net distortions this would be the only welfare effect.

3.3 Welfare measurement & approximations

Alas, utility and welfare cannot be measured directly. The expressions above, however, do contain, not quite accidentally, variables that can in principle be measured. Though measuring congestion externalities may be difficult, it is not impossible. Another issue remains problematic: the expressions above are based on small (infinitesimal) changes. For larger and discrete changes the differential changes of above should be integrated over an initial and a new situation.[11] Departing from equation (3.7) we have the following.[12]

$$\Delta W = \int dW = -\sum_h \sum_l \int x_{hl}^L (p_h^L, y_h) dp_{hl}^L + \sum_h \Delta y_h \tag{3.17}$$

This measure is not unique since it depends on the order of integration. Even if it is a first-order approximation, demand functions do depend on all prices. This is the problem of path dependency. On top of that the demand function will not completely be known. With a few 'observations' they may be approximated. An 'observation' of the new situation, i.e. after

[11] See f.i. Boadway and Bruce (1984), par. 7.2.

[12] Consumer surplus now appears as integrals over Marshallian demand functions.

investment, may be supplied by a traffic model. Such a model could be a partial equilibrium model. The values of the variables in the initial situation should be available.

We proceed with a second-order approximation of social welfare, using a linear approximation of the demand functions. Thus the so-called 'rule of half' will be derived.

The welfare function in (3.6) is expanded around the point of the initial situation. This implies that also the marginal utilities of income of the old situation are used to convert utility to a money measure.

$$dW = \sum_h \alpha_h \left(\sum_l \frac{\partial V_h}{\partial p_{hl}^L} \frac{dp_{hl}^L}{dI_l} dI_l + \frac{\partial V_h}{\partial y_h} \frac{dy_h}{dI_l} dI_l \right) + \\ + \frac{1}{2} \sum_h \alpha_h \sum_l \sum_{l'} \frac{\partial^2 V_h}{\partial p_{hl}^L \partial p_{hl'}^L} \frac{dp_{hl}^L}{dI_l} \frac{dp_{hl'}^L}{dI_{l'}} dI_l dI_{l'} + \\ + \frac{1}{2} \sum_h \alpha_h \sum_l \frac{\partial^2 V_h}{\partial p_{hl}^L \partial y_h} \frac{dp_{hl}^L}{dI_l} dI_l dy_h + \frac{1}{2} \sum_h \alpha_h \frac{\partial^2 V_h}{\partial {y_h}^2} \left(\frac{dy_h}{dI_l} dI_l \right)^2 \tag{3.18}$$

Roy's identity is applied making demand functions appear and the welfare weights disappear.

$$dW = -\sum_h \sum_l x_{hl}^L (p_h^L, y_h) dp_{hl}^L + \sum_h dy_h + \\ -\frac{1}{2} \sum_h \sum_l \sum_{l'} \frac{\partial x_{hl}^L (p_h^L, y_h)}{\partial p_{hl'}^L} dp_{hl}^L dp_{hl'}^L + \\ -\frac{1}{2} \sum_h \sum_l \frac{\partial x_{hl}^L (p_h^L, y_h)}{\partial y_h} dp_{hl}^L dy_h + \frac{1}{2} \sum_h \frac{\partial \lambda_h}{\partial y_h} (dy_h)^2 \tag{3.19}$$

Regrouping of terms gives the following.

$$dW = -\sum_h \sum_l x_{hl}^L (p_h^L, y_h) dp_{hl}^L + \sum_h dy_h + \\ -\frac{1}{2} \sum_h \sum_l \left\{ \sum_{l'} \frac{\partial x_{hl}^L (p_h^L, y_h)}{\partial p_{hl'}^L} dp_{hl'}^L + \frac{\partial x_{hl}^L (p_h^L, y_h)}{\partial p_{hl'}^L} dy_h \right\} dp_{hl}^L + \\ + \frac{1}{2} \sum_h \frac{\partial \lambda_h}{\partial y_h} (dy_h)^2 \tag{3.20}$$

The expression between accolades is exactly a linear approximation, i.e. first order, of demand with all cross price effects and the income effect.

$$dx_{hl}^{L} = \sum_{l'} \frac{\partial x_{hl}^{L}(p_h^L, y_h)}{\partial p_{hl'}^{L}} dp_{hl'}^{L} + \frac{\partial x_{hl}^{L}(p_h^L, y_h)}{\partial p_{hl'}^{L}} dy_h \tag{3.21}$$

Thus the expression for welfare change will hugely be simplified. In addition, it is common practice to assume the marginal utility of income to remain constant over the whole trajectory of utility change.[13] This makes the last term of (3.20) disappear. The final measure reads:

$$dW = -\sum_h \sum_l x_{hl}^{L} dp_{hl}^{L} - \frac{1}{2} \sum_h \sum_l dx_{hl}^{L} dp_{hl}^{L} + \sum_h dy_h \tag{3.22}$$

In practice one works with observations: $(x_{hl}^{L0}, p_{hl}^{L0}, h_h^0)$, $(x_{hl}^{L1}, p_{hl}^{L1}, h_h^1)$, where superscript *0* denotes the initial situation and superscript *1* the new one. Rewrite equation (3.22).

$$dW = -\sum_h \sum_l x_{hl}^{L0}(p_{hl}^{c1} - p_{hil}^{L0}) - \frac{1}{2} \sum_h \sum_l (x_{hl}^{L1} - x_{hl}^{L0})(p_{hl}^{L1} - p_{hl}^{L0}) + \sum_h (y_h^1 - y_h^0)$$

$$dW = -\frac{1}{2} \sum_h \sum_l (x_{hl}^{L1} + x_{hl}^{L0})(p_{hl}^{L1} - p_{hl}^{L0}) + \sum_h (y_h^1 - y_h^0) \tag{3.23}$$

This then is the rule of half contained in a measure for total welfare change.

It is important to realize that the same expression could have been derived in terms of routes. The rule of half for routes obviously has the same form as the one expressed in links. Under the assumption of linear additive link costs the approximation will have exactly the same value.

We demonstrate this with observations in routes: $(x_{hr}^{R0}, p_{hr}^{R0}, h_h^0)$, $(x_{hr}^{R1}, p_{hr}^{R1}, h_h^1)$ and rewriting (3.23) using the route-link relations.

$$\begin{aligned} dW \quad &= -\frac{1}{2} \sum_h \sum_l (x_{hl}^{L1} + x_{hl}^{L0})(p_{hl}^{L1} - p_{hl}^{L0}) + \sum_h (y_h^1 - y_h^0) \\ &= -\frac{1}{2} \sum_h \sum_l (p_{hl}^{L1} - p_{hl}^{L0}) \sum_r M_{lr} (x_{hr}^{R1} + x_{hr}^{R0}) + \sum_h (y_h^1 - y_h^0) \\ &= -\frac{1}{2} \sum_h \sum_l \sum_r (p_{hl}^{L1} - p_{hl}^{L0}) M_{lr} (x_{hr}^{R1} + x_{hr}^{R0}) + \sum_h (y_h^1 - y_h^0) \\ &= -\frac{1}{2} \sum_h \sum_r (p_{hr}^{R1} - p_{hr}^{R0})(x_{hr}^{R1} + x_{hr}^{R0}) + \sum_h (y_h^1 - y_h^0) \end{aligned} \tag{3.24}$$

[13] With this assumption our final measure and compensation and equivalent variation will be the same.

4 Practical issues

4.1 The rule-of-a-half for a new link

In principle, one might apply the ROH in case of a new link by simply assuming that the link already did exist, but that the costs of using the link were prohibitively large. E.g. if the new link is a road crossing fields, the costs of crossing these fields by car in the absence of the new link were incredibly high and the number of cars actually crossing the fields was zero. In fact it suffices to assume that the cost of crossing the fields was just a little bit higher than the cost of using the closest substitute.

There can still be doubts about the appropriateness of applying the ROH in this case. The ROH is a good approximation of user benefits only in case of small curvature of demand and little variation of perceived costs (Jara-Díaz, 2007, p. 88). Looking at figure 4.1a, the linearization by ROH would do well for price drops from p^0 to p^1, p^1 to p^2 or p^2 to p^3. But in case of a price drop from p^0 to p^2 ROH would overestimate, and in case of a price drop from p^1 to p^3 underestimate the user benefits.

Figure 4.1a A demand curve with strong curvature

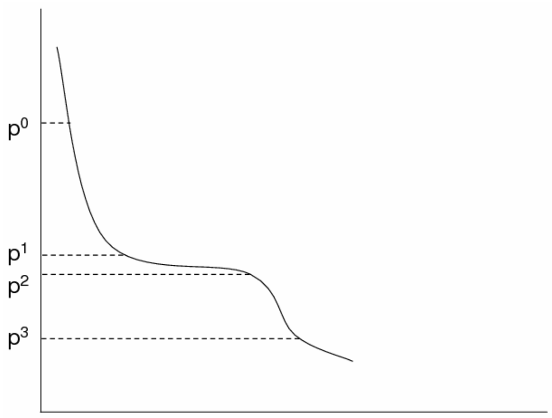
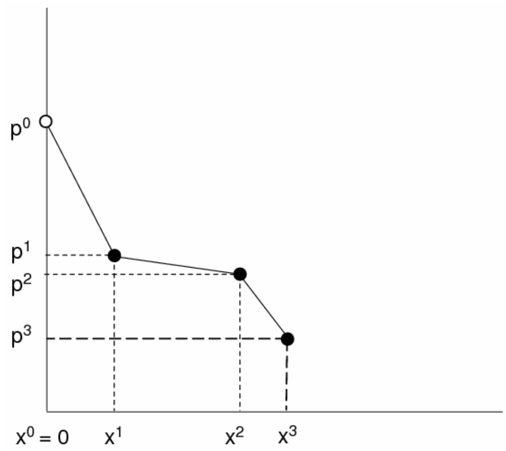


Figure 4.1b: A step-by-step approach for the ROH



The Marshallian demand curve depicted in figure 4.1a might apply to a new link being a close substitute for an existing link (or combinations of links), the cost of using that existing link laying somewhere between p^1 and p^2. If the cost of using the new link remains higher than p^1 hardly any users will be attracted. If the costs are reduced from p^1 to p^2 many users substitute the existing link for the new one. Further cost reductions beyond p^2 will not attract existing traffic anymore, but only generate some new traffic.

In case of doubt, Nellthorp and Hyman (2001) propose a procedure which might be called 'numerical integration'. Suppose (p^3, x^3) is the point that indicates the projected costs and projected use of the new link, while p^0 is a cost level that is definitely higher than the cost of using the closest substitute. In order to correctly calculate the user benefits one has to find the form of the demand curve between the points (p^0, $x^0 = 0$) and (p^3, x^3). One might ask the traffic analysts to provide some extra 'observations', i.e. projections for several more modest dimensions of the new link. Point (p^2, x^2) could represent the projection for a two lane road, rather than the proposed four lane road, and (p^1, x^1) the projection if additionally a speed limit would be imposed on the two lane road. Having these extra observations, one can calculate the total user benefits for this link as the sum of the results of the ROH calculations for, consecutively, the price drop from p^0 to p^1, from p^1 to p^2 and from p^2 to p^3. Kato et al (2003) did apply this procedure.

Notice, first, that this procedure of numerical integration is needed only for those links for which there are suspicions that the conditions for the ROH are not being met. For all other links, one step will suffice. Notice further that even in the case of a new link, the Marshallian demand curve could be much smoother than the one depicted in figure 2a and the one step calculation would suffice as well. E.g. in case there are a number of existing links that are physically substitutes in varying degrees for the new link, or in case consumers have heterogeneous preferences (including 'love of variety').

4.2 Aggregating routes into OD-relations

In practice, it is impossible to calculate user benefits for routes, simply because of the sheer endless number of routes. A simple and complete network of only 10 nodes has 90 links but no less than 10 million routes, even excluding those with cycles. Usually the number of routes is sized down to manageable numbers by aggregating them into origin-destination relations. First, all routes between a node of origin and a node of destination are aggregated by summing all the traffic flows over the routes concerned and by calculating the weighted average of the user costs. Then, all nodes in a zone are aggregated likewise in order to arrive at a manageable number of OD-pairs. As pointed out by Kidokoro (2004) in an appendix to his paper, the user benefits calculated on the basis of the OD-matrix data are usually not equal to the true user benefits calculated on the basis of the use of the separate routes.

However, if the Wardrop principle applies, the calculated benefits on the basis of aggregated routes do correspond to the true benefits. The Wardrop principle assumes that all used routes concerned are perfect substitutes and thus have the same user costs, while all not-used routes are more costly. Goods that are perfect substitutes can, from an economic point of view, always be aggregated. So, if the traffic forecasts are produced by a model employing the Wardrop principle, and if the preferences of the people making trips on each OD-relation can be assumed to be homogeneous, no harm is done by calculating benefits on the basis of the OD-matrix data.

Notice, however, that some models that do employ the Wardrop principle in the route choice module, revert thereafter to a stochastic technique for assigning the traffic flows to the various parts of the physical network. Stochastic assignment implicitly assumes less than perfect substitutability and/or some degree of heterogeneity of preferences. Then again, the resulting OD-matrix data do not produce a correct figure of the true user benefits.

If less than perfect substitutability of routes and/or some degree of heterogeneity in preferences are considered to be predominant, one will from the outset opt for employing a logit model. In fact there are many reasons why routes are imperfect substitutes, such as differences in reliability, comfort, safety, scenery etc. Even in the case of freight the degree of heterogeneity is considerable (De Jong, 2000). The logit model is a very rich and powerful tool for dealing with these differences. And besides calculating user benefits from the changes in the use of the separate routes and/or links, one might calculate them for the so-called 'logsums' (De Jong et al, 2006). But for welfare assessments one should not use the OD-matrix data resulting from the logit model, since all the relevant information about the differences in prices and preferences is being lost in the process of aggregation.

We can illustrate the possible errors by looking at the CBA of the high speed rail track Amsterdam-Brussels (Van Hasselen & Van Schijndel-Pronk, 1994). First, the change to be expected in traffic flows (by road, air, conventional train and high speed rail) was analyzed by a dedicated logit model. Then, the user benefits were calculated from the resulting OD-matrix data for 25 zones. Separate OD-matrices for business and non-business, domestic and international travel were calculated, but all modes of transport (all 'routes') were aggregated. This clearly resulted in errors. According to the traffic analysis half a million business people would move on a yearly basis from air transport to train, for their trips between Amsterdam on the one hand and London or Paris on the other. They would trade the relative advantages of air transport for the relatively cheaper ticket price of the train. Since the CBA was based on OD-matrix data, the savings on out-of-pocket cost was among the user benefits, the loss of comfort was neglected. Consequently, the total benefits for this group of travelers were calculated as the equivalent of 117 euro per trip, while applying the ROH to each and every route apart would have resulted in a benefit of only 20 euro. This figure is much lower since, implicitly, the ROH takes all reasons why people might have preferred air travel over rail into account. And the fact that a large number of business travelers were expected to stick to air travel, notwithstanding the higher out-of-pocket costs, does point clearly to unobserved advantages of air transport.

5 Concluding remarks

We showed that for a quite general economic model and under fairly general assumptions regarding the properties of the network, user benefits could be derived in two equivalent ways. Welfare changes could be derived either from changes in door-to-door journeys, which we called the route approach, or from changes in the use of each and every separate part of the physical networks, which we called the link approach. The route approach, championed in textbooks and CBA guidelines, fits in nicely with the theory of consumer demand. However, the links approach maintains a tight relation to the actual policy measures which usually entail a change in the capacity or the user costs of some specific links of a network.

Both approaches confront the CBA practitioner with a measurement problem. The link approach might give less precise outcomes in case of a new link. In that case, we propose a step-by-step approach for the new link and the most affected competing links. The routes approach lacks precision since routes always have to be aggregated into an OD-matrix, while the conditions that allow for aggregation are hardly ever being met. It is recommended to do the welfare calculations at the lowest possible level of aggregation, i.e. on the basis of the most detailed OD-matrix. From a real-life case we showed that aggregation of journeys which are clearly not perfect substitutes yields considerable measurement errors.

From a practical point of view there are some additional considerations that plead for the link approach as a supplement to, or in some cases even instead of, the route approach.

The most important advantage of the link approach is that it shows, on a map, where benefits of the change in costs or capacity occur. Even more important, it shows where new bottlenecks emerge. This is useful information for the design of the project. It will suggest how one might optimize the design by adding some extra capacity at specific points elsewhere in the network.

The link approach is neatly connected with the actual traffic predictions, since it uses these changes in traffic flows as inputs. This down-to-earth approach makes it easier to unravel seemingly implausible CBA outcomes if they occur.

The number of calculations for the CBA can be kept to a minimum. While for the OD-matrix approach in principle all traffic flows should be considered, since a part of each OD-flow might use links in the project area, the link approach can be limited to changes in the project area and its direct surroundings. Particularly for a small local project, one might safely assume that the user costs on links farther away will not change.

As long as the length of links is not altered, one might either use the number of trips or the number of kilometers traveled over the link as the unit of measurement. Doing al the calculations in terms of the number of kilometers traveled is convenient, since other items in the CBA such as the change in pollution, use these data as well.

The link approach is particularly suitable for ex post evaluations. The evaluation of the Stockholm congestion charge by Eliasson (2009) illustrates this. Using the link approach, Eliasson only had to collect data on actual changes in traffic flows and speeds on the links in and around the project area, without having to ask the motorists for the origin and destination of their journey.

By deriving the appropriateness of the link approach from a quite general economic model and under fairly general assumptions regarding the network, we showed in fact that the CBA practitioner does not have to know the model that generates the changes in the traffic flows. This point was made already by Kidokoro (2004) and stressed more forcefully in Kidokoro (2006). Not only for ex post evaluations, also for ex ante assessments it suffices that the CBA practitioner is told what changes actually will take place on the network, not what drives them. Thus, the division of labor one encounters in practice, between the traffic engineer and the CBA economist, is justified.

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